Homework 12 Project Question

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3/27

For this week's paired project question, we summarize a few sources and explain what our Laser Wakefield Acceleration (LWFA) simulation will consist of based on last week's feedback and timeline.

In [1], the authors study the electron beam characteristics as the plasma density changes using 3D particle-in-cell simulations. In [2], the validity of linear-fluid theory is studied with particle-in-cell simulations. Bi-Gaussian beam formulas are derived from the assumption of linear wakefield theory and the conditions of validity are written in terms of the beam density.

These references are particularly useful for implementing our model. We will implement a basic simulation of an electron accelerating from LWFA. The model will consist of one LWFA 'bubble' which consists of a ring of electrons surrounding a region of positive charge. A spatial profile for the laser will be defined by when the first bubble region should form. Based off the references, the shape of the bubble will be elliptical (wider on the horizontal axis).

In chapter 9 of Griffiths he explores dispersive behavior for electromagnetic fields as they interact with matter. The simple spring-mass model that is assumed will make for an accurate enough representation of the behavior we would like to simulate. Therefore, we will begin by exploring the interaction of plane waves with loosely bound charges. The electric field of the charge distribution formed from passing a Gaussian wave packet through a plasma will act as the primary cause for charged particle acceleration in the region. If time is available we will investigate deeper and examine focusing effects of the electromagnetic field from the laser on our dynamic particle.

4/03

We have established an electron oscillation model as a function of z and t:

$$F_{driving} + F_{spring} = F_{tot} \tag{1}$$

.

$$m\frac{d^2x}{dt^2} = -kx - eE_0(z)cos(\omega t) \tag{2}$$

In EQN 2. we assume the spring model that was presented in class. The driving force is the electric field component of an incident EM plane wave.

$$x(z,t) = \frac{qEcos(kz)}{m(\omega^2 - \omega_0^2)}cos(\omega t) \tag{3}$$

In EQN 3. we introduce the electric field amplitude as a function of z.

A simulation in arbitrary units over a time interval of 1 second was calculated and is provided as a Jupyter notebook on Github. The programming was done in an object oriented fashion to facilitate the simulation of an arbitrary number of electrons. Currently, we look to extend the code to include object orientation for an assortment of independent EM waves to form a wave packet. The interaction of the wave packet with a sufficient number of electrons will result in the charged cavity that is needed to accelerate our hypothetical particle. Therefore, our goal for the coming week is to explore how the dispersive behavior of EM waves in matter can form a cavity capable of accelerating an electron.

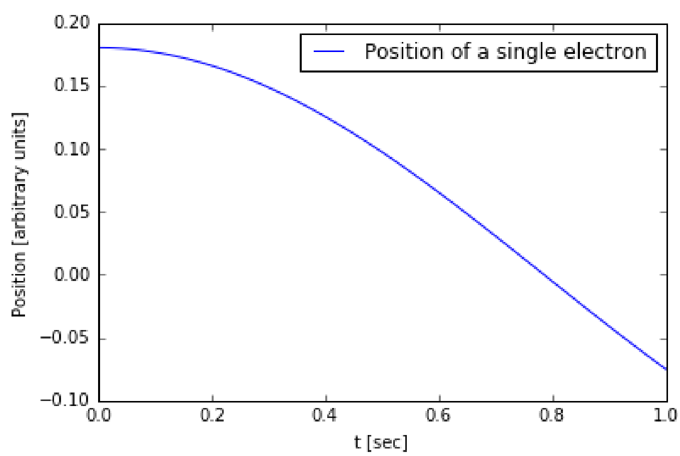

Figure 1: Single Electron Dispersion of incident plane wave over an interval of 1 second.

4/09

Using the code produced from last week, we created a few pictorial representations of a single plasma bubble in LWFA. The plasma bubble consists of an ellipse of electrons surrounding a section of positive charge in the middle where the bubble shape is formed from the incident laser. In figure 2, we depict the outer electrons in a single bubble evolving with time. Note, figure 2 does not account for the positive charge in the center of the bubble. This figure will be useful for describing the dynamics of LWFA. For now, consider one electron at the beginning of the bubble (z=0). The laser forms the bubble, the loose electron is attracted to the positive charge in the middle. The bubble then starts to close, reversing the time dynamics seen in figure 2. This accelerates the electron to the end of the bubble as it closes. This figure will be near the beginning of our poster to help explain the basic concept of LWFA.

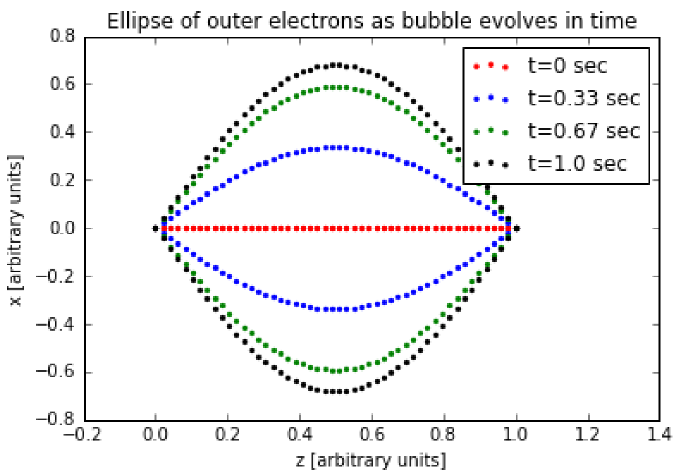

Figure 2: Time evolution of many electrons oriented along the z-axis after turning on the incident laser. The points correspond to the position of the outer electrons along the plasma bubble. Initially, the ellipse of electrons expand outward. Eventually the electrons fall back into their initial position after some period of time.

After establishing the formation of the bubble region under the influence of a single plane wave as we see in figure 2, we looked to reference [1] for characterization of the laser light that we plan to have interact with our plasma. In figure 3 we see the complete formation of the bubble region for a gaussian wave packet characterized for as a 35 fs wave pulse. For now the trial packet is composed of 3 waves with frequencies uniformly distributed between -pi/2 to pi/2. For next week it should be simple to incorporate a more specific distribution of THz frequencies as used in [1]. With the inclusion of negative and positive symbols to demonstrate the placement of charges amongst the packet one may find it clear as to how the electric field in such a region would accelerate a negative charge. For next week we plan to incorporate an electric field calculation that can be used to accelerate a free electron. The tentative coding approach for next week is to use a velocity vertlet method to calculate the motion of a negatively charged electron through the bubble region. To examine the, closer to life, time dependent behavior of the electromagnetic wave dispersion we plan to let the bubble region evolve over time along side the motion of our free electron.

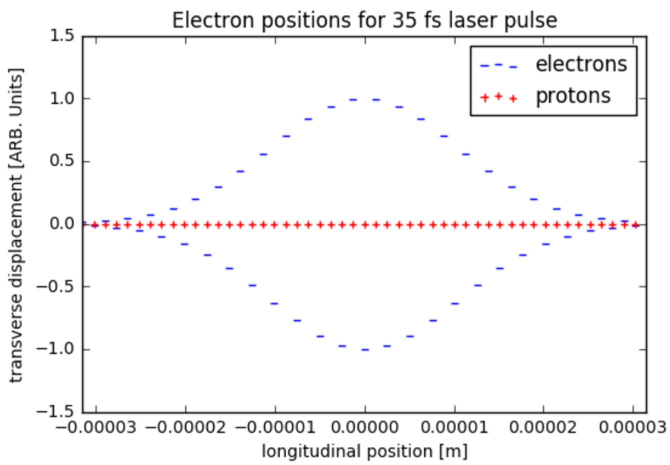

Figure 3: dispersion of a gaussian wave packet with a temporal width of 35 fs. This particular wave has a uniform distribution of frequencies between - pi/2 and pi/2 split between 3 waves. Blue '-' denote electron placement and red '+' denote proton placement.

References

[1] S.P.D. Mangles et al., *Proton-driven plasma wakefield acceleration: a path to the future of high- energy particle physics.* Plasma Physics and Controlled Fusion, 56(8):084013, 2014.
[2] W. Lu et al., *Limits of linear plasma wakefield theory for electron or positron beams.* Physics of Plasmas, 12(6):063101, 2005.
[3] Griffiths, D., *Introduction to Electrodynamics, 4th Edition* Pearson, October 6th 2012. Ch. 9