

Homework 11 Project Question

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For this week's paired project question, we summarize a few sources and explain what our Laser Wakefield Acceleration (LWFA) simulation will consist of based on last week's feedback and timeline.

In [1], the authors study the electron beam characteristics as the plasma density changes using 3D particle-in-cell simulations. In [2], the validity of linear-fluid theory is studied with particle-in-cell simulations. Bi-Gaussian beam formulas are derived from the assumption of linear wakefield theory and the conditions of validity are written in terms of the beam density.

These references are particularly useful for implementing our model. We will implement a basic simulation of an electron accelerating from LWFA. The model will consist of one LWFA 'bubble' which consists of a ring of electrons surrounding a region of positive charge. A spatial profile for the laser will be defined by when the first bubble region should form. Based off the references, the shape of the bubble will be elliptical (wider on the horizontal axis).

In chapter 9 of Griffiths he explores dispersive behavior for electromagnetic fields as they interact with matter. The simple spring-mass model that is assumed will make for an accurate enough representation of the behavior we would like to simulate. Therefore, we will begin by exploring the interaction of plane waves with loosely bound charges. The electric field of the charge distribution formed from passing a Gaussian wave packet through a plasma will act as the primary cause for charged particle acceleration in the region. If time is available we will investigate deeper and examine focusing effects of the electromagnetic field from the laser on our dynamic particle.

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We have established an electron oscillation model as a function of z and t :

$$F_{driving} + F_{spring} = F_{tot} \tag{1}$$

$$m \frac{d^2 x}{dt^2} = -kx - eE_0(z) \cos(\omega t) \quad (2)$$

In EQN 2. we assume the spring model that was presented in class. The driving force is the electric field component of an incident EM plane wave.

$$x(z, t) = \frac{qE \cos(kz)}{m(\omega^2 - \omega_0^2)} \cos(\omega t) \quad (3)$$

In EQN 3. we introduce the electric field amplitude as a function of z .

A simulation in arbitrary units over a time interval of 1 second was calculated and is provided as a Jupyter notebook on Github. The programming was done in an object oriented fashion to facilitate the simulation of an arbitrary number of electrons. Currently, we look to extend the code to include object orientation for an assortment of independent EM waves to form a wave packet. The interaction of the wave packet with a sufficient number of electrons will result in the charged cavity that is needed to accelerate our hypothetical particle. Therefore, our goal for the coming week is to explore how the dispersive behavior of EM waves in matter can form a cavity capable of accelerating an electron.

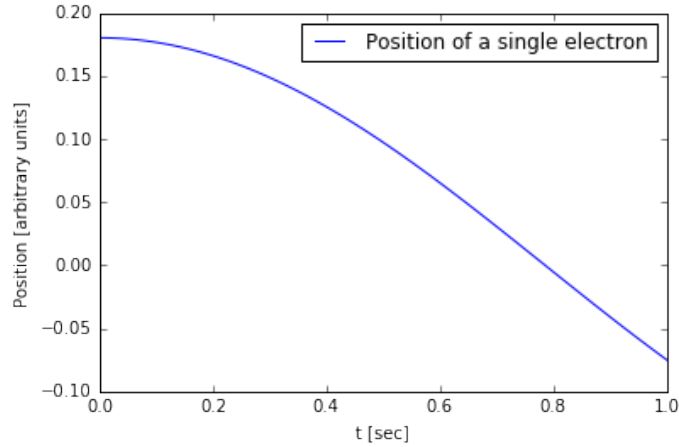


Figure 1: Single Electron Dispersion of incident plane wave over an interval of 1 second.

References

- [1] S.P.D. Mangles et al., *Proton-driven plasma wakefield acceleration: a path to the future of high- energy particle physics*. Plasma Physics and Controlled Fusion, 56(8):084013, 2014.
- [2] W. Lu et al., *Limits of linear plasma wakefield theory for electron or positron*

beams. Physics of Plasmas, 12(6):063101, 2005.

[3] Griffiths, D., *Introduction to Electrodynamics*, 4th Edition Pearson, October 6th 2012. Ch. 9