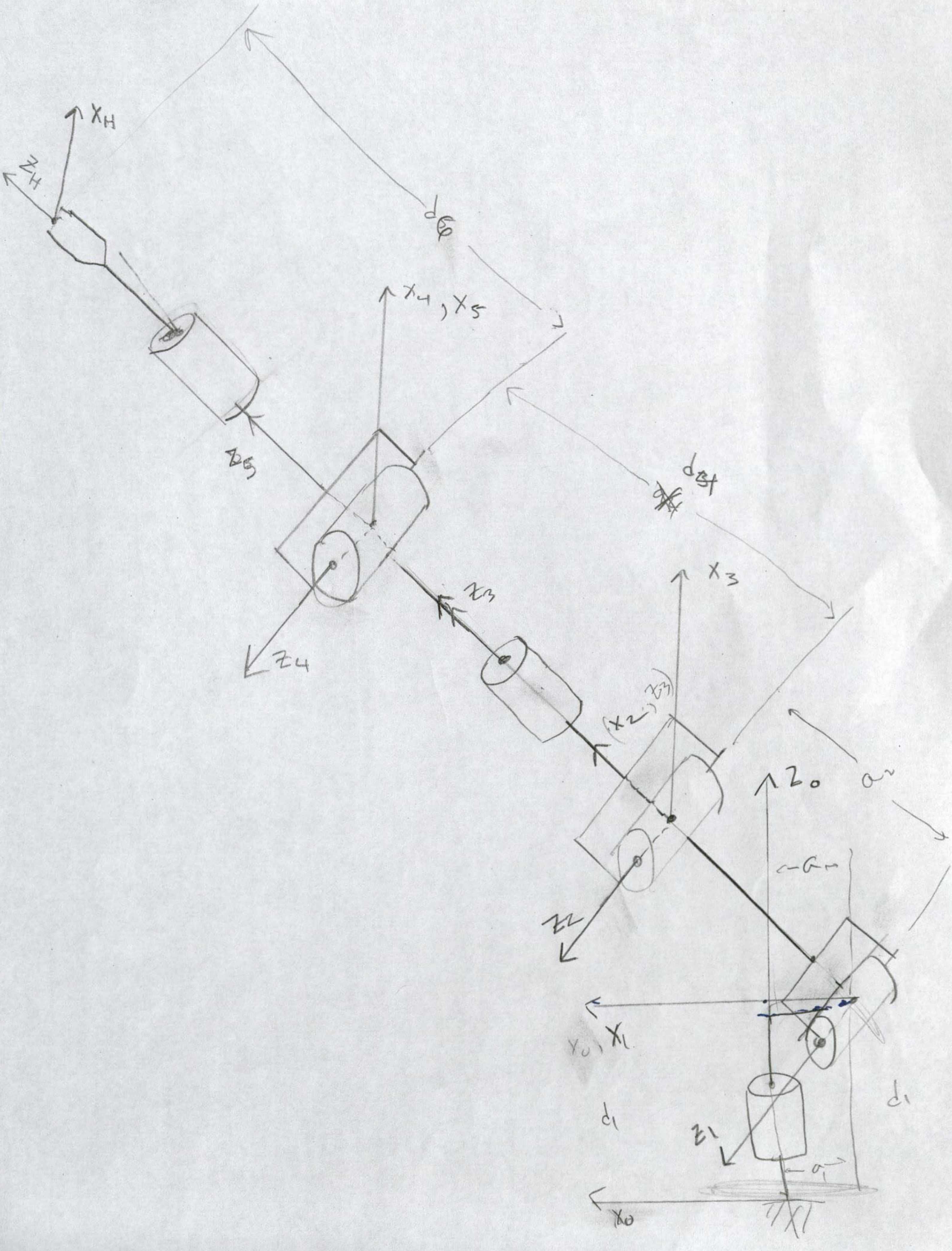

X_H
Z_H
d_6
X_4, X_5
Z_5
d_{34}
Z_3
X_3
Z_4
(X_2, Z_3)
Z_0
a_2
Z_2
Y_0, X_1
d_1
d_1
Z_1
a_1
X_0
X_1

#	θ	d	a	α
0-1	$\theta_{1,var}$	d_1	a_1	$-90°$
1-2	$\theta_{2,var}$	0	a_2	$0°$
2-3	$\theta_{3,var}$	0	0	$-90°$
3-4	$\theta_{4,var}$	d_4	0	$90°$
4-5	$\theta_{5,var}$	0	0	$-90°$
5-~~4~~6	$\theta_{6,var}$	d_6	0	$0°$

all θ's are variables

$${}^{n}A_{n+1} = Rot(z, \theta_{n+1})\, Trans(0, 0, d_{n+1})\, Trans(a_{n+1}, 0, 0)\, Rot(x, \alpha_{n+1})$$

$${}^{0}A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(-90°) & -S(-90°) & 0 \\ 0 & S(-90°) & C(-90°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(-90°) & -S(-90°) & 0 \\ 0 & S(-90°) & C(-90°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{0}A_1 = \begin{bmatrix} C_1 & -S_1 & 0 & a_1C_1 \\ S_1 & C_1 & 0 & a_1S_1 \\ 0 & 0 & 1 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{{}^{0}A_1 = \begin{bmatrix} C_1 & 0 & -S_1 & a_1C_1 \\ S_1 & 0 & C_1 & a_1S_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$
{}^{1}A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & 0 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(0) & -S(0) & 0 \\ 0 & S(0) & C(0) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (1) & (0) & 0 \\ 0 & (0) & (1) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\boxed{{}^{1}A_2 = \begin{bmatrix} C_2 & -S_2 & 0 & a_2 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}
$$

$$
{}^{2}A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(-90) & -S(-90) & 0 \\ 0 & S(-90) & C(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}A_3 = \begin{bmatrix} C_3 & -S_3 & 0 & 0 \\ S_3 & C_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
\boxed{{}^{2}A_3 = \begin{bmatrix} C_3 & 0 & -S_3 & 0 \\ S_3 & 0 & C_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}
$$

$$^{3}A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(90) & -S(90) & 0 \\ 0 & S(90) & C(90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{3}A_4 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{^{3}A_4 = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$^{4}A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(-90) & -S(-90) & 0 \\ 0 & S(-90) & C(-90) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{4}A_5 = \begin{bmatrix} C_5 & -S_5 & 0 & 0 \\ S_5 & C_5 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\boxed{^{4}A_5 = \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

$$ {}^{5}A_{\cancel{H}\,6} = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C(0^\circ) & -S(0^\circ) & 0 \\ 0 & S(0^\circ) & C(0^\circ) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{5}A_{\cancel{H}\,6} = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (1) & (0) & 0 \\ 0 & (0) & (1) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \boxed{ {}^{5}A_{\cancel{H}\,6} = \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} } $$

$$ \boxed{ {}^{0}A_{\cancel{H}\,6} = {}^{0}A_{1}\,{}^{1}A_{2}\,{}^{2}A_{3}\,{}^{3}A_{4}\,{}^{4}A_{5}\,{}^{5}A_{\cancel{H}\,6} } $$

$$
{}^{0}A_1\,{}^{1}A_2 = \begin{bmatrix} c_1 & 0 & -s_1 & a_1 c_1 \\ s_1 & 0 & c_1 & a_1 s_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_2 & -S_2 & 0 & a_2 \\ S_2 & C_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}A_1\,{}^{1}A_2 = \begin{bmatrix} C_1 C_2 & -C_1 S_2 & -S_1 & a_2 C_1 + a_1 C_1 \\ S_1 C_2 & -S_1 S_2 & C_1 & a_2 S_1 + a_1 S_1 \\ -S_2 & -C_2 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}A_1\,{}^{1}A_2\,{}^{2}A_3 = \begin{bmatrix} C_1 C_2 & -C_1 S_2 & -S_1 & C_1(a_1+a_2) \\ S_1 C_2 & -S_1 S_2 & C_1 & S_1(a_1+a_2) \\ -S_2 & -C_2 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_3 & 0 & -S_3 & 0 \\ S_3 & 0 & C_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} C_1 C_2 C_3 - C_1 S_2 S_3 & S_1 & -C_1 C_2 S_3 - C_1 S_2 C_3 & C_1(a_1+a_2) \\ S_1 C_2 C_3 - S_1 S_2 S_3 & -C_1 & -S_1 C_2 S_3 - S_1 S_2 C_3 & S_1(a_1+a_2) \\ -S_2 C_3 - C_2 S_3 & 0 & S_2 S_3 - C_2 C_3 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{0}A_1\,{}^{1}A_2\,{}^{2}A_3 = \begin{bmatrix} C_1 C_2 C_3 - C_1 S_2 S_3 & S_1 & -C_1 C_2 S_3 - C_1 S_2 C_3 & C_1(a_1+a_2) \\ S_1 C_2 C_3 - S_1 S_2 S_3 & -C_1 & -S_1 C_2 S_3 - S_1 S_2 C_3 & S_1(a_1+a_2) \\ -S_2 C_3 - C_2 S_3 & 0 & S_2 S_3 - C_2 C_3 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$ {}^{3}A_{4}\,{}^{4}A_{5} = \begin{bmatrix} C_4 & 0 & S_4 & 0 \\ S_4 & 0 & -C_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_5 & 0 & -S_5 & 0 \\ S_5 & 0 & C_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}A_{4}\,{}^{4}A_{5} = \begin{bmatrix} C_4 C_5 & -S_4 & -C_4 S_5 & 0 \\ S_4 C_5 & C_4 & -S_4 S_5 & 0 \\ S_5 & 0 & C_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}A_{4}\,{}^{4}A_{5}\,{}^{5}A_{6} = \begin{bmatrix} C_4 C_5 & -S_4 & -C_4 S_5 & 0 \\ S_4 C_5 & C_4 & -S_4 S_5 & 0 \\ S_5 & 0 & C_5 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C_6 & -S_6 & 0 & 0 \\ S_6 & C_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{3}A_{4}\,{}^{4}A_{5}\,{}^{5}A_{6} = \begin{bmatrix} C_4 C_5 C_6 - S_4 S_6 & -C_4 C_5 S_6 - S_4 C_6 & -C_4 S_5 & -d_6 C_4 S_5 + 0 \\ S_4 C_5 C_6 + C_4 S_6 & -S_4 C_5 S_6 + C_4 C_6 & -S_4 S_5 & -d_6 S_4 S_5 + 0 \\ S_5 C_6 & -S_5 S_6 & C_5 & d_6 C_5 + d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$(1,1)\quad c_1c_2c_3 - c_1s_2s_3 \Rightarrow c_1(c_2c_3 - s_2s_3) = c_1\Big[\frac{1}{2}[C(\theta_2-\theta_3)+C(\theta_2+\theta_3)]$
$-\frac{1}{2}[C(\theta_2-\theta_3) - C(\theta_2+\theta_3)]\Big]$

$= c_1(c_{23})$

$(1,3)\quad -c_1(c_2s_3+s_2c_3) \Rightarrow -c_1\Big[\frac{1}{2}[S(\theta_2+\theta_3)-S(\theta_2-\theta_3)] + \frac{1}{2}[S(\theta_2+\theta_3)+S(\theta_2-\theta_3)]\Big]$

$= -c_1[S_{23}] \Rightarrow -c_1S_{23}$

$(3,1)\quad -(s_2c_3+c_2s_3) \Rightarrow -\Big[\frac{1}{2}[S(\theta_2+\theta_3)+S[\theta_2-\theta_3]] + \frac{1}{2}[S(\theta_2+\theta_3)-S(\theta_2-\theta_3)]\Big]$

$= -(S_{23})$

$(3,3):\ (s_2s_3 - c_2c_3) \Rightarrow \frac{1}{2}[C(\theta_2-\theta_3) - C(\theta_2+\theta_3)] - \frac{1}{2}[C(\theta_2-\theta_3)+C(\theta_2+\theta_3)]$

$\Rightarrow -C_{23}$

$$
{}^0A_3 = \begin{bmatrix} c_1(c_2c_3 - s_2s_3) & s_1 & -c_1(c_2s_3 + s_2c_3) & c_1(a_1 + a_2) \\ s_1(c_2c_3 - s_2s_3) & -c_1 & -s_1(c_2s_3 + s_2c_3) & s_1(a_1 + a_2) \\ -(s_2c_3 + c_2s_3) & 0 & s_2s_3 - c_2c_3 & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\boxed{{}^0A_3 = \begin{bmatrix} c_1c_{23} & s_1 & -c_1s_{23} & c_1(a_1 + a_2) \\ s_1c_{23} & -c_1 & -s_1s_{23} & s_1(a_1 + a_2) \\ -s_{23} & 0 & -c_{23} & d_1 \\ 0 & 0 & 0 & 0 \end{bmatrix}}
$$

${}^0A_3\,{}^3A_6$ position column "p"

$$
\begin{Bmatrix} c_1c_{23}\{-d_6c_4s_4\} + s_1\{-d_6s_4s_5\} - c_1s_{23}\{d_6c_5 + d_4\} + c_1(a_1 + a_2) \\ s_1c_{23}\{-d_6c_4s_4\} - c_1\{-d_6s_4s_5\} - s_1s_{23}\{d_6c_5 + d_4\} + s_1(a_1 + a_2) \\ -s_{23}\{-d_6c_4s_4\} + 0 - c_{23}\{d_6c_5 + d_4\} + d_1 \\ 0 + 0 + 0 + 1 \end{Bmatrix}
$$

position "p" of ${}^0A_3\,{}^3A_6$

$$
\begin{bmatrix} -d_6c_1c_{23}c_4s_4 - d_6s_1s_4s_5 - d_6c_1c_{23}c_5 - d_4c_1s_{23} + c_1(a_1 + a_2) \\ -d_6s_1c_{23}c_4s_4 - d_6c_1s_4s_5 - d_6s_1s_{23}c_5 - d_4s_1s_{23} + s_1(a_1 + a_2) \\ -d_6s_{23}c_4s_4 - d_6c_{23}c_5 - d_4c_{23} + d_1 \\ 1 \end{bmatrix}
$$