Lecture16

Tuesday, March 19, 2019 9:33 AM

<u>Test problems</u>

- Simplest: shocktube in Minkowski
  - pure hydro
  - fully non-linear
- Analytic solutions are few. Some are:
  - TOV equilibrium: can be challenging to maintain balance. Spacetime should <u>not</u> evolve.
  - Oppenheimer-Snyder collapse: nice analytic solution; requires some coordinate transforms
- tests of conservation:
  - ADM mass, angular momentum
  - circulation (Kelvin-Helmholtz theorem)

Imperfect gasses

- inclusion of viscosity, conductivity, radiation
- needed for most realistic astrophysical problems

problems

Viscosity

- viscous part of S-E tensor:

$$T^{ab}_{visc} = -2\eta\sigma^{ab} - \zeta\Theta P^{ab}$$

$\eta \geq 0$ ... coeff. dynamic/shear viscosity

$\zeta \geq 0$ ... coeff. of bulk viscosity

$\Theta = \nabla_a u^a$ ... shear

$P^{ab} = g^{ab} + u^a u^b$ ... expansion tensor

$\sigma^{ab} = \frac{1}{2}(P^{ac}\nabla_c u^b + P^{bc}\nabla_c u^a) - \frac{1}{3}\Theta P^{ab}$

... projection tensor

- w/ viscosity, energy equation,

$$\partial_t(\gamma^{1/2} E_*) + \partial_j(\gamma^{1/2} E_* v^j) = 0$$

becomes,

$$\boxed{\partial_t(\gamma^{1/2} E_*) + \partial_j(\gamma^{1/2} E_* v^j) = \frac{\alpha\gamma^{1/2}}{\Gamma}\left(\frac{E_*}{w}\right)^{(1-\Gamma)}(2\eta\sigma^{ab}\sigma_{ab} + \zeta\Theta^2)}$$

→ implies entropy generation rate,

$$\rho_0 T \frac{ds}{d\tau} = (2\eta\sigma^{ab}\sigma_{ab} + \zeta\Theta^2)$$

- momentum eqn. becomes,

$$\partial_t(\gamma^{1/2} S_i) + \partial_j(\gamma^{1/2} S_i v^j) =$$
$$-\alpha\gamma^{1/2}\left(\partial_i P + \frac{S_a S_b}{2\alpha S^t}\partial_i g^{ab}\right) + \alpha\gamma^{1/2}\eta\sigma_{ab}\partial_i g^{ab}$$

$$+ 2\partial_a\left(\alpha\gamma^{1/2}\eta\sigma_{ib}g^{ab}\right) + \frac{1}{2}\alpha\gamma^{1/2}\zeta\Theta P_{ab}\partial_i g^{ab}$$
$$+ \partial_a\left(\alpha\gamma^{1/2}\zeta\Theta P_{ib}g^{ab}\right)$$

$\rightarrow$ relativistic Navier-Stokes eqns.

- these eqns. then are non-conservative
- presence of time derivatives of u_a + σ_{ab} on RHS make these eqns. tricky
- often solved "operator-split"

Heat/Radiation diffusion

- heat flux conduction S-E:

$$T^{ab}_{heat} = u^a q^b + u^b q^a$$

$$q^a = -\lambda_{th} P^{ab}\left(\nabla_b T + T a_b\right) \quad \text{... heat flux}$$

T ... temperature

$a^a = u^b \nabla_b u^a$ ... 4-acceleration of fluid

λ_{th} ... coeff. of thermal conduction

- for diffusion approx. of thermal radiation,

$$\lambda_{th} = \frac{4}{3}\frac{b_R T^3}{\bar{\chi}}$$

b_R ... radiation constant (depends on partcle) photons: $b_R = b = 8\pi^5 k_B^4 / (15 c^3 h^3) = 7.565 \times 10^{-15}$ erg/cm^3/deg^4

non-degenerate neutrinos: $b_R = \frac{7}{16} b$

(in equilibrium, chemical pot. $= 0$)

$\bar{\chi}$ ... Rosseland mean opacity

Radiation hydro

- if gas is neither optically-thick or -thin, becomes a transport problem.
- radiation S-E tensor:

$$T_{rad}^{ab} = \iint d\nu \, d\Omega \, I_\nu N^a N^b$$

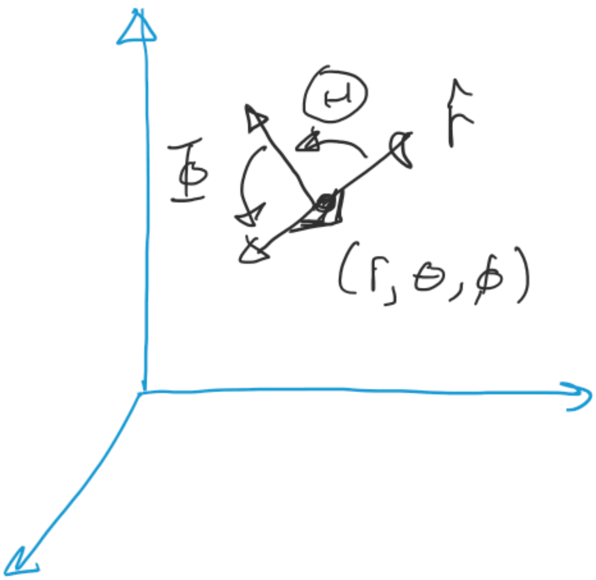
$I_\nu = I(x^a; N^i; \nu)$ ... specific intensity of radiation at x^a w/ frequency ν and 4-mom. p^a moving in direction $N^a \equiv p^a / h\nu$ [$d\Omega$ is solid angle]

- frame matters! I_ν, ν, $d\Omega$ all in frame of observer w/ 4-vel. u^a, $\therefore h\nu = -p_a u^a$

- radiation phase-space distribution function:

$$f = \frac{c^2}{h^4} \left(\frac{I_\nu}{\nu^3} \right)$$

- Lorentz invariant



- in spherical symmetry, radiation field at (r, θ, ϕ) depends only on angle from radial direction, Θ, not azimuth, Φ
  - $\rightarrow$ define $\mu = \cos\Theta$
  - $\Rightarrow I_\nu = I_\nu(t, r, \mu)$

- in spherical coords (t, r, θ, ϕ), moments of specific intensity:

$$I = \int d\nu \, I_\nu \quad \text{... mean intensity}$$

$$E(t,r) = 2\pi \iint d\nu \, d\mu \, I_\nu \quad \text{... radiation energy density}$$

$$F(t,r) = 2\pi \iint d\nu \, d\mu \, \mu \, I_\nu \quad \text{... radiation energy flux}$$

$$P(t,r) = 2\pi \iint d\nu \, d\mu \, \mu^2 I_\nu \quad \text{... radiation pressure}$$

- defining basis vectors $\{e^a_{(\hat{t})}, e^a_{(\hat{r})}, e^a_{(\hat{\theta})}, e^a_{(\hat{\phi})}\}$ components of S-E tensor are

$$T^{\hat{a}\hat{b}}_{rad} = \begin{pmatrix} E & F & 0 & 0 \\ F & P & 0 & 0 \\ 0 & 0 & \frac{1}{2}(E-P) & 0 \\ 0 & 0 & 0 & \frac{1}{2}(E-P) \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 & 2(E-P) \end{pmatrix}$$

- radiation moments evolve according to

$$\nabla_b T^{ab}_{rad} = -G^a$$

$$G^a = \iint d\nu \, d\Omega \left[\chi_\nu I_\nu - \eta_\nu \right] N^a$$

.... radiation 4-force density

- fluid evolution equation:

$$\nabla_b T^{ab}_{fluid} = -\nabla_b T^{ab}_{rad} = G^a$$

→ what comes out of fluid must go into radiation & vice versa

- must solve fluid & radiation moment evolution simultaneously along w/ transport eqn. to determine I_ν
- radiation also contributes to spacetime curvature.