M/CS 375 HW 16

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Problem 4

Solve the system by finding the PA = LU factorization and then carrying out the two-step back substitution.

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ \textcircled{1} & 4 & 2 \\ \textcircled{\frac{1}{2}} & 2 & 3 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ \textcircled{1} & 2 & 2 \\ \textcircled{\frac{1}{2}} & \textcircled{\frac{1}{2}} & 2 \end{bmatrix} \tag{2}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ \textcircled{\frac{1}{4}} & 1 & 0 \\ \textcircled{\frac{1}{2}} & \textcircled{\frac{1}{2}} & 1 \end{bmatrix} \quad U = \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \tag{3}$$

$$PA = LU$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{4} & 1 & 0 \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 4 & 2 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 2 \end{bmatrix} \tag{4}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 4 \\ \frac{1}{2} & \frac{1}{2} & 1 & 6 \end{array}\right] \rightarrow \overrightarrow{y} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix} \tag{5}$$

$$\left[\begin{array}{ccc|c} 4 & 2 & 0 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 4 \end{array}\right] \rightarrow \overrightarrow{x} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} \tag{6}$$