CSCI 305 Assignment 1

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Problem 1

```
1   for i = 1 to n
2       j = 1
3       while j<= i do
4           j = 2.5 * j
```

cost	time
C_1	$n+1$
C_2	n
C_3	$(n+1)\log_{2.5}$
C_4	$n\log_{2.5} n$

Detailed Runtime:

$$C_1 \cdot (n+1) + C_2 \cdot n + C_3 \cdot \lceil (n+1)\log_{2.5} \rceil + C_4 \cdot \lceil n \log_{2.5} n \rceil$$

Asymptotic Runtime:

$$\Theta(n \log n)$$

Problem 2

```
0   def factorial(n):
1       x = 1
2       while n > 1:
3           x = x * n
4           n = n - 1
5       return x
```

cost	time
C_1	1
C_2	n
C_3	$n-1$
C_4	$n-1$
C_5	n

Detailed Runtime:

$$C_1 \cdot 1 + C_2 \cdot n + C_3 \cdot (n-1) + \cdot C_4 \cdot (n-1) + C_5 \cdot n$$

Asymptotic Runtime:

$$O(n)$$

Problem 3

```
0  def Binary_InsertionSort(A):
1      for j = 2 to n:
2          key = A[j]
3          // insert A[j] into the
               sorted sequence A[1..j-1]
4          left = 1
5          right = j - 1
6          while right > left:
7              mid = floor((left + right
                   ) / 2)
8              if key > A[mid]:
9                  left = mid + 1
10             else right = mid
11         if key > A[left]:
12             left = left + 1
13         for i = j downto (left + 1):
14             A[i] = A[i - 1]
15         A[left] = key
```

best	worst
n	n
$n-1$	$n-1$
$n-1$	$n-1$
$n-1$	$n-1$
$n-1$	$n-1$
$\log_2$	

1. Note that regardless of input, the for loop will always run n times, with the contents of the loop running $n-1$ times.

   Worst Case occurs when the array is reverse sorted. The *while* loop will run $\log_2 j$ times, and the *for* loop will run $j-1$ times.

   $$\sum_{j=2}^{n} log_2(j)$$

   Worst Case occurs when the array is sorted in reverse order, as the left and right bounds will need to be updated $\log_2 j$ times.
   Best Case occurs when the array is already sorted, as the left and right bounds will only need to be updated once.

2. **Worst Case** for the *while* block is the same in the best and worst case scenario, running $\lceil \log_2 j \rceil$ times.