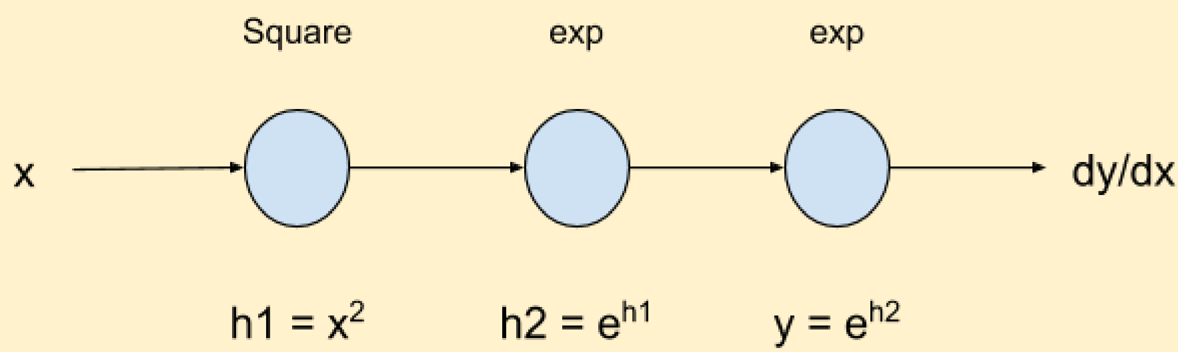
PadhAI: Backpropagation - the light math version

One Fourth Labs

Revisiting Basic Calculus

Let's do a quick recap of some basic calculus concepts

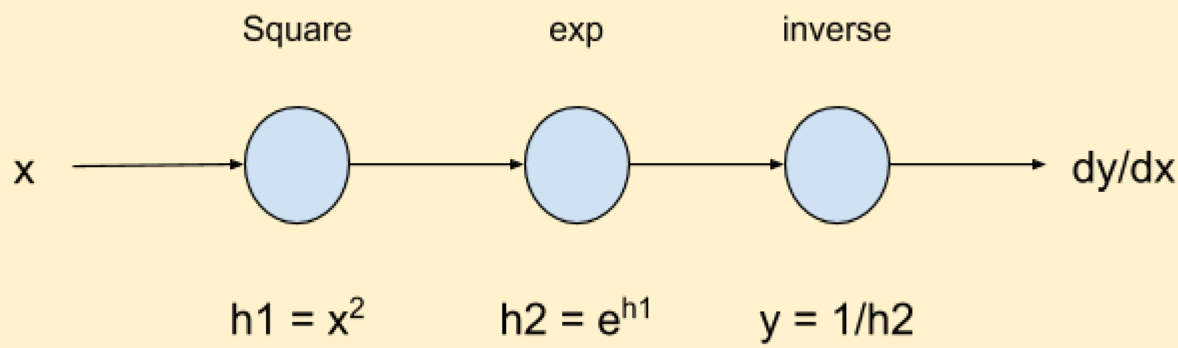
1. Here are some examples of simple derivatives
   a. $\frac{de^x}{dx} = e^x$
   b. $\frac{dx^2}{dx} = 2x$
   c. $\frac{d(\frac{1}{x})}{dx} = -\frac{1}{x^2}$
2. Now, let's look at a slightly more complicated derivative
   a. a $\frac{de^{x^2}}{dx}$
   b. Here, we break it into two parts
      i. h = x^2
      ii. y = $e^{(term)}$
   c. Therefore, $\frac{de^{x^2}}{dx} = \frac{dy}{dh}\frac{dh}{dx}$
   d. $\frac{dh}{dx} = 2x$
   e. $\frac{dy}{dh} = e^h$
   f. $\frac{de^{x^2}}{dx} = \frac{dy}{dh}\frac{dh}{dx} = (e^h).(2x) = (e^{x^2}).(2x) = 2xe^{x^2}$
   g. Here, the output is a composite function of the input. This process of breaking the equation into parts and solving them sequentially is known as **Chain Rule**
   h. Consider another example $\frac{de^{e^{x^2}}}{dx}$
   i. Here is the flow diagram of chain rule applied to the above equation



   j. $\frac{de^{e^{x^2}}}{dx} = \frac{dy}{dh2}\frac{dh2}{dh1}\frac{dh1}{dx} = (e^{h2}).(e^{h1}).(2x) = (e^{e^{x^2}}).(e^{x^2}).(2x) = 2xe^{e^{x^2}}e^{x^2}$
   k. Another example $\frac{d(\frac{1}{e^{x^2}})}{dx}$

---

l. Flow diagram of chain rule



m. $\frac{d(\frac{1}{e^{x^2}})}{dx} = \frac{dy}{dh2}\frac{dh2}{dh1}\frac{dh1}{dx} = (\frac{-1}{(h2)^2}).(e^{h1}).(2x) = (\frac{-1}{(e^{x^2})^2}).(e^{x^2}).(2x) = (\frac{-1}{(e^{x^2})^2}).2xe^{x^2}$