Utilisation of transitory methods of resolution for the strongly nonlinear quasi-static problems

Résumé:
This document presents the use of methods of transitory resolution (implicit or explicit) for computational simulation of quasi-static problems presenting of strong non-linearities materials. One only seeks to obtain solutions where the ram effects are negligible.
These methods of resolution (based on the use of `DYNA_NON_LINE`) are conceived as an alternative to the usual quasi-static approaches of *Code_Aster* (based on `STAT_NON_LINE`) when they prove to be unable to converge in an acceptable time. That can be the case for applications implementing damaging materials.
The methods which are the object of this document are of two types:

1) implicit transitory resolution,
2) resolution clarifies pseudo-dynamics.

The choice will be depend on the nature of the problem to treating, in particular, in restrain with the scales of time considered. The use of these strategies being more delicate than the quasi-static approach, they are to be held for the most problematic cases.
The reading of U2.06.13 documentation strongly constitutes pre-necessary advised: the general advices of use of `DYNA_NON_LINE` remain relevant.

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1Introduction

Of many problems of mechanics require the taking into account, on the level material, of damaging behaviors: structures out of reinforced concrete, models of grounds…
The behavior model can then present softening and the known implicit quasi-static methods of resolution test difficulties of converging (the tangent operator of stiffness becomes singular). In certain cases, even the recourse to very powerful strategies as mixed linear search or control is insufficient.
In order to be able to free itself from these limitations, there exist alternative strategies which is based on methods inspired by the tools of the direct transient analysis [bib1].
It is specified clearly that it is not question of wanting here to simulate a dynamic response of vibratory type or with wave propagation: one seeks to obtain solutions in slow evolution, therefore in coherence with the assumption of quasi-staticity. The usual dynamic methods must thus be adapted to this frame and the solution thus obtained will have to check the assumptions of sufficiently slow evolution.
Lastly, it is advisable well to keep in mind that these transitory approaches, of share their specificities, must be used as a last resort, when all the parades available in `STAT_NON_LINE` failed.
The precondition to the use of the methods presented in this documentation is thus to have already developed and have exhaustively tested the options available in `STAT_NON_LINE` for the application considered. It goes without saying a good knowledge of `STAT_NON_LINE` and `DYNA_NON_LINE` is also strongly recommended just as the reading of corresponding documentations: R5.03.01, R5.05.05 and especially U2.06.13.
In particular, the general advices of use of operator `DYNA_NON_LINE` given in U2.06.13 documentation remain valid and they thus constitute an essential precondition to the good implementation of the methods which are this documentation object.

2Functionalities available into quasi-static and absent out of transient

One will approach here, mainly, the problems of the models endommageantes, in local version. Indeed, in version 9, the nonlocal approaches, like `GRAD_VARI` or `GRAD_EPSI` are not available in dynamics. It will thus be advisable to pay particularly attention to be defined a size of minimal mesh adapted not to observe excessive phenomena of localization.

Then, the methods of the type seeks linear (mixed or not) are not authorized in dynamics. This lack is to be relativized, knowing that the attempts at applications of these methods on reinforced concrete structure studies in dynamics did not put forward significant contribution on convergence, as opposed to what one observes into quasi-static. Let us announce, nevertheless, that no theoretical argument would prohibit the use of these methods in dynamics.

Lastly, the techniques of control available in `STAT_NON_LINE` (length of arc, for example) are prohibited in dynamics because they then do not have a meaning.

3Passage d' a quasi-static computation with a Loin de

transient computation limit yourself to override term STAT_NON_LINE by DYNA_NON_LINE in a command file (like defining the densities, *has minimum*…), the transition of quasi-static with the transient must be accompanied by a certain number of precautions essential, under penalty of strongly degrading the quality of the numerical solution obtained.
These adaptations, described in detail in U2.06.13 documentation, relate to:

- the regularization in time of the boundary conditions,
- the definition of initial conditions which do not introduce numerical oscillations.

In complement of these general aspects, the user will have to pay attention to other more specific adaptations.

•Definition of the densities :

from a physical point of view, it is necessary, obviously, to define the density in any point of the model. If models it understands discrete elements, will have to be attached to them a discrete mass. The operator of mass assembled must be invertible. Artifices sometimes employed into quasi-static as of the stiffenings on areas of the model (anchorages…) with materials having very large Young moduli (or very large specific stiffness) are to be handled with precaution. Indeed, these very stiff areas will generate in dynamics of the disturbances high frequencies. Moreover, with an explicit time scheme, these very stiff areas are likely to make fall the value of the critical time step (*cf* R5.05.05).

•Definition of the sizes of mesh and the time steps :

like precondition to transient computation, it is strongly recommended to carry out a modal computation (for example with MODE_ITER_SIMULT) to obtain modal information which will make it possible to qualify the quality of the model in dynamics and to adjust certain parameters. The objective not being to re-enter in the details of the modal analysis, one can nevertheless point out some rules.

Solutions low frequencies are sought, therefore only the first modes are relevant. Their good representation can give indications on the sizes of meshes to be used, besides the considerations already taken into account for preceding quasi-static computations. Approximately, about ten meshes by the smallest wave length is sufficient.

The modal analysis as will make it possible to check as models it is free from problems like contributions nondefinite to inertia or the stiffness.

Lastly, the modal analysis is essential for the use of modal damping in DYNA_NON_LINE or to readjust the damping of Rayleigh, as one will see it in what follows.

•Definition of damping :

the user will have to also put the question of intrinsic damping to the model which he wants to use.

In DYNA_NON_LINE, apart from the discrete elements, one can introduce a total of Rayleigh type or modal damping. Since one seeks to simulate slow evolutions, one can be tried to use values of damping higher than for conventional dynamic computations. A compromise however remains to be found, on a case-by-case basis, between an insufficiently damped problem (which will have oscillations) and a too damped problem (even supercritical critical damping).

One thus advises to start by implementing a “realistic” damping (thus of value identical so that one meets in transient dynamics). Then, if this damping is considered to be insufficient, increase it gradually.

On various applications [bib1], one could note that a damping of the Rayleigh type, readjusted on an equivalent modal damping of about a 20%, even 30% was suitable.

4In numeric work This

chapter bets will approach the choices to be privileged on the level as of numerical methods of DYNA_NON_LINE [bib2].
Generally, one advises to follow following logic:

1) once the possibilities of quasi-static resolution through exhausted STAT_NON_LINE (including linear search and control),
2) to begin the dynamic approaches with an implicit resolution,
3) in the event of failure of the transitory implicit strategies (including by combining various depreciation like Rayleigh and a diagram of the type HHT [bib2] and [bib3]), one tilts into explicit (while having checked as a preliminary that total damping does not depend on the stiffness matrix).

For each transitory approach, it is advisable to start with less possible damping, therefore, in particular, with nondissipative time schemes (like Newmark or the central differences [bib4]).

4.1 Discretization in Contrairement

time with quasi-static, time has a physical meaning. Its discretization in is all the more sensitive.
One can state some rules:

- the evolution of the imposed loadings must be sampled in a sufficiently fine way (between 5 with 10 pas de time per the shortest period of the signals considered),
- the modal behavior of structure must be well represented (like above, one must have between 5 and 10 pas de time per the weakest period of the modes considered).

Being given the character low frequency of the problems which one wants to tackle here, these two rules are in general not very penalizing. Nevertheless, compared to time step of preceding quasi-static computations, the dynamic time steps can be rather definitely weaker.

Into explicit, it is moreover necessary to observe the Flow condition (CFL *cf* [bib5], [bib2] and R05.05.05) under penalty of numerical divergence. For a diagram of integration of the central differences type, the critical time step is worth $2/\omega$ with ω which is the highest own pulsation of the system.
One can calculate this pulsation with MODE_ITER_SIMULT by choosing option "PLUS_PETITE" and by reversing the roles of the mass matrix and of stiffness. Indeed the modal operators of *Code_Aster* directly do not offer a computation option of the high frequency, which is indeed of a restricted use for the current structural analyses.
For more details, the player will be able to refer to U2.06.13 documentation.

For most structures, the Flow condition is very penalizing: the celerity of the waves being often about a few thousands of m/s, one arrives at time steps of less than 10-5 S.

4.2 Choix of the diagrams of integration in time

One can classify the implicit schemes in two categories (one puts side, voluntarily, the diagrams order 1 and/or of velocity which are more specifically adapted to the very irregular problems):

- average acceleration (NEWMARK [bib4]) of order 2 and which does not bring numerical dissipation: to use in first,
- HHT complete (MODI_EQUI = "OUI" [bib3]) which remains of order 2, contrary to the case of the modified average acceleration (MODI_EQUI = "NON", default option). This diagram is specifically developed to introduce a numerical damping high frequency and thus not to disturb the physical response low frequency. Damping is directly controlled by parameter ALPHA of the diagram.

If one observes oscillations high frequencies in solution numerical (approximately, of the oscillations the period is about some time steps), one can choose complete diagram HHT, to start with a value about - 0.1 for parameter ALPHA. A value of - 0.3 constitutes a high limit still usable.

If one wishes more damping on average frequency, then the diagram of average acceleration modified can be employed.

Into explicit, one has two diagrams:

- central differences (DIFF_CENT [bib4]) which is nondissipative,
- Tchamwa-Wielgosz (TCHAMWA [bib6]) which is dissipative, in a way comparable with HHT.

Here still, one recommends to start by using a nondissipative diagram.

4.3 Models of damping

the order of introduction and use of dissipation in the discretized model is the following:

1) intrinsic dissipation related on the behavior models nonlinear, connections (friction),
2) total dissipation of standard damping structural (Rayleigh or modal),
3) numerical dissipation of the time scheme.

Ideally, the first category should be sufficient, but in practice, for reasons of simplification of the model, it is often essential to add structural damping, the damping of the diagram being the last recourse.

We will approach here only the use of structural damping, within the meaning of Rayleigh, and that related to the diagram.

Just let us point out that the more one will multiply the sources of dissipation, the more their control and their physical interpretation will be difficult.

4.3.1 Damping of Rayleigh

This model makes it possible to define the total matrix of damping C as being a linear combination of the stiffness matrixes and of mass (to have a diagonal damping matrix on the basis as of usual dynamic modes):

$$\boldsymbol{C} = \alpha \boldsymbol{K} + \beta \boldsymbol{M}$$

U2.06.13 documentation presents it in detail.

The damping coefficients of Rayleigh are defined, on the level as of characteristics of the material (command DEFI_MATERIAU), by parameters AMOR_ALPHA and AMOR_BETA. The values to force to obtain damping desired Ξ . in the interval of the eigen frequencies f1 and f2 are deducted of the following equations:

Equation 1: $$\alpha = \frac{\xi}{\pi(f1 + f2)}$$

Equation 2: $$\beta = \frac{4\xi\pi\ f1\, f2}{f1 + f2}$$

Where f1 and f2 are the two eigen frequencies limiting the interval of study considered. In the frame of this document, one seeks solutions low frequencies, therefore the frequencies f1 and f2 will be associated with the first frequencies of the model, whose modes are coherent with the imposed loading.

4.3.2 Damping due to the time scheme [bib2]

Les documentations R05.05.05 and U2.06.13 present this aspect. One here will restrict oneself to recall the main tendencies of them.
To summarize, one can recall that, among the implicit schemes:

- the diagram of average acceleration does not dissipate,
- only complete diagram HHT does not disturb the field low frequency,
- for the same value of parameter ALPHA the modified average acceleration introduced much more dissipation than diagram HHT.

In order to put forward the influence of damping high frequency of the implicit schemes, U2.06.13 documentation presents sample applications.

Lastly, with regard to the explicit diagrams, the pace of the damping of the diagram of Tchamwa is qualitatively close to that of the modified average acceleration.

4.4 Adaptation of the explicit methods

the conditional stability of the explicit diagrams returns them very little adapted to the simulation of slow phenomena. The explicit methods of resolution are not used here to collect fast phenomena like the wave propagation, but their use should be perceived as a particular solver whom one adapts for slow problems.
In order to be able to increase the critical time step [bib2], one can increase the density of the structure (what cause a drop in the celerity of the waves proportionally to its square root):

$$c_p = \sqrt{E/\rho}$$

It should however be done gradually.
Indeed, two risks exist:

- if the time step becomes too large, the computed solution will be able to miss certain phenomena like the appearance of tapes of shears and to go until forking towards a branch very different from the expected response,
- the increase in the density can be restricted by the bad conditioning of the mass matrix.

As indication, the maximum time step into explicit (and thus maximum density) can be of the same order of magnitude as the time step necessary to implicit transient computation, in any case, it must remain lower than the quasi-static time step. In fact coarse tendencies do not exempt parametric study on the explicit time step.

If the model has strong heterogeneities of stiffness (definition of several materials), it can be relevant to modify the densities separately, so as to have a relatively homogeneous Flow condition between the areas having different materials.

Notice important

If one imposes boundary conditions in displacement which evolve in the course of time, it should be held account owing to the fact that these conditions in fact are imposed in acceleration into explicit (because it is the primal unknown factor). That means that one must enter DYNA_NON_LINE the derivative second of the signal in displacement which one wants to impose. This evolution of imposed displacement must thus be differentiable at least twice in time…

To finish, it is recommended to use a diagonal mass matrix (lumped), which is obtained by the key word MASS_DIAG = "OUI" of DYNA_NON_LINE. This computation option not being available for all the finite elements, the user can be constrained to use the consistent mass, if necessary, as into implicit.

5Quality control of the computed solutions

5.1 Quantités of interest exits of quasi-static

Comme for quasi-static computations, one can classify the relevant quantities for postprocessing in two categories [bib1]:

1) evolutions of the type forces / displacement: quantities allowing D ` to interpret the total response of structure,
2) isovaleurs of fields like the damage or the cumulated plastic strain.

5.2 Dynamic complementary quantities to analyze

En plus de these analyses, for dynamic computations, it is essential to check that the assumption of slow evolution is respected. For that, it should be made sure that the inertia forces remain weak in front of the other forces in the system (external forces and interiors). A simple way to have a rating of the evolution of the inertia forces during computation consists in observing the field of acceleration in the course of time.

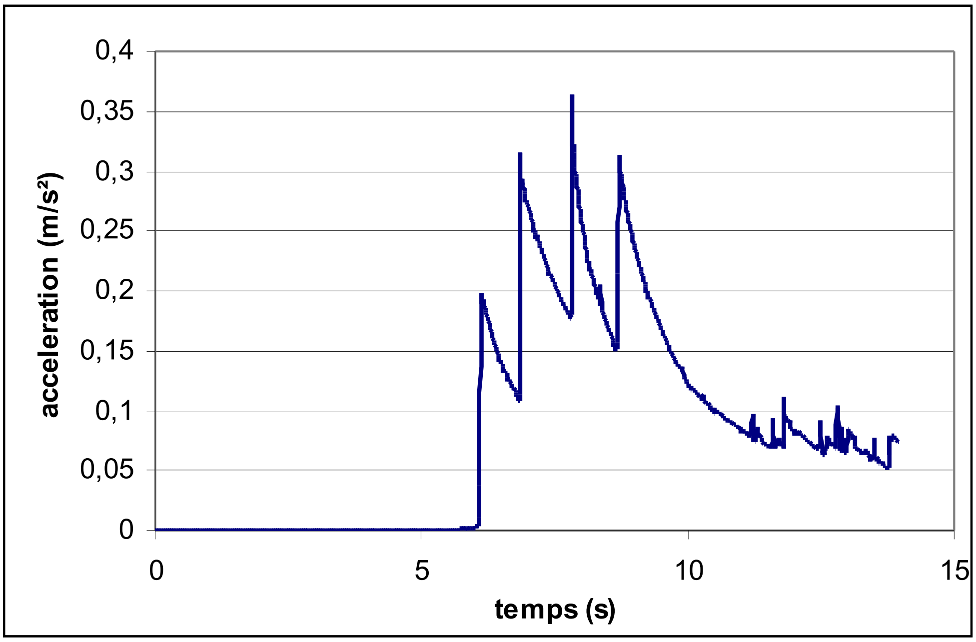
A first simple criterion can be based on a norm (infinite) of acceleration at every moment. In the event of notable and durable increase in acceleration (for example beyond 1 m/s2 on several time steps of continuation), that means that dynamic phenomena (thus not taken into account by a quasi-static resolution) occur:

- if one is in a physical frame, therefore with a realistic density, the computed solution is thus subjected to considerable dynamic phenomena;
- if one is in an explicit frame where the density at summer multiplied to increase the critical time step, then the noted effects of inertia are the sign that the method of resolution is not adapted. It is then imperative to modify the parameters, like lowering the density, modifying damping…

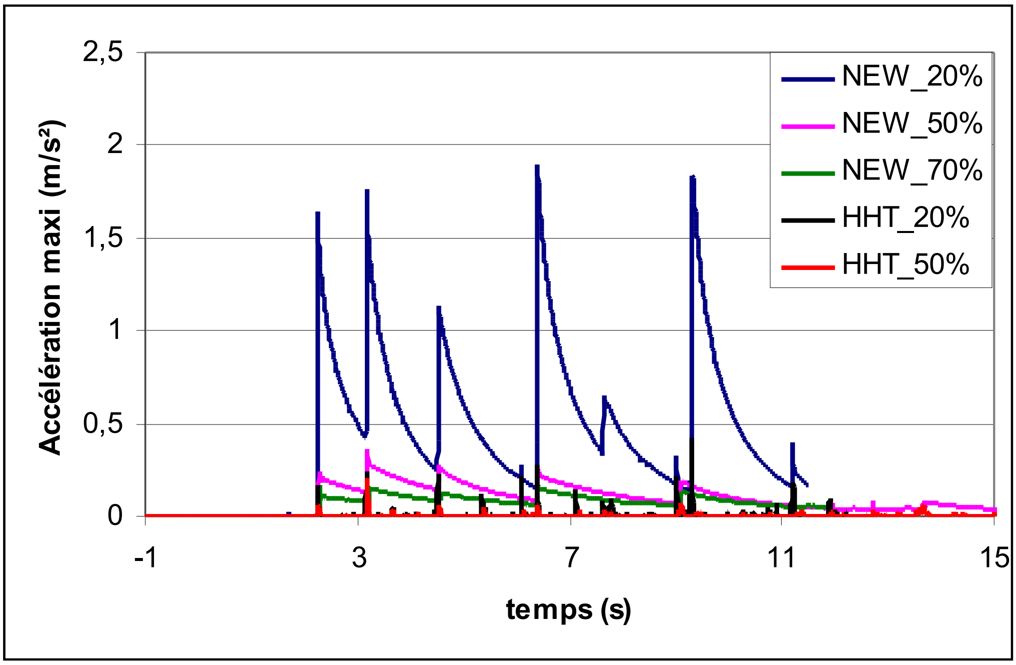
In the first case, one can try to start again a computation with a pitch slightly finer and possibly a damping of Rayleigh a little more raised, even with a standard diagram HHT. If, even with all these amendments, the solution still presents effects of inertia, then any quasi-static approach is unsuited.

5.3 Examples of Sur

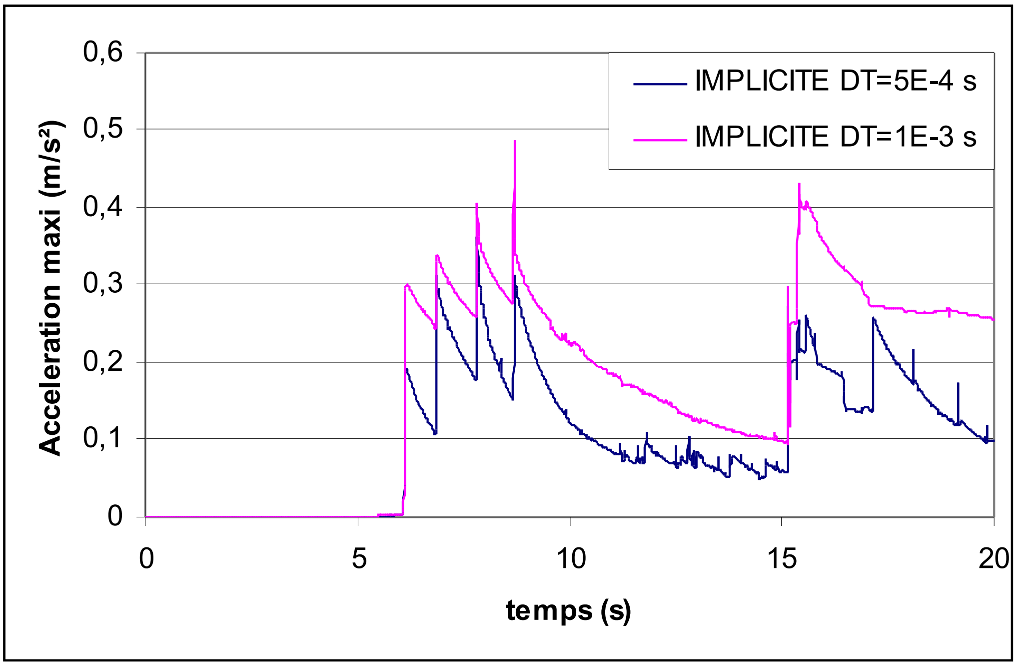
analysis the graph below (reinforced concrete small structure), one presents a case where acceleration will become considerable, from 6 S. the maximum value remains lower than 0.4 m/s2 and we are in a case with realistic density. Moreover, acceleration will become again weak after 10 S: from all that one can conclude that computation remains compatible with the assumption of slow evolution.



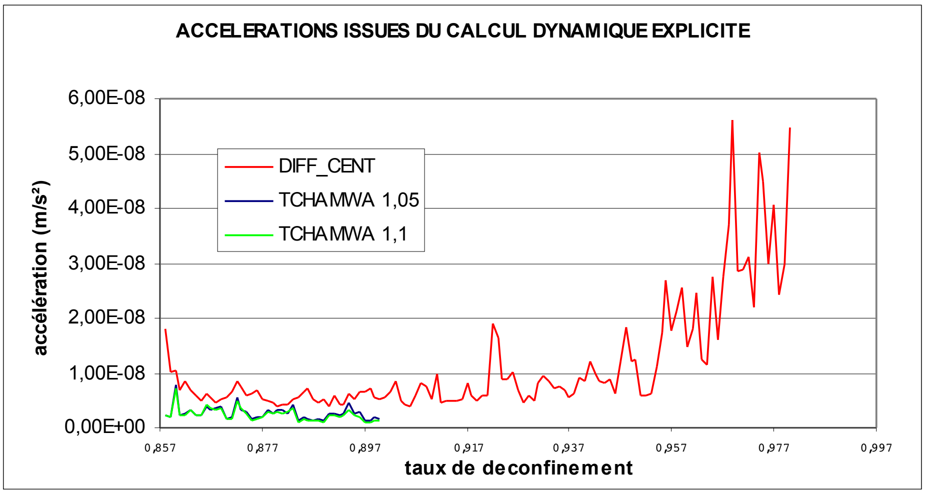
On the following graph, one can quantify the influence of damping (due to Rayleigh and the diagram) on maximum acceleration. All in all, on this example, the use of a diagram of the type HHT with a damping of Rayleigh fixed on a modal value equivalent of 20% is well adapted (curved black). On the other hand the diagram of average acceleration (Newmark) will make it possible to control acceleration only if one couples it with a damping of very important Rayleigh: 50% of equivalent modal damping. The solution computed risk then to be too dissipative.



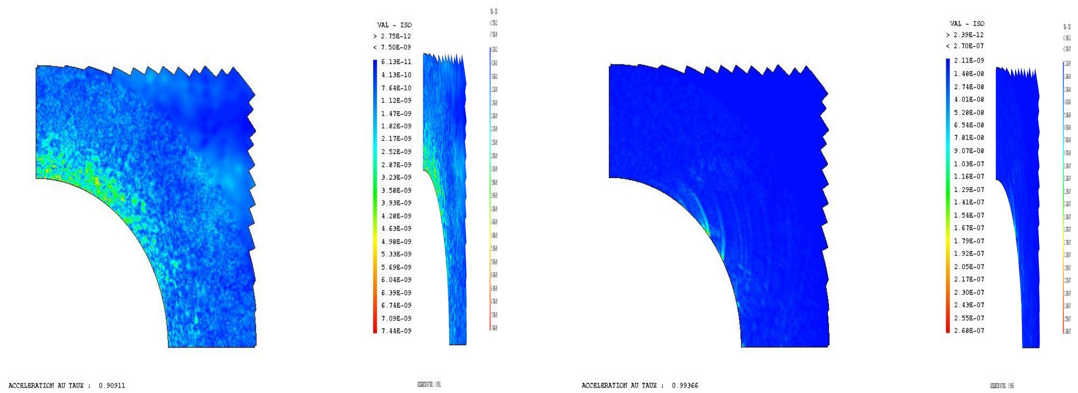
On the following figure, one can judge influence of a parameter of the discretization: the time step, in term of control of maximum acceleration. The value of 10-3 S for the time step is not quite selected and it is then preferable to divide the time step by two. To analyze this behavior, it is essential to undertake a parametric study on the time step.



In the case of a resolution clarifies (on an example very different from the precedents: excavation of a circular gallery [bib1]) one compares various time schemes: central differences (thus without induced damping) and the diagram of Tchamwa for two values of parameter `PHI`. The larger this parameter is, the more one introduces damping high frequency and this damping becomes null for `PHI=1`.



In complement as of preceding curves, it is instructive to visualize some isovaleurs of norm of acceleration (for the same example of excavation into explicit):



On the figure of left, one is in a weak phase of acceleration (non-linearity related to the rate of déconfinement is still low). The pace of the field of acceleration is rather random: one cannot perceive “reason” betraying a real dynamic phenomenon.
On the figure of straight line, non-linearity is established and one observes a distribution of very different acceleration: one sees taking shape the pace of tapes of shears on the circumference of the excavation. But even in this case, the maximum values of acceleration remain very low (including by taking account of the factor of increase in the density). The solution obtained explicitly is thus relevant within the meaning of a slow evolution.

6Dans

performance optimization most case it is recommended to carry out computation into quasi-static when non-linearities are moderated, then, as soon as the iteration count to convergence for the equilibrium increases significantly, to toggle in dynamics (implicit, then clarifies if need be). More precisely one advises to toggle before the appearance of strong non-linearities, so that dynamic computation is initiated on a “regular” evolution still enough.

Into quasi-static, it is not rare to have to carry out more than 10 iterations to have convergence within the meaning of the residue in equilibrium. In implicit dynamics this value of 10 iterations constitutes, in general, a good starting value for parameter ITER_GLOB_MAXI of CONVERGENCE. If one cannot converge in less than 10 to 20 iterations, it is then preferable to decrease the time step rather than to increase the authorized maximum number of iterations.

Into explicit, there are no iterations for the equilibrium, the cost of computation of each time step will be thus constant, whatever the level of non-linearity (except, possibly, the local checking of the behavior). In the case of a quasi-static computation where non-linearity is growing with time, one can thus find a point of crossing, on the level of the time CPU, for which the effectiveness of the explicit methods (even implicit according to the cases) becomes larger than to continue into quasi-static. The gain brought by a larger time step into quasi-static is cancelled by the need for making more and more iterations with each pitch. In the same way, the cost of computation in explicit constant remainder, one can also evaluate rather precisely the total CPU time necessary to the resolution of the complete case, whereas into quasi-static, the iteration count to each pitch being very variable and also being able to involve an unknown number of subdivision of the pitch, the total CPU time is sometimes very difficult to predict.

The use, even current, of the explicit methods thus seems very tempting within sight of the time CPU which remains controlled. It is however necessary to moderate this optimism while keeping well with the spirit which one deprives of the parapet which is the precise checking of the equilibrium and which, consequently, the quality of the explicit solution obtained must be analyzed with more precautions. The explicit algorithm will not diverge (if the Flow condition is observed), but the solution obtained is not guaranteed by a criterion of checking of the equilibrium. In particular a parametric study on the time step is essential because the pace of the solution can strongly vary when this pitch becomes too large.

This obligation of check out of the explicit solution is all the more large as one seeks in more to check the fundamental assumption of slow evolution.

Conclusion

This document presents the use of methods transient dynamics for simulation of slow and strongly nonlinear evolutions [bib1]. The dynamic operators thus employed for the resolution can be seen like particular solvers allowing, in certain cases, to obtain solutions for which the quasi-static algorithmy available in *Code_Aster* (around operator `STAT_NON_LINE`) shows great difficulties of convergence (within the meaning of the checking of the equilibrium).
These dynamic methods are to be used as a last resort because their control is more delicate than the quasi-static frame. It is even more outstanding with the explicit methods since the quality of the solution is not guaranteed by a precise checking of the equilibrium at every moment.
The first stage is the adaptation of the model with the dynamic methods. It is mainly a question of making sure of the good regularity of the imposed conditions and the correct definition of the density and total damping (Rayleigh). The nonlocal continuation methods, approaches and linear search are not usable in dynamics. Moreover, it is necessary to introduce a model of total damping (Rayleigh or modal) whose level is stronger than in conventional dynamics (usually 20% of equivalent modal damping).
Then, it is recommended to start by using an implicit transitory method (`DYNA_NON_LINE` with a time scheme of the type `NEWMARK`, or `HHT`). From a tweaking point of view of the CPU time, one recommends to carry out all the phases slightly nonlinear into quasi-static and not to toggle out of transient (implicit to start) only during the appearance of notable non-linearities (that can result in a clear increase amongst iterations with convergence, even a subdivision of the time step).
In the event of failure, including with a complete diagram HHT and a structural damping relatively extremely (until towards 30% of equivalent modal damping), the user can tilt into explicit. Other adaptations should then be made, like increasing the density to obtain a sufficiently large critical time step and modifying damping.
In all the dynamic cases, it is essential to analyze the evolution of acceleration in the course of time, in order to make sure of the validity of the assumption of slow evolution. The effects of inertia must remain weak.

7Bibliography

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