CSCI 305 HW 3

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Problem 1

Show that $5n + 12 = O(n)$ directly from the definition.

Proof. $5n + 12 = O(n) \implies 5n + 12 \leq cn$
Suppose $c = 6$, then

$$5n + 12 \leq 6n$$
$$12 \leq n$$

which shows us that $5n + 12 = O(n)$ for $n_0 = 12, c = 6$. $\square$

Problem 2

Show that $2n + 100 \log n = \Omega(n)$

Proof. $2n + 100 \log n = \Omega(n) \implies 2n + 100 \log n \geq cn$
Suppose $c = 2$, then

$$2n + 100 \log n \geq 2n$$
$$100 \log n \geq 0$$

which holds true for all $n > 1$, showing us that $2n + 100 \log n = \Omega(n)$ for $n_0 = 1, c = 2$. $\square$

Problem 3

Show that $\lceil 2n \rceil + 5/n = \Theta(n)$.

Proof.

$$\lceil 2n \rceil + 5/n = \Theta(n) \implies$$
$$\lceil 2n \rceil + 5/n = O(n)$$
$$\wedge \lceil 2n \rceil + 5/n = \Omega(n)$$

Part 1

$\lceil 2n \rceil + 5/n = O(n) \implies \lceil 2n \rceil + 5/n \leq cn$ for some c.
Suppose $c = 3$, then

$$\lceil 2n \rceil + 5/n < 3n \rightarrow \lceil 2n \rceil + 5/n \leq 2n + 1 + 5/n$$
$$2n + 1 + 5/n \leq 3n$$
$$1 + 5/n \leq n$$

which holds true for all $n \geq 5$, showing us that $\lceil 2n \rceil + 5/n = O(n)$ for $n_0 = 5, c = 3$.

Part 2

$\lceil 2n \rceil + 5/n = \Omega(n) \implies \lceil 2n \rceil + 5/n \geq cn$ for some c.
Suppose $c = 1$, then

$$\lceil 2n \rceil + 5/n > n \rightarrow \lceil 2n \rceil + 5/n \geq 2n + 1 + 5/n$$
$$2n + 1 + 5/n \geq n$$
$$n + \frac{5}{n} \geq -1$$

which holds true for all $n \geq 0$, showing us that $\lceil 2n \rceil + 5/n = \Omega(n)$ for $n_0 = 0, c = 1$. $\square$