Reimbursing Consumers' Switching Costs in Network Industries

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Abstract

Prior literature finds that switching costs incentivize firms to harvest their locked-in consumers rather than price aggressively for market dominance, resulting in a lower market concentration. Using a dynamic duopoly model with switching costs and network effects, we show that this result is overturned if firms have the option to reimburse consumers' switching costs. The larger firm reimburses a larger proportion of the switching cost than the smaller firm does, leading to a bigger market share for the larger firm. Correspondingly, an increase in the switching cost increases market concentration. Compared to the case without reimbursement, firms' option to reimburse switching costs increases consumer surplus and total surplus, and increases producer surplus when network effects are strong. Switching costs decrease consumer surplus if firms do not have the option to reimburse switching costs, but leave consumer surplus largely unchanged if firms have that option.

Keywords: network goods, switching costs, reimbursement, consumer welfare.

1 Introduction

In network industries, the product's value to a consumer increases in the number of its users. Examples of network industries include airline services, banking services and credit cards, cable television, computer hardware and software, dating platforms, health and legal services, music and video players, radio, telecommunications, and many more (Shy, 2001; Farrell and Klemperer, 2007). Switching costs—the costs in terms of money, time, and/or effort that consumers incur when switching between products—are a fundamental characteristic of network industries (Shy, 2001, p.

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1). Switching health insurance plans often requires consumers to also switch primary care providers (Strombom et al., 2002). Switching lenders requires clearing old debts and closing old accounts in order to open new accounts (Stango, 2002) and entails losing relationship-based benefits such as easier access to credit and lower interest rates (Barone et al., 2011). Similar stories are found in the television network industry (Shcherbakov, 2016) and the telephone/wireless industry (Viard, 2007; Cullen and Shcherbakov, 2010; Park, 2011), among many others.

In this paper, we investigate a growth strategy commonly used by firms in network industries: reimbursing the switching costs incurred by consumers switching from rivals. For example, in the wireless industry, Verizon Wireless offers consumers switching from rival carriers up to \$350 to pay off early termination fees (Goldman, 2015). The other major wireless carriers AT&T and T-Mobile offer similar reimbursement policies. In the cable television industry, Spectrum offers switching consumers a "contract buyout" that reimburses up to \$500 in early termination fees (Spectrum, 2021). The reimbursement of switching costs applies to non-monetary switching costs as well. Computer hardware and software companies may provide settings and data migration, file conversion, and training services to new users who switch from rival products, reducing or eliminating their switching costs (Dell, 2021).[1]

While switching costs have been extensively studied (see for example the survey in Farrell and Klemperer (2007)), switching cost reimbursement has remained unexplored in the literature, despite being used by prominent firms such as Verizon, AT&T, and Dell and affecting millions of consumers. In this paper, we provide a first study of how this common practice by firms affects the market outcome, particularly market concentration and welfare.

Our study is also motivated by the many implemented or proposed public policies aimed at reducing switching costs in network industries. For example, mobile phone number portability has been implemented in over 100 countries in the past two decades, with over 40% of numbers ported in many countries (XConnect, 2021). The US Federal Communications Commission considered limiting the early termination fees charged by wireless carriers (German, 2008). The European Competition Authorities proposed providing switching facilities for retail banking and payment systems and implementing bank account number portability (ECAFSS, 2006). In the software industry, common standards that enhance compatibility across different products, such as the Open Document Format (ODF), are being adopted by various governments (Casson and Ryan, 2006). In order for policymakers to develop informed policies, they need to understand, among other things,

[1] The wireless industry features network effects derived from cell tower network coverage and on-net call discounts. The cable television industry features network effects associated with the quality and variety of television programming. The computer hardware and software industries feature network effects resulting from complementary application software and file sharing.

firms' endogenous response to the policies. Since a change in switching costs alters firms' choice set with respect to their reimbursement strategy, research into how firms' reimbursement and price choices are affected by a change in switching costs is much needed.

The reimbursement of switching costs is effectively a form of behavior-based price discrimination—charging different prices to consumers with different purchase histories.[2] Specifically, the effective price charged to a switching consumer, defined as the nominal price minus the switching cost reimbursement, is below the price charged to a non-switching consumer. In many markets, overt behavior-based price discrimination may face significant backlash[3], and switching cost reimbursement may be the firms' only (or most substantial) channel of price discrimination based on past purchases. An important feature in such cases is that the extent to which firms can price discriminate using this channel changes when the magnitude of switching costs changes due to public policies (such as phone number portability, bank account number portability, and limits on early termination fees) or technological developments (such as enhanced compatibility across products), leading to important policy and managerial implications.

To study the above issues, we build an infinite-horizon duopoly model using the Ericson and Pakes (1995) dynamic computational approach.[4] There are network effects and switching costs, and each firm makes both price and reimbursement decisions in each period. Firms dynamically optimize. Consumers are myopic in the main specification, and we study forward-looking consumers in an extension. We use the model to investigate when a firm chooses to reimburse consumers' switching costs and how such reimbursements affect competition and the market outcome.

Before tackling the dynamic specification, we analyze a simplified static version in order to better understand the intuition and derive analytical results. In the simplified model, one firm sets its price and makes a 0-1 decision on switching cost reimbursement. The competing product is nonstrategic, with its price normalized to zero and no reimbursements. Such an environment serves two purposes. First, it captures an important aspect of the software industry, namely competition between proprietary and open source software (Sacks, 2015). Second, it provides an analytically tractable benchmark with cleaner intuition for understanding firms' pricing and reimbursement decisions while abstracting from the various dynamic effects.

In the simplified static model, we find that when reimbursing the switching cost, the firm also increases its price, but by less than the amount of the switching cost. Hence, when switching

[2]See Fudenberg and Villas-Boas (2006) for an excellent survey of the literature on behavior-based price discrimination.

[3]For example, in a highly publicized case, due to the outcry against Amazon's price discrimination that charged regular Amazon customers higher prices, the company had to publicly apologize and refund customers who had paid higher prices (Ramasastry, 2005).

[4]See Doraszelski and Pakes (2007) for an excellent survey of the literature using this approach.

costs are being reimbursed, non-switching consumers' price goes up while switching consumers' effective price (nominal price minus switching cost reimbursement) goes down. Furthermore, with strong network effects, the firm chooses to reimburse switching costs when it is sufficiently large and will not reimburse when small. This finding is supported by anecdotal evidence. For instance, in the US wireless industry, the largest three carriers (AT&T, Verizon, and T-Mobile) reimburse consumers' switching costs, while the fourth (US Cellular, much smaller than the top three) does not. Moreover, this pattern that we obtain in the simplified static model similarly persists in the dynamic specification we examine below, where we find that the larger firm reimburses a larger proportion of the switching cost than the smaller firm does.

For the full dynamic model, we solve for a symmetric Markov perfect equilibrium using numerical methods. We use this equilibrium to investigate how the option to reimburse consumers' switching costs influences long-run industry dynamics and how these dynamics depend on the magnitude of the switching cost and the strength of the network effect. We build our analysis by studying two regimes. In the no reimbursement (NR) regime, the firms do not have the ability to reimburse consumers' switching costs. The NR regime serves as a benchmark that allows us to later assess how the market outcome is changed when the firms have the option to reimburse switching costs. In terms of the elements included in the model, the NR regime corresponds to the models in Keller et al. (2010), Suleymanova and Wey (2011), Doganoglu and Grzybowski (2013), and Chen (2016), all of which model network effects and switching costs but not switching cost reimbursement. In the endogenous reimbursement (ER) regime, each firm simultaneously chooses its price and the proportion of consumers' switching cost it reimburses.

Under the interaction of network effects, switching costs, and switching cost reimbursements, across different parameterizations two qualitatively distinct types of equilibrium emerge. The first is a *tipping equilibrium* in which the market tips in favor of one firm. In the long run, a single firm dominates the market. This equilibrium satisfies the property of market dominance discussed in the network goods literature.[5] The second is a *splintered equilibrium* in which the firms split the market with nearly symmetric shares. This equilibrium satisfies the property of reversion to the mean (the opposite of market dominance) discussed in Cabral (2011).

Under the NR regime, we find that switching costs tend to reduce market concentration. In particular, an increase in the switching cost can transform the market from a highly concentrated tipping equilibrium to a less concentrated splintered equilibrium. This concentration-reducing effect of switching costs under the NR regime is consistent with the existing switching cost literature for both non-network goods (Beggs and Klemperer, 1992; Chen and Rosenthal, 1996; Taylor, 2003)

[5] For more details, see Farrell and Klemperer (2007), Cabral (2011), and the citations therein.

and network goods (Suleymanova and Wey, 2011; Doganoglu and Grzybowski, 2013; Chen, 2016).[6]

Under the ER regime, we find that the switching cost reimbursement offered by the larger firm is greater than the one offered by the smaller firm. This difference in the two firms' reimbursement policies gives the larger firm an advantage in attracting switching consumers and increasing its market share. Correspondingly, an increase in the switching cost—which increases the magnitude of reimbursement that's possible—leads to a higher market concentration rather than transforming the market from a tipping equilibrium to a splintered equilibrium as under the NR regime. Thus the option to reimburse consumers' switching costs overturns the aforementioned concentration-reducing effect of switching costs commonly found in the literature on switching costs without the reimbursement option. Furthermore, when network effects are large, producer surplus is greater under the ER regime than under the NR regime, indicating that with exogenous switching costs, introducing the option to reimburse switching costs does not induce a prisoner's dilemma for the firms (unlike the typical case of an advertising game).

Comparing consumer surpluses under the ER and NR regimes for any given level of switching costs, we find that consumers receive greater surplus under the ER regime even though the market is more concentrated, as they benefit from the network externality of a larger network and/or lower effective prices. This result shows that in markets for network goods characterized by switching costs, a higher market concentration need not correspond to a less desirable outcome for the consumers. In fact, consumers prefer the ER regime over the NR regime even though the latter results in a market in which the competing firms have more symmetric market shares. This result is not obvious. Prices are not set independently of reimbursements. As illustrated in the analytical results of the simplified static model, prices are higher in the presence of reimbursement as firms finance the reimbursement of their rivals' consumers by charging higher prices to locked-in consumers. Our results indicate that consumers' gains from both the increase in network size and the lower effective price paid by switching consumers outweigh the loss in surplus from the higher price paid by repeat customers. Consequently, antitrust analysis that relies on the level of market concentration must be extra careful in network industries with consumer switching costs and reimbursement of such costs.

Additionally, we find that the effects of switching costs on consumer surplus critically depend on the availability of the reimbursement option: switching costs decrease consumer surplus if firms do not have the option to reimburse switching costs, but leave consumer surplus largely unchanged

[6] In the literature on endogenous market dominance, Budd et al. (1993) use a dynamic duopoly model to study whether the larger firm becomes increasingly dominant. They similarly suggest that switching costs make price cuts more costly for the larger firm than for the smaller firm and may overcome the gravitation towards asymmetric market shares and result in a "catch-up" equilibrium.

if firms have that option.

Furthermore, our findings are shown to be robust in robustness checks using an extensive set of parameterizations, as well as in an extension incorporating forward-looking consumers. The extension also provides additional insights into how consumers' forward-looking behavior affects the market outcome.

The remainder of the paper is structured as follows. The next section discusses in more detail how this paper relates to prior studies. The main model is presented in Section 3. Section 4 considers a simplified static version of the model to derive analytical results and to better understand the intuition. Section 5 discusses the dynamic equilibrium of the full model, and Section 6 assesses how switching cost reimbursement affects the market outcome. Section 7 examines the robustness of the findings, and Section 8 considers an extension in which consumers, in addition to firms, are forward-looking. Section 9 concludes.

2 Related Literature

This paper is related to the literatures on switching costs and network effects. The literature on switching costs can be traced back to von Weizsäcker (1984). Shortly thereafter, markets with switching costs were examined in Klemperer (1987*a*), Klemperer (1987*b*), and Klemperer (1987*c*). Since then, much headway has been made. A thorough summary through 2007 is given in Farrell and Klemperer (2007). For more recent surveys see Villas-Boas (2015) and the literature review in Cabral (2016). We make several departures from the existing literature.

An important finding in the literature on switching costs is that switching costs incentivize larger firms to harvest their locked-in consumers rather than price aggressively for market dominance, and therefore switching costs tend to make markets less concentrated. See, for example, Beggs and Klemperer (1992), Chen and Rosenthal (1996), Taylor (2003), Farrell and Klemperer (2007), and Chen (2016). Contributing to this literature, the current paper shows that if firms have the option to reimburse consumers' switching costs, then an increase in switching costs actually increases market concentration, reversing the finding in the previous literature.

Chen (1997) and Shaffer and Zhang (2000) are among the first papers to study discriminatory pricing in the context of switching costs. Using a two-period homogeneous-good duopoly model, Chen (1997) finds that firms play a "bargain-then-ripoff" strategy, where the prices in the first period are below marginal cost while prices in the second period are above marginal cost. When engaging in discriminatory pricing, firms are worse off and consumers are not necessarily better off, leading to deadweight losses. The paper finds that discriminatory price is not a function of the firm's market share, which follows from the model having a finite time horizon. In the present paper, we consider an infinite-horizon model in which firms set both price and switching cost reimbursement. As a result, we find that both the pricing and reimbursement decisions depend explicitly on the firm's market share.

Shaffer and Zhang (2000) study the properties of price discrimination in a static model with switching costs. They show that when demand is symmetric, a firm charges a lower price to its rival's consumers (paying to switch). However, when demand is asymmetric, a firm charges a lower price to its own consumers (paying to stay). In the present paper, we incorporate dynamic competition into the analysis and show that in the dynamic equilibrium, each firm reimburses a portion (potentially all) of consumers' switching costs, in essence charging a lower price to its rival's consumers and a higher price to its own consumers.

Several recent papers, including Dube et al. (2009), Arie and Grieco (2014), Rhodes (2014), Fabra and García (2015), and Cabral (2016), study switching costs in infinite-horizon dynamic models of price competition. Except for Cabral (2016), those papers have focused on the case in which firms cannot distinguish between locked-in and not locked-in consumers. Cabral (2016), along with two-period models in Fudenberg and Tirole (2000), Gehrig et al. (2011), and Bouckaert et al. (2012), also relates to the literature on behavior-based price discrimination. In addition to allowing switching cost reimbursement, a special form of behavior-based price discrimination, the present paper also incorporates network effects. Complementing those existing studies, we focus on the comparison between the case with switching cost reimbursement and the case without such reimbursement. We show that the market outcome crucially depends on whether firms have the option to reimburse consumers' switching costs.

Cabral (2016) studies the effects of switching costs in a dynamic competitive environment in which sellers can discriminate between locked-in and not locked-in consumers. He shows that if markets are very competitive to begin with, then switching costs make them even more competitive; whereas if markets are not very competitive to begin with, then switching costs make them even less competitive. In the present paper, we consider the effects of switching costs on market competitiveness from a different angle, and show that if firms have the option to reimburse consumers' switching costs, then switching costs make the market less competitive, otherwise they make the market more competitive.

Our paper is also related to the literature on network effects. Recent surveys on that literature include Farrell and Klemperer (2007), Birke (2009), and Shy (2011); also see Shy (2001) for a book on network effects. Prior studies that have emphasized the dynamic nature of network effects include Doganoglu (2003), Mitchell and Skrzypacz (2006), Markovich (2008), Markovich and Moenius

(2009), Chen et al. (2009), and Cabral (2011), among others. In recent years, a few papers have emerged that study network effects and switching costs jointly, including Keller et al. (2010), Suleymanova and Wey (2011), Doganoglu and Grzybowski (2013), and Chen (2016, 2018). Those papers show that the interaction between switching costs and network effects plays an important role in determining industry dynamics and market outcomes, but they do not consider switching cost reimbursement. To our knowledge, the present paper is the first to model both network effects and switching costs and allow for the reimbursement of switching costs.

3 Model

This section describes a dynamic duopoly model of network industries in which consumers face switching costs and firms have the option to reimburse such switching costs. The model uses the Ericson and Pakes (1995) dynamic computational approach. It builds on prior dynamic analyses of network effects and switching costs, particularly Chen (2016), and adds the reimbursement of switching costs.

Our objective is to provide some general insights into the effects of switching cost reimbursement in network industries. Towards that end, we do not tailor the model to any specific type of switching costs or reimbursement. Instead, we work with a generic model to capture the key features of many markets characterized by network effects, switching costs, and the reimbursement of switching costs.

3.1 Supply Side

The model is cast in discrete time with an infinite horizon. There are two single-product firms indexed by $j = 1, 2$. The firms' products are referred to as the *inside goods*. There is also an *outside good*, indexed by 0. The goods are durable and subject to stochastic death.

Firm j is described by its state $b_j \in \{0, 1, \ldots, M\}$, subject to $b_1 + b_2 \leq M$, where M is the number of consumers. A firm's state indicates the installed base of its product at the beginning of a period, from which the network effect is derived. Each firm goes up or down the "installed base ladder" based on product sales and depreciation.[7] By making a sale, a firm adds to its installed base, whereas depreciation of installed base takes place when existing consumers' products die and they return to the market to make new purchases. Product sales and depreciation are detailed in the next subsection. Denote by $b_0 = M - b_1 - b_2$ the outside good's installed base. The industry state

[7]The installed base ladder in our model is analogous to the quality ladder in the dynamic quality ladder models starting with Pakes and McGuire (1994), with the difference that in our model there is an upper bound M on the sum of the two firms' installed bases (where the upper bound comes from the size of the consumer population), while in quality ladder models there is an upper bound on each firm's quality.

is given by $b = (b_1, b_2)$, and the state space is $\Omega = \{(b_1, b_2) \mid 0 \le b_j \le M,\ j = 1, 2;\ b_1 + b_2 \le M\}$.

The firms compete in selling to a sequence of buyers with unit demands, one in each period. In each period, given b, the firms simultaneously decide what price to charge and what proportion of the switching cost to reimburse. Let p_j denote the price for good j and let $p = (p_1, p_2)$ denote the vector of prices. The outside good's price p_0 is normalized to zero. Let $d_j \in [0, 1]$ denote the reimbursement choice of firm j, where $d_j = 1$ corresponds to "fully reimburse consumers' switching costs," $d_j = 0$ corresponds to "do not reimburse," and an intermediate value corresponds to "reimburse $d_j \times 100\%$." Let $d = (d_1, d_2)$.

3.2 Demand Side

In each period, out of M consumers, one random consumer's product dies and she returns to the market to purchase one of the three (inside or outside) goods. This approach of modeling demand as coming from a random consumer in each period follows prior dynamic models of network goods, such as Chen et al. (2009) and Cabral (2011). Whereas in those papers, in each period a random old consumer dies and is replaced with a new consumer, here we assume that the random consumer doesn't die but instead her product dies, which allows us to model the consumer's switching costs.

In our model, a consumer is *attentive*, i.e. making a purchasing decision, only when her product dies, and remains *inattentive* otherwise. This assumption is supported by anecdotal evidence. For instance, Tom Meyvis, Professor of Marketing at NYU's Stern School of Business, was quoted by the radio program *Marketplace* as saying: "We're lazy, we don't want to think too much. So as long as things are going OK, we tend not to change" (Schwab, 2020).

Denote by $r \in \{0, 1, 2\}$ the good that the attentive consumer is loyal to, i.e. the good that she previously owns. With stochastic product death, the probability that the attentive consumer is loyal to good j is

$$\Pr(r = j|b) = \frac{b_j}{M},\ j = 0, 1, 2. \tag{1}$$

The utility a consumer loyal to good r gets from purchasing good j is given by

$$u_{rj} = v_j - p_j - \mathbf{1}(r \neq 0, j \neq 0, r \neq j)(1 - d_j)k + \mathbf{1}(j \neq 0)\theta g(b_j) + \epsilon_j. \tag{2}$$

Below we explain the notation and rationale for the expressions in Eq. (2).

v_j is the intrinsic product quality, which is fixed over time and common across firms: $v_j = v$, $j = 1, 2$. As the intrinsic quality parameter affects demand only through the quality differential $v - v_0$, we set $v = 0$ without loss of generality and consider different values for v_0.

p_j is the price for good j, with the outside good's price p_0 normalized to zero, as described in the previous subsection.

A consumer incurs a switching cost $k \geq 0$ if she switches from one inside good to the other, and a portion of her switching cost is reimbursed by the firm if $d_j > 0$. In this paper, we focus on exogenous switching costs and abstract from endogenous ones chosen by firms. Incorporating endogenous switching costs in the analysis, which involves adding a third choice variable of the firms besides price and reimbursement, is an interesting though challenging avenue for future research.

The firms do not observe r when they make their price and reimbursement decisions, though they do know its probability distribution (Eq. (1)). Rather than charging different prices based on whether the consumer is a switching consumer or a repeat consumer, each firm announces a single price and a switching cost reimbursement policy. Hence, price discrimination occurs only through the reimbursement channel. This modeling approach is guided by the real-world examples discussed in the Introduction, where overt behavior-based price discrimination is impractical and instead each firm sets a single price and a reimbursement policy.

The increasing function $\theta g(b_j)$ captures the network effect, where $\theta \geq 0$ controls the strength of the network effect. We assume the outside good exhibits no network effects. The main results reported are based on linear network effects, where $g(b_j) = b_j/M$. We have also allowed $g(b_j)$ to be convex, concave, and S-shaped. The main results are robust to those alternative specifications, which are presented in Section 7.

Note that we model the network effect as based on b_j, the installed base at the beginning of the period before the random product death and the attentive consumer's purchasing decision. The motivation for this specification is that network effects often come from a complementary stock which enhances the value of the network, such as apps for a smartphone ecosystem or video game titles for a video game console. As it takes time for the developers to build up this complementary stock, it is reasonable for the size of the complementary stock to be proportional to the size of the product's customer base with a lag.

When a consumer makes a purchasing decision, she chooses the good that offers the highest current utility. We are then assuming that consumers make myopic decisions. This parsimonious representation of consumers' decision-making allows rich modeling of firms' decisions with respect to price and reimbursement. In some real-world markets such as the printer and ink cartridge market (Miao, 2010), there is evidence that consumers behave myopically, while in some other markets such as the car market (Busse et al., 2013), there is evidence that consumers are forward-looking. We therefore consider in an extension, reported in Section 8, a more general setting that encompasses the case of forward-looking consumers and varies the degree to which consumers are forward-looking to explore how the results are affected.

ϵ_j is the consumer's idiosyncratic preference shock, distributed type I extreme value, independent across products, consumers, and time. Therefore, the probability that a consumer who is loyal to good r buys good j is given by the logit choice probability

$$\phi_{rj}(b, d, p) \equiv \frac{\exp\left(\bar{u}_{rj}\right)}{\sum_{h=0}^{2} \exp\left(\bar{u}_{rh}\right)}, \tag{3}$$

where $\bar{u}_{rj} \equiv u_{rj} - \epsilon_j$ is the deterministic component of u_{rj}. The expected demand for firm j's product, without observing r, is then $\mathbb{E}_r\left(\phi_{rj}(b, d, p)\right)$, where the expectation is taken over the probability distribution of r given in Eq. (1).

Let $s \in \{0, 1, 2\}$ denote the attentive consumer's product choice. The industry state then transitions based on the joint outcome of the installed base depreciation (product death) and the attentive consumer's purchasing decision:

$$b' = B(b, r, s) = (\underbrace{b_1 - \mathbf{1}(r = 1) + \mathbf{1}(s = 1)}_{b'_1}, \underbrace{b_2 - \mathbf{1}(r = 2) + \mathbf{1}(s = 2)}_{b'_2}). \tag{4}$$

3.3 Bellman Equation

Let $V_j(b)$ denote the expected net present value of current-period and future cash flows to firm j in state b. Firm j's Bellman equation is given by

$$\begin{aligned} V_j(b) = \max_{p_j, d_j} \mathbb{E}_r \Bigg[& \phi_{rj}\left(b, d_j, d_{-j}(b), p_j, p_{-j}(b)\right)\left(p_j - \mathbf{1}\left(r \neq 0, r \neq j\right) d_j k\right) \\ & + \beta \sum_{h=0}^{2} \phi_{rh}\left(b, d_j, d_{-j}(b), p_j, p_{-j}(b)\right) V_j(b') \Bigg], \end{aligned} \tag{5}$$

where $p_{-j}(b)$ is the equilibrium price charged by firm j's rival, $d_{-j}(b)$ is the equilibrium proportion of the switching cost reimbursed by firm j's rival, the constant marginal cost of production is normalized to zero, $\beta \in [0, 1)$ is the firms' common discount factor, and the next-period industry state b' at the end of the equation is given by Eq. (4).

3.4 Equilibrium

We focus attention on symmetric Markov perfect equilibrium (MPE), where symmetry requires that agents with identical states behave identically, i.e., for any $\tilde{b}, \hat{b} \in \{0, 1, ..., M\}$, firm 2's MPE strategy in state $\left(\hat{b}, \tilde{b}\right)$ is identical to firm 1's MPE strategy in state $\left(\tilde{b}, \hat{b}\right)$. Existence of a symmetric MPE follows from the proof given in Doraszelski and Satterthwaite (2010). In general, there may exist multiple dynamic equilibria, so we use a selection rule in the dynamic games literature where we compute the limit of the equilibrium of a finite-horizon game as the horizon grows to infinity (Chen et al., 2009). Computation of the MPE via value function iteration is carried out using MATLAB and the solver KNITRO in the TOMLAB optimization environment.

4 Simplified Static Model

Before analyzing the full dynamic model, we first derive analytical results using a simplified version of the model to better understand the intuition, focusing on the question of whether a large firm or a small firm would find it profitable to reimburse consumers' switching costs. We make the following simplifications. First, we set $\beta = 0$ and consider a static point in time for a given state b. Second, we extend the consumer population to a continuum with unit mass, reinterpreting b_j as the share of consumers in firm j's installed base. Third, we eliminate the outside option as a consideration by taking $v_0 \to -\infty$, so $b_0 = 0$ and $b_2 = 1 - b_1$. Fourth, we model firm 2 as a non-strategic actor by setting $d_2 = 0$ and $p_2 = 0$ (its product still offers network effect), which allows us to focus squarely on firm 1's pricing and reimbursement decisions. Fifth, we restrict reimbursement to a binary choice, $d_1 \in \{0, 1\}$, so firm 1 chooses to either fully reimburse or not reimburse at all. As firm 2's strategic choices are eliminated, the model is fully identified by firm 1's decisions, so we replace $\phi_{rj}(b, d, p)$ with $\phi_{rj}(b_1, d_1, p_1)$ for $r, j = 1, 2$.

In addition to offering analytical tractability and cleaner intuition, studying the simplified model described above is also a useful analysis on its own, capturing the key features of competition between open source and proprietary software (Sacks, 2015). Switching between different software is costly: downloading and installing the software onto all machines, calibrating the software for the user's needs, and ensuring compatibility with hardware all require time and effort. The open source community makes its software available for free and offers no assistance to offset consumers' switching costs, whereas the proprietary software developer charges a price for its software and may choose to perform the installation and calibration services for switching consumers or compensate the consumers in the form of a switching cost reimbursement.

For a given reimbursement decision d_1, let $p_1(d_1)$ represent firm 1's profit maximizing price:

$$p_1(d_1) = \arg\max_{p_1} \underbrace{b_1 p_1 \phi_{11}(b_1, d_1, p_1) + (1 - b_1)(p_1 - d_1 k)\phi_{21}(b_1, d_1, p_1)}_{\text{firm 1's profits}}. \tag{6}$$

The following proposition characterizes the relationship between the firm's profit-maximizing prices with and without reimbursement.

Proposition 1 *For all $b_1 \in (0,1)$, $0 < p_1(1) - p_1(0) < k$, with $p_1(1) - p_1(0) = k$ at $b_1 = 0$ and $p_1(1) - p_1(0) = 0$ at $b_1 = 1$.*

The proofs of Propositions 1 and 2 (below) can be found in Appendix A1. The top panel of Figure 1 illustrates the relationship among switching cost k, price without reimbursement $p_1(0)$, and price under reimbursement $p_1(1)$ summarized in Proposition 1. When b_1 is interior, compared to no reimbursement, under reimbursement the firm heightens the price, but the increase is less than the switching cost which it now reimburses. Consequently, consumers switching to the firm pay a lower effective price while the firm's repeat consumers pay a higher price. Thus the locked-in consumers subsidize the consumers who switch from the competing good.

When $b_1 = 0$, we can interpret firm 1 as a new entrant into the market. In this case firm 1 completely finances the cost of reimbursement through a higher price ($p_1(1) - p_1(0) = k$), and consumers' benefits from reimbursement are entirely offset by the accompanying increase in price. When firm 1 completely covers the market ($b_1 = 1$) and the non-strategic (open source) competitor is the entrant, there are no consumers to reimburse. Hence, the firm's profit-maximizing price is independent of the reimbursement decision. In either case, reimbursement yields firm 1 identical profits to no reimbursement.

We next consider firm 1's reimbursement decision. Given the binary reimbursement choices, the firm compares the maximum profits with $d_1 = 1$ and $d_1 = 0$, respectively. Let $\Delta\pi_1(b_1)$ denote firm 1's profits given $(d_1, p_1) = (1, p_1(1))$ minus its profits given $(d_1, p_1) = (0, p_1(0))$. Then the firm's reimbursement decision is determined by the sign of $\Delta\pi_1(b_1)$: $\Delta\pi_1(b_1) > 0 \implies d_1^*(b_1) = 1$, and $\Delta\pi_1(b_1) < 0 \implies d_1^*(b_1) = 0$. When $\Delta\pi_1(b_1) = 0$, the firm is indifferent between reimbursing and not reimbursing.

Proposition 2 *There exist cutoff values $0 < \underline{b} \le \bar{b} < 1$ such that for sufficiently large θ, $d_1^*(b_1) = 0$ if $b_1 \le \underline{b}$ and $d_1^*(b_1) = 1$ if $b_1 \ge \bar{b}$.*

To illustrate the pattern of $\Delta\pi_1(b_1)$ that underlies firm 1's reimbursement choices described in Proposition 2, the bottom panel of Figure 1 plots $\Delta\pi_1(b_1)$ against b_1 for $k = 2$ and $\theta \in \{2, 3, 4\}$ as examples. The panel shows that as b_1 increases between $(0,1)$, $\Delta\pi_1(b_1)$ is initially negative but turns positive for large values of b_1. For example, with $k = 2$ and $\theta = 3$, for $0 < b_1 < 0.60$, $\Delta\pi_1(b_1) < 0$ and correspondingly firm 1 chooses not to reimburse, whereas for $0.60 < b_1 < 1$, $\Delta\pi_1(b_1) > 0$ and correspondingly firm 1 chooses to reimburse.

Proposition 2 says that in industries with strong network effects, it is large firms rather than small firms who reimburse consumers' switching costs. Here's our intuition. A small firm chooses not to offer switching cost reimbursement as it cannot finance such reimbursement through higher prices charged to its locked-in consumers as effectively as a large firm, because it has fewer locked-in consumers, and because consumers in the small network are more easily enticed to pay the switching cost to join the larger network due to network effects. A large firm, on the other hand, can benefit from the reimbursement strategy. With a large network, it is relatively easy for the large firm to attract new consumers by offering reimbursement, and the firm is able to finance such reimbursement through raising the price charged to its large base of locked-in consumers who are benefiting from the large network.

As we will see in the next section, the above pattern regarding firms' reimbursement choices which we obtained in a static setting carries over to the dynamic environment, and it underpins further results regarding market concentration and welfare. In particular, the pattern that the larger firm reimburses a larger proportion of the switching cost than the smaller firm does will contribute to the higher market concentration under endogenous reimbursement than under no reimbursement.

Our finding regarding the larger firm and the smaller firm's asymmetric reimbursement choices is supported by anecdotal evidence from real-world industries. For example, among the four largest carriers in the US wireless industry, the top three (AT&T, Verizon, and T-Mobile) reimburse consumers' switching costs, whereas the fourth (US Cellular, much smaller than the top three) does not, consistent with the pattern that large firms rather than small firms reimburse consumers' switching costs.[8]

5 Dynamic Equilibrium

We now study the full dynamic model. The model incorporates realistic features such as firms' dynamic optimization over an infinite horizon, competition between firms, and an outside option, but the insight obtained from the simplified static model regarding the firms' asymmetric incentives toward reimbursement still applies, as will be discussed below.

In the dynamic model, across different parameter values, two qualitatively distinct types of MPE occur: the tipping equilibrium and the splintered equilibrium. In the tipping equilibrium, the firm that obtains an initial advantage is able to build on that advantage and dominate the market in the long run, resulting in a high level of market concentration. In the splintered equilibrium, starting from any industry state the market converges to a symmetric outcome, and in the long run the two firms share the market about evenly. For the remainder of this section, we will be referring

[8]The list of the four largest wireless carriers in the US is based on market share data for the fourth quarter of 2020, available at `https://www.statista.com/statistics/199359/market-share-of-wireless-carriers-in-the-us-by-subscriptions/`, accessed April 21, 2021.

to firm 1 without loss of generality, as the two firms' policy functions are symmetric to each other, given that we study a symmetric MPE.

5.1 Parametrization

For the baseline specification, we set the quality of the outside good $v_0 = -5$, which represents a scenario in which the inside goods' intrinsic quality ($v = 0$) is significantly higher than the outside good's. Consequently, the market is not completely covered by the inside goods (a reasonable approximation of real-world examples such as the wireless industry and the banking industry). We set the number of consumers $M = 20$, which, given our modeling of the random depreciation of installed bases described above, corresponds to a depreciation rate of $1/20 = 0.05$. For the baseline specification, we investigate the following values of the network effect and the switching cost: $\theta \in \{0, 0.5, ..., 5\}$ and $k \in \{0, 0.25, ..., 3\}$, for a total of $11 \times 13 = 143$ (θ, k) combinations. We fix the firms' discount factor at $\beta = 1/1.05$, which corresponds to a yearly interest rate of 5%.

While our model is not intended to fit any particular industry, some market characteristics emerging from the baseline parameterizations that we consider are reasonable compared to empirical findings. For example, the own-price elasticity of demand for the firms' products in the baseline parameterizations range from -1.02 to -0.37. These numbers are in the same ballpark as the own-price elasticities reported in Clements and Ohashi (2005) (ranging from -2.15 to -0.18 for video game consoles), Dick (2008) (ranging from -0.87 to -0.12 for banking services), and Gandal et al. (2000) (-0.54 for CD players, computed from the results reported in the paper). Additionally, the combined market share of the inside goods in the baseline parameterizations range from 89.4% to 99.8%. These numbers are in line with, for instance, the percentage of U.S. households with a bank account, at 93% in 2013 (Furman, 2016).

Beyond the above baseline parameterizations, we also run the model for a wide range of parameter values to assess the robustness of our findings. Details of those robustness checks are presented in Section 7. They show that our findings are robust.

5.2 The No Reimbursement (NR) Regime

First suppose firms are unable to reimburse consumers' switching costs (e.g. if price discrimination is made illegal). In an industry featuring network effects, there is a tipping equilibrium when the switching costs are low, illustrated in Figure 2 ($\theta = 2$, $k = 1$), and there is a splintered equilibrium when the switching costs are high, illustrated in Figure 3 ($\theta = 2$, $k = 2$). The results in this subsection are qualitatively similar to Chen (2016) and serve as a benchmark against which we will compare the Endogenous Reimbursement (ER) regime presented later.

Low Switching Costs. Consider the tipping equilibrium at low switching costs. Panels 5 and 6 of Figure 2 show the evolution of the industry structure over time, plotting the 15-period transient distribution of the industry state (the probability distribution of the industry state after 15 periods, starting from state $(0, 0)$ in the first period) and the limiting distribution (the probability distribution of the industry state as the number of periods approaches infinity), respectively. The transient distribution and the limiting distribution are bimodal and indicate that over time, the industry moves towards asymmetric states. Panel 4 plots the resultant forces, which report the expected movement of the state from one period to the next. The panel shows that even the smallest of advantages leads to convergence towards the modal state in which the initial leader dominates the market.

There are two factors that can induce consumers to switch: the price and the network effect. Panel 1 presents firm 1's price policy function and Panel 3 shows the probability that a consumer loyal to firm 1 switches to firm 2. Examination of these two panels allows us to better understand the two factors at work.

Panel 1 shows that when the firms' market shares are even they engage in fierce price competition, hoping to gain an initial advantage which would propel them to become a dominant firm. This results in a deep trench along and around the diagonal of the state space, representing very low prices, often below the marginal cost (normalized to 0). When one firm gains an advantage, market tipping takes place and the smaller firm gives up the fight (to the left of the diagonal, firm 1 is the smaller firm, and to the right of the diagonal, firm 1 is the larger firm). Correspondingly, the smaller firm increases its price substantially. In response, the larger firm also increases its price, but keeps it lower than the smaller firm's until it has achieved a significant installed base advantage over its smaller rival. Therefore, during most of the market evolution, both the network effect factor and the price factor work in favor of the larger firm, as it has a larger network and charges a lower price. As a result, in Panel 3 we see that the larger firm's customers rarely switch to the smaller firm, whereas the smaller firm's customers frequently switch to the larger firm. This pattern in the consumers' switching behavior allows the larger firm to achieve and maintain market dominance. This pricing strategy resembles the harvesting and investing effects described in Chen (1997), Shaffer and Zhang (2000), Farrell and Klemperer (2007), and the citations therein.

High Switching Costs. We now turn to the splintered equilibrium at high switching costs. Panel 1 of Figure 3 shows that a firm's price is always strictly above marginal cost (normalized to 0) except when it has little-to-no installed base, at which point it drops its price to marginal cost. While prices are, in general, increasing with the firm's own installed base, there is a spike when the market is fully covered and evenly split. The high switching costs incentivize the firms to harvest their locked-in consumers (Farrell and Klemperer, 2007) rather than price aggressively for market dominance (Cabral, 2011, p. 84). As a result, the market settles on a symmetric outcome (Panel 6) with high prices in the long run. This result is consistent with much of the literature on switching costs, with or without network effects (e.g. Beggs and Klemperer (1992) and Suleymanova and Wey (2011)).

When the smaller firm has little-to-no installed base, it prices at or near marginal cost while the larger firm charges a much higher price to harvest its locked-in consumers. Correspondingly, the probability of the smaller firm's customer switching is close to zero while the probability of the larger firm's customer switching is higher at around 0.2 (Panel 3), implying a transition towards a more symmetric outcome. This result indicates a form of reversion to the mean discussed in Cabral (2011), where the larger firm is both more likely to have its consumers on the market due to the constant hazard rate and its consumers more likely to switch. Consequently, Panel 4 shows that there is global convergence of the industry state towards the symmetric modal state, resulting in a low level of market concentration.

5.3 The Endogenous Reimbursement (ER) Regime

Now suppose firms simultaneously choose price and the proportion of switching costs to reimburse. We again divide the analysis into two cases: when the switching costs are low, and when the switching costs are high.

Low Switching Costs. First consider $\theta = 2$ and $k = 1$. Like the NR regime at these parameter values, a tipping equilibrium occurs. The price policy function, resultant forces and limiting distribution also remain similar from the NR regime and are depicted in Figure 4, Panels 1, 4, and 6, respectively.

Panel 2 plots the reimbursement policy. When the two firms have equal market shares, only a modest proportion of the switching cost is reimbursed. If firm 1 is able to obtain an advantage in installed base, it sharply increases the proportion of switching cost it reimburses to 100% or near 100% in an effort to attract its competitor's customers and propel itself to a dominant position. Given the strength of the network effect, firm 2 is unable to compete with firm 1 on this margin and thus has no incentive to increase its reimbursement. Instead, firm 2 chooses to reimburse a small proportion of the switching cost, around 20%. A larger proportion of consumers switch from the smaller firm to the larger firm than in the opposite direction (Panel 3), and correspondingly, the market converges to a highly asymmetric outcome (Panel 4).

Although the pricing policy function is similar to the NR regime, there is an important distinction with the interpretation of the pricing strategy when coupling the pricing policy with the reimbursement policy. Rather than an invest-then-harvest strategy, we can view the larger firm's strategy as invest-and-harvest. Utilizing reimbursements, the firm can simultaneously harvest loyal consumers while offering reimbursement to invest in switching consumers.

High Switching Costs. When the switching cost is high ($\theta = 2$, $k = 2$), the results run in stark contrast to the NR regime and Chen (2016). Unlike the NR regime, the larger firm can achieve and maintain its dominance even at a high switching cost, and the market remains in a tipping equilibrium. That is, by using the switching cost reimbursement as a strategic instrument, the large firm is able to counteract the effects of the increased switching cost.

The first thing to note is that the price policy function (Panel 1 of Figure 5) maintains the same general shape as in the other instances of tipping equilibrium (Panel 1 of Figures 2 and 4). However, the trench is lower and the peak is higher. For instance, under the ER regime, the lowest price along the trench in firm 1's price policy function is -3.72 when $k = 1$ (Panel 1 of Figure 4), and is much lower at -5.58 when $k = 2$ (Panel 1 of Figure 5). Also, the peak in firm 1's price policy function, which occurs at state $(b_1, b_2) = (20, 0)$, is 2.64 when $k = 1$ (Panel 1 of Figure 4), and is much higher at 3.12 when $k = 2$ (Panel 1 of Figure 5). This indicates that given endogenous reimbursement of switching costs, instead of foiling the preemption race and market tipping, an increase in the switching cost actually makes them more pronounced: the firms price more aggressively when they are neck and neck, and the eventual market winner enjoys a higher price markup.

Panel 2 plots the reimbursement policy and shows that the larger firm reimburses a much higher proportion of the switching cost than the smaller firm does, which gives the larger firm an additional advantage in attracting consumers from the rival firm.

Here is the intuition for the above patterns. Compared to the smaller firm, the larger firm has a larger pool of locked-in consumers, so it has a stronger incentive to charge high prices in order to "harvest" its locked-in consumers. An increase in the switching costs strengthens the degree of consumer lock-in and therefore strengthens the larger firm's harvesting incentive. If there isn't the option to reimburse rival's customers' switching costs, the larger firm's ability to charge high prices to its locked-in consumers is restricted, as a high price charged to all consumers would make it less likely that the larger firm can attract new customers and expand its installed base. The option to reimburse rival's customers' switching costs solves this dilemma for the larger firm. Its strategy is then to charge a higher price to its locked-in consumers to take advantage of the higher switching costs, while compensating new customers using switching cost reimbursement. Such a strategy, combined with the fact that the larger firm has a larger network and therefore a better product, means that the larger firm can both harvest its locked-in consumers and attract new consumers at the same time. Hence our finding above that under the ER regime, the dominant firm enjoys a higher price markup when switching costs are higher. Correspondingly, the reward to being the dominant firm is greater, and this greater reward in turn makes the initial preemption race even fiercer, which explains the other finding above that the firms price more aggressively when they are neck and neck.

The remaining panels in Figure 5 ($k = 2$) resemble their counterparts in Figure 4 ($k = 1$). A comparison between Panel 6 of Figure 5 and Panel 6 of Figure 4, which plot the respective limiting distributions, shows that when the switching cost increases from 1 to 2, the industry spends more time in more asymmetric states, indicating that under the ER regime, a higher switching cost makes market tipping and winner-takes-most more pronounced. This is in contrast to what we find earlier under the NR regime, where an increase in the switching cost from 1 to 2 changes the market from a tipping equilibrium to a splinter equilibrium and lowers market concentration.

6 The Role of Switching Cost Reimbursement

This section examines the effects of switching cost reimbursement in markets characterized by network effects and switching costs. We focus on several elements of the market outcome: market concentration, network benefit, average price, and welfare measures including consumer surplus, producer surplus, and total surplus. They are shown in Figures 6-8, where each panel plots one variable of interest for different combinations of θ (the network effect) and k (the switching cost).

Market Concentration. First consider long-run market concentration, measured as the larger firm's share of installed base, i.e. b_L/M where b_L denotes the larger firm's installed base, averaged across all industry states using the probabilities in the limiting distribution as weights.[9] Panel 1 of Figure 6 shows that in markets with strong network effects ($\theta \geq 2.5$), an increase in the switching cost can lead to a significant drop in the level of market concentration under NR. For example, with $\theta = 2$, an increase from $k = 1$ (corresponding to the parameterization depicted in Figure 2) to $k = 2$ (Figure 3) causes the market concentration to drop from 0.70 to 0.57. As discussed above, in this case the increase in the switching cost changes the type of equilibrium that occurs in the market, from a tipping equilibrium at low switching costs to a splintered equilibrium at high switching costs.

In contrast, Panel 3 shows that under ER, an increase in the switching cost increases the level of market concentration, sometimes dramatically so. For example, with $\theta = 2$, an increase from $k = 1$ (corresponding to the parameterization depicted in Figure 4) to $k = 2$ (Figure 5) causes the

[9] In addition to the larger firm's share of installed base, we have also used other measures of market concentration, including the Herfindahl-Hirschman Index (HHI) with respect to the firms' installed bases and the difference between the two firms' installed bases, and the results are very similar.

market concentration to increase from 0.72 to 0.76 (both are tipping equilibria). With $\theta = 1$, an increase from $k = 1$ to $k = 2$ causes the market concentration to increase from 0.61 (a splintered equilibrium) to 0.70 (a tipping equilibrium).

This pattern is due to the fact that under ER, a higher switching cost gives the larger firm an additional edge, in that the larger firm, by reimbursing a bigger proportion of consumers' switching cost, is able to expand its advantage in attracting switching customers.

Panel 5 plots the difference in the market concentration between the NR and ER regimes. Consistent with the above discussion, we see that for every (θ, k) combination, ER results in a higher market concentration than NR. The difference is particularly large when both θ and k are large—in this area of the parameter space, NR results in a splintered equilibrium, whereas ER results in a tipping equilibrium.

Network Benefit. We next consider *network benefit*, measured as the expected network externality enjoyed by consumers in one period, aggregated over all consumers (both attentive and inattentive) and then averaged across all industry states using the probabilities in the limiting distribution as weights.

In a tipping equilibrium, the dominant firm offers a large network and its product is used by the majority of consumers. In contrast, in a splintered equilibrium, neither firm offers a particularly large network as the two firms split the market about evenly. Therefore, under NR, we expect the network benefit to be larger in a tipping equilibrium than in a splintered equilibrium. Panel 2 of Figure 6 confirms this intuition and shows a drop in network benefit when the market changes from a tipping equilibrium to a splintered equilibrium as the switching cost increases.

We see a different picture for ER. Under ER, an increase in the switching cost increases market concentration and therefore results in a larger network enjoyed by the majority of the consumers. Consequently, under ER, the network benefit is expected to increase in the switching cost. This intuition is confirmed in Panel 4, which shows that as the switching cost increases, the network benefit increases.

Turning to the difference between the NR and ER regimes, Panel 6 shows that compared to NR, ER results in a larger network benefit for the consumers, corresponding to our previous finding that ER results in a higher market concentration than NR. The difference is particularly large when both the network effect and the switching cost are large. The larger network benefit that consumers enjoy under ER will contribute to their higher consumer surplus under ER, which we examine later in this section.

Average Price. Here the term *average price* refers to the average effective price charged to the attentive consumer by the firms, weighted by the probabilities of the attentive consumer's loyalty (Eq. (1)), the two firms' expected sales, and the probabilities of the industry state in the limiting distribution. For switching consumers, the effective price is equal to the nominal price minus the switching cost reimbursement which the consumer receives from the firm.

Panels 1 and 3 in Figure 7 plot the average price for the NR regime and the ER regime, respectively. An increase in the switching cost increases the average price for the most part, except in the following three cases: (1) under NR when both the network effect and the switching cost are modest ($\theta \leq 1.5$ and $k \leq 0.5$), (2) under NR when a small increase in k results in the equilibrium switching from tipping to splintered (for example when $\theta = 4$ and k increases from 2 to 2.25), and (3) under ER when both the network effect and the switching cost are modest ($\theta \leq 1.5$ and $k \leq 1.75$).[10]

Turning to the comparison between NR and ER, Panel 5 shows that the change in the average price from NR to ER depends on the magnitude of the network effect. Inspection of the data shows that relative to NR, ER increases the average price when the network effect is large ($\theta \geq 3$) and decreases the average price otherwise. In both cases (large network effect or small network effect), the difference between NR and ER becomes larger as the switching cost becomes larger, consistent with the fact that the magnitude of the switching cost represents an upper bound on the firms' endogenous reimbursement under ER.

Welfare. First consider consumer surplus, measured as the expected consumer surplus in one period aggregated over all consumers (both attentive and inattentive) and then averaged across all industry states using the probabilities in the limiting distribution as weights.

Panels 2 and 4 in Figure 7 plot consumer surplus for the NR regime and the ER regime, respectively. Inspection of the data shows that for all the values of network effect that we consider, when switching cost increases from 0 to 3, under NR consumer surplus decreases, whereas under ER consumer surplus remains largely unchanged. For example, for $\theta = 2$, an increase of k from 0 to 3 substantially lowers consumer surplus from 22.1 to 9.9 under NR, but leaves consumer surplus little changed (from 22.1 to 22.3) under ER.

As will be shown in the next section, the above pattern is robust across a wide range of parameterizations. In particular, under ER, as switching cost increases, the market becomes more concentrated, and the larger network benefit and the reimbursement of switching costs that consumers receive offset the higher average price that they pay, leaving consumer surplus largely unchanged. This result highlights the important role that switching cost reimbursement plays in

[10] Our finding that when both the network effect and the switching cost are modest, an increase in the switching cost reduces the average price is consistent with the finding in some recent studies on switching costs without network effects, such as Cabral (2009) and Arie and Grieco (2014), who find that small switching costs lead to lower prices.

determining the effects of switching costs on consumer welfare. A public policy that reduces consumers' switching costs, such as phone number portability and bank account number portability, would increase consumer welfare if firms do not have the option to reimburse switching costs, but would have little effect on consumer welfare if firms have that option.

Panel 6, which plots the change in consumer surplus from NR to ER, shows that ER results in higher consumer surplus than NR, particularly when the switching cost is high. Examination of Panel 6 of Figure 6 and Panels 5 and 6 of Figure 7 indicates that at high switching costs, when the network effect is strong, the increased consumer surplus under ER comes primarily from a higher network benefit, and when the network effect is weak, the increased consumer surplus under ER comes primarily from a lower average price.

These results have useful policy implications, as they show that with endogenous reimbursement of switching costs, even though the market is more concentrated, the network benefit is greater and the average price is often lower than a less concentrated market without switching cost reimbursement, thus benefiting consumers. Therefore an antitrust authority relying on the market concentration for its antitrust analysis must exercise caution in industries with switching costs. If the firms are practicing price discrimination via the reimbursement channel, then consumer surplus is actually greater in these markets than it would be if such price discrimination were to be restricted by the authority.

Next consider producer surplus, measured as the expected firm profits in one period aggregated over both firms and then averaged across all industry states using the probabilities in the limiting distribution as weights. The left column of panels in Figure 8 plot the producer surplus under NR, the producer surplus under ER, and the change in producer surplus from NR to ER, respectively. Notice that these panels are similar to their counterparts in the left column in Figure 7, which plot the average price. The reason is that in this paper we focus on industries in which the outside good is inferior to the inside goods (in the baseline parameterization, the inside goods' quality is $v = 0$ while the outside good's quality is $v_0 = -5$), so that most of the market is covered by the firms, as is the case in industries such as mobile phone services and banking services. As a result, the firms' combined profits, which equal the sum of the firms' expected sales times the average price (recall that the marginal cost has been normalized to 0), are close to the average price, because the sum of the firms' expected sales is close to 1 due to the inferiority of the outside good.

Consequently, the results regarding producer surplus parallel those regarding the average price. An increase in the switching cost increases producer surplus for the most part, except in the following three cases: (1) under NR when both the network effect and the switching cost are modest ($\theta \leq 1.5$ and $k \leq 0.5$), (2) under NR when a small increase in k results in the equilibrium changing from tipping to splintered (for example when $\theta = 4$ and k increases from 2 to 2.25), and (3) under ER when both the network effect and the switching cost are modest ($\theta \leq 1.5$ and $k \leq 1.75$). Furthermore, relative to NR, ER increases producer surplus when the network effect is large ($\theta \geq 3$) and decreases producer surplus otherwise, with the difference between NR and ER becoming larger as the switching cost becomes larger.

Combining the results regarding consumer surplus and those regarding producer surplus, we find that when the network effect is large, both consumers and firms prefer ER over NR, but when the network effect is small, consumers and firms have opposing preferences, with consumers still in favor of ER but firms in favor of NR.

Lastly, we consider total surplus, the sum of consumer surplus and producer surplus. A comparison between the right column of panels in Figure 7 (consumer surplus) and the left column of panels in Figure 8 (producer surplus) shows that across the different (θ, k) combinations, the variation in consumer surplus is much larger than the variation in producer surplus, therefore consumer surplus is the main driving force for the variation in total surplus. Correspondingly, Panels 2 and 4 in Figure 8, which plot total surplus for the NR regime and the ER regime, respectively, show patterns similar to those with consumer surplus. Inspection of the data shows that for all the values of network effect that we consider, when switching cost increases from 0 to 3, under NR total surplus decreases, whereas under ER total surplus remains largely unchanged. For example, for $\theta = 2$, an increase of k from 0 to 3 substantially lowers total surplus from 23.5 to 12.7 under NR, but leaves total surplus little changed (from 23.5 to 24.5) under ER. Additionally, Panel 6 plots the change in total surplus from NR to ER and shows that ER results in higher total surplus, particularly when both the network effect and the switching cost are high.

Our results thus show that firms' reimbursement of consumers' switching cost is total welfare-enhancing, and such welfare gains are particularly large in industries with strong network effects and switching costs, providing support for public policies that allow or even promote switching cost reimbursement in such industries.

7 Robustness of Findings

Our main findings regarding firms' reimbursement strategies and the comparison between the NR and ER regimes can be summarized as follows:

Result 1 (Reimbursement). Under ER, the larger firm reimburses a larger proportion of the switching costs than the smaller firm does.

Result 2 (Market Concentration). Compared to NR, ER results in higher market concentration.

An increase in switching costs decreases market concentration under NR but increases it under ER.

Result 3 (Welfare). Compared to NR, ER results in higher consumer surplus and higher total surplus. It results in lower producer surplus when network effects are weak and higher producer surplus when network effects are strong. Switching costs decrease consumer surplus and total surplus under NR but leave them largely unchanged under ER.

The above findings were obtained by varying network effect θ and switching cost k while using the baseline parameter values of $v_0 = -5$, $M = 20$, and the linear network effect function. In this section, we examine the robustness of those findings by considering a wide range of parameterizations: $\theta \in \{1, 2, 3, 4, 5\}$, $k \in \{1, 2, 3\}$, $v_0 \in \{-7, -6, -5, -4, -3\}$, $M \in \{12, 14, ..., 24\}$, shape of the network effect function $\in$ {Linear, Convex, Concave, S-shaped} (defined below), and regime $\in$ {NR, ER}, for a total of $5 \times 3 \times 5 \times 7 \times 4 \times 2 = 4,200$ parameterizations. We compute the equilibrium for each parameterization in this set and examine whether Results 1-3 continue to hold. Below we first describe our rationales for running those robustness parameterizations, then present our findings from them.

Quality of the Outside Good. v_0 is the outside good's intrinsic quality, while the inside goods intrinsic quality v is normalized to 0. By varying v_0 between -7 and -3, we allow the quality differential between the inside goods and the outside good to vary considerably, which results in a wide range of market size (i.e. the two firms' combined installed base as a proportion of M, ranging from 76.63% to 99.99% across all the robustness parameterizations), while maintaining the assumption that the outside good is inferior to the inside goods.

Product Death and Consumer Inertia. In our model, in each period, one out of M consumers experiences product death and becomes attentive, while the other $M - 1$ consumers are inattentive and keep their existing products, exhibiting consumer inertia (see Dube et al. (2010), Handel (2013), and Hortacsu et al. (2017) for examples of consumer inertia in consumer packaged goods markets, health insurance markets, and residential electricity markets, respectively). In the robustness checks, we vary the rate of product death (products die at the rate of $1/M$ in each period) and the degree of consumer inertia (a fraction $1/M$ of the consumers are attentive in each period) by varying the value of M, and assess the robustness of the results.

Shape of the Network Effect Function. In the main analysis of this paper, we have worked with a linear network effect function $g(b_j) = b_j/M$. In the robustness checks we consider three different shapes of the network effect function in addition to the linear one, to assess the robustness of the results.[11] Following Chen et al. (2009), we make use of the convexity-concavity

[11] For example, Swann (2002) explores functional forms of network effects in a model of a telephone network and

property of the sine function and consider the following additional shapes of the network effect function: (1) *Convex*: $g(b_j) = \sin\left(\frac{b_j}{M} \times \frac{\pi}{2} + \frac{3\pi}{2}\right) + 1$, (2) *Concave*: $g(b_j) = \sin\left(\frac{b_j}{M} \times \frac{\pi}{2}\right)$, and (3) *S-shaped*: $g(b_j) = \left(\sin\left(\frac{b_j}{M} \times \pi + \frac{3\pi}{2}\right) + 1\right)/2$.

7.1 Findings from Robustness Checks

Figures 9a and 9b provide a succinct summary of the robustness checks results as they relate to Results 1-3 described above. Each panel in Figures 9a and 9b is a scatter plot, i.e. a plot which displays data as a collection of points, each point having its position on the horizontal axis determined by the value of one variable and its position on the vertical axis determined by the value of another variable.

Reimbursement. Panel 1 of Figure 9a relates to Result 1 (Reimbursement) and compares the larger firm's reimbursement with the smaller firm's reimbursement under ER. In this panel, each point corresponds to a $(\theta, k, v_0, M, shape)$ combination, where *shape* denotes the shape of the network effect function (so there are $5 \times 3 \times 5 \times 7 \times 4 = 2100$ points in this plot). For each point, the vertical coordinate is the larger firm's reimbursement under ER in that parameterization, while the horizontal coordinate is the smaller firm's reimbursement under ER in that parameterization. The larger firm's reimbursement is defined as a firm's average reimbursement across all the states in which its installed base is larger than its rival's (using the states' probabilities in the limiting distribution as the weights when computing the average). The smaller firm's reimbursement is defined analogously. For comparison purposes, the 45-degree line is also drawn.

Notice that all the points in Panel 1 are located above the 45-degree line, indicating that for each parameterization that we consider, under ER the larger firm reimburses a larger proportion of the switching costs than the smaller firm does, confirming Result 1. Specifically, across all the parameterizations considered, the larger firm's reimbursement ranges from 0.61 to 1.00, while the smaller firm's ranges from 0.07 to 0.58, and the difference between the two ranges from 0.13 to 0.93.

Furthermore, inspection of the data shows that for many parameterizations, the larger firm chooses to fully reimburse the switching cost, indicating that for the larger firm, the upper bound of reimbursement at 100% is often binding. This pattern illustrates a point we made earlier in the Introduction. In markets where overt behavior-based price discrimination may face significant backlash, switching cost reimbursement may be the firms' only (or most substantial) channel of price discrimination based on past purchases, and the extent to which firms can price discriminate

suggests that in theoretical models with network effects, the character of the results depends on the functional form of network effects.

using this channel changes when the magnitude of switching costs changes due to public policies or technological developments. Therefore, in order to make well-informed decisions, policymakers and firm managers alike need to take into consideration this impact of switching costs changes on firm behavior.

Market Concentration. Panels 2-4 relate to Result 2 (Market Concentration). Panel 2 compares the market concentration under ER with the market concentration under NR. In this panel, each point corresponds to a $(\theta, k, v_0, M, shape)$ combination. For each point, the vertical coordinate is the market concentration (as defined previously) under ER, while the horizontal coordinate is the market concentration under NR. Notice that although the points in Panel 2 span a wide range of locations, they are all located above the 45-degree line, indicating that for the same parameterization, ER results in a higher market concentration than NR does.

Panel 3 compares the market concentration at high switching costs with the market concentration at low switching costs under NR. In this panel, each point corresponds to a $(\theta, v_0, M, shape)$ combination (so there are $5 \times 5 \times 7 \times 4 = 700$ points in this plot). For each point, the vertical coordinate is the market concentration when $k = 3$ under NR, while the horizontal coordinate is the market concentration when $k = 1$ under NR. The points in Panel 3 are located below the 45-degree line, indicating that under NR, an increase of switching cost k from 1 to 3 lowers the market concentration.

Panel 4 is similar to Panel 3 but plots for ER rather than NR. In contrast to Panel 3, the points in Panel 4 are located above the 45-degree line, indicating that under ER, an increase of switching cost k from 1 to 3 increases the market concentration. Thus Panels 2-4 are consistent with Result 2.

Welfare. Panels 5-9 relate to Result 3 (Welfare). Panel 5 compares consumer surplus (CS) under ER with CS under NR. In this panel, each point corresponds to a $(\theta, k, v_0, M, shape)$ combination. For each point, the vertical coordinate is the CS under ER, while the horizontal coordinate is the CS under NR. The points in Panel 5 are located above the 45-degree line, indicating that for the same parameterization, ER results in a higher CS than NR does. Panel 6 plots total surplus (TS) rather than CS. The points in Panel 6 are also located above the 45-degree line, indicating that for the same parameterization, ER results in a higher TS than NR does. Panel 7 plots producer surplus (PS). In this panel, some points are located above the 45-degree line, while some points are located below it, indicating that in general, firms' option to reimburse consumers' switching costs under ER has an ambiguous effect on producer surplus.

Panels 8 and 9 also plot PS, but focus on the subsets of parameterizations with weak network effects and strong network effects, respectively. Specifically, while the overall plot in Panel 7 has $\theta \in \{1, 2, 3, 4, 5\}$ (2100 points in total), Panel 8 plots for weak network effects with $\theta = 1$ (420 points) and Panel 9 plots for strong network effects with $\theta \in \{4, 5\}$ (840 points). The points in Panel 8 are located below the 45-degree line, and in contrast, the points in Panel 9 are located above it, indicating that compared to NR, ER results in lower producer surplus when network effects are weak and higher producer surplus when network effects are strong. Thus Panels 5-9 are consistent with Result 3.

Figure 9b report additional welfare results from the robustness checks. Panel 1 compares consumer surplus at high switching costs with consumer surplus at low switching costs under NR. In this panel, each point corresponds to a $(\theta, v_0, M, shape)$ combination. For each point, the vertical coordinate is consumer surplus when $k = 3$ under NR, while the horizontal coordinate is consumer surplus when $k = 1$ under NR. The points in Panel 1 are located below the 45-degree line, indicating that under NR, an increase of switching cost k from 1 to 3 lowers consumer surplus.

Panel 2 is similar to Panel 1 but plots for ER rather than NR. In contrast to Panel 1, the points in Panel 2 are located on or near the 45-degree line, indicating that under ER, an increase of switching cost k from 1 to 3 leaves consumer surplus largely unchanged. Thus Panels 1-2 are consistent with Result 3.

Panels 3-6 are similar to Panels 1-2 but plot for producer surplus and total surplus instead. Panels 3-4 show that an increase of switching cost k from 1 to 3 has an ambiguous effect on producer surplus under NR, and tends to increase producer surplus under ER. Panels 5-6 show that an increase of switching cost k from 1 to 3 lowers total surplus under NR, but leaves total surplus largely unchanged under ER, consistent with Result 3.

In summary, while the large set of parameterizations we consider here result in a wide range of market outcomes in terms of firms' reimbursement choices, market size, market concentration, and welfare measures, those market outcomes continue to be consistent with Results 1-3 described above, illustrating those results' robustness to variations in parameter values.

8 Extension: Forward-Looking Consumers

In this section, we consider an extension in which consumers—in addition to firms—are forward-looking by introducing an additional parameter, the consumers' discount factor $\beta_c \in [0, 1)$, into the model. We examine the results as we vary β_c. The model with myopic consumers corresponds to $\beta_c = 0$. When $\beta_c > 0$, consumers are forward-looking and value not only their current-period utility but also their discounted future utilities. The other assumptions of the model remain unchanged, including the assumption that consumers are attentive only when their existing products die. The firms continue to be forward-looking with discount factor β. The details of this modified version of the model are presented in Appendix A2, which describes both consumers' and firms' Bellman equations as well as the equilibrium conditions for Markov perfect equilibrium of this infinite-horizon dynamic game with forward-looking agents on both sides of the market.

The results from this extension show that our earlier findings are robust and furthermore shed light on the effects of consumers' forward-looking behavior, as illustrated by Figure 10. The figure plots for $v_0 = -5$, $M = 20$, $\theta = 0.4$, $k \in \{0, 0.2, ..., 2\}$, and $\beta_c \in \{0, 0.1, 0.3, 0.5, 0.7, 0.9\}$. $\beta_c = 0$, the myopic case, is included for comparison purposes. In the figure, the left column of panels plot the market size (the two firms' combined installed base as a proportion of M), and the right column of panels plot the market concentration (the larger firm's installed base as a proportion of the two firms' combined installed base). We discuss three findings.

First, when consumers are myopic, increases in the switching cost k don't cause market size to shrink much (see $\beta_c = 0$ in Panels 1 and 3 of Figure 10), as consumers do not internalize the switching cost that they would incur in the future if they choose an inside good now and decide to switch to the other inside good later. This pattern changes when consumers are forward-looking under NR ($\beta_c > 0$ in Panel 1). In this case consumers take into consideration future switching costs, and as a result increases in k lower the attractiveness of the inside goods relative to the outside good, thereby increasing the outside good's market share and shrinking the market size. Under ER, however, firms are able to reimburse consumers' switching costs, and as a result increases in k continue to have little impact on market size even when consumers are forward-looking ($\beta_c > 0$ in Panel 3). Compared to NR, ER results in a higher market size except when $k = 0$, in which case the option to reimburse doesn't matter and the two regimes are the same. The difference in market size between NR and ER is particularly large when both β_c and k are large (Panel 5).

Second, the right-column of panels shows that our previous findings regarding market concentration when consumers are myopic continue to hold when consumers are forward-looking: switching costs reduce market concentration under NR (Panel 2) and increase market concentration under ER (Panel 4). Compared to NR, ER results in a higher market concentration (Panel 6) except when $k = 0$, in which case the two regimes are the same. The difference in market concentration between NR and ER is particularly large when both β_c and k are large.

Third, forward-looking consumers internalize future network benefits that they would enjoy if they choose one of the inside goods (recall that the products are durable and consumers make purchasing decisions only when their existing products die). Therefore, consumers' forward-looking behavior amplifies network effects and tends to lead to a tipping equilibrium, especially under ER.

Under NR (Panel 2), a tipping equilibrium occurs when β_c is 0.9 and k is less than or equal to 0.6; further increases in k change the equilibrium from tipping to splintered, a pattern that we saw previously in the case with myopic consumers and larger network effects (see Panel 1 of Figure 6). Under ER (Panel 4), when k is increased, there is a splintered equilibrium throughout for small values of β_c ($\beta_c = 0$ or 0.1) and there is a tipping equilibrium throughout when $\beta_c = 0.9$. For intermediate values of β_c, there is a splintered equilibrium at low switching costs, which is then changed to a tipping equilibrium at high switching costs (a pattern that we saw previously in Panel 3 of Figure 6 when $\theta \in \{1, 1.5\}$).

Note that if consumers are attentive in every period, then when $k = 0$, whether consumers are forward-looking or myopic won't make a difference, because when there are no switching costs, each consumer can costlessly re-optimize in every period, thus for forward-looking consumers, their dynamic decision problem boils down to a period-by-period optimization problem that has no inter-temporal linkages and is no different from the decision problem facing myopic consumers. However, in our model, consumers make purchasing decisions infrequently (they are attentive only when their existing products die), and so consumers' forward-looking behavior (indexed by β_c) makes a difference even when $k = 0$, as can be seen most clearly in Panels 2 and 4 of Figure 10.

Additional results (not shown) show that our previous findings continue to hold in the new runs with forward-looking consumers: the larger firm reimburses a larger proportion of the switching cost than the smaller firm does, and firms' option to reimburse consumers' switching costs increases consumer surplus and total surplus while its effect on producer surplus is ambiguous.

9 Concluding Remarks

This paper develops a dynamic duopoly model of price competition with switching costs and network effects, where firms have the ability to reimburse consumers' switching costs. We use the full dynamic model and a simplified static version to investigate firms' pricing and reimbursement strategies and how competition and welfare are affected by these strategies. This setup yields several interesting results.

When firms cannot reimburse consumers' switching costs, an increase in the switching cost causes a transition from a tipping equilibrium in which one firm dominates the market to a splintered equilibrium in which the firms split the market about evenly. Introducing the ability to reimburse switching costs benefits the larger firm and facilitates market tipping and winner-takes-most. A consequence is that the economy remains in a tipping equilibrium even at high switching costs. Even though the market is more concentrated, consumer welfare is higher than the case in which switching costs cannot be reimbursed. This finding has useful antitrust and consumer welfare policy implications, illustrating that in such industries, policy analysis relying heavily on the market concentration may be problematic.

In addition, we find that compared to the case without reimbursement, firms' option to reimburse switching costs increases consumer surplus and total surplus, and increases producer surplus when network effects are strong. Switching costs decrease consumer surplus if firms do not have the option to reimburse switching costs, but leave consumer surplus largely unchanged if firms have that option. These results demonstrate that the welfare outcome in the market critically depends on whether firms have the option to reimburse consumers' switching costs.

Lastly, we reiterate the caveat that in this paper, we have abstracted from some issues such as endogenous switching costs and endogenous product quality. Nonetheless, an unambiguous finding that emerges from our analysis is the important role that switching cost reimbursement plays in determining the market outcome including market concentration and consumer welfare, as well as the importance of taking such reimbursement into account when designing public policies. The model and results we have presented in this paper can hopefully serve as one benchmark and aid future research in this and related areas.

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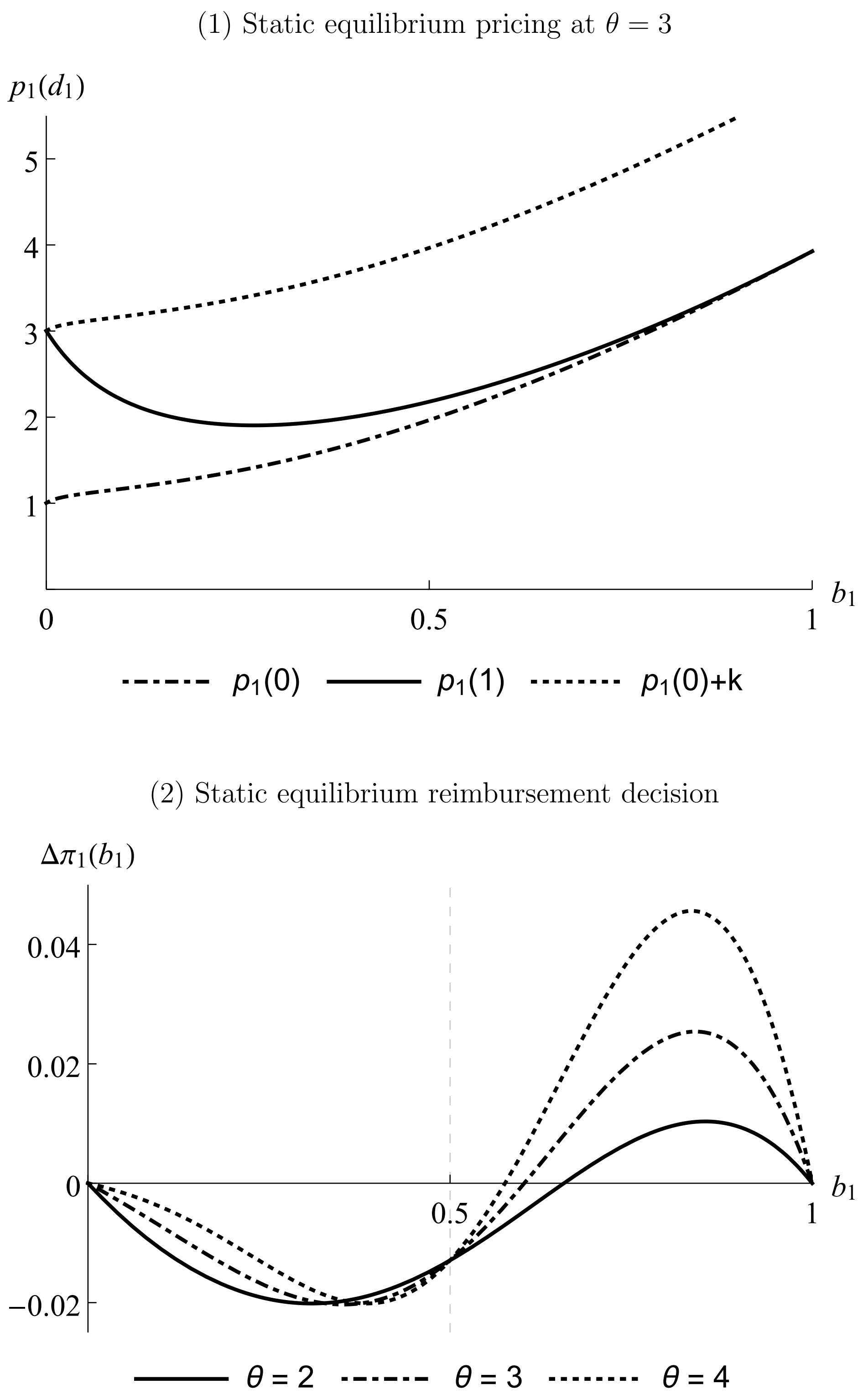

Figure 1: Simplified static equilibrium pricing and reimbursement decisions. $k = 2$.

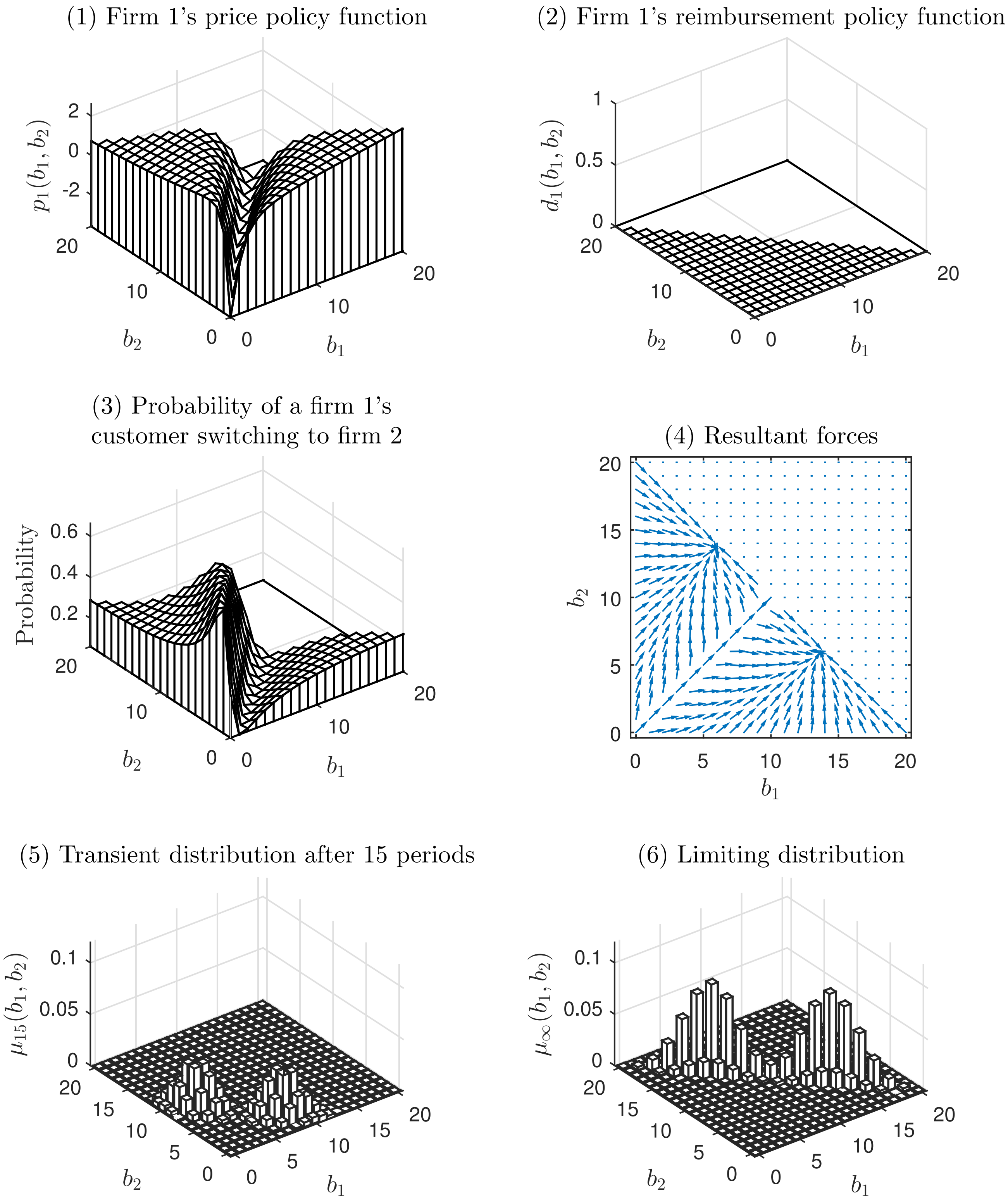

Figure 2. No reimbursement (NR): Tipping equilibrium at low switching cost. $v_0 = -5$, $M = 20$, $\theta = 2$, $k = 1$.

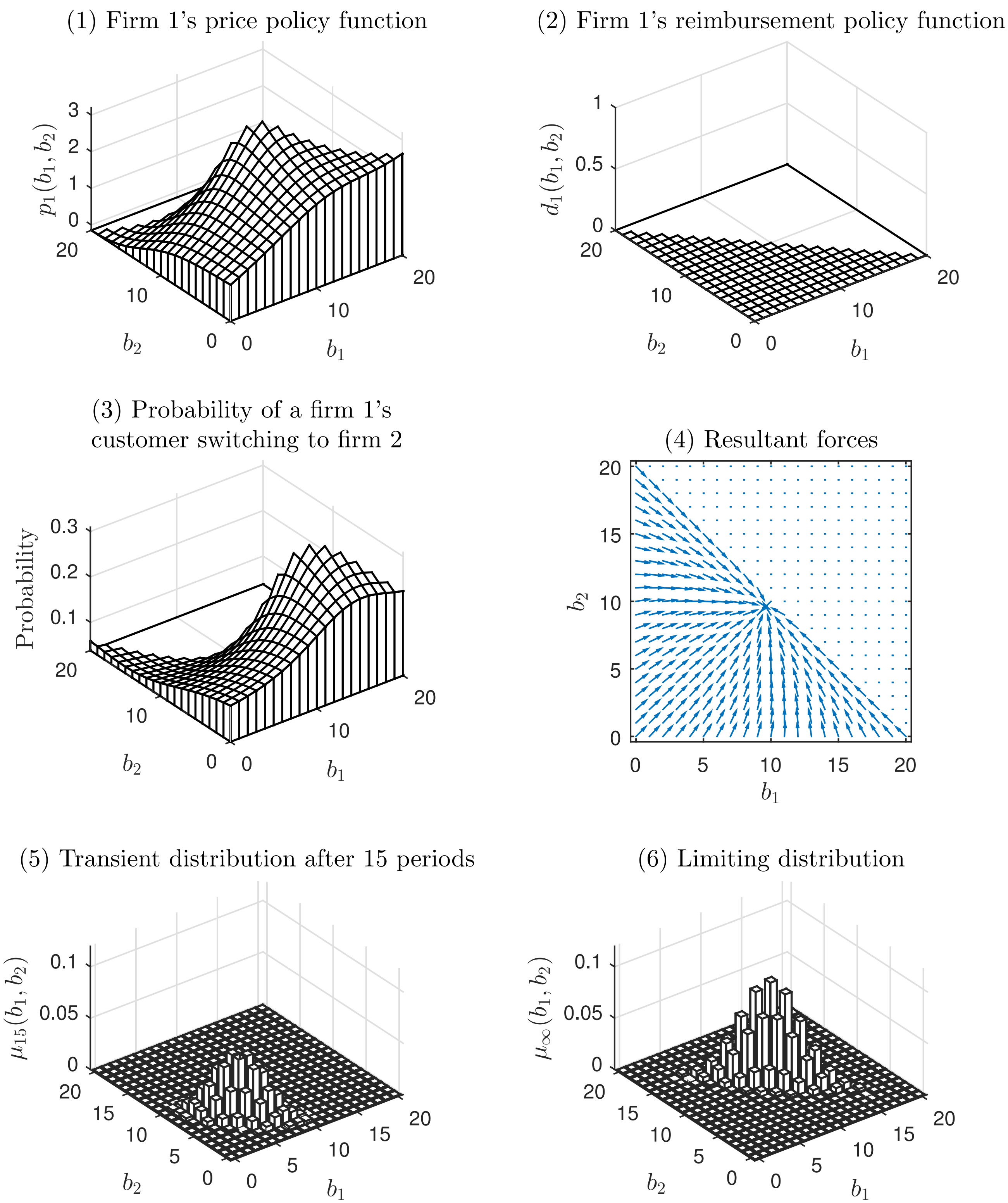

Figure 3. No reimbursement (NR): Splintered equilibrium at high switching cost. $v_0 = -5$, $M = 20$, $\theta = 2$, $k = 2$.

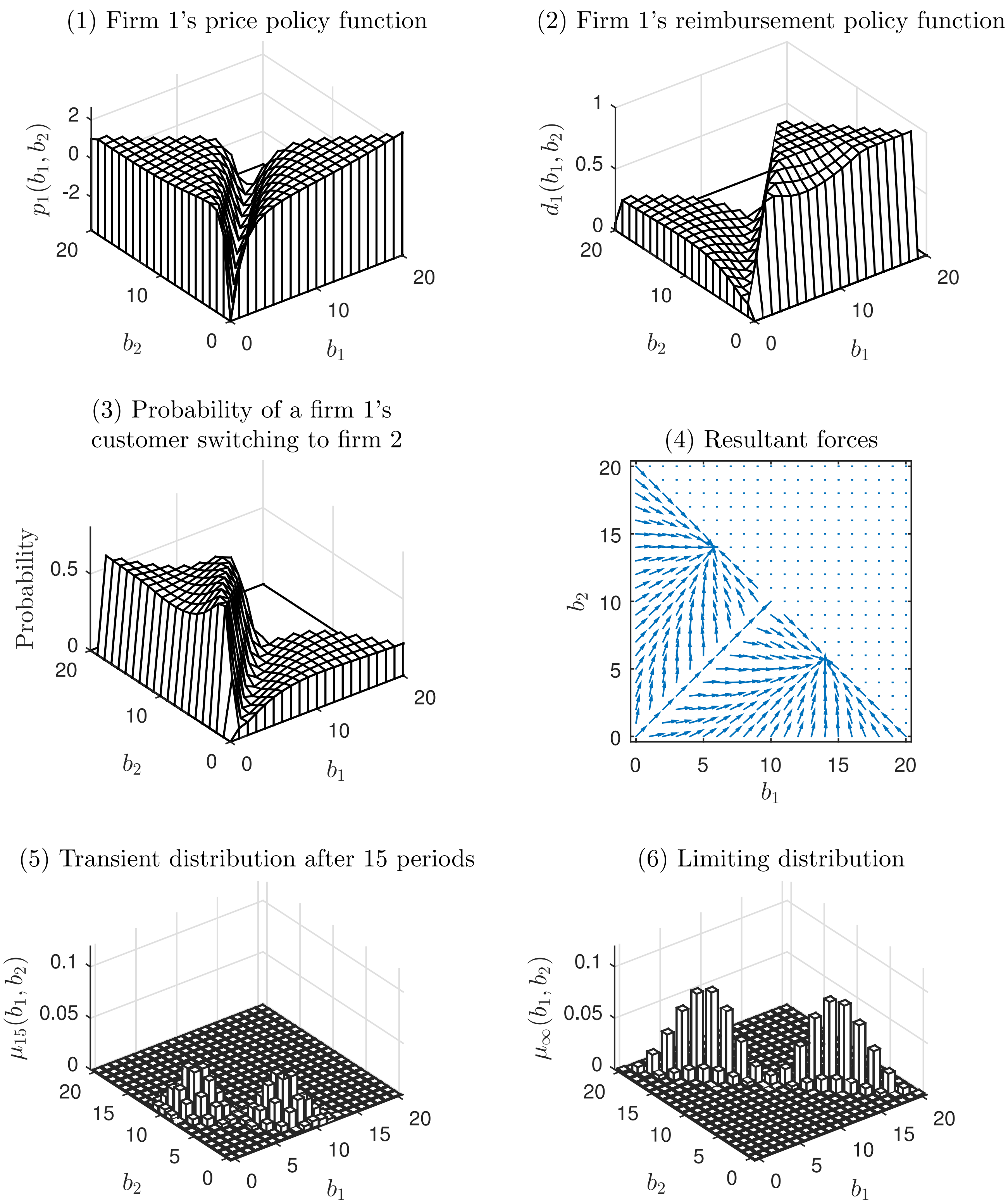

Figure 4. Endogenous reimbursement (ER): Tipping equilibrium at low switching cost. $v_0 = -5$, $M = 20$, $\theta = 2$, $k = 1$.

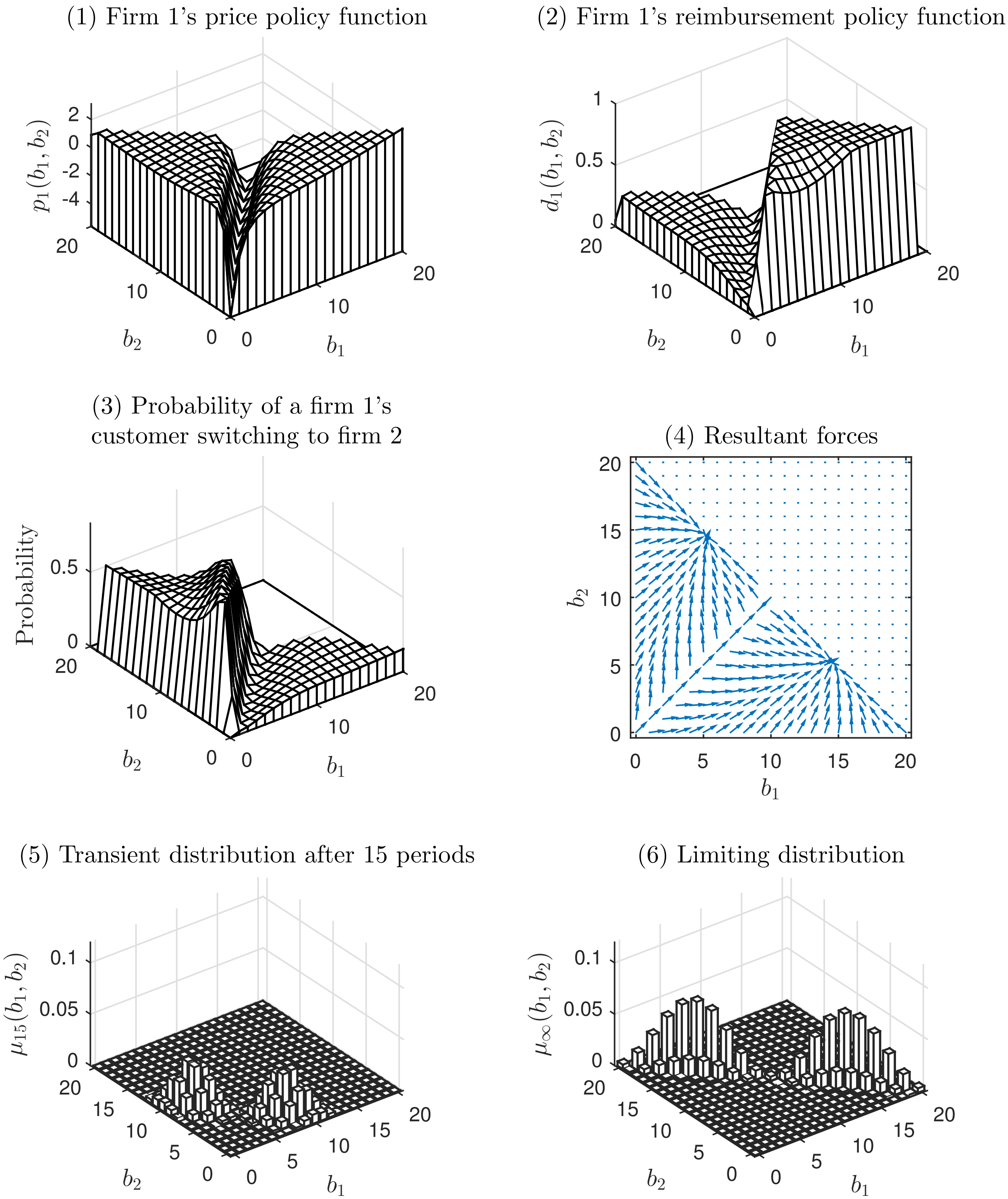

Figure 5. Endogenous reimbursement (ER): Tipping equilibrium at high switching cost. $v_0 = -5$, $M = 20$, $\theta = 2$, $k = 2$.

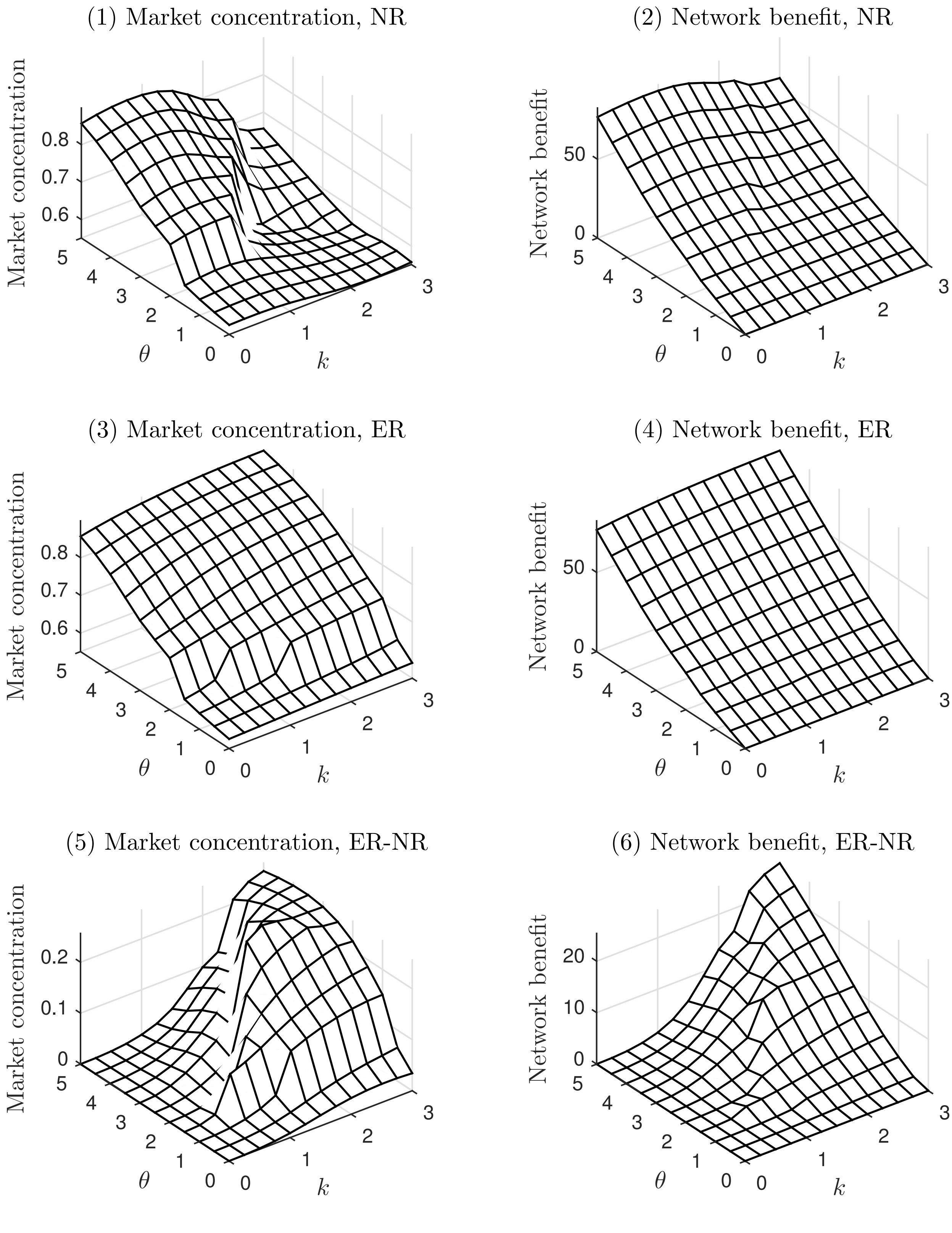

Figure 6. Market concentration and network benefit. $v_0 = -5$, $M = 20$.
NR: No reimbursement. ER: Endogenous reimbursement.

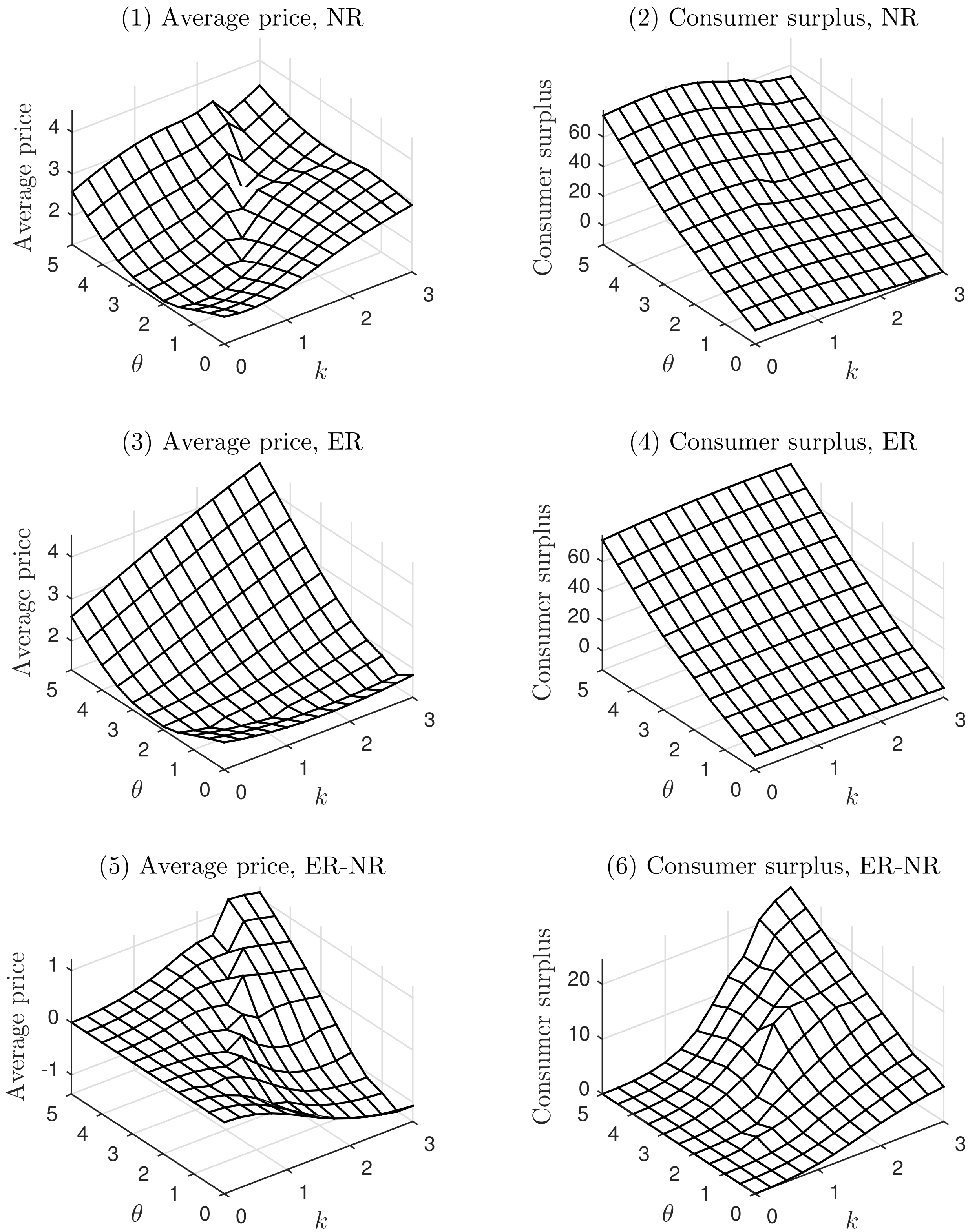

Figure 7. Average price and consumer surplus. $v_0 = -5$, $M = 20$.
NR: No reimbursement. ER: Endogenous reimbursement.

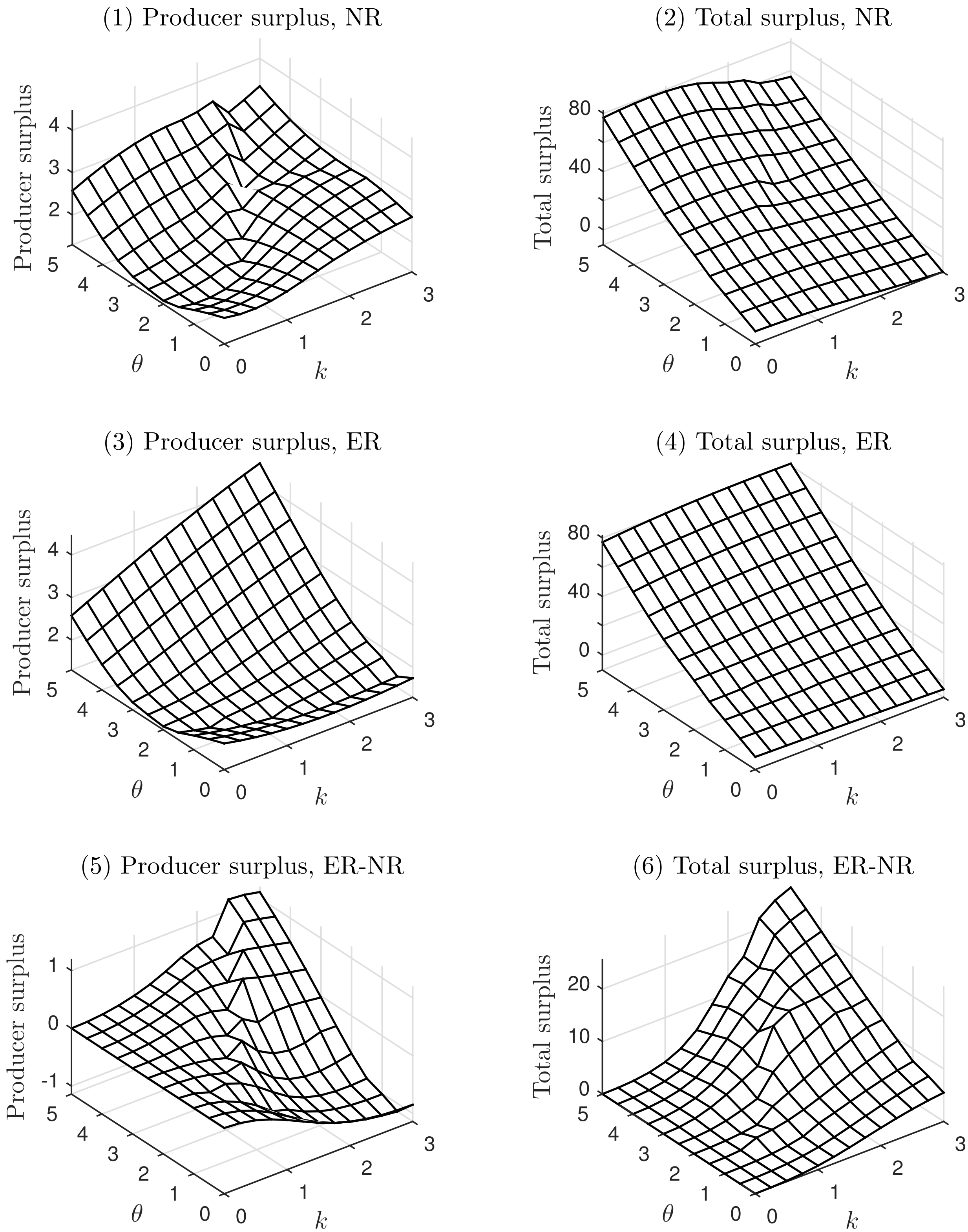

Figure 8. Producer surplus and total surplus. $v_0 = -5$, $M = 20$.
NR: No reimbursement. ER: Endogenous reimbursement.

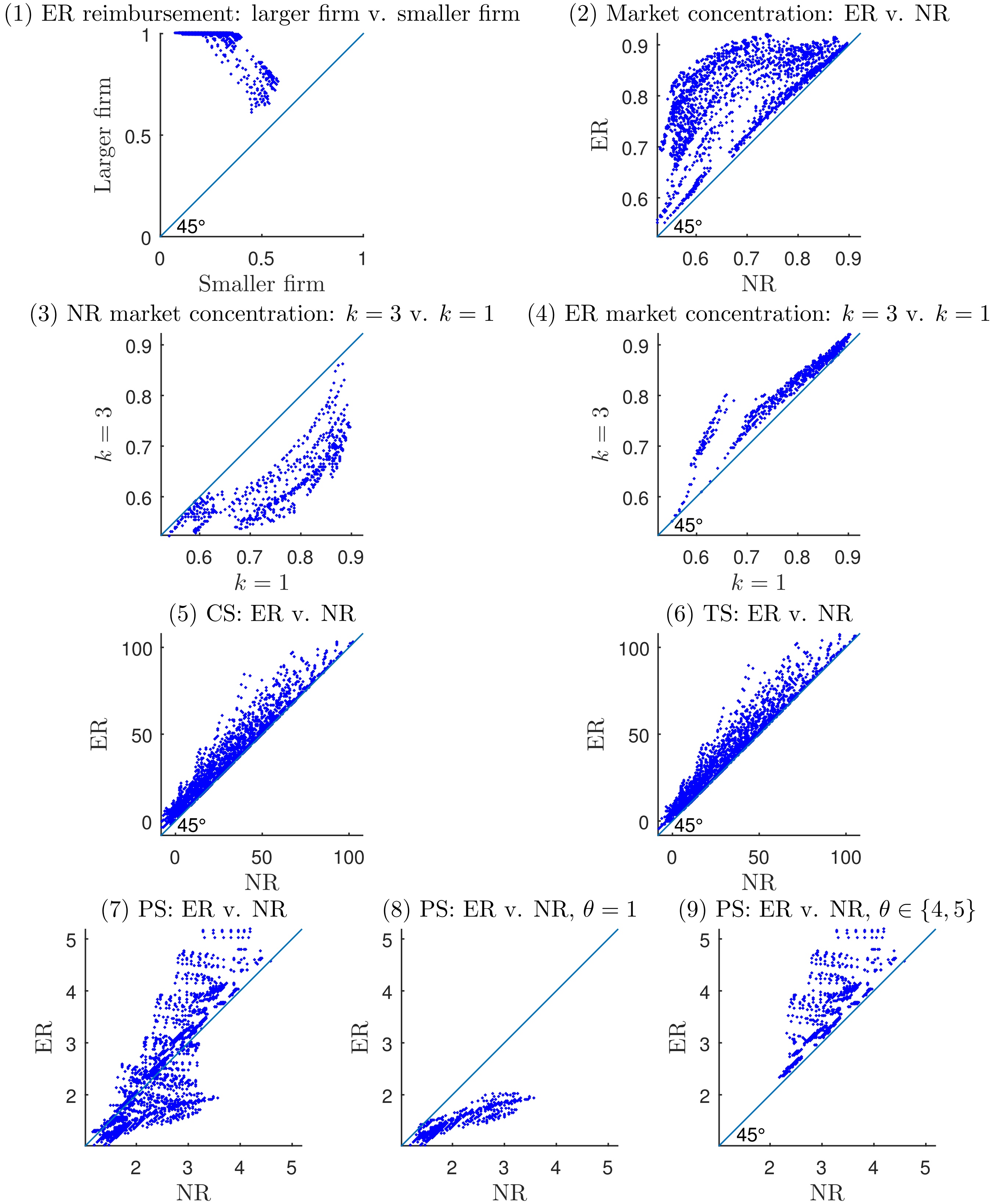

Figure 9a. Scatter plots of robustness checks results. Each point corresponds to a parameterization. $\theta \in \{1, 2, 3, 4, 5\}$, $k \in \{1, 2, 3\}$, $v_0 \in \{-7, -6, -5, -4, -3\}$, $M \in \{12, 14, ..., 24\}$, shape of network effect function $\in$ {Linear, Convex, Concave, S-shaped}.

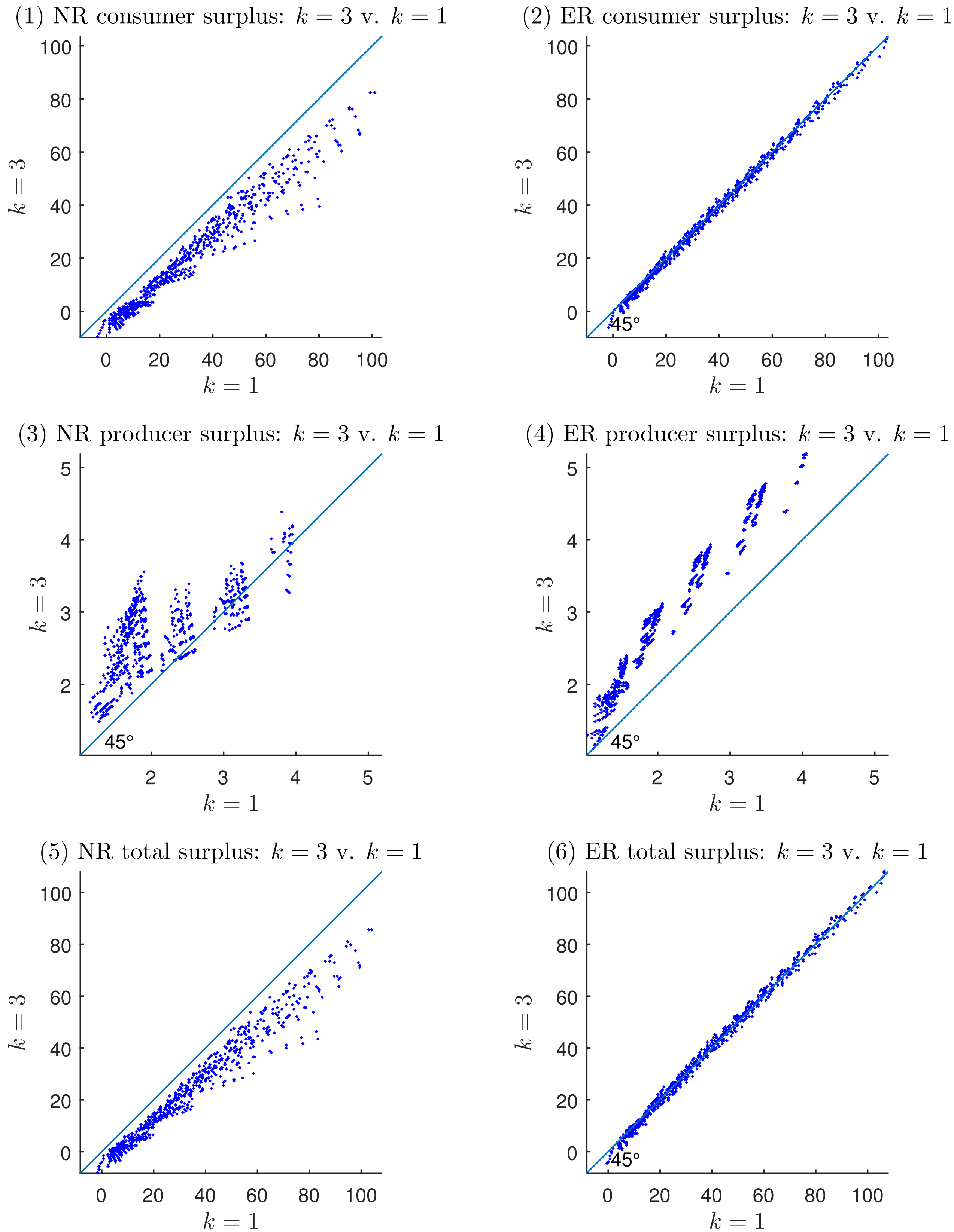

Figure 9b. Scatter plots of robustness checks results. Each point corresponds to a parameterization. $\theta \in \{1, 2, 3, 4, 5\}$, $k \in \{1, 2, 3\}$, $v_0 \in \{-7, -6, -5, -4, -3\}$, $M \in \{12, 14, ..., 24\}$, shape of network effect function $\in$ {Linear, Convex, Concave, S-shaped}.

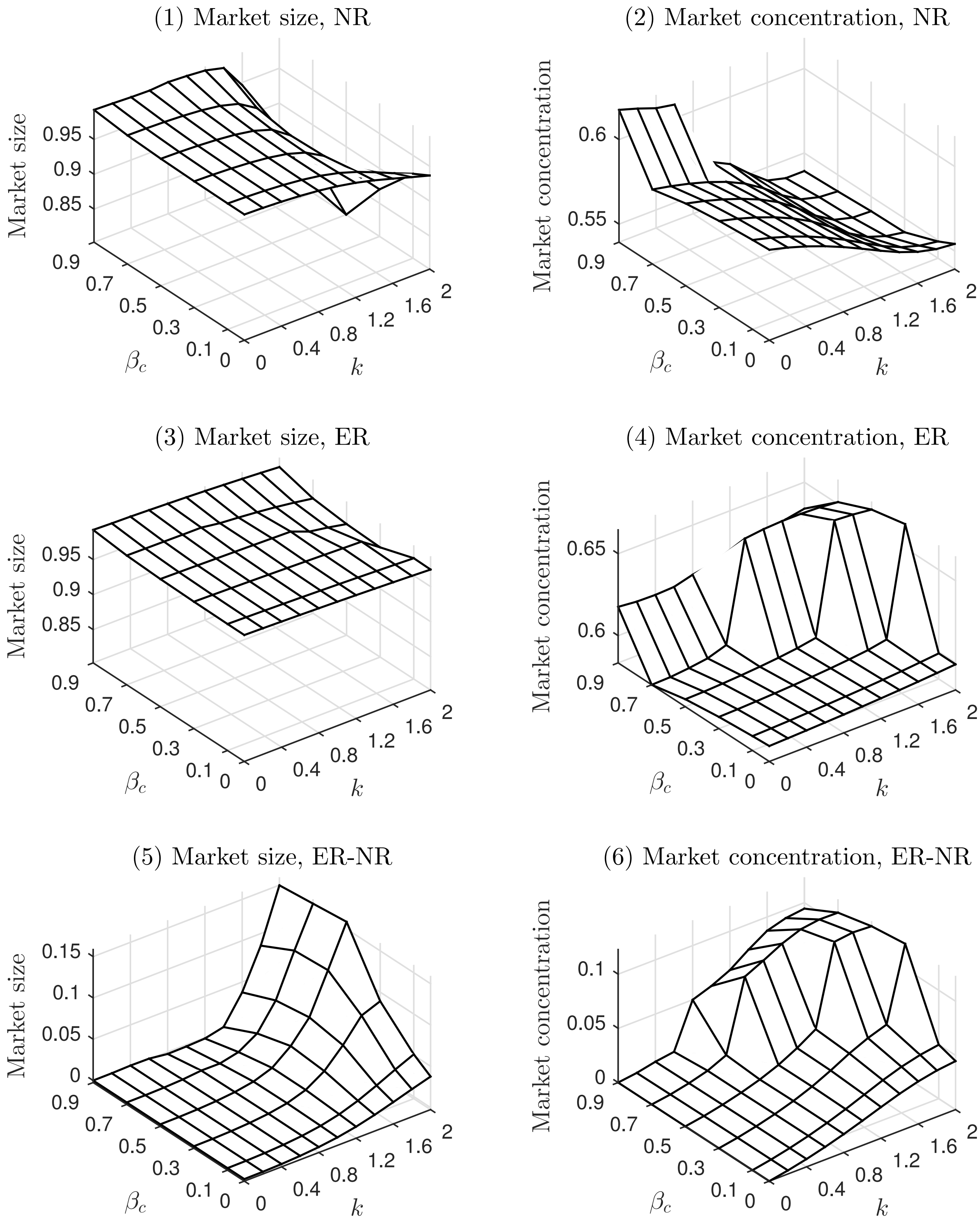

Figure 10. Market size and market concentration: forward-looking consumers. $v_0 = -5$, $M = 20$, $\theta = 0.4$. NR: No reimbursement. ER: Endogenous reimbursement.

Online Appendix

A1 Proofs of Propositions

Before proving the results, it is useful to display the consumers' choice probabilities of purchasing from firm 1, as we will be applying various manipulations to them. From Eq. (3), they are

$$\phi_{11}(b_1, d_1, p_1) = \frac{e^{-p_1+\theta b_1}}{e^{-p_1+\theta b_1} + e^{-k+\theta b_2}} = \frac{e^{-p_1}}{e^{-p_1} + e^{-k+\theta(1-2b_1)}} = \frac{1}{1 + e^{p_1-k+\theta(1-2b_1)}}$$
$$\phi_{21}(b_1, d_1, p_1) = \frac{e^{-p_1-(1-d_1)k+\theta b_1}}{e^{-p_1-(1-d)k+\theta b_1} + e^{\theta b_2}} = \frac{e^{-p_1-(1-d_1)k}}{e^{-p_1-(1-d_1)k} + e^{\theta(1-2b_1)}} = \frac{1}{1 + e^{p_1+(1-d_1)k+\theta(1-2b_1)}}.$$

The equalities follow from setting $b_2 = 1 - b_1$ and algebraic manipulations.

Also note that firm 1's profit maximizing price is the solution to

$$p_1(d_1) = \frac{b_1\phi_{11}(b_1,d_1,p_1(d_1))+(1-b_1)\phi_{21}(b_1,d_1,p_1(d_1))[1+d_1k(1-\phi_{21}(b_1,d_1,p_1(d_1)))]}{b_1\phi_{11}(b_1,d_1,p_1(d_1))(1-\phi_{11}(b_1,d_1,p_1(d_1)))+(1-b_1)\phi_{21}(b_1,d_1,p_1(d_1))(1-\phi_{21}(b_1,d_1,p_1(d_1)))}, \tag{A1}$$

which is obtained by differentiating firm 1's profits (given in Eq. (6)) with respect to p_1, setting the resulting derivative equal to zero, and solving for p_1. We also utilize $\partial\phi_{rj}(b_1, p_1, d_1)/\partial p_1 = -\phi_{rj}(b_1, p_1, d_1)(1 - \phi_{rj}(b_1, p_1, d_1))$, which follows from Eq. (3).

Proof of Proposition 1. Note that Proposition 1 is equivalent to $p_1(0) < p_1(1) < p_1(0) + k$, which is the statement we prove. Given reimbursement decision d_1, firm 1's maximum profits are

$$b_1p_1(d_1)\phi_{11}(b_1, d_1, p_1(d_1)) + (1 - b_1)(p_1(d_1) - d_1k)\phi_{21}(b_1, d_1, p_1(d_1)), \tag{A2}$$

where $p_1(d_1)$ is given by Eq. (A1). We first show that at $b_1 = 0$, $p_1(1) = p_1(0) + k$. We then show that at $b_1 = 1$, $p_1(1) = p_1(0)$. Lastly, we show the intermediate result, that for $b \in (0, 1)$, $p_1(1) \in (p_1(0), p_1(0) + k)$.

At $b_1 = 0$, (A2) becomes $(p_1(d_1) - d_1k)\phi_{21}(0, d_1, p_1(d_1))$, where substituting the expression of $\phi_{21}(0, d_1, p_1(d_1))$ into the maximum profits yields

$$\frac{p_1(d_1) - d_1k}{1 + e^{p_1(d_1)+(1-d_1)k+\theta}}.$$

Note that $\Delta\pi_1(0) = 0$ if

$$\frac{p_1(1) - k}{1 + e^{p_1(1)+\theta}} = \frac{p_1(0)}{1 + e^{p_1(0)+k+\theta}},$$

which is true if and only if $p_1(1) = p_1(0) + k$. Hence, at $b_1 = 0$, the firm can guarantee itself identical profits regardless of its reimbursement decision. If $\Delta\pi_1(0) \neq 0$, then it must be that either $p_1(1)$ or $p_1(0)$ is not a maximizer. If $\Delta\pi_1(0) < 0$, then $p_1(1) \neq p_1(0) + k$ cannot be a maximizer, as firm 1 can increases its profits given $d_1 = 1$ by setting $p_1 = p_1(0) + k$ and if $\Delta\pi_1(0) > 0$, then $p_1(0)$ cannot be a maximizer, as firm 1 can increase its profits given $d_1 = 0$ by setting $p_1 = p_1(1) - k$. Therefore, at $b_1 = 0$, $p_1(1) = p_1(0) + k$.

At $b_1 = 1$, (A2) becomes $p_1(d_1)\phi_{11}(1, d_1, p_1(d_1))$. Substituting this value into the maximum profits yields

$$\frac{p_1(d_1)}{1+e^{p_1(d_1)-k-\theta}}.$$

Note that $\Delta\pi_1(1)=0$ if

$$\frac{p_1(1)}{1+e^{p_1(1)-k-\theta}}=\frac{p_1(0)}{1+e^{p_1(0)-k-\theta}},$$

which is true if and only if $p_1(1)=p_1(0)$. By an analogous argument to the above, there can be no maximizers in which $p_1(1)\neq p_1(0)$, so $p_1(1)=p_1(0)$ at $b_1=1$.

Next, we establish that $p_1(1)>p_1(0)$ for all $b_1<1$. To do so, note the respective first-order conditions for the profit-maximizing price when $d_1=1$ and $d_1=0$:

$$\begin{aligned}
d_1=0: \quad & b_1\phi_{11}(b_1,1,p_1)+(1-b_1)\phi_{21}(b_1,1,p_1)-b_1p_1\phi_{11}(b_1,1,p_1)[1-\phi_{11}(b_1,1,p_1)]\\
&\qquad -(1-b_1)(p_1-k)\phi_{21}(b_1,1,p_1)[1-\phi_{21}(b_1,1,p_1)]=0,\\
d_1=1: \quad & b_1\phi_{11}(b_1,0,p_1)+(1-b_1)\phi_{21}(b_1,0,p_1)-b_1p_1\phi_{11}(b_1,0,p_1)[1-\phi_{11}(b_1,0,p_1)]\\
&\qquad -(1-b_1)p_1\phi_{21}(b_1,0,p_1)[1-\phi_{21}(b_1,0,p_1)]=0
\end{aligned}$$

Given that $p_1(1)>p_1(0)$ at $b_1=0$ and $p_1(1)=p_1(0)$ at $b_1=1$, for $p_1(1)\leq p_1(0)$, both of the above first-order conditions must be satisfied at the same price $\hat{p}$ for at least one $b\in(0,1)$. Combining the two first-order conditions yields

$$\begin{aligned}
&b_1\phi_{11}(b_1,1,\hat{p})+(1-b_1)\phi_{21}(b_1,1,\hat{p})-b_1\hat{p}\phi_{11}(b_1,1,\hat{p})[1-\phi_{11}(b_1,1,\hat{p})]\\
&\quad -(1-b_1)(\hat{p}-k)\phi_{21}(b_1,1,\hat{p})[1-\phi_{21}(b_1,1,\hat{p})]=b_1\phi_{11}(b_1,0,\hat{p})+(1-b_1)\phi_{21}(b_1,0,\hat{p})\\
&\qquad -b_1\hat{p}\phi_{11}(b_1,0,\hat{p})[1-\phi_{11}(b_1,0,\hat{p})]-(1-b_1)\hat{p}\phi_{21}(b_1,0,\hat{p})[1-\phi_{21}(b_1,0,\hat{p})], \quad \text{(A3)}
\end{aligned}$$

where $\phi_{11}(b_1,0,\hat{p})=\phi_{11}(b_1,1,\hat{p})$ and $\phi_{21}(b_1,0,\hat{p})<\phi_{21}(b_1,1,\hat{p})$ for all $b_1\in(0,1)$ and $\hat{p}\geq 0$. Hence, Eq. (A3) simplifies to

$$\phi_{21}(b_1,1,\hat{p})-(\hat{p}-k)\phi_{21}(b_1,1,\hat{p})[1-\phi_{21}(b_1,1,\hat{p})]=\phi_{21}(b_1,0,\hat{p})-\hat{p}\phi_{21}(b_1,0,\hat{p})[1-\phi_{21}(b_1,0,\hat{p})],$$

which is false for all $b_1<1$. Therefore, $p_1(1)>p_1(0)$ for all $b_1\in[0,1)$.

Lastly, we establish that $p_1(1)<p_1(0)+k$ for all $b_1>0$. We follow a similar approach to the above. For $p_1(1)>p_1(0)+k$, there must exist a $b_1>0$ such that for maximizer $p_1(0)$, $p_1(1)=p_1(0)+k$. That is,

$$\begin{aligned}
d_1=1: \quad & b_1\phi_{11}(b_1,1,p_1(0)+k)+(1-b_1)\phi_{21}(b_1,1,p_1(0)+k))\\
&\quad -b_1(p_1(0)+k)\phi_{11}(b_1,1,p_1(0)+k)[1-\phi_{11}(b_1,1,p_1(0)+k)]\\
&\quad -(1-b_1)p_1(0)\phi_{21}(b_1,1,p_1(0)+k)[1-\phi_{21}(b_1,1,p_1(0)+k)]=0,\\
d_1=0: \quad & b_1\phi_{11}(b_1,0,p_1(0))+(1-b_1)\phi_{21}(b_1,0,p_1(0))\\
&\qquad -b_1p_1(0)\phi_{11}(b_1,0,p_1(0))[1-\phi_{11}(b_1,0,p_1(0))]\\
&\qquad -(1-b_1)p_1(0)\phi_{21}(b_1,0,p_1(0))[1-\phi_{21}(b_1,0,p_1(0))]=0.
\end{aligned}$$

Note that $\phi_{21}(b_1,1,p_1(0)+k)=\phi_{21}(b_1,1,p_1(0)+k)$ while $\phi_{11}(b_1,1,p_1(0)+k)<\phi_{11}(b_1,1,p_1(0))$.

Using these properties and setting the above to first-order conditions equal to each other yields

$$\begin{aligned} b_1\phi_{11}(b_1,1,p_1(0)+k) - b_1(p_1(0)+k)\phi_{11}(b_1,1,p_1(0)+k)[1-\phi_{11}(b_1,1,p_1(0)+k)] \\ = b_1\phi_{11}(b_1,0,p_1(0)) - b_1p_1(0)\phi_{11}(b_1,0,p_1(0))[1-\phi_{11}(b_1,0,p_1(0))], \end{aligned}$$

which is false for all $b_1 > 0$. Therefore, $p_1(1) < p_1(0) + k$ for all $b_1 \in (0,1]$, completing the proof. ■

Proof of Proposition 2. In proving Proposition 1, we showed that $\Delta\pi_1(0) = \Delta\pi_1(1) = 0$. To prove the existence of cutoffs $0 < \underline{b} \leq \overline{b} < 1$, we show that $\frac{d\Delta\pi_1(b_1)}{db_1} < 0$ as both $b_1 \to 0$ and $b_1 \to 1$. The result then follows from the continuity of $\Delta\pi_1(b_1)$ on $b_1 \in [0,1]$, which follows from the continuity of $p_1(d_1)$ and $\phi_{rj}(b_1, d_1, p_1(d_1))$. With some algebraic manipulation,

$$\begin{aligned} \frac{d\Delta\pi_1(b_1)}{db_1} &= p_1(1)\phi_{11}(b_1,1,p_1(1)) - (p_1(1)-k)\phi_{21}(b_1,1,p_1(1)) - p_1(0)\phi_{11}(b_1,0,p_1(0)) \\ &\quad + p_1(0)\phi_{21}(b_1,0,p_1(0)) + 2\theta[b_1\phi_{11}(b_1,1,p_1(1)) + (1-b_1)\phi_{21}(b_1,1,p_1(1))] \\ &\quad - 2\theta[b_1\phi_{11}(b_1,0,p_1(0)) + (1-b_1)\phi_{21}(b_1,0,p_1(0))]. \end{aligned} \tag{A4}$$

Taking the limit of Eq. (A4) as $b_1 \to 0$ yields

$$\begin{aligned} \frac{d\Delta\pi_1(0)}{db_1} &= p_1(1)\phi_{11}(0,1,p_1(1)) - (p_1(1)-k)\phi_{21}(0,1,p_1(1)) - p_1(0)\phi_{11}(0,0,p_1(0)) \\ &\quad + p_1(0)\phi_{21}(0,0,p_1(0)) + 2\theta[\phi_{21}(0,1,p_1(1)) - \phi_{21}(0,0,p_1(0))]. \end{aligned} \tag{A5}$$

By the proof of Proposition 1, at $b_1 = 0$, $p_1(1) = p_1(0) + k$, which implies that $\phi_{21}(0,1,p_1(1)) = \phi_{21}(0,1,p_1(0)+k) = \phi_{21}(0,0,p_1(0))$. Hence, Eq. (A5) simplifies to

$$\begin{aligned} \frac{d\Delta\pi_1(0)}{db_1} &= p_1(1)\phi_{11}(0,1,p_1(1)) - (p_1(1)-k)\phi_{21}(0,1,p_1(1)) - p_1(0)\phi_{11}(0,0,p_1(0)) + p_1(0)\phi_{21}(0,0,p_1(0)) \\ &= (p_1(0)+k)\phi_{11}(0,1,p_1(1)) - p_1(0)\phi_{21}(0,1,p_1(1)) - p_1(0)\phi_{11}(0,0,p_1(0)) + p_1(0)\phi_{21}(0,0,p_1(0)) \\ &= (p_1(0)+k)\phi_{11}(0,1,p_1(1)) - p_1(0)\phi_{11}(0,0,p_1(0)). \end{aligned}$$

Therefore, $\frac{d\Delta\pi_1(0)}{db_1} < 0 \Leftrightarrow (p_1(0)+k)\phi_{11}(0,1,p_1(1)) < p_1(0)\phi_{11}(0,0,p_1(0))$. Plugging in the values for $\phi_{11}(0,1,p_1(1))$ and $\phi_{11}(0,0,p_1(0))$ using Eq. (3) and $p_1(1) = p_1(0) + k$ yields

$$\begin{aligned} (p_1(0)-k)\frac{e^{-p_1(0)-k}}{e^{-p_1(0)-k} + e^{-k+\theta}} &< p_1(0)\frac{e^{-p_1(0)}}{e^{-p_1(0)} + e^{-k+\theta}} \\ \frac{(p_1(0)-k)e^{-k}}{e^{-p_1(0)-k} + e^{-k+\theta}} &< \frac{p_1(0)}{e^{-p_1(0)} + e^{-k+\theta}}. \end{aligned}$$

Cross-multiplying and simplifying yields

$$k\left(e^{-p_1(0)-\theta} + e^{-k}\right) < p_1(0)\left(1 - e^{-k}\right). \tag{A6}$$

By Eq. (A1), $p_1(0)$ is given by the solution to

$$\begin{aligned} p_1(0) &= \frac{1}{1 - \phi_{11}(0,0,p_1(d_1))} \\ &= \frac{e^{-p_1(0)} + e^{-k+\theta}}{e^{-k+\theta}}. \end{aligned}$$

Thus, as $\theta \to \infty$, $p_1(0) \to 1$. Taking the limit of both sides of (A6) as $\theta \to \infty$ yields

$$\begin{aligned} ke^{-k} &< 1 - e^{-k} \\ (1+k)e^{-k} &< 1, \end{aligned}$$

which is true for all $k > 0$. Thus, for θ sufficiently large, $\frac{d\Delta\pi_1(0)}{db_1} < 0$.

Next, we show that $\frac{d\Delta\pi_1(b_1)}{db_1} < 0$ as $b_1 \to 1$. Taking the limit of Eq. (A4) as $b_1 \to 1$ yields

$$\begin{aligned} \frac{d\Delta\pi_1(1)}{db_1} = p_1(1)\phi_{11}(1,1,p_1(1)) - (p_1(1)-k)\phi_{21}(1,1,p_1(1)) - p_1(0)\phi_{11}(1,0,p_1(0)) \\ + p_1(0)\phi_{21}(1,0,p_1(0)) + 2\theta[\phi_{11}(1,1,p_1(1)) - \phi_{11}(1,0,p_1(0))]. \end{aligned} \tag{A7}$$

By the proof of Proposition 1, at $b_1 = 1$, $p_1(1) = p_1(0)$, which implies that $\phi_{11}(0,1,p_1(1)) = \phi_{11}(0,1,p_1(0)) = \phi_{11}(0,0,p_1(0))$. Hence, Eq. (A7) simplifies to

$$\begin{aligned} \frac{d\Delta\pi_1(1)}{db_1} &= p_1(1)\phi_{11}(1,1,p_1(1)) - (p_1(1)-k)\phi_{21}(1,1,p_1(1)) - p_1(0)\phi_{11}(1,0,p_1(0)) + p(0)\phi_{21}(1,0,p_1(0)) \\ &= p_1(0)\phi_{11}(1,1,p_1(0)) - (p_1(0)-k)\phi_{21}(1,1,p_1(0)) - p_1(0)\phi_{11}(1,0,p_1(0)) + p_1(0)\phi_{21}(1,0,p_1(0)) \\ &= -(p_1(0)-k)\phi_{21}(1,1,p_1(0)) + p_1(0)\phi_{21}(1,0,p_1(0)). \end{aligned}$$

Therefore, $\frac{d\Delta\pi_1(1)}{db_1} < 0 \Leftrightarrow -(p_1(0)-k)\phi_{21}(1,1,p_1(0)) + p_1(0)\phi_{21}(1,0,p_1(0)) < 0$. Rearranging yields

$$p_1(0)[\phi_{21}(1,1,p_1(0)) - \phi_{21}(1,0,p_1(0))] > k\phi_{21}(1,1,p_1(0)). \tag{A8}$$

By Eq. (3),

$$\phi_{21}(1,1,p_1(0)) = \frac{e^{-p_1(0)}}{e^{-p_1(0)} + e^{-\theta}} > \frac{e^{-p_1(0)-k}}{e^{-p_1(0)-k} + e^{-\theta}} = \phi_{21}(1,0,p_1(0)).$$

Thus, both the left-hand and right-hand side of (A8) are positive. By inspection of the above, as $\theta \to \infty$, $\phi_{21}(1,1,p_1(0)) \to 1$, $\phi_{21}(1,0,p_1(0)) \to 1$, and $\phi_{21}(1,1,p_1(0)) - \phi_{21}(1,0,p_1(0)) \to 0$. However, note that at $b_1 = 1$, $p_1(0)$ is given by the solution to

$$1 - p_1(0)[1 - \phi_{11}(1,0,p_1(0))] = 0.$$

By the implicit function theorem,

$$\frac{dp_1(0)}{d\theta} = \frac{p_1(0)\phi_{11}(1,0,p_1(0))}{1 + p_1(0)\phi_{11}(1,0,p_1(0))} > 0.$$

Further differentiating the above with respect to θ yields

$$\frac{d^2p_1(0)}{d\theta^2} = \frac{\left[\frac{dp_1(0)}{d\theta}\phi_{11}(1,0,p_1(0)) + p_1(0)\phi_{11}(1,0,p_1(0))(1-\phi_{11}(1,0,p_1(0)))\right]}{(1 + p_1(0)\phi_{11}(1,0,p_1(0)))^2}.$$

Given that $\frac{dp_1(0)}{d\theta} > 0$, $\frac{d^2p_1(0)}{d\theta^2} > 0$ and as $\theta \to \infty$, $p_1(0) \to \infty$. Furthermore, this implies that

$$\lim_{\theta\to\infty} p_1(0)[\phi_{21}(1,1,p_1(0)) - \phi_{21}(1,0,p_1(0))] \to \infty.$$

Hence, the left-hand side of (A8) tends to ∞ as $\theta \to \infty$ while the right-hand side is bounded above by k. Thus, (A8) is satisfied and $\frac{d\Delta\pi_1(1)}{db_1} < 0$ for θ sufficiently large.

We have shown that at $b_1 \to 0$ and $b_1 \to 1$, $\frac{d\Delta\pi_1(b_1)}{db_1} < 0$. Combining this with $\Delta\pi_1(0) = \Delta\pi_1(1) = 0$ and the continuity of $\Delta\pi_1(b_1)$ on $[0, 1]$ implies the existence of cutoff values $0 < \underline{b} \leq \overline{b} < 1$ such that $\Delta\pi_1(b_1) \leq 0$ for $b_1 \leq \underline{b}$ and $\Delta\pi_1(b_1) \geq 0$ for $b_1 \geq \overline{b}$. ■

A2 A Model with Forward-Looking Consumers

In this appendix, we modify the main model in Section 3 to incorporate forward-looking consumers. In what follows, we will use $i \in \{1, 2, ..., M\}$ to index consumers, and use $j \in \{0, 1, 2\}$ to index goods and firms.

A2.1 Consumers' Problem

Let $r_i \in \{0, 1, 2\}$ denote the good that consumer i has at the beginning of the period. Below we will use superscript 0 to indicate expressions for an inattentive consumer, and use superscript 1 to indicate expressions for an attentive consumer.

Single-Period Utility Consumer i's single-period utility when she is inattentive is

$$u^0(b, r_i, \epsilon_i) = v_{r_i} + \mathbf{1}(r_i \neq 0)\theta g(b_{r_i}) + \epsilon_{i,r_i}, \tag{A9}$$

and her single-period utility when she is attentive and chooses to purchase good j is

$$u^1(b, p, d, r_i, \epsilon_i, j) = v_j + \mathbf{1}(j \neq 0)\theta g(b_j) - p_j - \mathbf{1}(r_i \neq 0, j \neq 0, r_i \neq j)(1 - d_j)k + \epsilon_{ij}. \tag{A10}$$

The definitions of the variables in Eqs. (A9) and (A10) are the same as those in the main model, described in Subsection 3.2.

Bellman Equation Let $W(\cdot)$ denote consumer i's value function at the beginning of a period <u>before</u> knowing whether her product dies in that period. Her Bellman equation is

$$W(b, p, d, r_i, \epsilon_i) = (1 - \frac{1}{M})W^0(b, p, d, r_i, \epsilon_i) + \frac{1}{M}W^1(b, p, d, r_i, \epsilon_i), \tag{A11}$$

where $W^0(b, p, d, r_i, \epsilon_i)$ is consumer i's value if her product doesn't die and she is therefore inattentive in this period:

$$W^0(b, p, d, r_i, \epsilon_i) = u^0(b, r_i, \epsilon_i) + \beta_c \mathbb{E}_{b', \epsilon_i'} \left[W(b', P(b'), D(b'), r_i, \epsilon_i') \right], \tag{A12}$$

and $W^1(b, p, d, r_i, \epsilon_i)$ is consumer i's value if her product dies and she is therefore attentive in this period:

$$W^1(b, p, d, r_i, \epsilon_i) = \max_{j \in \{0,1,2\}} \left\{ u^1(b, p, d, r_i, \epsilon_i, j) + \beta_c \mathbb{E}_{\epsilon_i'} \left[W(b', P(b'), D(b'), j, \epsilon_i') \right] \right\}. \tag{A13}$$

In the above expressions for the consumer's value function, $\beta_c \in [0, 1)$ is the consumers' discount factor, $P(\cdot) = (P_1(\cdot), P_2(\cdot))$ and $D(\cdot) = (D_1(\cdot), D_2(\cdot))$ denote the two firms' price and reimbursement policy functions, respectively, $b' = (b_1', b_2')$ is the next-period industry state, and $\epsilon_i' = (\epsilon_{i0}', \epsilon_{i1}', \epsilon_{i2}')$ is consumer i's next-period idiosyncratic preference shocks. The expectation on the right-hand side of Eq. (A12) is taken over both b' and ϵ_i', where the expectation of b' is based on the probabilities of the attentive consumer's loyalty (Eq. (1)) and her choice probabilities (Eq. (A18) below). In comparison, the expectation on the right-hand side of Eq. (A13) is taken over ϵ_i' only, as in that equation b' is pinned down given b, r_i, and j; see the state transition function Eq. (A19) below.

Note that the Bellman equation in Eq. (A11) involves the stochastic ϵ_i and therefore cannot be directly used for value function iteration. Let $\tilde{W}(b, p, d, r_i) = \mathbb{E}_{\epsilon_i} W(b, p, d, r_i, \epsilon_i)$ denote consumer i's expected value function where the expectation is taken over ϵ_i. We can then take the expectation of both sides of Eq. (A11) with respect to ϵ_i to obtain the consumer's Bellman equation in expected value function:

$$\tilde{W}(b, p, d, r_i) = \left(1 - \frac{1}{M}\right) \tilde{W}^0(b, p, d, r_i) + \frac{1}{M} \tilde{W}^1(b, p, d, r_i), \tag{A14}$$

where

$$\tilde{W}^0(b, p, d, r_i) = \tilde{u}^0(b, r_i) + \beta_c \mathbb{E}_{b'} \left[\tilde{W}(b', P(b'), D(b'), r_i)\right] \tag{A15}$$

and

$$\tilde{W}^1(b, p, d, r_i) = \log \left[\sum_{j=0}^{2} \exp\left\{\tilde{u}^1(b, p, d, r_i, j) + \beta_c \tilde{W}(b', P(b'), D(b'), j)\right\}\right]. \tag{A16}$$

In the above, $\tilde{u}^0(\cdot)$ and $\tilde{u}^1(\cdot)$ denote the deterministic part of $u^0(\cdot)$ and $u^1(\cdot)$, respectively, the mean of ϵ_i is normalized to 0, and Eq. (A16) is obtained using the property of logit demand that the expected value of the best choice among a set of choices is the so-called *log-sum term* (Train (2009, p. 56)).

The consumer's Bellman equation in expected value function does not involve the stochastic ϵ_i and is used in the value function iteration in our algorithm to solve for the dynamic equilibrium.

Choice Probabilities of the Attentive Consumer Consider the attentive consumer in the current period, consumer a, whose original product r_a dies and who returns to the market to make a purchasing decision. Let

$$\psi(b, p, d, r_a, j) = \tilde{u}^1(b, p, d, r_a, j) + \beta_c \tilde{W}(b', P(b'), D(b'), j) \tag{A17}$$

denote this consumer's expected value associated with choosing good j. In the above expression, the next-period industry state b' is pinned down given b, r_a, and j; see the state transition function Eq. (A19) below. Using the logit choice probability formula, we can write the probability that this consumer chooses good j as

$$\phi(b, p, d, r_a, j) = \frac{\exp\left[\psi(b, p, d, r_a, j)\right]}{\sum_{h=0}^{2} \exp\left[\psi(b, p, d, r_a, h)\right]}. \tag{A18}$$

State Transition Given our assumption that in every period, one random consumer out of the M consumers experiences product death and becomes attentive, the probability distribution of r_a—the attentive consumer's original product—is given by $\Pr(r_a = j|b) = b_j/M$, for $j = 0, 1, 2$.

Let $s_a \in \{0, 1, 2\}$ denote the attentive consumer's product choice. The industry state then transitions based on the joint outcome of the installed base depreciation (product death) and the attentive consumer's purchasing decision:

$$b' = B(b, r_a, s_a) = (\underbrace{b_1 - \mathbf{1}(r_a = 1) + \mathbf{1}(s_a = 1)}_{b_1'}, \underbrace{b_2 - \mathbf{1}(r_a = 2) + \mathbf{1}(s_a = 2)}_{b_2'}). \tag{A19}$$

A2.2 Firms' Problem

Firm j chooses its price p_j and reimbursement d_j in each period. Let $V_j(b)$ denote the expected net present value of current-period and future cash flows to firm j in state b. Firm j's Bellman equation is given by

$$V_j(b) = \max_{p_j, d_j} \mathbb{E}_{r_a} \Bigg[\phi\left(b, (p_j, P_{-j}(b)), (d_j, D_{-j}(b)), r_a, j\right) \left(p_j - \mathbf{1}\left(r_a \neq 0, r_a \neq j\right) d_j k\right) \\ + \beta \sum_{h=0}^{2} \phi\left(b, (p_j, P_{-j}(b)), (d_j, D_{-j}(b)), r_a, h\right) V_j(b') \Bigg], \quad \text{(A20)}$$

where $\beta \in [0, 1)$ is the firms' discount factor, the (constant) marginal cost of production is normalized to zero, $P_{-j}(b)$ is the equilibrium price charged by firm j's rival, $D_{-j}(b)$ is the equilibrium proportion of the switching cost reimbursed by firm j's rival, and the next-period industry state b' at the end of the equation is $b' = B(b, r_a, h)$ according to the state transition function Eq. (A19).

A2.3 Equilibrium

In equilibrium, from consumers' point of view, both p and d are functions of the industry state b based on the firms' equilibrium price and reimbursement policy functions: $p = P(b) = (P_1(b), P_2(b))$ and $d = D(b) = (D_1(b), D_2(b))$. Therefore, we can rewrite consumers' expected value function $\tilde{W}(b, p, d, r_i)$ as a function of b and r_i only, by substituting p and d with $P(b)$ and $D(b)$, respectively. Consequently, consumers' Bellman equation Eq. (A14) can be rewritten as an equation that involves only two variables, b and r_i:

$$\tilde{W}(b, r_i) = \left(1 - \frac{1}{M}\right) \left\{ \tilde{u}^0(b, r_i) + \beta_c \mathbb{E}_{b'} \left[\tilde{W}(b', r_i) \right] \right\} \\ + \frac{1}{M} \log \left[\sum_{j=0}^{2} \exp \left\{ \tilde{u}^1(b, P(b), D(b), r_i, j) + \beta_c \tilde{W}(B(b, r_i, j), j) \right\} \right]. \quad \text{(A21)}$$

The Markov perfect equilibrium of the infinite-horizon dynamic game described above consists of the following equilibrium functions: the firms' price and reimbursement policy functions $P_j(b)$ and $D_j(b)$, the firms' value function $V_j(b)$, and the consumers' expected value function $\tilde{W}(b, r_i)$, $r_i = 0, 1, 2$. In equilibrium, those functions jointly satisfy the following conditions for every industry state b:

1. $(P_j(b), D_j(b))$ is the solution to firm j's maximization problem on the right-hand side of Eq. (A20).
2. $V_j(b)$ satisfies the recursive equation in Eq. (A20).
3. $\tilde{W}(b, r_i)$, $r_i = 0, 1, 2$ satisfies the recursive equation in Eq. (A21).

We use value function iteration based on the above conditions to solve for the Markov perfect equilibrium.