M/CS 375 Project 1

Isaac Boaz

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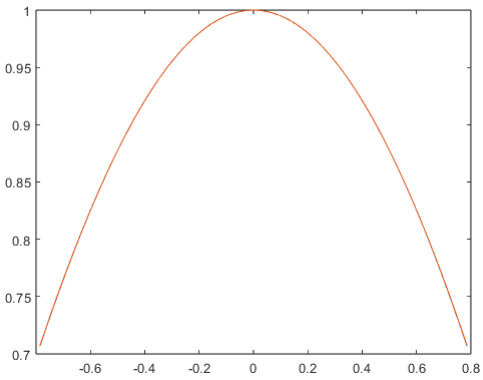
1. Taylor Polynomials

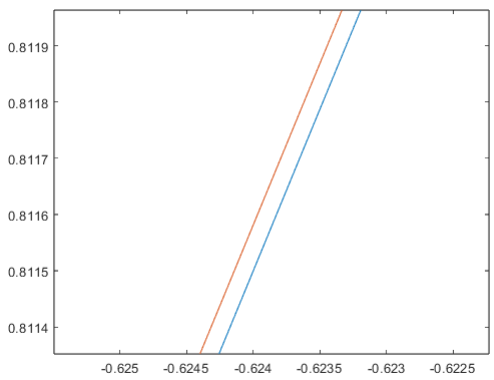
   (a) Degree 5 Taylor Polynomial for $f(x) = cosx$ where $x_0 = 0$.

$$\begin{aligned} P_5(x) &= \sum_{k=0}^{5} \frac{f^{(k)}(x_0)}{k!}(x - x_0)^k \\ &= cos(0) - sin(0)x - \frac{cos(0)x^2}{2} + \frac{sin(0)x^3}{6} - \frac{cos(0)x^4}{24} + \frac{sin(0)x^5}{120} \\ &= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \end{aligned}$$

   (b) Worst-case error for x in $[-\frac{\pi}{4}, \frac{\pi}{4}]$

$$\begin{aligned} E_{5+1} &= \frac{1}{6!} f^{(6)}(c)(x)^6 \\ &= \frac{1}{720} - cos(c)(x)^6 \\ &= \frac{\frac{\pi}{4}^6}{720} cos(c) \\ &< \frac{0.2397}{720} < 3 \times 10^{-4} \end{aligned}$$





2. MatLab

   (a) $f(x) = 230x^4 + 18x^3 + 9x^2 - 221x - 9;\ [0, 1]$

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Iterating 19 times
 k    A           B           c           f(c)
 0:   0.0000000   1.0000000   0.5000000  -100.6250000
 1:   0.5000000   1.0000000   0.7500000   -89.3203125
 2:   0.7500000   1.0000000   0.8750000   -48.6040039
 3:   0.8750000   1.0000000   0.9375000   -15.7762756
 4:   0.9375000   1.0000000   0.9687500     4.2870235
 5:   0.9375000   0.9687500   0.9531250    -6.0654713
 6:   0.9531250   0.9687500   0.9609375    -0.9707179
 7:   0.9609375   0.9687500   0.9648438     1.6376179
 8:   0.9609375   0.9648438   0.9628906     0.3283365
 9:   0.9609375   0.9628906   0.9619141    -0.3224666
10:   0.9619141   0.9628906   0.9624023     0.0026157
11:   0.9619141   0.9624023   0.9621582    -0.1600052
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12:  0.9621582  0.9624023  0.9622803  -0.0787147
13:  0.9622803  0.9624023  0.9623413  -0.0380545
14:  0.9623413  0.9624023  0.9623718  -0.0177207
15:  0.9623718  0.9624023  0.9623871  -0.0075528
16:  0.9623871  0.9624023  0.9623947  -0.0024686
17:  0.9623947  0.9624023  0.9623985   0.0000735
18:  0.9623947  0.9623985  0.9623966  -0.0011976
19:  0.9623966  0.9623985  0.9623976  -0.0005620

ans =

   0.962397575378418
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(b) $f(x) = 1 + ln(1 + x^2);\ [0, 1]$

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1.000000 and 1.693147 must be opposite signs
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(c) $f(x) = e^x + 2^{-x} + 2cosx - 6;\ [1, 2]$

```
Iterating 19 times
k   A          B          c          f(c)
0:  1.0000000  2.0000000  1.5000000  -1.0232831
1:  1.5000000  2.0000000  1.7500000  -0.3045877
2:  1.7500000  2.0000000  1.8750000   0.1943790
3:  1.7500000  1.8750000  1.8125000  -0.0682744
4:  1.8125000  1.8750000  1.8437500   0.0596374
5:  1.8125000  1.8437500  1.8281250  -0.0051567
6:  1.8281250  1.8437500  1.8359375   0.0270289
7:  1.8281250  1.8359375  1.8320312   0.0108835
8:  1.8281250  1.8320312  1.8300781   0.0028502
9:  1.8281250  1.8300781  1.8291016  -0.0011565
10:  1.8291016  1.8300781  1.8295898   0.0008460
11:  1.8291016  1.8295898  1.8293457  -0.0001554
12:  1.8293457  1.8295898  1.8294678   0.0003453
13:  1.8293457  1.8294678  1.8294067   0.0000949
14:  1.8293457  1.8294067  1.8293762  -0.0000303
15:  1.8293762  1.8294067  1.8293915   0.0000323
16:  1.8293762  1.8293915  1.8293839   0.0000010
17:  1.8293762  1.8293839  1.8293800  -0.0000146
18:  1.8293800  1.8293839  1.8293819  -0.0000068
19:  1.8293819  1.8293839  1.8293829  -0.0000029

ans =

    1.829382896423340
```

3. Interest Rate

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Iterating 12 times
k   A          B          c          f(c)
0:  0.0000000  0.0500000  0.0250000  -535.4359178
1:  0.0250000  0.0500000  0.0375000  436.7535239
2:  0.0250000  0.0375000  0.0312500  -72.5564497
3:  0.0312500  0.0375000  0.0343750  176.0219417
4:  0.0312500  0.0343750  0.0328125   50.2486600
5:  0.0312500  0.0328125  0.0320312  -11.5206196
6:  0.0320312  0.0328125  0.0324219   19.2718057
7:  0.0320312  0.0324219  0.0322266    3.8526063
8:  0.0320312  0.0322266  0.0321289   -3.8397450
9:  0.0321289  0.0322266  0.0321777    0.0049950
10:  0.0321289  0.0321777  0.0321533   -1.9177338
11:  0.0321533  0.0321777  0.0321655   -0.9564591
12:  0.0321655  0.0321777  0.0321716   -0.4757545

ans =

    0.032171630859375
```