Three-Dimensional EMHD Simulation Studies of Non-linear Magnetic Structures in Magnetized Plasmas

* Mathematical Model

The numerical simulation is based on the nonlinear EMHD equation, which is deduced from Faraday's & Ampere's Laws

$$\frac{1}{c}\frac{\partial \vec{B}}{\partial t} = -\nabla\times\vec{E} \quad [1]$$

and

$$\nabla\times\vec{B} = -\frac{4\pi e n_e \vec{v}_e}{c} \quad [2]$$

These together with the electron momentum equation

$$m_e\left(\frac{\partial}{\partial t} + \vec{v}_e\cdot\nabla\right)\vec{v}_e = e\left(\vec{E} + \frac{\vec{v}_e\times\vec{B}}{c}\right) \quad [3]$$

where $\vec{B}$ & $\vec{E}$ are the magnetic and electric fields, $\vec{v}_e$ is the electron fluid velocity and n_e is the electron number density, and m_e is the electron mass and e is the magnitude of the electron charge and c is the speed of light in vacuum.

Equation [3] is true whenever $\partial/\partial t$ >> the electron-ion collision frequency. Furthermore, since the divergence of the electron flux is zero, there are insignificant density fluctuations associated with the whistlers. The wave-equation interaction is absent in our equations. By eliminating $\vec{E}$ and $\vec{v}_e$ and noting that $(\vec{v}_e\cdot\nabla)\vec{v}_e = -\vec{v}_e\times(\nabla\times\vec{v}_e) + \nabla v_e^2/2$, we obtain the nonlinear EMHD

$$\frac{\partial}{\partial t}(\vec{B} - \lambda_e^2\nabla^2\vec{B}) = \frac{c^2}{4\pi e n_0}\nabla\times\left((\vec{B} - \lambda_e^2\nabla^2\vec{B})\times(\nabla\times\vec{B})\right), [4]$$

which is the desired equation for computer simulations

Set up for simulation

The initial conditions for equation [4] are

$\vec{B} = B_0\hat{z} + \nabla\times\vec{A} + B_{tor}\hat{\phi}$. In cylindrical coordinates the vector potential $\vec{A}$ is taken to be of the form

$$\vec{A} = [A_{forward}(r,z) + A_{reverse}(r,z)]\hat{\phi}$$

Here the forward field is

$$A_{forward}(r,z) = A_0\frac{r}{D}\exp\left[\frac{-(r-r_0)^2 + z^2}{D^2}\right]$$

is centered at zero ($z=0$) and the reverse vector potential is

$$A_{reverse}(r,z) = -A_0\frac{r}{D}\exp\left[\frac{-(r-r_0)^2 + (z-5)^2}{D^2}\right]$$

is centered at $z=5$cm. The toroidal magnetic field is chosen to be of the form

$$B_{tor}(r,z) = -B_{0,tor}\frac{r}{D}\exp\left[\frac{-(r-r_0)^2 + (z-5)^2}{D^2}\right]$$