MAGNETO-OPTICAL TRAP THEORY

We use a one dimensional magneto-optical force derived in [1]:

$$\begin{aligned} f_{MOT}(z, v_z) &= L \cdot (\beta z + k v_z) \\ L &= \hbar k \Gamma \Delta \frac{\Omega^2/2}{(\Delta^2 + (\Gamma/2)^2 + \Omega^2/2)^2)} \end{aligned} \tag{1}$$

Where $\Delta = \omega - \omega_0$, $\Omega = d\epsilon_0/\hbar$ is the Rabi frequency, Γ is the decay rate, βz is the zeeman term and kv is the doppler term. From this we consider an atom shifted off of the z axis. The laser follows a Gaussian beam profile, so we can tack on a normalized Gaussian to the force to account for the beam intensity:

$$F_{MOT}(z) = f(z) \cdot G(x, y) \tag{2}$$

$$G(x, y) = \frac{e^{-(x^2/\sigma_x^2 + y^2/\sigma_y^2)}}{2\pi\sqrt{\sigma_x \sigma_y}} \tag{3}$$

In 3 dimensions, the total force is given by:

$$F_{MOT}(x, y, z) = f(x) \cdot G(y, z) + f(y) \cdot G(x, z) + f(z) \cdot G(x, y) \tag{4}$$

The potential for an atom moving along a path in the x direction is given by:

$$\begin{aligned} U_x &= -\int F_{MOT} \cdot dx \\ &= -L \cdot G(y, z) \cdot (\beta \int x \cdot dx + k \int v_x \cdot dx) = -L \cdot G(y, z) \cdot (\beta x^2/2 + k v_x x) \end{aligned} \tag{5}$$

$$\begin{aligned} v_i = v_{i-1} + F_{MOT}(x_{i-1}, y_{i-1}, z_{i-1}) t/m \\ t = t_i - t_{i-1} \end{aligned} \tag{6}$$

[1] http://www.iqst.ca/media/pdf/publications/PantitaPalittapongarnpim.pdf