CSCI 305 HW 6

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$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2i\pi}{N} kn}$$

Problem 1

Compute the DFT of ...

A) $[0, 1, 0, -1]$

k=0

$$\begin{aligned}\sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4} 0n} &= \sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4} 0n} \\ &= 0 + e^{\frac{-2i\pi}{4} 0 \cdot 1} + 0 - e^{\frac{-2i\pi}{4} 0 \cdot 3} \\ &= 0 + 1 + 0 - 1 \\ &= 0\end{aligned}$$

k=1

$$\begin{aligned}\sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4} 1n} &= \sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4} 1n} \\ &= 0 + e^{\frac{-2i\pi}{4} 1 \cdot 1} + 0 - e^{\frac{-2i\pi}{4} 1 \cdot 3} \\ &= 0 - i + 0 - i \\ &= 0 - 2i\end{aligned}$$

k=2

$$\begin{aligned}\sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4} 2n} &= \sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4} 2n} \\ &= 0 + e^{\frac{-2i\pi}{4} 2 \cdot 1} + 0 - e^{\frac{-2i\pi}{4} 2 \cdot 3} \\ &= 0 - 1 + 0 + 1 \\ &= 0\end{aligned}$$

k=3

$$\begin{aligned}\sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4} 3n} &= \sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4} 3n} \\ &= 0 + e^{\frac{-2i\pi}{4} 3 \cdot 1} + 0 - e^{\frac{-2i\pi}{4} 3 \cdot 3} \\ &= 0 + i + 0 + i \\ &= 0 + 2i\end{aligned}$$

$$DFT([0, 1, 0, -1]) = [0, -2i, 0, 2i]$$

B) $[1, 1, 1, 1]$ (In-Class Exercise) DFT([1, 1, 1, 1]) = [4, 0, 0, 0]

C) $[0, -1, 0, 1]$

k=0

$$\begin{aligned}\sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4}0n} &= \sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4}0n} \\ &= 0 - e^{\frac{-2i\pi}{4}0\cdot 1} + 0 + e^{\frac{-2i\pi}{4}0\cdot 3} \\ &= 0 - 1 + 0 + 1 \\ &= 0\end{aligned}$$

k=1

$$\begin{aligned}\sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4}1n} &= \sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4}1n} \\ &= 0 - e^{\frac{-2i\pi}{4}1\cdot 1} + 0 + e^{\frac{-2i\pi}{4}1\cdot 3} \\ &= 0 + i + 0 + i \\ &= 0 + 2i\end{aligned}$$

k=2

$$\begin{aligned}\sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4}2n} &= \sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4}2n} \\ &= 0 - e^{\frac{-2i\pi}{4}2\cdot 1} + 0 + e^{\frac{-2i\pi}{4}2\cdot 3} \\ &= 0 + 1 + 0 - 1 \\ &= 0\end{aligned}$$

k=3

$$\begin{aligned}\sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4}3n} &= \sum_{n=0}^{3} x_n e^{-\frac{2i\pi}{4}3n} \\ &= 0 - e^{\frac{-2i\pi}{4}3\cdot 1} + 0 + e^{\frac{-2i\pi}{4}3\cdot 3} \\ &= 0 - i + 0 - i \\ &= 0 - 2i\end{aligned}$$

$$DFT([0, -1, 0, 1]) = [0, 2i, 0, -2i]$$

D) $[0,1,0,-1,0,1,0,-1]$

k=0

$$\begin{aligned}\sum_{n=0}^{7} x_n e^{-\frac{2i\pi}{4}0n} &= \sum_{n=0}^{7} x_n e^{-\frac{2i\pi}{8}0n} \\ &= 0 + e^{\frac{-2i\pi}{8}0\cdot1} + 0 - e^{\frac{-2i\pi}{8}0\cdot3} + 0 + e^{\frac{-2i\pi}{8}0\cdot5} + 0 - e^{\frac{-2i\pi}{8}0\cdot7} \\ &= 0 + 1 + 0 - 1 + 0 + 1 + 0 - 1 \\ &= 0\end{aligned}$$

k=1

$$\begin{aligned}\sum_{n=0}^{7} x_n e^{-\frac{2i\pi}{4}1n} &= \sum_{n=0}^{7} x_n e^{-\frac{2i\pi}{8}1n} \\ &= 0 + e^{\frac{-2i\pi}{8}1\cdot1} + 0 - e^{\frac{-2i\pi}{8}1\cdot3} + 0 + e^{\frac{-2i\pi}{8}1\cdot5} + 0 - e^{\frac{-2i\pi}{8}1\cdot7} \\ &= 0 + e^{-\frac{i\pi}{4}} + 0 - e^{-\frac{3i\pi}{4}} + 0 + e^{\frac{3i\pi}{4}} + 0 - e^{\frac{i\pi}{4}} \\ &= 0\end{aligned}$$

k=2

$$\begin{aligned}\sum_{n=0}^{7} x_n e^{-\frac{2i\pi}{4}2n} &= \sum_{n=0}^{7} x_n e^{-\frac{2i\pi}{8}2n} \\ &= 0 + e^{\frac{-2i\pi}{8}2\cdot1} + 0 - e^{\frac{-2i\pi}{8}2\cdot3} + 0 + e^{\frac{-2i\pi}{8}2\cdot5} + 0 - e^{\frac{-2i\pi}{8}2\cdot7} \\ &= 0 - i + 0 - i + 0 - i + 0 - i \\ &= 0 - 4i\end{aligned}$$

k=3

$$\begin{aligned}\sum_{n=0}^{7} x_n e^{-\frac{2i\pi}{4}3n} &= \sum_{n=0}^{7} x_n e^{-\frac{2i\pi}{8}3n} \\ &= 0 + e^{\frac{-2i\pi}{8}3\cdot1} + 0 - e^{\frac{-2i\pi}{8}3\cdot3} + 0 + e^{\frac{-2i\pi}{8}3\cdot5} + 0 - e^{\frac{-2i\pi}{8}3\cdot7} \\ &= 0 + e^{-\frac{3i\pi}{4}} + 0 - e^{-\frac{i\pi}{4}} + 0 - e^{\frac{i\pi}{4}} + 0 - e^{\frac{3i\pi}{4}} \\ &= 0\end{aligned}$$

$$DFT([0,-1,0,1]) = [0,2i,0,-2i]$$