CSCI 305 HW 8

Isaac Boaz

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Use the loop invariant 'at step $i, k = \sum_{m=1}^{i} 2^m$ after line 6 evaluates' to show that the code is a correct algorithm to compute $\sum_{m=1}^{n} 2^m$.

Proof. By Induction.
Base Case $n = 1$
We observe that the given code returns 2 which is $= \sum_{m=1}^{2} 2^m$.

Inductive Step Suppose $k_{i-1} = \sum_{m=1}^{i-1} 2^m$. Then

$$\begin{aligned} k_i &= k_{i-1} + \ell(i) \\ &= \sum_{m=1}^{i-1} 2^m + 2^i \\ &= \sum_{m=1}^{i} 2^m \end{aligned}$$

Conclusion By induction, the code is a correct algorithm to compute $\sum_{m=1}^{n} 2^m$ where $i = n$. $\square$