First-Order Predicate Logic

Introduction to Intelligent Systems

First-Order (Predicate) Logic

- Propositional logic assumes the world contains facts
- Predicate logic (like natural language) assumes the world contains:
  - Objects: people, houses, numbers, colors, baseball games, wars, …
  - Relations: red, round, prime, brother of, bigger than, part of, comes between, …
  - Functions: father of, best friend, one more than, plus, …

Syntax of Predicate Calculus

Symbols represent constants, variables, functions, or predicates:

- Constants begin with a lower-case letter
  - bill, john, etc.
  - The constants ‘true’ and ‘false’ are reserved as truth symbols
- Variables begin with an upper-case letter
  - For example, X, is a variable that can represent *any* constant
- Functions have input and produce an output
  - plus(5,4)
  - The *arity* of a function is the number of arguments
  - For example, ‘plus’ has an arity of 2
- Predicates are similar to functions, but return true
  - A predicate names a relationship between objects in the world
  - For example, father(bill, john) means that “bill is the father of john”
  - father(bill, X) refers to any child of bill

Semantics of Predicate Calculus

- Quantification of variables is important
- When a variable appears in a sentence, that variable serves as a *placeholder*
  - For example, the “X” in likes(george, X)
  - Any constant allowed under the interpretation can be substituted for it in the expression
  - Substituting “kate” or “susie” for “X” forms the statement likes(george, kate) or likes(george, susie)

Sentences in Predicate Calculus

- Sentences are created by combining predicates using logical operators (the same as used for propositional calculus) and *quantifiers*
- Examples:

  likes(george, kate) $\wedge$ friend(X, george)

  $\neg$helps(X, X)

  has_children(ben, plus(2, 3))

  friend(father_of(david, X), father_of(andrew, Y))
- *likes*, *friend*, *helps* and *father_of* are predicates
- *plus* is a function

Sentences in Predicate Calculus

- When a variable appears in a sentence, it refers to unspecified objects in the domain. First order predicate calculus includes two additional symbols, the variable quantifiers $\forall$ and $\exists$, that constrain the meaning of a sentence:

  $\exists$Y friend(Y, peter)

  $\forall$X likes(X, ice_cream)

Quantifiers $\forall$ and $\exists$

- $\forall$ The universal quantifier

  $\forall X\ p(X)$ is read "For all X, p(X) is true"

- $\exists$ The existential quantifier

  $\exists X\ p(X)$ is read "There exists an X such that p(X) is true"

- Relationship between the quantifiers:

  $\exists X\ p(X) \equiv \neg(\forall X)\neg p(X)$

  "There exists an X for which p(X) is true" is equivalent to "it is not true that for all X p(X) does not hold"

Universal Quantification

- Typically, $\rightarrow$ is the main connective with $\forall$

  Example: "Everyone at RIT is smart":

$$\forall X \ at(X, rit) \rightarrow smart(X)$$

- Common mistake: using $\wedge$ as the main connective with $\forall$:

$$\forall X \ at(X, rit) \wedge smart(X)$$

  means "Everyone is at RIT and everyone is smart"

Existential Quantification

- Typically, $\wedge$ is the main connective with $\exists$
  Example: "Someone at RIT is smart":

$$\exists X \text{ at}(X, rit) \wedge \text{smart}(X)$$

- Common mistake: using $\rightarrow$ as the main connective with $\exists$:

$$\exists X \text{ at}(X, rit) \rightarrow \text{smart}(X)$$

This means:

"If RIT has a student then that student is smart."

Properties of Quantifiers

- $\forall X \, \forall Y$ is the same as $\forall Y \, \forall X$
- $\exists X \, \exists Y$ is the same as $\exists Y \, \exists X$
- $\exists X \, \forall Y$ is not the same as $\forall Y \, \exists X$
- $\exists X \, \forall Y$ loves(X,Y)
  - "There is a person who loves everyone"
- $\forall Y \, \exists X$ loves(X,Y)
  - "Everyone is loved by someone"
- Quantifier duality: each can be expressed using the other:
  - $\forall X$ likes(X, iceCream)= $\neg\exists X \, \neg$likes(X, iceCream)
  - $\exists X$ likes(X, broccoli) = $\neg\forall X \, \neg$likes(X, broccoli)

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object
- For example, the definition of *sibling* in terms of *parent* is:

  $\forall X,Y\ (\neg(X = Y) \wedge\ \exists M,F\ \neg(M = F) \wedge \text{parent}(M,X) \wedge \text{parent}(F,X) \wedge \text{parent}(M,Y) \wedge\ \text{parent}(F,Y) \rightarrow \text{sibling}(X,Y))$

Using First-Order Logic

First-order rules for the kinship domain:

- Brothers are male siblings
  $\forall X,Y\ (male(X) \wedge sibling(X,Y) \rightarrow brother(X,Y))$

- “sibling” is symmetric
  $\forall X,Y\ (sibling(Y,X) \rightarrow sibling(X,Y))$

- One's mother is one's female parent
  $\forall M,C\ (female(M) \wedge parent(M,C) \rightarrow mother(M,C))$

Using First-Order Logic

For example, given the following facts:

mother(eve,abel)
mother(eve,cain)
father(adam,abel)
father(adam,cain)

And the following rules:

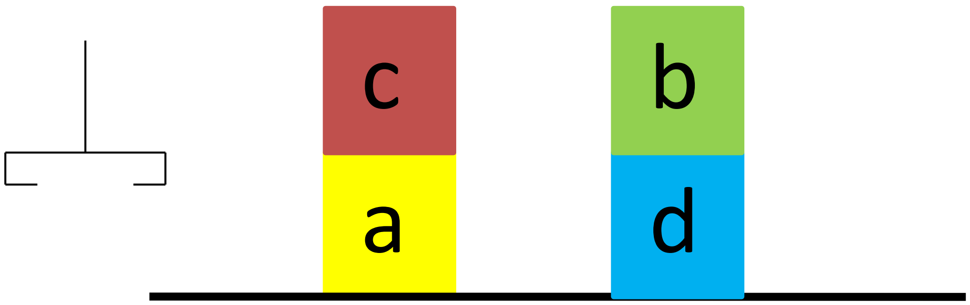
$\forall X,Y\ (father(X,Y) \vee mother(X,Y) \rightarrow parent(X,Y))$
$\forall X,Y,Z\ (parent(X,Y) \wedge parent(X,Z) \rightarrow sibling(Y,Z))$

One can conclude that cain and abel are siblings (or half-siblings)

Examples of First-Order Predicates

- "If it doesn't rain on Monday, Tom will go to the mountains."
  $\neg$weather(rain, monday) $\rightarrow$ go(tom, mountains)
- "Emma is a Doberman pinscher and a good dog."
  is_a(emma,doberman) $\wedge$ good_dog(emma)
- "All basketball players are tall."
  $\forall$ X (basketball_player(X) $\rightarrow$ tall(X))
- "Some people like anchovies."
  $\exists$ X (person(X) $\wedge$ likes(X, anchovies))
- "If wishes were horses, beggars would ride."
  equal(wishes, horses) $\rightarrow$ ride(beggars)
- "Nobody likes taxes."
  $\neg\exists$ X likes(X, taxes)

Blocks World Example



A collection of logical clauses describes the important properties and relationships in a Blocks World:

on(c,a)
on(b,d)
onTable(a)
onTable(d)
clear(b)
clear(c)
hand_empty

Suppose you want to define a test to determine whether all blocks are clear (have nothing stacked on top of them):

$\forall X (\neg \exists Y \text{ on } (Y, X)) \rightarrow \text{clear}(X).$

"For all X, if there does not exist a Y such that Y is on X, then X is clear."