M/CS 375 HW 8

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February 24, 2023

Problem 14

Which fixed point iteration converges to $\sqrt{2}$?

(A) $x \to \frac{1}{2}x + \frac{1}{x}$

$$\frac{d}{dx}\left(\frac{1}{2}x + \frac{1}{x}\right) = \frac{1}{2} - \frac{1}{x^2}$$
$$x'(\sqrt{2}) = \frac{1}{2} - \frac{1}{2} = 0$$

(B) $x \to \frac{2}{3}x + \frac{2}{3x}$

$$\frac{d}{dx}\left(\frac{2}{3}x + \frac{2}{3x}\right) = \frac{2}{3} - \frac{2}{3x^2}$$
$$x'(\sqrt{2}) = \frac{2}{3} - \frac{2}{6} = \frac{1}{3}$$

(C) $x \to \frac{3}{4}x + \frac{1}{2x}$

$$\frac{d}{dx}\left(\frac{3}{4}x + \frac{1}{2x}\right) = \frac{3}{4} - \frac{1}{2x^2}$$
$$x'(\sqrt{2}) = \frac{3}{4} - \frac{1}{4} = \frac{1}{2}$$

All three fixed point iterations converge to $\sqrt{2}$, with A being the quickest, B being in the middle, and C being slowest.