Optimal Rays

-9	0	1	0	0	0
2	1	1	1	-1	0
6	0	1	2	-1	1

Since x_4 has $c_k = 0, \forall a_{ik} \leq 0$, we know there is an optimal ray.

1. x_2, x_3 are non-basic and not our optimal ray column, so set those to 0 and solve for the basic variable.

$$2 = x_1 + x_2 - x_4 \to x_1 = 2 + x_4$$
$$6 = x_2 + 2x_3 - x_4 \to x_5 = 6 + x_4$$

2. Set $x_4 = t, t \geq 0$

3. We get

$$\begin{aligned} \overline{x}(t) &= [2+t, 0, 0, t, 6+t], t \geq 0 \\ &= [2,0,0,0,6] + t[1,0,0,1,1], t \geq 0 \end{aligned}$$

which is an **optimal ray**.

Pivoting in x_3:

-9	0	1	0	0	0
2	1	1	1	-1	0
6	0	1	2	1	1

We *can* still pivot on x_4, but first our BFS $\overline{x}_1^* = [0,0,2,0,2]$.
Now, pivoting gets us

-9	0	1	0	0	0
4	-1	0	1	0	1
2	-2	-1	0	1	1

This BFS $\overline{x}_2^* = [0,0,4,2,0]$, which is $\neq \overline{x}_1^*$. We also have an *optimal ray* here, so following the process above...
Set $x_2 = x_5 = 0$, solve for x_3 and x_4 respectively.

$$x_3 = 4 + x_1$$
$$x_4 = 2 + 2x_1$$

Set $x_1 = t, t \geq 0$

$$\begin{aligned} \overline{x}(t) &= [t, 0, 4+t, 2+2t, 0], t \geq 0 \\ &= [0,0,4,2,0] + t[1,0,1,2,0], t \geq 0 \end{aligned}$$

So we've found all possible BFS and optimal rays. Thus, our **optimal set** is *all possible convex combinations of pair of points from edges* $[\overline{x}_0^*, \overline{x}_1^*]$ and $[\overline{x}_1^*, \overline{x}_2^*]$ and optimal rays $\overline{x}_0^* + t[1,0,0,1,1]$ and $\overline{x}_2^* + t[1,0,1,2,0]$ with $t \geq 0$.

Graphing

*	*	*	*	*	*	*
2	0	2	1	1	0	-1
8	0	4	-2	0	1	1
6	1	1	2	0	0	1

1. Pick non-basic var to use as vert axis.
2. Solve equations for basic variable.
3. Set that equation to $= 0$.

$$2 = 2x_2 + x_3 + x_4 - x_6$$
$$x_4 = 2 - 2x_2 - x_3 + x_6 = \mathbf{0}$$
$$x_6 = 2x_2 + x_3 + x_4 - 2$$
$$\vdots$$
$$x_6 = 8 - 4x_2 + 2x_3$$
$$x_6 = 6 - x_2 - 2x_3$$

Good Luck! - Isaac Boaz June 7, 2023

Subproblem Technique

1. Pivot to get I_m with 0s in the OF row.
   - If you get a row of 0s, delete and continue
   - If you get a row with $b_i \neq 0$ and $a_{ik} = 0$ for all k - Infeasible Form 1

   Once you have I_m with 0s in the OF row, you're done with Step 0.
   Either you're in CF or not.
   - If CF, done with step 1.
   - If not, some $b_i < 0$.
2. Try to get all $b_i \geq 0$
   If *all* $b_i < 0$, pick *any negative* entry in any row and pivot. You will now have some $b_i > 0$.
   - If some $b_i < 0$ and all $a_{ik} \geq 0$ - Infeasible Form 2
3. Now you should have at least one row with $b_i \geq 0$ and at least one row with $b_i < 0$. Form the subproblem and pivot to either optimal/unbounded form or until b_i becomes non-negative. Make sure you zero-out the entire column when you pivot, so at the end you still have I_m with 0s in OF row.
   - If optimal form for subproblem and $b_i < 0$ - Infeasible Form 2
   - If optimal form for subproblem and $b_i \geq 0$, you have one fewer $b_i < 0$. Form a new subproblem for any $b_i < 0$, otherwise you're now in CF.
   - If unbounded form for subproblem and $b_i < 0$, then pivot on a *negative* entry of the subproblem OF in an unbounded column.

Artificial Variables

1. Get $\overline{b} \geq \overline{0}$
   Multiply any rows with $b_i < 0$ by -1.
2. Get I_m
   - Add **artificial vars** y_i as need to get I_m.
   - Zero out the artificial OF row.
   - Perform simplex algorithm on artificial problem.
   - If optimal artificial OF value ¿ 0, then infeasible.
   - If optimal artificial OF value ¡ 0, you did something wrong.
   - If optimal artificial OF value = 0, there are two cases:
     (i) All artificial vars are non-basic.
     Delete artificial OF row and replace original OF row.
     Delete columns corresponding to artificial vars. **Zero out the original OF row.**
     (ii) At least one artificial var is basic.
     Pivot in any x_j column in the row corresponding to the var with non-zero entry. (If no such column exists, delete the row).
     Now you have one fewer basic artificial var. Delete the corresponding artificial column and repeat until you are in case (i).