CSCI 305 Assignment 5

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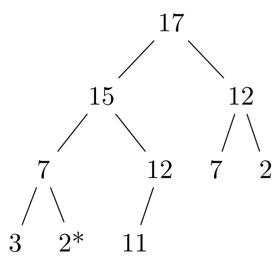
1. Recreating the code in Python and running for all $x \in [1, 10]$ shows us that x could be $3, 4, 5, 6, 7, 8, 9$. We execute 3 swaps for $x \in [3, 7]$, and 4 swaps for $x \in \{8, 9\}$.

2. Let's first re-create the heap visually.



   We can visually see that the parent of the 2* node is 7, so it *definitely* needs to be > 7. Keeping in mind one swap would put the new value as the child of 15, having another swap would require it to be > 15. Thus, we should do **4. Change 2 to 16**.

3. Assuming $h(x)$ uniformly distriutes amongst m slots, the likelihood of any element being inserted into any specific slot is $\frac{1}{m}$. With n distinct keys, we get

$$\frac{n}{m} \text{ expected collisions}$$

4.
   1. Since X represents the number of comparisons, all possible values of X means $[1, 4]$. Let's make a table for each possible $x \in X$.

x	$P(x)$	$x \cdot P(x)$
1	1/10	0.1
2	2/10	0.4
3	4/10	1.2
4	3/10	1.2

   2. Summing up all the values in the last column, we get $E[X] = 2.9$.
   3. Calculating $Pr(K)$ just involves us seeing which unique values from $[1, 30]$ are in the tree. Seeing as there are no duplicates, we know there are 10 values in the tree, and thus there is a $\frac{10}{30} = \frac{1}{3}$ chance of any value (from $[1, 30]$) being in the tree.
      Thus, $Pr(K) = \frac{1}{3} \rightarrow Pr(\overline{K}) = 1 - Pr(K) = \frac{2}{3} \rightarrow Pr(K) + Pr(\overline{K}) = 1$.
   4. No. X is *number of comparisons required to find a node ...* ***from the set of nodes in the tree*** *...* . Whereas Y is the case that the key is quered uniformly from {1, 2, ..., 30}. Since K is given, that is simply rephrasing "what is the probability that we do y comparisons **given y is in the tree**", which is the same as X. **ie** $Pr(Y = y|K) = Pr(X)$.
   5.

Value	Comparisons	Value	Comparisons
2	4	20	3
4	3	21	3
5	3	22	3
6	3	23	3
8	4	25	3
10	4	26	3
11	4	26	3
13	4	28	3
15	4	29	3
16	4	30	3

Converting this table to an expected value table:

y	$Pr(y)$	$y \cdot Pr(y)$
3	13/20	1.95
4	7/20	1.4

6. $E[Y|\overline{K}] = 3.35$

7.

$$
\begin{aligned}
E[Y] &= E[Y = y|K]Pr(K) + E[Y = y|\overline{K}]Pr(\overline{K}) \\
&= 2.9 \cdot \frac{1}{3} + 3.35 \cdot \frac{2}{3} \\
&= 3.2
\end{aligned}
$$