CSCI 305 HW 3

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October 9, 2023

Problem 1

Show that $5n + 12 = O(n)$ directly from the definition.

Proof. $5n + 12 = O(n) \implies 5n + 12 < cn$
Suppose $c = 6$, then

$$\begin{aligned} 5n + 12 &< 6n \\ 12 &< n \end{aligned}$$

which shows us that $5n + 12 = O(n)$ for $n_0 = 12, c = 6$. $\square$

Problem 2

Show that $2n + 100 \log n = \Omega(n)$

Proof. $2n + 100 \log n = \Omega(n) \implies 2n + 100 \log n > cn$
Suppose $c = 2$, then

$$\begin{aligned} 2n + 100 \log n &> 2n \\ 100 \log n &> 0 \end{aligned}$$

which holds true for all $n > 1$, showing us that $2n + 100 \log n = \Omega(n)$ for $n_0 = 1, c = 2$. $\square$

Problem 3

Show that $\lceil 2n \rceil + 5/n = \Theta(n)$.

Proof.

$$\begin{aligned} \lceil 2n \rceil + 5/n &= \Theta(n) \implies \\ \lceil 2n \rceil + 5/n &= O(n) \\ \wedge \lceil 2n \rceil + 5/n &= \Omega(n) \end{aligned}$$

Part 1

$\lceil 2n \rceil + 5/n = O(n) \implies \lceil 2n \rceil + 5/n < cn$ for some c.
Suppose $c = 3$, then

$$\begin{aligned} \lceil 2n \rceil + 5/n < 3n &\to \lceil 2n \rceil + 5/n \leq 2n + 1 + 5/n \\ 2n + 1 + 5/n &< 3n \\ 1 + 5/n &< n \end{aligned}$$

which holds true for all $n \geq 5$, showing us that $\lceil 2n \rceil + 5/n = O(n)$ for $n_0 = 5, c = 3$.

Part 2

$\lceil 2n \rceil + 5/n = \Omega(n) \implies \lceil 2n \rceil + 5/n > cn$ for some c.
Suppose $c = 1$, then

$$\begin{aligned} \lceil 2n \rceil + 5/n > n &\to \lceil 2n \rceil + 5/n \geq 2n + 1 + 5/n \\ 2n + 1 + 5/n &> n \\ n + \frac{5}{n} &> -1 \end{aligned}$$

which holds true for all $n \geq 0$, showing us that $\lceil 2n \rceil + 5/n = \Omega(n)$ for $n_0 = 0, c = 1$. $\square$