

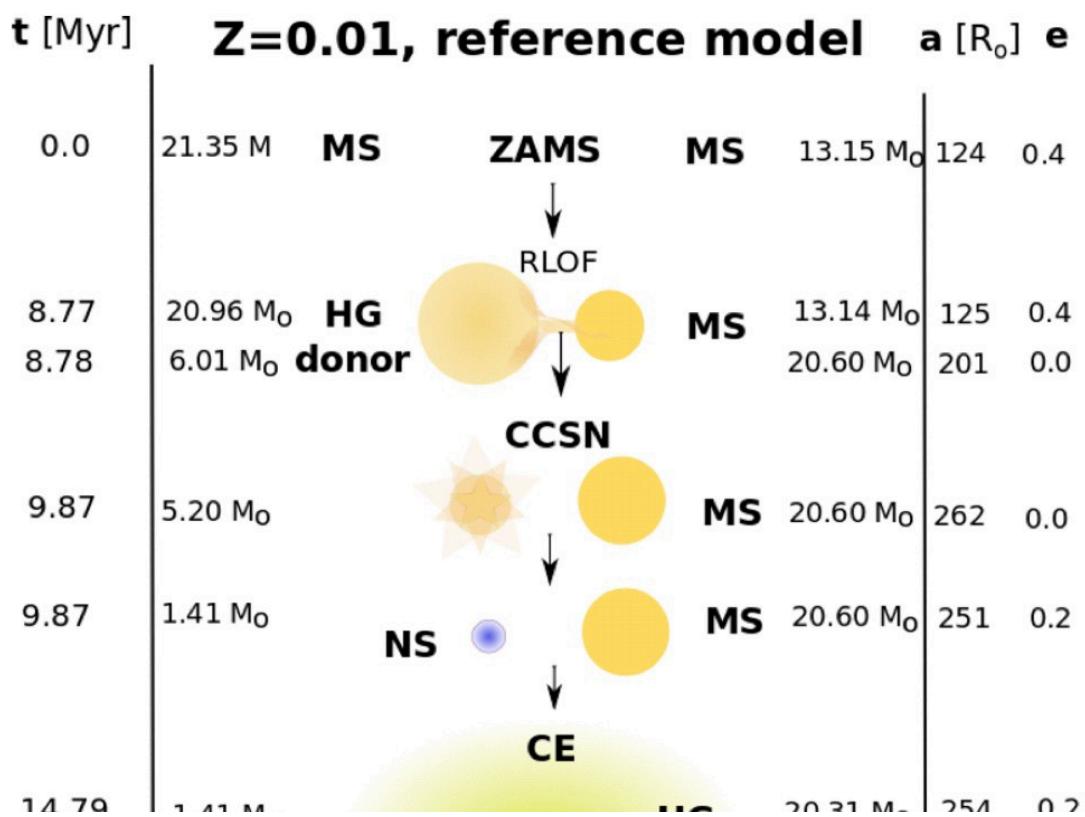
# Lecture 24

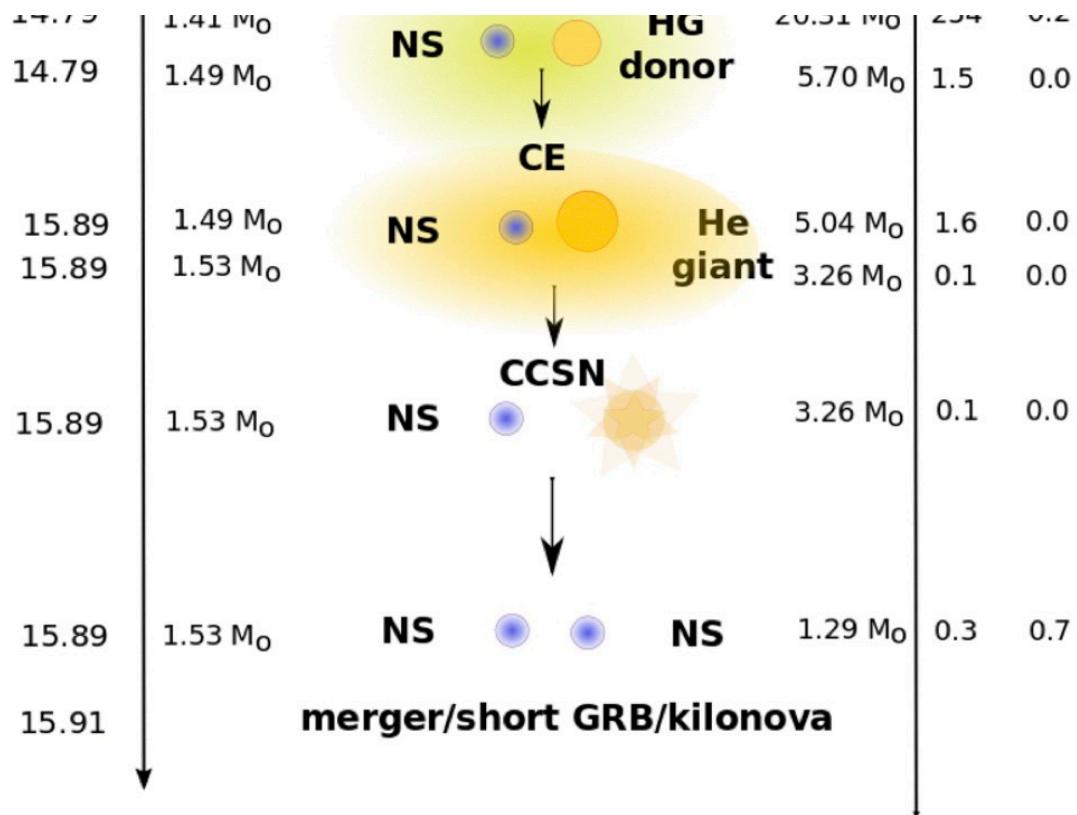
Tuesday, April 23, 2019

10:19 AM

## Binary Neutron Star Mergers

- see review by Baiotti & Regassa (2017, Reports on Progress in Physics, 80, 096901)
- 2 possible channels for producing BNS:
  - binary stellar evolution
  - dynamical capture in dense stellar environments

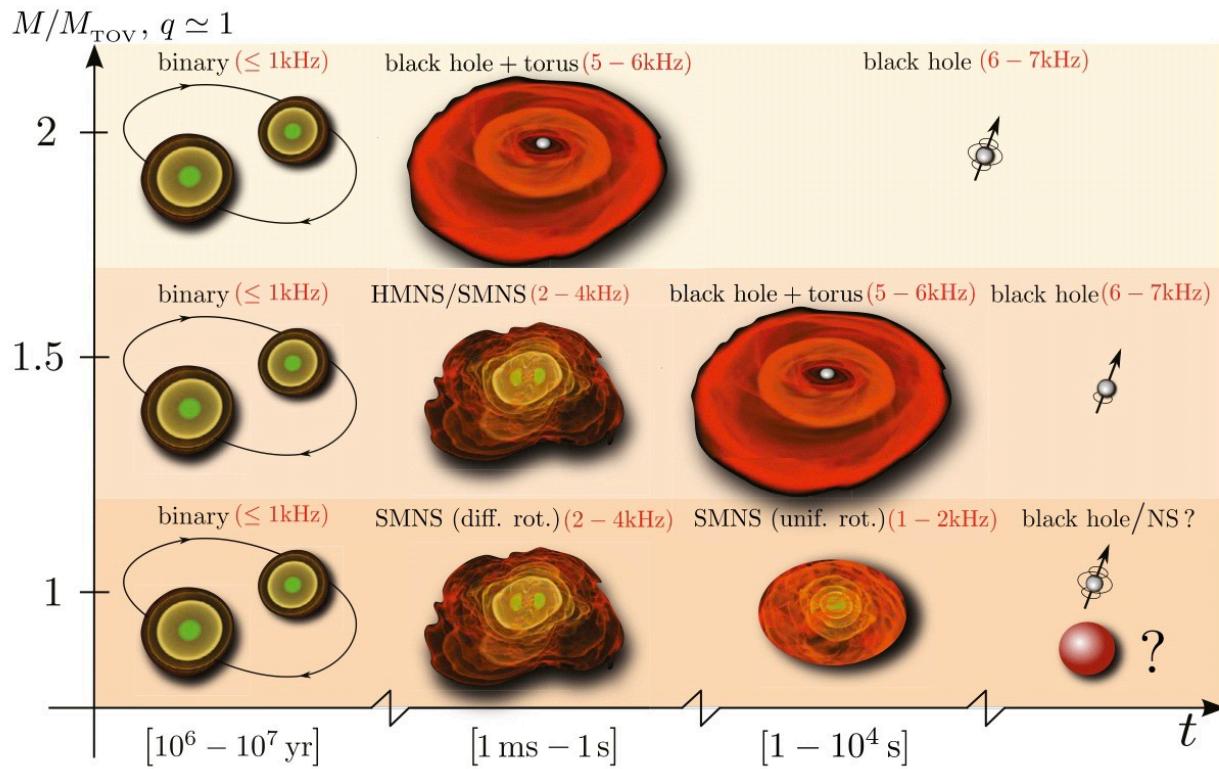




→ from Chruslinska et al. (2018, MNRAS)

- strong connection to short gamma-ray bursts (sGRBs)
- associated cyclical emission generally referred to as **kilonova** : 1000x brighter than a typical nova (NOT super-nova)
- likely site for production of heavy  $\sigma$ -process elements
- overview of evolution of BNS to

# merger (Baiotti & Rezzolla):



- HMNS: hypermassive NS:  $M >$  minimum for uniformly rotating NS  
→ collapse to BH delayed by differential rotation support
- SMNS: supramassive NS: rotating NS w/  $M > M_{\text{TOV}}$  → the mass limit for non-rotating NS

Numerical simulation of BNS

- numerical simulation

- consequences
  - must be 3D
  - needs dynamical spacetime
  - microphysical nuclear EOS
  - magnetohydrodynamics
  - neutrino transport
  - Oh my!
- BSSN a common approach  
(see previous lecture 23)

gauge choice for BNS sims

- hyperbolic singularity avoidance  
"1+log" slicing conditions:  

$$(\partial_t - \beta^i \partial_i) \alpha = -f(\alpha) \alpha^2 (K - K_0)$$

where  $f(\alpha) > 0$  &  $K_0 \equiv K(t=0)$
- "Gamma-drive" shift conditions:  

$$\partial_t^2 \beta^i = F \partial_t \tilde{\gamma}^i - \eta \partial_t \beta^i$$

$F$  &  $\eta$ : positive functions of space & time

$\rightarrow$  drives  $\tilde{P}^i$  to be constant

- first-order implementation of these gauge conditions:

$$\partial_t \beta^i - \beta^j \partial_j \beta^i = \frac{3}{4} \alpha B^i$$

$$\partial_t B^i - \beta^j \partial_j B^i = \partial_t \tilde{P}^i - \beta^j \partial_j \tilde{P}^i - \eta B^i$$

$B^i$ : auxiliary variable (NOT mag. field)

$\eta$ : acts as a damping coefficient  
to avoid oscillations in  $\beta^i$

- '1+log' + 'Gamma-driver' gauge conditions ensure:
  - singularities are avoided
  - coordinate distortions due to large spatial curvature are minimized

### Spacetime evolution w/ CCZ4 & Z4c

- alternatives to BSSN

$\wedge \quad \square \quad - \quad \nabla \quad A \quad \partial \partial - H$

→ combine some advantages of both  
BSSN and generalized harmonic  
coordinates

- CCZ4: conformal + covariant "Z4"
  - in Z4, elliptic constraints converted into algebraic conditions for a new 4-vector  $Z_\mu$  that measures the deviation from Einstein field eqns.
  - derived from covariant Lagrangian:

$$\mathcal{L} = g^{\mu\nu} [R_{\mu\nu} + 2\nabla_\mu Z_\nu]$$

- $Z_\mu = 0$  amounts to satisfying both Hamiltonian & momentum constraints
- damped formulation of Z4:
 
$$R_{\mu\nu} + \nabla_\mu Z_\nu + \nabla_\nu Z_\mu + \kappa_1 [n_\mu Z_\nu + n_\nu Z_\mu - (1 + \kappa_2) g_{\mu\nu} n_\sigma Z^\sigma] = 8\pi (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T)$$
  - ... Z4 field eqns.
- in CCZ4, energy-momentum constraints

become evolution eqns. for  $Z_\mu$   
 $\rightarrow "3+1"$  goes from weakly to  
 strongly hyperbolic

- CCZ 4 eqns.:

$$(\partial_t - \mathcal{L}_{\vec{\beta}}) \gamma_{ij} = -2\alpha K_{ij}$$

$$\begin{aligned} (\partial_t - \mathcal{L}_{\vec{\beta}}) K_{ij} &= -\nabla_i \alpha_j + \alpha [R_{ij} + 2\nabla_{(i} Z_{j)}] \\ &\quad - 2K_i^l K_{lj} + (K - 2\Theta) K_{ij} - K_1(1 + K_2)\Theta \gamma_{ij} \\ &\quad - 8\pi \alpha [S_{ij} - \frac{1}{2}(S - E)\gamma_{ij}] \end{aligned}$$

$$\begin{aligned} (\partial_t - \mathcal{L}_{\vec{\beta}}) \Theta &= \frac{\alpha}{2} [R + 2\nabla_j Z^j + (K - 2\Theta) K - K^{ij} K_{ij} \\ &\quad - 2\frac{Z^i \partial_j \alpha}{\alpha} - 2K_1(1 + K_2)\Theta - 16\pi E] \end{aligned}$$

$$\begin{aligned} (\partial_t - \mathcal{L}_{\vec{\beta}}) Z_i &= \alpha [\nabla_j (K_i^j - \delta_i^j K) + \partial_i \Theta - 2K_i^j Z_j \\ &\quad - \Theta \frac{\partial_i \alpha}{\alpha} - K_1 Z_i - 8\pi S_i] \end{aligned}$$

where,

$$E \equiv n_\alpha n_\beta T^{\alpha\beta}$$

$$S_i \equiv -\gamma_{i\alpha} n_\beta T^{\alpha\beta}$$

$$\hookrightarrow \dots \hookrightarrow \alpha^\beta$$

$$\gamma_{ij} \equiv \gamma_{ia} \gamma_{jb} / \gamma$$

$$\Theta \equiv n_\mu Z^\mu = \alpha Z^0 \dots \text{projection}$$

→ use same gauge conditions  
as for BSSN

### Conformally-flat spacetime

- "Isenberg-Wilson-McWilliams" approach
  - $\gamma_{ij} = \phi^4 \hat{\gamma}_{ij} ; \hat{\gamma}_{ij} = \eta_{ij}$
- simplifies things a lot
- results in system in which constraints are solved at every step (elliptic)
- does not admit gravitational radiation! Evolution of BNS orbit put in by hand via a post-Newtonian approx.

### Initial Data

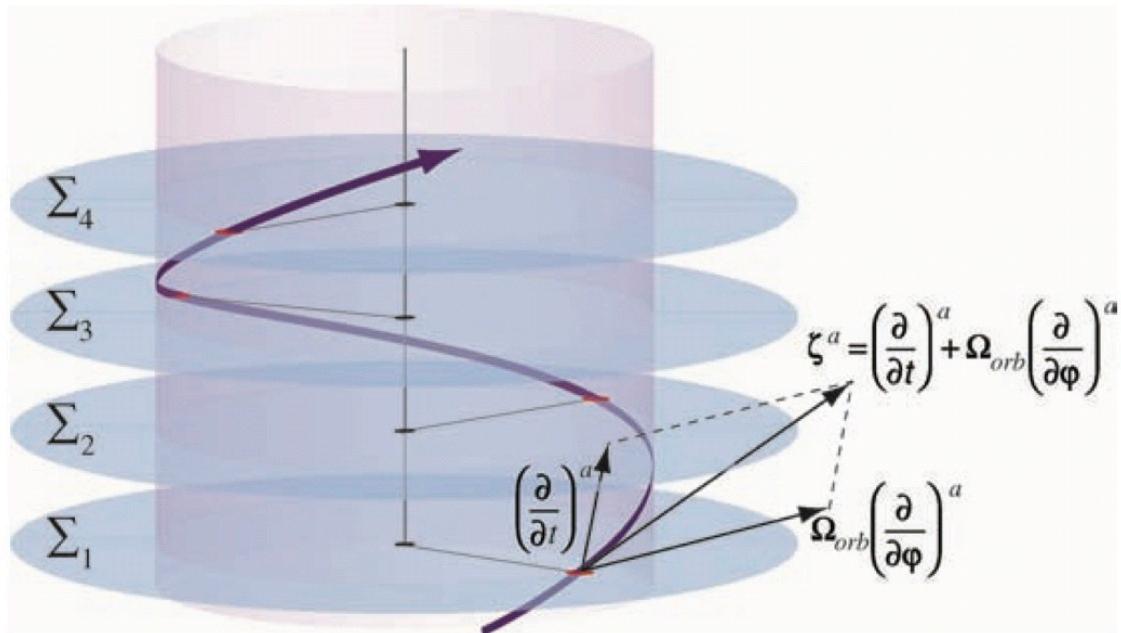
- must satisfy field eqns. including

Hamiltonian & momentum Constraints:

$$\mathcal{H} \equiv {}^{(3)}R + K^2 - K_{ij}K^{ij} - 16\pi E = 0$$

$$\mathcal{M}^i \equiv D_j(K^{ij} - \delta^{ij}K) - 8\pi S^i = 0$$

- conformal flatness usually used to solve this
- also assume initially helical symmetry:
  - ie, a corotating frame in which the BNS is stationary
  - mathematically equivalent to the existence of helical Killing vector:



**Figure 12.4** Illustration of a helical Killing vector generating a circular binary orbit.

- the LORENE code incorporates the above requirements & is publicly-available: [Www.lorene.obspm.fr](http://Www.lorene.obspm.fr)
- must choose to construct either irrotational or corotational BNS systems
  - the latter likely don't exist:  
too fast rotation!
- a problem in initial data is spurious eccentricity. Arises from assumptions of conformal flatness and circular binary orbit initially