PadhAI: The Convolution Operation

One Fourth Labs

The 1D convolution operation

What does the convolution operation do?

1. Let's approach this with a real world example
2. Consider a flight from Chennai to Delhi
   a. We measure the distance of the flight from Chennai at regular intervals,
      i. $x_0 \; at \; t_0$
      ii. $x_1 \; at \; t_1$
      iii. $x_2 \; at \; t_2$
   b. In general, to calculate the overall speed, we would take the average speed at these measured points i.e $\frac{1}{3}(x_0 + x_1 + x_2)$.
   c. However, let us try giving the most importance to the current reading, and a progressively decreasing level of importance to every reading preceding the current one.
   d. Let's assign different weights to each of these reading points
      i. $x_0 \dashrightarrow w_0$ (0 indicates current reference point)
      ii. $x_1 \dashrightarrow w_{-1}$ (1 reading before reference point)
      iii. $x_2 \dashrightarrow w_{-2}$ (1 readings before reference point)
   e. So the new overall speed would be calculated by $w_{-2}x_0 + w_{-1}x_1 + w_0x_2$ where the weights are decreasing from w_0
3. The formula could be written as follows
   a. $s_t = \sum_{a=0}^{\infty} w_{-a}x_{t-a} = (x * w)_t$
   b. Where t refers to reference point
   c. a is the index of the weight, ranging from 0 for reference point to ∞
4. In practice, we wouldn't want to take the reading up till $-\infty$, thus we can simply say that those unwanted weights are all 0.
5. Consider the following table

	w_{-6}	w_{-5}	w_{-4}	w_{-3}	w_{-2}	w_{-1}	w_0					
W	0.01	0.01	0.02	0.02	0.04	0.04	0.05					
X	1.00	1.10	1.20	1.40	1.70	1.80	1.90	2.10	2.20	2.40	2.50	2.70
S							1.80					

6. In the above table, w_{-7} to $w_{-\infty}$ are all consider to be 0
7. Here, $s_6 = x_6w_0 + x_5w_{-1} + x_4w_{-2} + x_3w_{-3} + x_2w_{-4} + x_1w_{-5} + x_0w_{-6}$