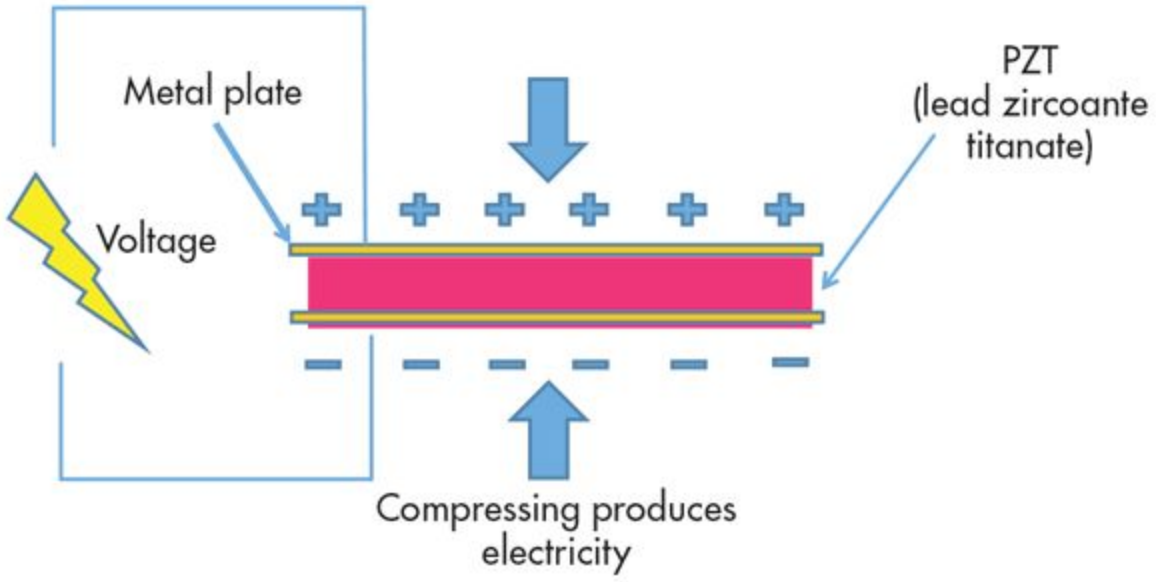
Team Catastrophe
Piezoelectricity

Central Questions

What is the general mathematical model for piezoelectrics?
How is this effect different for unique materials?
Why do we use lithium niobate for SAW in graphene? (And how does it work.)

Theory

Piezoelectricity is defined as the electric charge which accumulates in crystalline materials in response to mechanical stress on the lattice. The idea behind this is that a crystal has a certain configuration of polarizations, and the application of mechanical stress changes this configuration of dipoles. Piezoelectricity may vary the polarization strength or direction, or both. This is dependent on the mechanical stress applied and the crystal itself. This process is reversible, and the inverse process of an electric field deforming the mechanical configuration of the crystal is the converse piezoelectric effect.



Mathematical Model Overview

Linear piezoelectric materials combine two mathematical models to produce a pair of coupled equations. The two models are Hooke's law relating strain to stress by a constant [Eq. 1], and that linear materials have a displacement field that is proportional to the electric field [Eq. 2].

$$\sigma_{ij} = C_{ijkl}\epsilon_{kl} \quad [1]$$

$$\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0(1 + \chi_e)\vec{E} = \varepsilon\vec{E} \quad [2]$$

In the first equation the σ is the stress and ε is the strain, while C is the fourth-order tensor relating the two. The second equation shows D as the displacement field, E as the electric field, and ε being the permittivity. These two equations when coupled give rise to the following:

$$\begin{bmatrix} \mathbf{S} \\ \mathbf{D} \end{bmatrix} = \begin{bmatrix} \mathbf{s}^E & \mathbf{d}^t \\ \mathbf{d} & \boldsymbol{\varepsilon}^T \end{bmatrix} \begin{bmatrix} \mathbf{T} \\ \mathbf{E} \end{bmatrix}$$

Here S is stress, D is the displacement field, T is the strain, E is the electric field, and the middle 2x2 matrix describes the constants of piezoelectricity, permittivity, and strain. We will be reducing this equation to describe a thin plate, since that is the piezoelectric material we use in our SAW set-up (we use a thin film of Lithium Niobate).