CSCI 301 HW 11

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April 11, 2023

Problem 1

Let

$$\begin{aligned} |R| &= \text{Rock} & &= 150 \\ |C| &= \text{Classical} & &= 100 \\ |J| &= \text{Jazz} & &= 75 \end{aligned}$$

$$\begin{aligned} |R \cap C| &= 30 \\ |R \cap J| &= 20 \\ |C \cap J| &= 10 \\ |R \cap C| \land J &= 5 \end{aligned}$$

a

$$\begin{aligned} \text{Rock Music Only} = |R| &= 150 \\ \text{Classical Music Only} = |C| &= 100 \\ \text{Jazz Music Only} = |J| &= 75 \end{aligned}$$

b

$$\begin{aligned} \text{Rock and Classical} = |R \cap C| &= 30 \\ \text{Rock and Jazz} = |R \cap J| &= 20 \\ \text{Classical and Jazz} = |C \cap J| &= 10 \end{aligned}$$

Problem 2

Alice committed the burglary	Bob committed the burglary	Carol comitted the burglary	The burglar was wearing gloves	The burglar was wearing a hat	If the burglar was wearing gloves, then it was not Bob.	The thief was wearing a hat, only if it was not Carol.	If the burglar was not wearing gloves, then it was not Alice.
A	B	C	G	H	$G \implies \tilde{B}$	$\tilde{C} \implies H$	$\tilde{G} \implies \tilde{A}$
F	T	T	F	F	T	T	T
F	T	T	F	T	T	T	T
F	T	T	T	F	F	T	T
F	T	T	T	T	F	T	T
T	F	T	F	F	T	T	F
T	F	T	F	T	T	T	F
T	F	T	T	F	T	T	T
T	F	T	T	T	T	T	T
T	T	F	F	F	T	F	F
T	T	F	F	T	T	T	F
T	T	F	T	F	F	F	T
T	T	F	T	T	F	T	T
T	T	F	F	F	T	F	F
T	**T**	**F**	**T**	**F**	**F**	**F**	**T**

Steps Taken

1. I first generated all possible 5-long combinations of Truth and Falsehood.
2. Since we are told there are two burglars, I filtered any combination that did not have two Truths in the first three positions.
3. I then did the logic for each potential combination.
4. Since we are told that only the innocent person told the truth (ie only one statement should be true), I filtered combinations that had more than 1 truthful statement.
5. This leaves us with the bottom two possible combinations.
6. Lastly, since we know the innocent person told the truth, I cross-referenced which person had the truthful statement with which person is marked innocent.
7. This gives us the conclusion that Alice and Bob are the burglars and that the burglar was wearing gloves (but not a hat).

Problem 3

1) (a) $\forall a, b \in \mathcal{Z}, E(ab) \wedge E(a+b) \implies E(a) \wedge E(b)$

   (b) $\exists a, b \in \mathcal{Z}, \neg(E(ab) \vee E(a+b)) \implies \neg(E(a) \vee E(b))$

   (c) $(E(ab) \vee E(a+b)) \vee \neg(E(a) \vee E(b))$

2) (a) $\forall a, b \in \mathcal{Z}, 4 \mid (a^2+b^2) \implies \neg O(a) \wedge \neg O(b)$

   (b) $\exists a, b \in \mathcal{Z}, \neg(4 \mid (a^2+b^2)) \implies O(a) \vee O(b)$

   (c) $\exists a, b \in \mathcal{Z}, (4 \mid (a^2+b^2)) \vee O(a) \vee O(b)$