PadhAl: The Convolution Operation

One Fourth Labs

The 2D convolution operation

What about 2D inputs? What are the neighbors that we consider?

- 1. In a nutshell, the convolution operation boils down to taking a given input and re-estimating it as a weighted average of all the inputs around it.
- 2. The above definition is easy to visualise in 1D, but what about 2D?
- 3. In 2D, we would consider neighbors along the rows and columns, using the following formula

4.
$$s_{ij} = (I * K)_{ij} = \sum_{a=0}^{m-1} \sum_{b=0}^{n-1} I_{i+a,j+b} * K_{a,b}$$

a. K refers to kernel or weights and I refers to the input. And * refers to the convolution operation

K₀₀ K₀₁ K₁₁

h = cols

- b. Let a be the number of rows and b be the number of columns
- c. m & n specify the size of the matrix, in this case we consider them to be 2 each. So it's a 2x2 matrix. Therefore a & b range from 0-1 each.
- d. Now, to calculate the new value at a particular pixel $I_{i,j}$, we simply need to fill in the values ino the formula.
- e. $s_{ij} = I_{i+0,j+0}K_{0,0} + I_{i+0,j+1}K_{0,1} + I_{i+1,j+0}K_{1,0} + I_{i+1,j+1}K_{1,1}$
- f. Here is a pictorial representation

| а | b | С | d |
|---|---|---|---|
| е | f | g | h |
| i | j | k | I |

w x y z

Convolution ->

| aw+bx+ey+fz | bw+cx+fy+gz | cw+dx+gy+hz |
|-------------|-------------|-------------|
| ew+fx+iy+jz | fw+gx+jy+kz | gw+hx+ky+lz |

Input

Kernel

Output

- g. This is how the convolutional operation looks like in 2D
- h. Instead of only choosing successive points, we must also consider previous points, on both

sides of the reference pixel. $s_{ij} = (I * K)_{ij} = \sum_{a=[-\frac{m}{2}]}^{[\frac{m}{2}]} \sum_{b=[-\frac{n}{2}]}^{[\frac{n}{2}]} I_{i-a,j-b} * K_{\frac{m}{2}+a,\frac{n}{2}+b}$

