Homework 2

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1. $E(Y)$

$= \frac{1}{42} + \frac{4}{42} + \frac{18}{42} + \frac{16}{42} + \frac{36}{42} + \frac{49}{42} + \frac{128}{42} = 6$

2. $f(x,y) = 12y^2$ for $0 \le y \le x \le 1$

$E(XY) = \iint XY f(x,y)\,dx\,dy = \iint_0^x 12xy^3\,dy\,dx$

$= \int_0^1 3x^5\,dx = [\frac{1}{2}x^6]_0^1 = \frac{1}{2}$

3. $E[(x_1 - 2x_2 + x_3)^2]$

$= E(x_1^2 - 4x_1x_2 + 4x_2^2 + 2x_1x_3 - 4x_2x_3 + x_3^2)$

$= E(x_1^2) - 4E(x_1x_2) + 4E(x_2^2) + 2E(x_1x_3) - 4E(x_2x_3) + E(x_3^2)$

$= 6E(x_1^2) - 6E(x_1x_2)$

$[\frac{1}{2}x_1^2x_2]_0^1$

$= 6\int_0^1 x_1^2\,dx_1 - 6\int_0^1\int_0^1 x_1x_2\,dx_1x_2$

$\frac{1}{2}x_2 \quad \frac{1}{4}x_2^2$

$= 6[\frac{1}{3}x_1^3]_0^1 - 6 \times \frac{1}{4}$

$= 2 - \frac{3}{2} = \frac{1}{2}$

4. $P(Y) = P(Y \le y) = P(e^{\frac{3}{4}x} \le y) = P(x \le \ln y^{\frac{4}{3}})$

So $P(Y) = \int_0^{\ln y^{\frac{4}{3}}} e^{-x}\,dx = [-e^{-x}]_0^{\ln y^{\frac{4}{3}}}$

$= 1 - y^{-\frac{4}{3}}$

So PDF $= +\frac{4}{3}y^{-\frac{7}{3}}$

$E(Y) = \int_1^\infty (\frac{4}{3})y^{-\frac{4}{3}}\,dy = 4$

5.

X	Y	
1	3	$\frac{1}{6}$
2	9	$\frac{1}{6}$
3	19	$\frac{1}{6}$

$Y = 2x^2 + 1$

4	33	$\frac{1}{6}$
5	51	$\frac{1}{6}$
6	73	$\frac{1}{6}$

So $E(Y) = \frac{94}{3}$

6. $P(Y) = P(y \le y) = P(2x+1 \le y) = P(x \le \frac{y-1}{2})$

$$= \int_0^{\frac{y-1}{2}} 2(1-x)\,dx$$

$$= [2x - x^2]_0^{\frac{y-1}{2}}$$

$$= 2 \times \frac{y-1}{2} - \frac{1}{4}(y-1)^2$$

$$= y-1 - \frac{1}{4}(y^2-2y+1) = -\frac{1}{4}y^2 + \frac{3}{2}y - \frac{5}{4}$$

So Y has PDF $f(y) = -\frac{1}{2}y + \frac{3}{2}$

$$E(y^2) = \int_1^3 (-\frac{1}{2}y + \frac{3}{2}y)dy = [-\frac{1}{6}y^3 + \frac{3}{4}y^2]_1^3 = -\frac{27}{6} + \frac{27}{4} + \frac{1}{6} - \frac{3}{4}$$

$$= 6 - \frac{13}{3} = \frac{5}{3}$$

7. $(ax+b)^n = \sum_{k=0}^{n} \binom{n}{k} (ax)^{n-k} b^k$

$$E[(ax+b)^n] = \sum_{i=0}^{n} \binom{n}{i} E(ax)^{n-i} b^i]$$

$$= \sum_{i=0}^{n} \binom{n}{i} a^{n-i} b^i E(x^{n-i})$$

8. $X \sim B(x; n, p)$ $\quad F(x; n,p) = \binom{n}{k} C_n^k p^k (1-p)^{n-k}$

$Y \sim B(x; n, 1-p)$ $\quad F(Y; n,p) = \binom{n}{i} C_n^i p^{n-i} (1-p)^i$

$$E(x-Y) = E(x) - E(Y) = np - n(1-p)$$

$$= 2np - n$$

Suppose Size is 20, p is 5%

$E(x-p) = 2 \times 20 \times 0.05 - 20 = -18$

This means the defective parts is less than good parts.