M/CS 375 HW 15

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March 8, 2023

Problem 4

Solve the system by finding the LU factorization and then carrying out the two-step back substitution.

$$\begin{bmatrix} 4 & 2 & 0 \\ 4 & 4 & 2 \\ 2 & 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ \boxed{1} & 4 & 2 \\ \boxed{\frac{1}{2}} & 2 & 3 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} 4 & 2 & 0 \\ 1 & 2 & 2 \\ \frac{1}{2} & (\frac{1}{2}) & 2 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix}
4 & 2 & 0 \\
1 & 2 & 2 \\
\frac{1}{2} & (\frac{1}{2}) & 2
\end{bmatrix}$$

$$L = \begin{bmatrix}
1 & 0 & 0 \\
\frac{1}{4} & 1 & 0 \\
\frac{1}{2} & (\frac{1}{2}) & 1
\end{bmatrix}$$

$$U = \begin{bmatrix}
4 & 2 & 0 \\
0 & 2 & 2 \\
0 & 0 & 2
\end{bmatrix}$$
(2)

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 4 \\ \frac{1}{2} & \frac{1}{2} & 1 & 6 \end{bmatrix} \rightarrow \overrightarrow{y} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$
 (4)

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 1 & 0 & 4 \\ \frac{1}{2} & \frac{1}{2} & 1 & 6 \end{bmatrix} \rightarrow \overrightarrow{y} = \begin{bmatrix} 2 \\ 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 2 & 0 & 2 \\ 0 & 2 & 2 & 2 \\ 0 & 0 & 2 & 4 \end{bmatrix} \rightarrow \overrightarrow{x} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$$

$$(5)$$