# Linear Algebra Questions ( Make sure to attend the Online session )

1. Given the matrices:

$$A = \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix}, \quad B = \begin{bmatrix} -6 & 2 & 10 \\ 3 & -3 & 4 \\ -5 & -11 & 9 \\ 1 & -1 & 9 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 2 & 9 & -5 & 7 \end{bmatrix}$$

$$D = \begin{bmatrix} -2 & 6 \\ -5 & 2 \end{bmatrix}, \quad E = [3], \quad F = \begin{bmatrix} 3 \\ 5 \\ -11 \\ 7 \end{bmatrix}, \quad G = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$$

a) What is the dimension of each matrix?

**A=2X3**
**B=4X3**
**C=1X5**
**D=2X2**
**E=1X1**
**F=4X1**
**G=3X3**

b) Which matrices are square?

**To be square it must be N of Row=N of column, the solution is:**
**D,E,G**

c) Which matrices are symmetric?

**To be Symmetric it Must be :**
**1-Square matrix**
**2-The elements on the main diagonal never change after Transpose the matrix.**
**So The solution is:**
**D,G,E**

d) Which matrix has the entry at row 3 and column 2 equal to -11?

**B**

e) Which matrices has the entry at row 1 and column 3 equal to 10?

**A,B**

f) Which are column matrices?

**Column matrix is the matrices or vector that have a 1 Column it called a column Vector also [Dimensions Is (N*1)]: F**

g) Which are row matrices?

**Row matrix is the matrices or vectors that have a 1 Row it called a Row Vector also [Dimensions Is (1*N)]: C**

H) Find AT,CT,ET,GT.     (T -> Transpose)

$$A = \begin{bmatrix} -1 & 23 & 10 \\ 0 & -2 & -11 \end{bmatrix} \longrightarrow A^{T} \begin{bmatrix} -1 & 0 \\ 23 & -2 \\ 10 & -11 \end{bmatrix}$$

$$C = \begin{bmatrix} -3 & 2 & 9 & -5 & 7 \end{bmatrix} \longrightarrow C^{T} = \begin{bmatrix} -3 \\ 2 \\ 9 \\ -5 \\ 7 \end{bmatrix}$$

$$E = \begin{bmatrix} 3 \end{bmatrix} \longrightarrow E^{T} = \begin{bmatrix} 3 \end{bmatrix}$$

$$G = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix} \longrightarrow G^{T} = \begin{bmatrix} -6 & -4 & 23 \\ -4 & -3 & 4 \\ 23 & 4 & 1 \end{bmatrix}$$

2. A, B, C, D and E are matrices given by:

$$A=\begin{bmatrix}-1 & 1 & -2\\ 0 & -2 & 1\end{bmatrix},\quad B=\begin{bmatrix}-1 & 2 & 0\\ 0 & -3 & 4\\ -1 & -2 & 3\end{bmatrix},\quad C=[-3\ \ 2\ \ 9\ \ -5\ \ 7$$

$$D=\begin{bmatrix}-2 & 6\\ -5 & 2\end{bmatrix},\quad E=\begin{bmatrix}3\\ 5\\ -11\end{bmatrix},\quad F=\begin{bmatrix}-1 & 0 & 2\\ -2 & -3 & 4\\ 1 & 4 & -3\end{bmatrix}$$

**Find if possible:**

**a) AB b) BC c) AD d) EF e) FE**

a) A B

$$\begin{bmatrix}-1 & 1 & -2\\ 0 & -2 & 1\end{bmatrix} \times \begin{bmatrix}-1 & 2 & 0\\ 0 & -3 & 4\\ -1 & -2 & 3\end{bmatrix}$$

A × B
2×3 3×3
2×3

$$\begin{bmatrix}(-1\times-1)+(1\times0)+(-2\times-1) & (-1\times2)+(1\times-3)+(-2\times-2) & (-1\times0)+(1\times4)+(3\times-2)\\ (0\times-1)+(-2\times0)+(1\times-1) & (0\times2+(-2\times-3)+(1\times2) & (0\times0)+(-2\times4)+(1\times3)\end{bmatrix}$$

$$=$$

$$AB=\begin{bmatrix}3 & -1 & -2\\ -1 & 4 & -5\end{bmatrix}$$

b) BC it's impossible to Multiplication

Because the N of Column of Matrix B $\neq$ the N of Rows of Matrix A

c) AD it's impossible Also !

d) EF it's impossible Also ☹ !

e) FE it's Work ☺ !

$$\begin{bmatrix}-1 & 0 & 2\\ -2 & -3 & 4\\ 1 & 4 & -3\end{bmatrix} \times \begin{bmatrix}3\\ 5\\ -11\end{bmatrix}$$

F E
3×3 = 3×1
3×1

$$FE=\begin{bmatrix}-25\\ -65\\ 56\end{bmatrix}$$

3. Find the determinant of the matrix M :

$$M = \begin{pmatrix} 15 & 10 \\ 3 & 2 \end{pmatrix} \qquad M = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 3 & 2 & -1 \end{pmatrix}$$

3) determinant of Matrix M:

$$M = \begin{pmatrix} 15 & 10 \\ 3 & 2 \end{pmatrix}$$

$$D \text{ of } M_{2\times 2} = (15 \times 2) - (10 \times 3) = 0$$

$$M = \begin{pmatrix} 2 & 3 & 1 \\ -1 & 2 & 3 \\ 3 & 2 & -1 \end{pmatrix}$$

$$D \text{ of } M_{3\times 3} = 2\begin{bmatrix} 2 & 3 \\ 2 & -1 \end{bmatrix} + 3\begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 3 & 2 \end{bmatrix} \cdot 1$$

$$= 0$$

4. Find the inverse matrix A-1 to the matrix A :

$$A=\begin{pmatrix}-3 & -2\\ 3 & 3\end{pmatrix} \qquad A=\begin{pmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}$$

4) $A^{-1}$ of

$$\underset{2\times 2}{A} = \begin{pmatrix}-3 & -2\\ 3 & 3\end{pmatrix} \rightarrow A^{-1} = \frac{1}{-9+6}\cdot\begin{pmatrix}3 & 2\\ -3 & -3\end{pmatrix}$$

$$A^{-1} = -\frac{1}{3}\cdot\begin{pmatrix}3 & 2\\ -3 & -3\end{pmatrix} = \begin{pmatrix}-1 & -\frac{2}{3}\\ 1 & 1\end{pmatrix}$$

$\underset{3\times 3}{A^{-1}}$ of

$$A = \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}, \quad A^{-1} = |A|\cdot Adj(A)$$

$$|A| = 1\begin{bmatrix}1 & 1\\ 1 & 1\end{bmatrix} + 0\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix} + 1\begin{bmatrix}0 & 1\\ 1 & 1\end{bmatrix} = -1$$

$$Adj(A) = \begin{bmatrix}+1 & -0 & +1\\ -0 & +1 & -1\\ +1 & -1 & +1\end{bmatrix} \rightarrow \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & -1\\ 1 & -1 & 1\end{bmatrix}$$

$$A^{-1} = \begin{bmatrix}1 & 0 & 1\\ 0 & 1 & -1\\ 1 & -1 & 1\end{bmatrix} \times -1$$

$$A^{-1} = \begin{bmatrix}-1 & 0 & -1\\ 0 & -1 & 1\\ -1 & 1 & -1\end{bmatrix}$$

5. What does it mean if three equations are linearly independent?

a. Two of the equations can be combined to come up with the third equation.
b. There is no way to combine any two equations to come up with the third equation.
c. The graphical representations of the equations are lines that do not intersect.
d. The graphical representations of the equations are lines that do intersect.

6. Let

$$f(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{x}^\top \boldsymbol{A}\boldsymbol{y} + \boldsymbol{x}^\top \boldsymbol{B}\boldsymbol{x} - \boldsymbol{C}\boldsymbol{y} + \boldsymbol{D}$$

with $\boldsymbol{x} \in \mathbb{R}^M, \boldsymbol{y} \in \mathbb{R}^N$, function $f : \mathbb{R}^M \times \mathbb{R}^N \to \mathbb{R}$.
Compute the dimensions of the matrices $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ for the function so that the mathematical expression is valid.

A has dimensions (M × N).
B has dimensions (M × M).
C has dimensions (1 × N).
D has dimensions (1 × 1).