CSCI 301 HW 11

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April 14, 2023

Problem 1

Let

$$\begin{aligned} |R| &= \operatorname{Rock} &= 150 \\ |C| &= \operatorname{Classical} &= 100 \\ |J| &= \operatorname{Jazz} &= 75 \end{aligned}$$

$$\begin{split} |R \cap C| &= 30 \\ |R \cap J| &= 20 \\ |C \cap J| &= 10 \\ |R \cap C| \wedge J &= 5 \end{split}$$

 \mathbf{a}

$$\label{eq:Rock Music Only} \begin{split} \text{Rock Music Only} &= |R| = 150 \\ \text{Classical Music Only} &= |C| = 100 \\ \text{Jazz Music Only} &= |J| = 75 \end{split}$$

b

Rock and Classical =
$$|R \cap C| = 30$$

Rock and Jazz = $|R \cap J| = 20$
Classical and Jazz = $|C \cap J| = 10$

Problem 2

The thief was wearing a hat, only if it was not Carol.	$ ilde{G} \Longrightarrow ilde{A}$	L	T	L	T	Ĺ	Ē	L	L	Ē	Ē	T	П	Ē	T
	$\tilde{C} \Longrightarrow H$	L	L	L	L	L	L	L	L	Ĺ	L	Ĺ	T	Ē	H
If the burglar was wearing gloves, then it was not Bob.	$G \Longrightarrow \tilde{B}$	L	L	ĹΤι	ഥ	L	L	L	L	L	L	ഥ	Ĺ	L	F
The burglar was wearing a hat	Н	ſΞı	H	ſΞ	L	ĹΤι	H	ſΞ	L	ſΞı	H	ſΞı	L	Έų	낸
The burglar was wearing gloves	5	Ŀ	Œ	L	T	Ŀ	Ŀ	L	L	Ŀ	Ŀ	П	L	Œ	Т
Carol comitted the burglary	C	L	L	Т	L	L	H	Т	L	ſΞı	ſΞı	ĹΤι	ſΞ	Έų	낸
Bob committed the burglary	В	L	L	Т	L	ĹΊ	ĹΤι	ъ	ĹΉ	H	H	L	L	T	T
Alice committed the burglary	A	ſΞų	ſΞų	ĹΤι	ĹΤι	L	Т	L	L	L	Т	Т	H	L	T

Steps Taken

- 1. I first generated all possible 5-long combinations of Truth and Falsehood.
- 2. Since we are told there are two burglars, I filtered any combination that did not have two Truths in the first three positions.
- 3. I then did the logic for each potential combination.
- 4. Since we are told that only the innocent person told the truth (ie only one statement should be true), I filtered combinations that had more than 1 truthful statement.
- 5. This leaves us with the bottom two possible combinations.
- 6. Lastly, since we know the innocent person told the truth, I cross-referenced which person had the truthful statement with which person is marked innocent.
- 7. This gives us the conclusion that Alice and Bob are the burglars and that the burglar was not wearing gloves or a hat.
- 8. Note: In the previous version of this assignment the resulting correct answer was the second possible solution, and the burglar was wearing gloves but not a hat.

Problem 3

1) (a)
$$\forall a, b \in \mathcal{Z}, E(ab) \wedge E(a+b) \implies E(a) \wedge E(b)$$

(b)
$$\exists a, b \in \mathcal{Z}, \neg(E(ab) \vee E(a+b)) \implies \neg(E(a) \vee E(b))$$

(c)
$$(E(ab) \vee E(a+b)) \vee \neg (E(a) \vee E(b))$$

2) (a)
$$\forall a, b \in \mathbb{Z}, 4 \mid (a^2 + b^2) \implies \neg O(a) \land \neg O(b)$$

(b)
$$\exists a, b \in \mathcal{Z}, \neg(4 \mid (a^2 + b^2)) \implies O(a) \lor O(b)$$

(c)
$$\exists a, b \in \mathcal{Z}, (4 \mid (a^2 + b^2)) \lor O(a) \lor O(b)$$