Rayleigh's Stingrays
Background

The basic theory behind Rayleigh scattering is electric dipole radiation. In general, the electron configuration of the air molecules in the atmosphere results in a small dipole moment. If the wavelength of light passing through the atmosphere is much larger than the size of the particles, the oscillating electric field of the wave will cause the tiny dipole to oscillate at the same frequency as the wave. Back on the surface of the earth, we see the radiation from the accelerating dipole as scattered light.

Consider two tiny spheres of opposite charge q_0 and $-q_0$, separated by a distance d. In the presence of an oscillating electric field, this physical dipole will also oscillate, resulting in a charge distribution that varies with time: $q(t) = q_0 \cos(\omega t)$. So the potential of the dipole is given by

$$V(\mathbf{r}, t) = \frac{1}{4\pi\epsilon_0}\left(\frac{q_0\cos(\omega t_+)}{\mathscr{r}_+} - \frac{q_0\cos(\omega t_-)}{\mathscr{r}_-}\right),$$

where $t_+ = t - \frac{\mathscr{r}_+}{c}$ and $t_- = t - \frac{\mathscr{r}_-}{c}$ accounts for the finite time delay caused by the speed of the EM wave. In the limit $d \ll r$, this physical dipole becomes a perfect dipole. If we also apply the Rayleigh limit $d \ll \lambda$ and assume we are interested in the resulting fields in the radiation zone $r \gg \lambda$, the potential simplifies to

$$V(r, \theta, t) = -\frac{q_0 d\omega}{4\pi\epsilon_0 c}\left(\frac{\cos\theta}{r}\right)\sin(\omega t_r),$$

where $t_r = t - \frac{r}{c}$. Now picture that the two spheres are connected by a thin wire. Then the oscillating charges cause an oscillating current to flow in the wire: $\mathbf{I}(t) = -q_0\omega\sin(\omega t)\hat{\mathbf{z}}$. So by applying the approximations stated above, we obtain the vector potential

$$\mathbf{A}(r, \theta, t) = -\frac{\mu_0 q_0 d\omega}{4\pi r}\sin(\omega t_r).$$

Now we just proceed to compute the electric and magnetic fields of the radiating dipole:

$$\mathbf{E} = -\nabla V - \frac{\partial \mathbf{A}}{\partial t} = -\frac{\mu_0 q_0 d\omega^2}{4\pi}\left(\frac{\sin\theta}{r}\right)\cos(\omega t_r)\hat{\theta}$$

$$\mathbf{B} = \nabla \times \mathbf{A} = -\frac{\mu_0 q_0 d\omega^2}{4\pi c}\left(\frac{\sin\theta}{r}\right)\cos(\omega t_r)\hat{\phi}$$

Finally, we obtain the intensity by time averaging the Poynting vector $\mathbf{S}$:

$$\langle \mathbf{S} \rangle = \left(\frac{\mu_0 q_0^2 d^2 \omega^4}{32\pi^2 c}\right)\frac{\sin^2\theta}{r^2}\hat{\mathbf{r}} = \left(\frac{\mu_0 \pi^2 q_0^2 d^2 c^3}{2\lambda^4}\right)\frac{\sin^2\theta}{r^2}\hat{\mathbf{r}},$$

where we have used $\omega = \frac{2\pi c}{\lambda}$. The most important aspect to note is that the intensity has a strong wavelength dependence, i.e. $I \propto \frac{1}{\lambda^4}$, which accounts for the blue-ish hue of the sky.