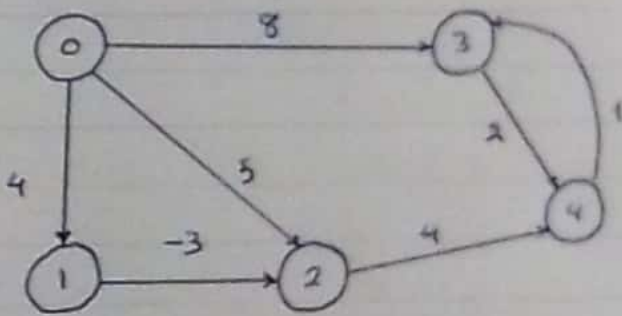


Ford Bellman

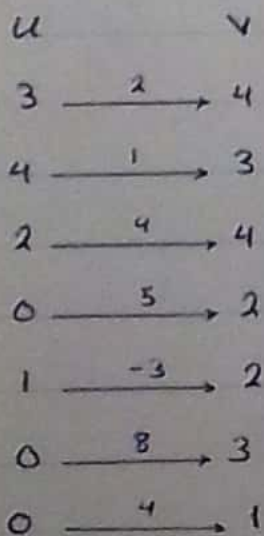
The graph may contain negative weight edges. Dijkstra's algorithm is a Greedy algorithm and time complexity is $O(V \log V)$. Dijkstra's doesn't work for Graphs with negative weight edges.

Bellman works for such graphs, and it's also simpler than Dijkstra and suits well for distributed systems. But time complexity of Ford Bellman is $O(VE)$, which is more than Dijkstra.



vertex	distance	parent
0	0	-
1	∞	-
2	∞	-
3	∞	-
4	∞	-
5	∞	-

Random order:



Relaxation Formula:

relax (Edge $u \rightarrow v$) {

if ($\text{distance}[v] > \text{distance}[u] + \text{weight}(u, v)$)

$\text{distance}[v] = \text{distance}[u] + \text{weight}(u, v)$

$\text{parent}[v] = u$

}

Complexity:

space complexity $O(V)$

Time complexity $O(V \cdot E)$

short path $O(V^2 \cdot E)$

short path in $O(V^4 \cdot E)$

Worst Case

Start: $v = 1$

$$\begin{array}{ccc} 3 & \xrightarrow{2} & 4 \\ \downarrow & & \downarrow \\ \infty + 2 & < & \infty \end{array}$$

X

$$\begin{array}{ccc} 0 & \xrightarrow{5} & 2 \\ 0 + 5 & < & 2 \end{array} \quad \checkmark \quad \begin{array}{ccc} \text{vertex} & \text{dis} & \text{parent} \\ 2 & 5 & 0 \end{array}$$

$$\begin{array}{ccc} 4 & \xrightarrow{2} & 3 \\ \downarrow & & \downarrow \\ \infty + 2 & < & \infty \end{array}$$

X

$$\begin{array}{ccc} 0 & \xrightarrow{8} & 3 \\ 0 + 8 & < & \infty \end{array} \quad \checkmark \quad \begin{array}{ccc} \text{vertex} & \text{dis} & \text{parent} \\ 3 & 8 & 0 \end{array}$$

$v = 2$

$$\begin{array}{ccc} 4 & \xrightarrow{1} & 3 \\ 10 + 1 & < & 8 \end{array}$$

X

$$\begin{array}{ccc} 3 & \xrightarrow{2} & 4 \\ 8 + 2 & < & \infty \end{array} \quad \checkmark \quad \begin{array}{ccc} \text{vertex} & \text{dis} & \text{parent} \\ 4 & 10 & 3 \end{array}$$

$$\begin{array}{ccc} 0 & \xrightarrow{5} & 2 \\ 0 + 5 & < & 8 \end{array}$$

X

$$\begin{array}{ccc} 2 & \xrightarrow{4} & 4 \\ 5 + 4 & < & 10 \end{array} \quad \checkmark \quad \begin{array}{ccc} \text{vertex} & \text{dis} & \text{parent} \\ 4 & 5 & 2 \end{array}$$

$$\begin{array}{ccc} 0 & \xrightarrow{8} & 3 \\ 0 + 8 & < & 8 \end{array}$$

X

$$\begin{array}{ccc} 1 & \xrightarrow{-3} & 2 \\ 4 - 3 & < & 5 \end{array} \quad \checkmark \quad \begin{array}{ccc} \text{vertex} & \text{dis} & \text{parent} \\ 2 & 1 & 1 \end{array}$$

$$\begin{array}{ccc} 0 & \xrightarrow{4} & 1 \\ 0 + 4 & < & 4 \end{array}$$

X

$$\begin{array}{ccc} 4 & \xrightarrow{1} & 3 \\ 3 + 1 & < & 8 \end{array} \quad \text{X}$$

$v = 3$

$$\begin{array}{ccc} 3 & \xrightarrow{2} & 4 \\ 8 + 2 & < & 3 \end{array}$$

X

$$\begin{array}{ccc} 2 & \xrightarrow{4} & 4 \\ 1 + 4 & < & 9 \end{array} \quad \checkmark \quad \begin{array}{ccc} \text{vertex} & \text{dis} & \text{parent} \\ 4 & 5 & 2 \end{array}$$

$$\begin{array}{ccc} 0 & \xrightarrow{5} & 2 \\ 0 + 5 & < & 1 \end{array}$$

X

$$\begin{array}{ccc} 1 & \xrightarrow{-3} & 2 \\ 4 - 3 & < & 1 \end{array} \quad \text{X}$$

$$\begin{array}{ccc} 0 & \xrightarrow{8} & 3 \\ 0 + 8 & < & 8 \end{array}$$

X

$$\begin{array}{ccc} 0 & \xrightarrow{1} & 4 \\ 0 + 1 & < & 5 \end{array} \quad \checkmark \quad \begin{array}{ccc} \text{vertex} & \text{dis} & \text{parent} \\ 4 & 5 & 0 \end{array}$$

$V = 7$

$3 \xrightarrow{2} 4$

$4 \xrightarrow{1} 3$

ver dis parent

X $8+2 < 5$

$5+1 < 8$ ✓

3 6 4

$2 \xrightarrow{4} 4$

$0 \xrightarrow{5} 2$

X $1+4 < 5$

$0+5 < 1$ X

$1 \xrightarrow{-3} 2$

$0 \xrightarrow{8} 3$

X $4-3 < 1$

$0+8 < 6$ X

$0 \xrightarrow{4} 1$

After finished All iteration (v):

$0+4 < 4$ X

vertex distance parent

0 0 Null

1 4 0

2 1 1

3 6 4

4 5 2

for Example : if you need Go to ③

③ $\xrightarrow{1}$ ④ $\xrightarrow{4}$ ② $\xrightarrow{-3}$ ① $\xrightarrow{4}$ ⑤ = 6

$d[s] \leftarrow 0$

For each $v \in V - \{s\}$

do $d[v] \leftarrow \infty$

For $i \leftarrow 1$ to $|V| - 1$

do for each edge $(u, v) \in E$

do if $d[v] > d[u] + w(u, v)$

Then $d[v] \leftarrow d[u] + w(u, v)$

for each edge $(u, v) \in E$

do if $d[v] > d[u] + w(u, v)$

Then report that negative weight cycle exists

At the end, $d[v] = \delta(s, v)$, if no negative weight cycles Time = $O(VE)$.