

M/CS 375 Project 1

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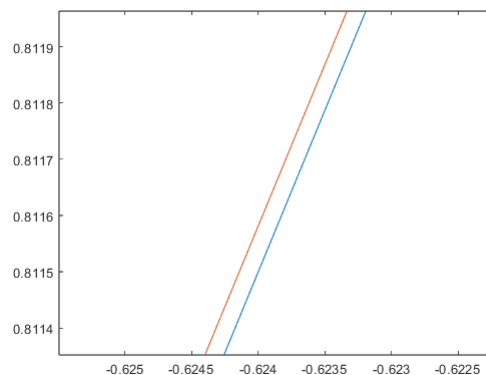
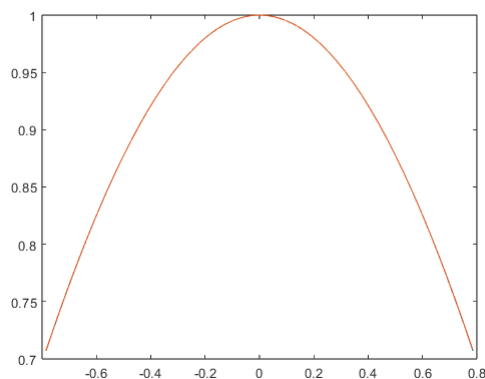
1. Taylor Polynomials

(a) Degree 5 Taylor Polynomial for $f(x) = \cos x$ where $x_0 = 0$.

$$\begin{aligned} P_5(x) &= \sum_{k=0}^5 \frac{f^{(k)}(x_0)}{k!} (x - x_0)^k \\ &= \cos(0) - \sin(0)x - \frac{\cos(0)x^2}{2} + \frac{\sin(0)x^3}{6} - \frac{\cos(0)x^4}{24} + \frac{\sin(0)x^5}{120} \\ &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \end{aligned}$$

(b) Worst-case error for x in $[-\frac{\pi}{4}, \frac{\pi}{4}]$

$$\begin{aligned} E_{5+1} &= \frac{1}{6!} f^{(6)}(c)(x)^6 \\ &= \frac{1}{720} - \cos(c)(x)^6 \\ &= \frac{\frac{\pi}{4}}{720} \cos(c) \\ &< \frac{0.2397}{720} < 3 \times 10^{-4} \end{aligned}$$



2. MatLab

(a) $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9$; $[0, 1]$

Iterating 19 times

k	A	B	c	f(c)
0:	0.0000000	1.0000000	0.5000000	-100.6250000
1:	0.5000000	1.0000000	0.7500000	-89.3203125
2:	0.7500000	1.0000000	0.8750000	-48.6040039
3:	0.8750000	1.0000000	0.9375000	-15.7762756
4:	0.9375000	1.0000000	0.9687500	4.2870235
5:	0.9375000	0.9687500	0.9531250	-6.0654713
6:	0.9531250	0.9687500	0.9609375	-0.9707179
7:	0.9609375	0.9687500	0.9648438	1.6376179
8:	0.9609375	0.9648438	0.9628906	0.3283365
9:	0.9609375	0.9628906	0.9619141	-0.3224666
10:	0.9619141	0.9628906	0.9624023	0.0026157
11:	0.9619141	0.9624023	0.9621582	-0.1600052

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12:  0.9621582  0.9624023  0.9622803  -0.0787147
13:  0.9622803  0.9624023  0.9623413  -0.0380545
14:  0.9623413  0.9624023  0.9623718  -0.0177207
15:  0.9623718  0.9624023  0.9623871  -0.0075528
16:  0.9623871  0.9624023  0.9623947  -0.0024686
17:  0.9623947  0.9624023  0.9623985   0.0000735
18:  0.9623947  0.9623985  0.9623966  -0.0011976
19:  0.9623966  0.9623985  0.9623976  -0.0005620

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ans =

0.962397575378418

(b) $f(x) = 1 + \ln(1 + x^2)$; $[0, 1]$

1.000000 and 1.693147 must be opposite signs

(c) $f(x) = e^x + 2^{-x} + 2\cos x - 6$; $[1, 2]$

Iterating 19 times

k	A	B	c	f(c)
0:	1.0000000	2.0000000	1.5000000	-1.0232831
1:	1.5000000	2.0000000	1.7500000	-0.3045877
2:	1.7500000	2.0000000	1.8750000	0.1943790
3:	1.7500000	1.8750000	1.8125000	-0.0682744
4:	1.8125000	1.8750000	1.8437500	0.0596374
5:	1.8125000	1.8437500	1.8281250	-0.0051567
6:	1.8281250	1.8437500	1.8359375	0.0270289
7:	1.8281250	1.8359375	1.8320312	0.0108835
8:	1.8281250	1.8320312	1.8300781	0.0028502
9:	1.8281250	1.8300781	1.8291016	-0.0011565
10:	1.8291016	1.8300781	1.8295898	0.0008460
11:	1.8291016	1.8295898	1.8293457	-0.0001554
12:	1.8293457	1.8295898	1.8294678	0.0003453
13:	1.8293457	1.8294678	1.8294067	0.0000949
14:	1.8293457	1.8294067	1.8293762	-0.0000303
15:	1.8293762	1.8294067	1.8293915	0.0000323
16:	1.8293762	1.8293915	1.8293839	0.0000010
17:	1.8293762	1.8293839	1.8293800	-0.0000146
18:	1.8293800	1.8293839	1.8293819	-0.0000068
19:	1.8293819	1.8293839	1.8293829	-0.0000029

ans =

1.829382896423340

3. Interest Rate

Iterating 12 times

k	A	B	c	f(c)
0:	0.0000000	0.0500000	0.0250000	-535.4359178
1:	0.0250000	0.0500000	0.0375000	436.7535239
2:	0.0250000	0.0375000	0.0312500	-72.5564497
3:	0.0312500	0.0375000	0.0343750	176.0219417
4:	0.0312500	0.0343750	0.0328125	50.2486600
5:	0.0312500	0.0328125	0.0320312	-11.5206196
6:	0.0320312	0.0328125	0.0324219	19.2718057
7:	0.0320312	0.0324219	0.0322266	3.8526063
8:	0.0320312	0.0322266	0.0321289	-3.8397450
9:	0.0321289	0.0322266	0.0321777	0.0049950
10:	0.0321289	0.0321777	0.0321533	-1.9177338
11:	0.0321533	0.0321777	0.0321655	-0.9564591
12:	0.0321655	0.0321777	0.0321716	-0.4757545

ans =

0.032171630859375