# Incentives for the Over-Provision of Public Goods *

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## Abstract

Economists typically worry about free riding and the under-provision of public goods. Yet a wide range of public goods, such as open source software, possess up to two often-ignored features: excludable and potentially rivalrous contribution benefits, e.g. status seeking, and nonexcludable and nonrival consumption costs, e.g. adoption costs. These additional features mitigate the well-known incentive problems in the provision of public goods, but introduce new ones. I show that, instead of under-provision, over-provision can occur via a negative congestion externality on the supply side. Status-seeking induces an increase in contributions to the benefit of each contributor but imposes a cost on all other consumers and contributors. Hence policies subsidizing the provision of public goods can therefore lower welfare instead of raising it. Efforts to maximize welfare by a community leader or social planner often involve transferring surpluses from consumers to producers.

**Keywords:** congestion, free riding, negative externalities, over-provision, positive externalities, public goods, under-provision.

**JEL classifications:** C72, D29, D62, D71, H41.

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# 1 Introduction

Public goods, whether provided by governments or privately, are often produced collaboratively. Open source software and Wikipedia are developed by voluntary contributors. Public policy is similarly crafted by groups consisting of bureaucrats, politicians, private citizens, and scientists. While all public goods feature nonexcludable and nonrival consumption benefits and private contribution costs, collaboratively-produced public goods often possess up to two additional features: nonexcludable and nonrival consumption costs and excludable and potentially rivalrous contribution benefits. Incorporating these additional features alters how public goods are provided and how policy to optimize provision should be conducted.

When developing open source software, individuals contribute for a variety of reasons beyond need of use, such as altruism, the joy of coding, seeking status, and signaling ability (Lerner and Tirole, 2002; Feller et al., 2005; Bitzer et al., 2007; Myatt and Wallace, 2008; Fang and Neufeld, 2009; Athey and Ellison, 2014).[1] Consumers also face costs from using open source software, including implementation costs (e.g. needing upgraded hardware), learning costs, and switching costs (Nagy et al., 2010; Sacks, 2015). The provision of public policy and regulations allows elected officials to develop political capital and allows companies to carve out valuable exemptions (Laffont and Tirole, 1991; Dal Bó, 2006). Incumbent politicians often solicit donations and campaign on their records. While public policy and regulations yield benefits to contributors and consumers (by internalizing externalities), they also often impose compliance costs (Nitzan et al., 2013; Hogan, 2019).

To develop a more complete understanding of the provision of public goods, I create a static model of the provision of a public good that allows the interaction of four critical components: consumption benefits, contribution costs, contribution benefits, and consumption costs. There is a finite homogeneous population of individuals that voluntarily contribute. Contributing is always costly, but may confer benefits that depend on either the amount an individual contributes (levels) or the amount an individual contributes relative to total contributions (shares). Contribution benefits are excludable regardless of whether they depend

[1] Altruism, both pure and impure (Andreoni, 1990), and signaling (Glazer and Konrad, 1996) have been studied as incentives to contribute to public goods more generally. Bekkers and Wiepking (2011) summarize the incentives underpinning charitable giving.

on levels or shares, but are also rivalrous only when they depend on shares.

I find that under-provision due to free riding persists in public goods with consumption costs. However, rather than the free-rider problem worsening as the population grows, it becomes less severe as the consumption costs temper the optimal level of provision, allowing the voluntary providers to catch up to the optimum. The founding developer of Linux, Linus Torvalds, once said, "given enough eyeballs, all bugs are shallow (Raymond, 1999, p. 29)." Dubbed 'Linus's law', this statement claims that increasing the number of contributors to an open source software project increases welfare by increasing the quality of the software. Linus's law is consistent with the provision of public goods with consumption costs. However, it may not be consistent with the provision of open source software more generally. For open source software projects whose contributions are solely motivated by need of use (consumption benefits), Linus's law holds; however, it need not hold when contributors contribute for other reasons, such as signaling ability.

When providing public goods with contribution benefits, over-provision occurs when the contribution benefits are prominent and depend on shares. Much like contests, contributors have an incentive to enter into an arms race in order to contribute the greatest share. In equilibrium, these efforts are negated as individuals contribute equal shares, though individuals still bear the full contribution costs of the arms race. This negative externality is a supply-side corollary to the well-known congestion externality in club goods. Referring back to open source software, this pattern is, to a degree, still consistent with Linus's law as the quality of the public good is increasing as contributions rise. Welfare is not increasing, though the free-riders do prefer over-provision.

While previous studies have shown over-provision to be a possibility, e.g., Brueckner (1979), Van de Kragt et al. (1983), Bayindir-Upmann (1998), and Bjorvatn and Schjelderup (2002), and Polborn (2008), competition (multiple jurisdictions) and heterogeneity are necessary conditions for over-provision in these studies. Gradstein and Nitzan (1990) also finds over-provision to be a possibility when contributions are binary, the marginal utility of consuming tends to zero as the number of contributors increases, and the population is sufficiently large. I show that neither binary contributions, competition, nor heterogeneity are necessary for the over-provision of public goods.

In the presence of both consumption costs and contribution benefits, over-provision occurs when the contribution benefits are prominent, regardless of their nature (levels or shares). In contrast to public goods with consumption benefits but no consumption costs, the costs of over-provision are primarily borne by consumers rather than contributors. Private incentives to increase contributions are successful, imposing a cost on all consumers that is not internalized by the contributors. This tension leads to inefficiently complex (low quality) public goods (public goods in which the marginal consumption costs exceed the marginal consumption benefits). There is a tradeoff between quality and quantity. Linus's law no longer holds. Instead, we can relate these results to an English proverb stating, "too many cooks spoil the broth." The claim is that too many contributors can have a deleterious effect on welfare.

This relationship is observed in a variety of public goods. Stamelos et al. (2002) show that the relationship between the size of a component of an open source application (aggregate contributions) is negatively related to user satisfaction for the application (quality) as measured by a four-point defect severity scale developed by IBM.[2] A similar tradeoff is shown in Francalanci and Merlo (2008). Quandt (1983) and Kearl (1983) show that there exist conditions such that there is an over-investment in regulation. This paper generalizes these results and broadens their applicability. Similarly, Nitzan et al. (2013) argue that the increasing volume and velocity of complexity in regulation over time leads to inefficiencies induced by implementing barriers to entry so only the largest established firms survive. The authors reference accounting regulations, corporate governance and securities regulations, and the market for corporate charters. Combining the results of Nitzan et al. (2013), Quandt (1983), Kearl (1983), and the citations therein with this paper leads to a more complete understanding of the effects of complexity in regulation via the quantity-quality tradeoff.[3]

[2] In cases where expert consumers reported only superficial errors at worst, the mean length of the program is strictly lower than in cases where (*a*) consumers reported either all major program functions are working but at least one minor function is disabled or incorrect, (*b*) at least one major function is disabled or incorrect, and (*c*) the program is inoperable. Their result is robust to other metrics, including the number of statements per module.

[3] For examples of regulatory complexity in finance, see Berglund (2014). For examples in IP, see Allison and Lemley (2002). Evidence of a negative impact of regulations is given in Murray and Stern (2007) and

Galasso and Schankerman (2015) find that whether or not patents hinder subsequent innovation depends on the complexity of the industry and that negative effects are concentrated in more complex industries. If it is relatively more costly to implement IP in more complex industries, then Galasso and Schankerman (2015) can be viewed as evidence in support of the quality-quantity tradeoff.

Open source software, like public policy and other collaboratively produced public goods, is not homogenous in nature. Depending on who is consuming and contributing, open source software (or policy) may or may not possess contribution benefits or consumption costs. Hence there is no uniform prediction regarding over-provision versus under-provision with regard to open source software, public policy, or many other public goods. With under-provision, the optimal policy entails subsidizing contributions while with over-provision, the optimal policy entails taxing contributions. As a result, no uniform policy can be applied to public goods, but only to classes of public goods. With specific types of public goods (e.g. open source software) belonging to multiple classes (based on the presence of contribution benefits and consumption costs), policies must be evaluated on a case-by-case basis, severely limiting the external validity of studies of specific cases.

As the provision of public goods, including open source software and public policy is not a free-for-all, I also consider the case where the public good is coordinated by a social planner, though individual participants are still bounded by their individual rationality constraints. The social planner selects who may contribute, which in a homogeneous population corresponds to the number of contributors. I find that a planner interested in maximizing total welfare favors transferring surpluses from consumers to contributors, which in the context of open source software involves inefficient complexity and in the context of public policy involves regulatory capture. Thus these phenomena are more general than their specific contexts and this sort of over-provision applies to a wide variety of public goods. To obtain a more equitable distribution of surpluses, policies need to be designed to maximize the quality of the public good rather than total welfare.

This paper contributes to several strands of literature, including the theory of public goods,

Williams (2013).

industrial organization, and the economics of innovation. The literature on the private provision of public goods began with Bernheim (1986) and Bergstrom et al. (1986). Summaries of the core theories are given by Cornes and Sandler (1996), Sandler and Tschirhart (1997), and Scotchmer (2002). Though some work has been conducted in the context of regulatory complexity, this paper is among the first to formally analyze the effects of consumption costs and contribution benefits on the provision of public goods.[4]

In addition to the provision of collaborative public goods, this paper is useful in understanding the structure and outcomes of industries whose interactions are characterized by collaborative production, such as the software industry. I show that collaborative groups will likely over-invest in technological investment, shedding light on the nature of research joint ventures by building upon Kamien et al. (1992) and the theories outlined in Caloghirou et al. (2003). Lastly, this paper offers an alternative mechanism for understanding over-investment in patent races (Gilbert and Newbery, 1982; Fudenberg et al., 1983; Shapiro, 1985; Harris and Vickers, 1987; Aoki, 1991; Baye and Hoppe, 2003).

The remainder of paper is structured as follows. Section 2 outlines the model. Section 3 presents the main results, first analyzing public goods with consumption costs, then public goods with contribution benefits, finishing with public goods with both consumption costs and contribution benefits. Section 4 presents the social planner's problem. Section 5 concludes.

# 2 The Model

There are $N > 2$ individuals and a single public good. Each individual $i$ has the option to either contribute $x_i \geq 0$ and consume or not consume.[5] Define $X = \sum_{j=1}^{N} x_j$ and $X_{-i} = X - x_i$. In what follows, I use capital letters to represent aggregate elements and lower-case letters to represent individual-level elements. All decisions are made simultaneously and independently.

[4] A general model of public goods incorporating non-monotonicity was developed in Mas-Colell (1980), where public projects are elements of a non-linear topological space. However, such a model is unable to capture the characteristics described above.

[5] Allowing individuals to contribute but not consume only strengthens the results of the paper. Contributors will not internalize the costs from consumption, further intensifying contributions.

Each individual's utility consists of four components. The first component is the benefit from consuming the public good, which I call the consumption benefit. The consumption benefit, $B(X)$, depends only on total contributions. I assume that $B(\cdot)$ is twice continuously differentiable, strictly increasing, and weakly concave with $B(0) = 0$. The second component is the consumption cost $\gamma C(X)$, which like the consumption benefit, depends only on the aggregate contributions. The parameter $\gamma \in \{0, 1\}$ is used isolate the effects of the consumption cost. I assume that $C(\cdot)$ is twice continuously differentiable, strictly increasing, and strictly convex with $C(0) = 0$. Define $B(X) - \gamma C(X)$ as the public good's *quality* and $\hat{X} \equiv \max_X \{B(X) - \gamma C(X)\}$ as the quality-maximizing contribution level.

The third component is the contribution benefit $\sigma b(x_i, X_{-i})$. I define $\sigma$ as the relative strength of the contribution benefits. If $\sigma = 0$, then the contribution benefits are turned off and contributing is solely a cost. If $\sigma > 0$, then individuals benefit not only from consuming the public good, but also from contributing to its production. I assume that the contribution benefit is twice continuously differentiable, strictly increasing in $x_i$, weakly decreasing in $X_{-i}$, and weakly concave with $b(0, X_{-i}) = 0$. Contribution benefits accrue in one of two ways: in terms of levels $x_i$ or in terms of shares $x_i/X$. In particular, I restrict $b(x_i, X_{-i})$ to one of two forms:

$$b(x_i, X_{-i}) = \begin{cases} b(x_i) & \text{if levels} \\ b\left(\frac{x_i}{x_i + X_{-i}}\right) & \text{if shares.} \end{cases}$$

These specifications imply that $\frac{\partial b(x, X_{-i})}{\partial x_i} = \frac{\partial b(x, X_{-j})}{\partial x_j} > 0$ for every $X_{-i}$ and $X_{-j}$ and that $\frac{\partial^2 b(x_i, X_{-i})}{\partial x_i^2} = \frac{\partial^2 b(x_i, X_{-i})}{\partial x_i \partial X_{-i}}$.

The fourth component is the contribution cost $c(x_i)$. I assume that $c(\cdot)$ is increasing, twice continuously differentiable and strictly convex with $c(0) = 0$. This cost can equivalently be interpreted as the opportunity cost of forgoing consumption of an unmodeled private good when individuals are resource-constrained.[6]

[6] More precisely, this setup is analogous to the setting in which there is both a public good and a private good, each individual is endowed with a sufficiently large (but finite) stock of resources, and the marginal utility of consuming the private good is infinite at zero consumption (Inada condition).

Combining these four components, individual $i$'s utility when contributing is

$$u\left(x_i, X_{-i}; \sigma\right) = B\left(X\right) - \gamma C\left(X\right) + \sigma b(x_i, X_{-i}) - c(x_i). \tag{1}$$

When not contributing, individual $i$ receives utility

$$u\left(0, X_{-i}; \sigma\right) = \max\{B(X) - \gamma C(X), 0\}.$$

# 3 Analysis

Denote by $x_i^*$ individual $i$'s (Nash) equilibrium contribution to the public good and by $X^*$ the aggregate equilibrium contribution level.

**Lemma 1.** *If $x_i^* > 0$ and $x_j^* > 0$, then $x_i^* = x_j^* = x^*$ for all $i$ and $j$.*

The proof, and all subsequent proofs, are contained in the appendix. Let $x'' > 0$ satisfy

$$\frac{dB\left(X_{-i}^* + x''\right)}{dX} - \gamma \frac{dC\left(X_{-i}^* + x''\right)}{dX} + \sigma \frac{\partial b(x'', X_{-i}^*)}{\partial x_i} - \frac{dc(x'')}{dx_i} = 0.$$

I order the individuals so if $M < N$ individuals contribute, individuals $1, \ldots, M$ are the contributors and individuals $M+1, \ldots, N$ are the non-contributors. There exists an equilibrium in which exactly $M$ individuals contribute if, for all $i = 1, \ldots, M$,

$$B\left(X^*\right) - \gamma C\left(X^*\right) + \sigma b(x^*, X_{-i}^*) - c(x_i^*) \geq \max\{B\left(X_{-i}^*\right) - \gamma C\left(X_{-i}^*\right), 0\}, \tag{2}$$

and if $M < N$,

$$\max\{B\left(X^*\right) - \gamma C\left(X^*\right), 0\} \geq B\left(X^* + x''\right) - \gamma C\left(X^* + x''\right) + \sigma b\left(x'', X_{-i}^*\right) - c(x'') \tag{3}$$

for all $i = M+1, \ldots, N$. Inequality (2) states that there is no profitable deviation in which an individual stops contributing and (3) states that there is no profitable deviation in which a free-rider starts contributing. Denote by $M^*(N)$ the values of $M$ such that $x^* = \mathbf{x}(M^*(N), \sigma)$ for all $i = 1, \ldots, M$ and $x^* = 0$ for all $i = M+1, \ldots, N$ constitutes a Nash equilibrium. By the implicit function theorem, $x^* = x(M, \sigma)$ and $X^* = X(M, \sigma)$ and by the theorem of the maximum, $\frac{\partial x(M,\sigma)}{\partial \sigma}$ exists and is well defined.[7]

[7]For $\sigma = 0$, define the derivative as $\lim_{\Delta \to 0^+} \frac{x(M,\Delta) - x(M,0)}{\Delta}$.

I also characterize the welfare-maximizing contribution levels in order to determine whether over-provision or under-provision occurs in equilibrium. I use $\tilde{x}_i$ and $\tilde{X}$ to denote the individual and aggregate welfare-maximizing contributions, which are defined according to

$$\max_{x_1,\ldots,x_N} \underbrace{\left\{ (N-M)\max\{B(X)-\gamma C(X),0\} + M\left[B(X)-\gamma C(X)\right] + \sum_{j=1}^{M}\left[\sigma b(x_j,X_{-j})-c(x_j)\right] \right\}}_{=\sum_{j=1}^{N} u(x_j,X_{-j};\sigma)}.$$

Denote by $\tilde{M}(N)$ the value(s) of $M$ that maximize welfare. In what follows, $M^*(N)$ and $\tilde{M}(N)$ respectively offer the number of equilibrium and welfare-maximizing contributors while $M$ represents an arbitrary number of contributors.

In what follows, I sequentially consider three types of public goods:

1. public goods with consumption costs but no contribution benefits ($\gamma = 1$ and $\sigma = 0$),
2. public goods with contribution benefits but no consumption costs ($\gamma = 0$ and $\sigma > 0$),
3. public goods with both consumption costs and contribution benefits ($\gamma = 1$ and $\sigma > 0$).

Each case includes consumption benefits $B(\cdot)$ and contribution costs $c(\cdot)$, as these components are necessary for the collaborative provision of all public goods. I omit the case of no consumption costs or contribution benefits ($\gamma = \sigma = 0$), which corresponds to the 'textbook' public good, as its properties are well known (a free-rider problem that worsens as the population size increases). This setup allows the model to cover the spectrum of collaboratively produced public goods and isolate how each component affects the equilibrium and optimal provision.

## 3.1 Public Goods with Consumption Costs

To isolate and identify the effects of consumption costs, I fix $\gamma = 1$ and $\sigma = 0$. Individual $i$'s utility becomes

$$u(x_i, X_{-i}; 0) = B(X) - C(X) - c(x_i).$$

Recall that $\hat{X}$ denotes the level of contributions that maximizes quality, $B(X) - C(X)$.

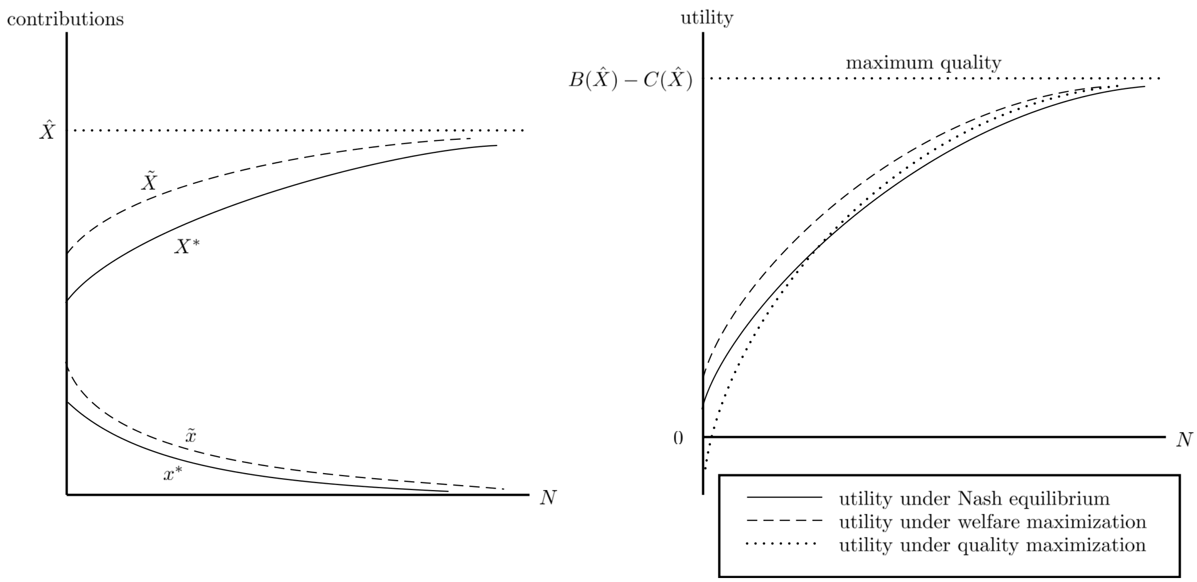


Figure 1: Contributions and utility when public goods have consumption costs.

**Proposition 1.** *$M^*(N) = \tilde{M}(N) = N$. For all $N$, $X^* < \tilde{X} < \hat{X}$. Lastly,* $\lim_{N\to\infty} X^* = \lim_{N\to\infty} \tilde{X} = \hat{X}$.

Figure 1 diagrammatically illustrates Proposition 1, which offers an immediate departure from the textbook treatment of public goods. While free-riding persists and neither the equilibrium nor the welfare-maximizing contributions maximize quality, both the equilibrium and welfare-maximizing contributions converge to the quality maximizer as the size of the population increases. The free-rider problem becomes less severe rather than more severe as the population grows. Proposition 1 provides the economic interpretation of Linus's law. Although the idea that "given enough eyeballs, all bugs are shallow" may not apply to open source software, it can be applied to public goods with consumption externalities.

I offer a simple analytical example to illustrate Proposition 1, which will be featured throughout the paper. Suppose that utility is given by

$$u(x_i, X_{-i}; \sigma) = X - \gamma \frac{1}{40} X^2 + \sigma \left( \mathbf{1}\{\text{shares}\} \frac{x_1}{X} + \mathbf{1}\{\text{levels}\} x_i \right) - \frac{1}{2} x^2,$$

where $\mathbf{1}\{\cdot\}$ is an indicator function determining the type of contribution benefits present (levels or shares). In keeping with this section, I set $\gamma = 1$ and $\sigma = 0$, so

$$u(x_i, X_{-i}; 0) = X - \frac{1}{40} X^2 - \frac{1}{2} x^2.$$

It follows that

$$X^* = N\underbrace{\left(\frac{20}{20+N}\right)}_{x^*}, \qquad \tilde{X} = N\underbrace{\left(\frac{20N}{20+N^2}\right)}_{\tilde{x}}, \qquad \hat{X} = 20.$$

Whenever $N > 1$, $\tilde{X} > X^*$ and as $N$ increases, both $X^*$ and $\tilde{X}$ converge to 20, which corresponds to $\hat{X}$.

In summary, consumption costs mitigate the free-rider problem, particularly in larger populations. While Linus's law is not entirely applicable to open source software, it does apply to the class of public goods with consumption costs, which includes goods such as Wikipedia and open source software projects where contributors' motivations are driven by necessity rather than incentives such as seeking status, signaling ability, or the joy of coding.

## 3.2 Public Goods with Contribution Benefits

To isolate the effects of contribution benefits like those mentioned above, I fix $\gamma = 0$ and assume that $\sigma > 0$. When contributing, individual $i$ receives utility

$$u(x_i, X_{-i}; \sigma) = B(X) + \sigma b(x_i, X_{-i}) - c(x_i).$$

Without consumption costs, the quality-maximizer is not well defined ($\hat{X} \to \infty$), so I focus attention on the relationship between the equilibrium contributions $X^*$ and the welfare-maximizing contributions $\tilde{X}$, as they will always fall below $\hat{X}$. There are two cases to consider: when contribution benefits depend on levels and when they depend on shares.

The nature of under-provision versus over-provision depends explicitly on whether benefits are determined by levels or shares.

**Proposition 2.** $M^*(N) = \tilde{N}(N) = N$. *If the contribution benefits depend on levels, then*

(i) $X^* < \tilde{X}$ *for all* $N$ *and* $\sigma$.

(ii) *Both* $X^*$ *and* $\tilde{X}$ *are increasing in* $\sigma$.

*If the contribution benefits depend on shares, then for every* $N$ *there exists a function* $\overline{\sigma}(N)$ *such that:*

(iii) $X^*$ *is increasing in* $\sigma$ *while* $\tilde{X}$ *is unaffected by* $\sigma$*.*

(iv) $X^* > \tilde{X}$ *if and only if* $\sigma > \overline{\sigma}(N)$*.*

When the contribution benefits depend on levels, the only externality present is the positive free-rider externality. Hence under-provision occurs. The net contribution benefits are $\sigma b(x_i) - c(x_i)$. Given the independence of contribution benefits from the contributions of others, the diminishing marginal returns to the contribution benefits $\frac{d^2 b(x)}{dx^2} \leq 0$, and the increasing marginal cost of contributing $\frac{d^2 c(x)}{dx^2} > 0$, the contribution benefits can be reinterpreted as subsidies to contributing. In particular, contributions up to a level $x' = \arg\max \sigma b(x_i) - c(x_i)$ are subsidised. Individuals then bear the full costs for contribution levels above $x'$. Given the first-order condition

$$\frac{dB\left(X^*\right)}{dX} + \sigma \frac{\partial b(x^*)}{\partial x_i} - \frac{dc(x^*)}{dx_i} = 0,$$

$x^* > x'$. Otherwise the left-hand side would be strictly positive, as at $x^* = x'$, $\sigma \frac{\partial b(x^*)}{\partial x_i} - \frac{dc(x^*)}{dx_i} = 0$ while $\frac{dB(X^*)}{dX} > 0$. Therefore, public goods with contribution benefits that depend on levels resemble the textbook public goods model, though both $x^*$ and $\tilde{x}$ are increasing in $\sigma$, much like . However, the severity of the inefficiency from under-provision still increases with the population size. Contribution benefits that accumulate via levels scale up provision, but less so that what is socially optimal.

Returning to the example introduced in Section 3.1, where $\gamma = 0$, $\sigma > 0$, and $b(x) = x$ implies that individual utility is

$$u(x_i, X_{-i}, \sigma) = X + \sigma x_i - \frac{1}{2} x_i^2.$$

Aggregate contributions are given by

$$X^* = N(\underbrace{1 + \sigma}_{x^*}), \qquad \tilde{X} = N(\underbrace{N + \sigma}_{\tilde{x}}).$$

Contributions are increasing in $\sigma$ and free-riding persists and worsens as the population size grows: $\tilde{X} - X^* = N(N - 1)$.

When the contribution benefits depend on shares, there is a tension between two externalities. The standard free-riding positive externality is present, where contributors do not

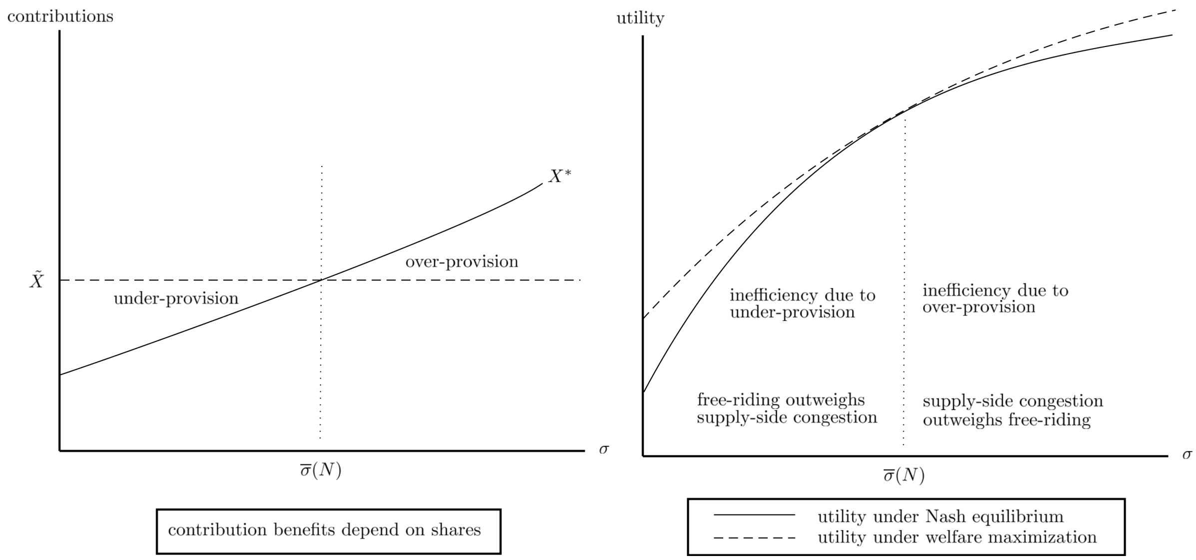


Figure 2: Contributions and utility when public goods have contribution benefits.

internalize the benefits their contributions impart on others. There is also a negative externality. Contributors enter an arms race where, in order to capture greater contribution benefits, each contributor seeks to contribute more than their peers. Given the diminishing marginal returns to contributing and the increasing marginal cost of contributing, each contributor imposes a negative congestion externality on the other contributors. Unlike the typical congestion externality, which is observed on the demand side, this is a supply-side congestion externality. In equilibrium, all contributors contribute identical amounts and thus equal shares. Could the contributors commit to lower (but still equal) contribution levels, costs would be shaved, which increases total welfare. When maximizing welfare, the value of $\sigma$ is irrelevant. Only that each individual contributes an equal share is relevant.

If the contribution benefits are small, then the incentive to free-riding outweighs supply-side congestion and there is inefficiency due to under-provision. If the contribution benefits are large, then supply-side congestion outweighs free-riding and there is inefficiency, but due to over-provision rather than under-provision. This tension is visualized in Figure 2. The effect of the population size on the cutoff value $\overline{\sigma}(N)$ depends explicitly on the shape of the benefits function.

If $b(\cdot)$ is strictly concave and $\frac{\partial b\left(\frac{1}{2}\right)}{\partial x}$ is sufficiently large relative to $\frac{dB(X)}{dX}$, for instance if

$\lim_{X\to\infty} \frac{dB(X)}{dX} = 0$, as in Gradstein and Nitzan (1990), which can be interpreted as technology constraints limiting the benefits of consuming a public good (such as open source software and limits to computing power). This restriction puts a cap on $\tilde{X}$, while $X^*$ is still strictly increasing in $N$. It follows that $\overline{\sigma}(N)$ is initially decreasing in $N$. The contribution benefits lead to large increases in $X^*$ relative to $\tilde{X}$, so $\sigma$ need not be so large. This pattern can continue for $N$ large, allowing $\overline{\sigma}(N)$ to fall for large $N$, inducing over-provision in large populations. If $b(\cdot)$ is linear, or the marginal utility from contributing is finite as $x_i \to 0$, i.e. $\lim_{N\to\infty} \frac{\partial b\left(\frac{1}{N}\right)}{\partial x} < \infty$, then under-provision occurs for sufficiently large populations.[8] This distinction is important, as this ambiguity makes it difficult to develop policy to elicit the optimal contribution levels. With over-provision, contributions should be taxed and with under-provision, they should be subsidized.

To illustrate these results more concretely, I return to the above example, but with $b\left(\frac{x}{X}\right) = \frac{x_i}{X}$. Individual $i$'s utility is

$$u(x_i, X_{-i}; \sigma) = X + \sigma\frac{x_i}{X} - \frac{1}{2}x_i^2.$$

The Nash equilibrium and welfare-maximizing contributions are

$$X^* = N\Bigg(\underbrace{\frac{1}{2} + \sqrt{\frac{1}{4} + \sigma\frac{N-1}{N^2}}}_{x^*}\Bigg), \qquad \tilde{X} = N \times \underbrace{N}_{\tilde{x}}.$$

As in statement (iii), $\tilde{X}$ is unaffected by $\sigma$ while $X^*$ is strictly increasing in $\sigma$. Eventually, $X^*$ must exceed $\tilde{X}$. Equating $X^*$ and $\tilde{X}$ and solving for $\sigma$ yields

$$\overline{\sigma}(N) = N^3,$$

which is increasing in $N$ as expected.

Before moving on to the class of public goods with both consumption costs and contribution benefits, it is important to consider the case in which only a subset of the population can contribute to the public good. For example, the typical software user does not have the skills necessary to contribute to open source software and most Wikipedia users do not contribute.

[8] Note that $\lim_{N\to\infty} \frac{\partial b\left(\frac{1}{N}\right)}{\partial x} = \infty$ is not sufficient for over-provision.

**Proposition 3.** *Suppose that there are $M' < N$ potential contributors, where $M' \geq 2$, and the contribution benefits depend on shares. Then $M^*(N) = \tilde{M}(N) = M'$. Furthermore, there exists a cutoff value $\overline{\overline{\sigma}}(M, N)$ such that $X^* > \tilde{X}$ if and only if $\sigma > \overline{\overline{\sigma}}(M', N)$.*

When only a subset of the population can contribute, the minimum $\sigma$ necessary to induce over-provision is smaller than the case in which the entire population can contribute. Thus for a given $\sigma$, over-provision is more likely when only a subset of the population contributes. This point is important, as most public goods have free-riders and evidence suggests that contribution benefits are more valuable when contributions are more visible (Glazer et al., 1997; Holmström, 1999; Andreoni and Petrie, 2004; Alpizar et al., 2008; Bekkers and Wiepking, 2011), which occurs when there are fewer contributors. When maximizing welfare, the optimal individual contribution is not a function of the number of contributors, but of the population as a whole. The equilibrium contribution, on the other hand, is a decreasing function of the number of contributors. With fewer contributors, each contributor contributes a greater amount. Under the supply-side congestion externality, this increase outweighs the losses in contributions from having fewer contributors. The welfare losses from over-provision are borne by the contributors. The $N - M'$ free-riders benefit from the increased provision while the supply-side congestion externality induces increased losses on the contributors through the increased contribution costs $c(\cdot)$ caused by the arms race to contribute the greatest share.

Referring again to the above example, but with $M'$ contributors,

$$X^* = M' \underbrace{\left( \frac{1}{2} + \sqrt{\frac{1}{4} + \sigma \frac{M'-1}{(M')^2}} \right)}_{x^*}$$

$$\tilde{X} = M' \times \underbrace{N}_{\tilde{x}}.$$

Equating these two values yields

$$\overline{\overline{\sigma}}(M', N) = \frac{(M')^2}{M'-1} N(N-1).$$

Note that,

$$\overline{\overline{\sigma}}(M', N) < \overline{\sigma}(N) \iff M' > \frac{N}{N-1},$$

which is true for $N \geq 3$ and $M' \geq 2$, the minimal conditions for a strict subset to be contributing and to call contributions collaborative, i.e. more than one contributor. Note that this relationship between the two cutoff values need not be true in general, as it depends on the slope of the benefits function as $M$ increases, so $\frac{1}{M}$ falls.

In summary, contribution benefits can induce over-provision when they depend on shares. When there is under-provision, the loss in utility stemming from the inefficiency is borne by the consumers. When there is over-provision, the loss in utility stemming from the inefficiency is borne by the contributors.

## 3.3 Public Goods with Consumption Costs and Contribution Benefits

Now that the effects of consumption costs and contribution benefits have been outlined when isolated, I consider public goods including both ($\gamma = 1$ and $\sigma > 0$). Contributor utility is as specified in (1). With consumption costs present, the quality-maximizer $\hat{X}$ becomes relevant again. When contributing, individual $i$' first-order condition is given by

$$\frac{dB\left(X_{-i}+x^{*}\right)}{dX}-\frac{dC\left(X_{-i}+x^{*}\right)}{dX}+\sigma\frac{\partial b(x^{*},X_{-i})}{\partial x_i}-\frac{dc(x^{*})}{dx_i}=0.$$

Looking at the above first-order condition, one of three properties will be satisfied in equilibrium.

**Property 1. (P1)** $\frac{dB(X^*)}{dX}-\frac{dC(X^*)}{dX}>0>\sigma\frac{\partial b(x^*,X^*_{-i})}{\partial x_i}-\frac{dc(x^*)}{dx_i}$.

**Property 2. (P2)** $\frac{dB(X^*)}{dX}-\frac{dC(X^*)}{dX}<0<\sigma\frac{\partial b(x^*,X^*_{-i})}{\partial x_i}-\frac{dc(x^*)}{dx_i}$.

**Property 3. (P3)** $\frac{dB(X^*)}{dX}-\frac{dC(X^*)}{dX}=0=\sigma\frac{\partial b(x^*,X^*_{-i})}{\partial x_i}-\frac{dc(x^*)}{dx_i}$.

Each of the properties relates the aggregate equilibrium contributions $X^*$ to the quality-maximizing contributions $\hat{X}$. Under Property 1, the equilibrium quality is suboptimal with too few contributions. Under Property 2, the equilibrium quality is suboptimal, but with too much being contributing. Under Property 3, quality is maximized in the equilibrium. Property 2 illustrates the quantity-quality tradeoff. Quality is decreasing in aggregate contributions. Yet the individual can benefit from contributing when the contribution benefits outweigh the contribution costs. If $\sigma$ is large enough, such an incentive is present in equilibrium. The following lemma formalizes the existence of such a $\sigma$. In addition, when quality is suboptimally low (high) in equilibrium, then there is under-provision (over-provision).[9] Characterizing which property holds in equilibrium is thus equivalent to identifying under-provision or over-provision.

With the interaction of consumption costs and contribution benefits, $M^*(N)$ need not equal $N$ as it does when only one is present (either $\gamma = 0$ or $\sigma = 0$). Denote by $M^U \in \mathbb{R}_+$ the $M$ that makes (2) hold with equality. The maximum number of contributors is given by $\lfloor M^U \rfloor$. If there are $M$ contributors and $\sigma < \overline{\sigma}(M)$, then (3) cannot be satisfied as both $B(x_i + X_{-i}) - C(x_i + X_{-i})$ and $\sigma b(x_i, X_{-i}) - c(x_i)$ are increasing for small $x_i$. Thus either $P2$ or $P3$ must be satisfied. If $\sigma \leq \overline{\sigma}(N)$, then all $N$ individuals contribute. If either $P2$ or $P3$ is satisfied, then the value of $M$ that makes (2) hold with equality ($M^U$) indicates the upper bound on the number of contributors in equilibrium. To see this relationship, note that under $P2$ and $P3$, $\sigma b(x^*, X^*_{-i}) - c(x^*)$ is decreasing in $M$ while $B\left(X^*_{-i}\right) - C\left(X^*_{-i}\right) - [B\left(X^*\right) - C\left(X^*\right)]$ is increasing in $M$, so (2) is violated for all $M > M^U$. The value of $M$, denoted by $M^L$, that makes (3) hold with equality indicates the lower bound on the number of contributors. It follows that (3) is violated for all $M < M^L$.

Consequently, the maximal number of contributors is determined by $\min\left\{\lfloor M^U \rfloor, N\right\}$. If $N > M^U$, then $N - \lfloor M^U \rfloor$ individuals do not contribute in equilibrium. The correspondence

$$M^*(N) : N \to \left\{Z \in \mathbb{N} \,|\, M^L \leq Z \leq \min\left\{M^U, N\right\}\right\}$$

defines the equilibrium number of contributors as a function of the population size. As welfare-maximization internalizes the externalities, $\tilde{M}(N) \leq M^*(N)$.

The relationships between the equilibrium contributions, welfare-maximizing contributions, and the quality-maximizing contributions are outlined in the following proposition.

**Proposition 4.** *There exists a function $\overline{\sigma}(N)$ such that the following holds.*

(i) *If $\sigma < \overline{\sigma}(N)$, then $X^* < \tilde{X} < \hat{X}$.*

[9] This statement is proven in Proposition 4.

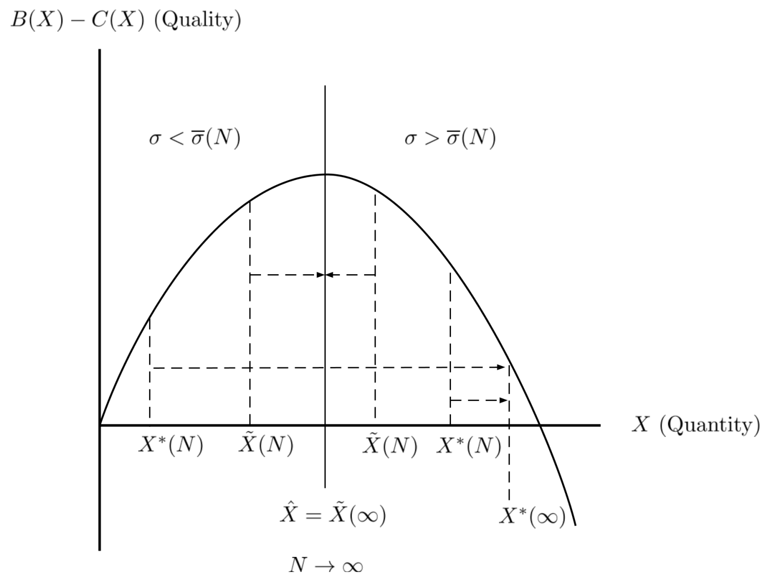


Figure 3: Equilibrium, welfare-maximizing, and quality-maximizing contributions.

(ii) *If* $\sigma > \overline{\sigma}(N)$*, then* $X^* > \tilde{X} > \hat{X}$*.*

(iii) $\lim_{N\to\infty} \overline{\sigma}(N) = 0$ *and* $\lim_{N\to\infty} \tilde{X} = \hat{X}$*, so* $\lim_{N\to\infty} X^* > \lim_{N\to\infty} \tilde{X}$ *for all* $\sigma > 0$*.*

*Moreover,* $\overline{\sigma}(N)$ *is strictly decreasing in* $N$ *if the contribution benefits depend on levels and* $\overline{\sigma}(N)$ *need not be monotonic in* $N$ *if the contribution benefits depend on shares.*

Figure 3 graphically illustrates the Proposition.

Like public goods with contribution benefits but no consumption costs, public goods with both consumption costs and contribution benefits suffer from over-provision when the contribution benefits are sufficiently large. However, there are a few critical differences with substantial implications. First, ass under-provision entails the equilibrium falling below both the welfare-maximizer and the quality-maximizer and over-provision entails the equilibrium falling above both the welfare-maximizer and quality-maximizer, consumers bear the a greater share of the utility losses stemming from the inefficiency than do contributors. Second, over-provision can occur whether the contribution benefits accrue in terms of levels or shares, whereas shares are required for over-provision when consumption costs are not present.

When the contribution benefits depend on levels, consumers bear the entire cost of the inefficiencies. With under-provision, consumers suffer from an inefficiently low quality public good. While the contributors suffer from this low quality as well, their suffering is mitigated via the private contribution benefits received. As those benefits rise, contributors are incentivized to contribute more until the public good becomes a public bad: $\frac{dB(X^*)}{dX} - \frac{dC(X^*)}{dX} < 0$. Additional contributions impose a negative externality on consumers, further depressing their utility. Over-provision becomes more likely as the population grows, as contributors do not impose a congestion externality on each other, inducing more individuals to contribute a greater amount to obtain those benefits. Thus the necessary $\sigma$ is decreasing in the population size.

When the contribution benefits depend on shares, the same general pattern emerges. They key distinction is not only do additional contributions eventually impose a cost on consumers, but additional contributions impose an additional cost on contributors as well. For small population sizes, $\overline{\sigma}(N)$ may be increasing. Observe the first-order condition for a contributor and the welfare-maximizing condition when all individuals consume:

$$\frac{dB(X^*)}{dX} - \frac{dC(X^*)}{dX} + \sigma \frac{\partial b\left(\frac{1}{N}\right)}{\partial x_i} \left(\frac{N-1}{Nx^*}\right) - \frac{dc(x^*)}{dx_i} = 0$$
$$N\left(\frac{dB(\tilde{X})}{dX} - \frac{dC(\tilde{X})}{dX}\right) - \frac{dc(\tilde{x})}{dx_i} = 0.$$

When $N$ is small, the free-riding externality is not significant, nor is the consumption cost $C(\cdot)$. Hence early contributions are favorable to all, as they impart consumption and contribution benefits. That is, both $X^*$ and $\tilde{X}$ are increasing in $N$ and the environment is similar to the case with no consumption costs, which implies an increasing $\overline{\sigma}(N)$. As the peak in quality $\hat{X}$ is approached, the welfare maximizing program suggests cutting back on contributions to not exceed the quality-maximizer so as to not harm consumers, particularly as $N$ grows and consumers become relatively more important. Yet as $N$ grows, the individual incentives are such that aggregate contributions $X^*$ increase, as $\frac{\partial b\left(\frac{1}{N}\right)}{\partial x_i}\left(\frac{N-1}{N}\right)$ is strictly increasing in $N$. The result is a smaller $\sigma$ yielding over-provision, or equivalently a decreasing $\overline{\sigma}(N)$.

Referring to the analytical example, individual $i$'s utility is given by

$$u(x_i, X_{-i}; \sigma) = X - \gamma \frac{1}{40} X^2 + \sigma \left( \mathbf{1}\{\text{shares}\} \frac{x_1}{X} + \mathbf{1}\{\text{levels}\} x_i \right) - \frac{1}{2} x^2,$$

leading to contributions of

$$X^* = \begin{cases} N\left(\frac{20(1+\sigma)}{20+N}\right) & \text{if levels} \\ N\left(\frac{2\left(5N+\sqrt{(5N)^2+\sigma(N+20)(N-1)}\right)}{N(20+N)}\right) & \text{if shares,} \end{cases} \qquad \tilde{X} = \begin{cases} N\left(\frac{20(N+\sigma)}{20+N^2}\right) & \text{if levels} \\ N\left(\frac{20N}{20+N^2}\right) & \text{if shares.} \end{cases}$$

The welfare maximizer always converges to $20 = \hat{X}$ as $N \to \infty$ while $X^*$ converges to $20+\sigma$ under levels and to $10 + 2\sqrt{5}\sqrt{5+\sigma} > 20$ under shares. Equating $X^*$ and $\tilde{X}$ yields

$$\overline{\sigma}(N) = \begin{cases} \frac{20}{N} & \text{if levels} \\ \frac{20}{N}\left(\frac{2\sqrt{5}N^2}{20+N^2}\right)^2 & \text{if shares.} \end{cases}$$

In both cases, $\lim_{N\to\infty} \overline{\sigma}(N) = 0$; however, when contribution benefits are derived from shares, $\overline{\sigma}(N)$ is increasing if $N \leq 7$ and decreasing if $N \geq 8$.

The critical distinctions between public goods with both consumption costs and contribution benefits and public goods with only contribution benefits is who bears the costs of over-provision and what happens when the population grows large. When there are no consumption costs, the lost utility from over-provision is isolated among the contributors. Consumers actually benefit with increased utility from free-riding. Over-provision is also less likely as the population grows, so the environment is more likely to resemble typical under-provision, where the costs are borne by consumers with suboptimal under-provision. When consumption costs and contribution benefits are present, much of the lost utility is transferred to the consumers rather than the contributors, particularly if the benefits accumulate based on levels. Consumers are faced with an inefficiently complex public good, so much so that they may opt to not even free-ride and instead not consume at all. While contributors are also faced with an inefficiently complex public good with low quality due to over-provision, they are more than compensated through their contribution benefits. Over-provision is more likely as the population grows, making this case the most severe and concerning when dealing with large populations.

Along the entire spectrum of collaboratively provided public goods studied here, equilibrium quality is monotonically increasing in the number of contributors whenever $\sigma = 0$. If $\sigma > 0$, then the relationship between quality and quantity is non-monotonic (the tradeoff) when consumption costs are present $\gamma = 0$. Over-provision can occur whenever contribution benefits are present except when there are no consumption costs and benefits depend on levels rather than shares. Collectively, the takeaway from these points cannot be understated. When constructing a policy regarding to optimize the provision of public goods, policies designed to increase contributions may backfire and lead to increased inefficiency. Moreover, the type of public good must be considered, particularly if a policymaker is not only interested in maximizing welfare, but also concerned with equity and the distribution of surpluses. The presence of consumption costs alters which side of the market bears a greater share of the inefficiency. In summary, there is no globally optimal policy and both policymakers and researchers alike must be very careful to consider the nature of the public good in question, carefully identifying the presence of consumption costs, contribution benefits, and the type of contribution benefits (levels or shares).

# 4 Inefficiently Complex Public Goods

Research suggests that regulation tends to be overly complex (Kearl, 1983; Quandt, 1983; Allison and Lemley, 2002; Berglund, 2014), leading to inefficiency (Murray and Stern, 2007; Williams, 2013). Similarly, more complex open source software projects tend to have lower user-satisfaction scores (Stamelos et al., 2002) and there tends to be less subsequent innovation in more complex industries (Galasso and Schankerman, 2015). Proposition 4 predicts such an outcome for public goods with both consumption costs and contribution benefits. Yet, many public goods including R&D innovation, open source software, and public policy, are not produced by independent individually-rational utility-maximizers. Rather, individual efforts are often coordinated by a leader or leadership team.

Open source software projects are guided by a leadership team, as are R&D teams. Public policy is often drafted by a small group of elected officials. While contributors act independently to maximize utility, they are not necessarily free to contribute as they see fit. In this section, I focus on a specific restriction: fixing the number of contributors. The leaders of open source software projects can choose whether or not to accept contributions. Similarly, drafters of legislation can pick and choose whose contributions to include. If this 'social planner' is interested in maximizing welfare, subject to contributing individuals satisfying individual rationality, how will this individual choose the number of contributors?

The objective of the social planner is given by

$$\max_{M}\Bigg\{(N-M)\max\{B(X)-C(X),0\} \\ +\sum_{j=1}^{M}\left[B(X)-C(X)+\sigma b(x_j,X_{-j})-c(x_j)\right]\Bigg\}, \quad (4)$$

subject to

$$\begin{aligned} &x_j = x(M,\sigma)=\arg\max\{B(X)-C(X)+\sigma b(x_j,X_{-j})-c(x_j)\} \quad \forall\, j\le M \\ &x_j=0 \quad \forall\, j>M \end{aligned} \quad (5)$$

Denote by $\hat{M}$ the solution to (4).

Firstly note that there can be no interior solution $\hat{M}<\overline{\sigma}^{-1}(\sigma)$ such that

$$\overline{\sigma}^{-1}(\sigma)<\frac{\hat{M}+(\hat{M}+1)}{2};$$

otherwise, adding one more contributor increases the net quality free-riders and both the quality and contribution benefit for the original $\hat{M}$ contributors while also increasing the utility of the new contributor by the marginal change in the quality plus the entirety of the contribution benefit, leading to a strict welfare gain.[10] This logic cannot be extended to the other side of $\overline{\sigma}^{-1}(\sigma)$. Suppose that $\hat{M}>\overline{\sigma}^{-1}(\sigma)$ and there exists a value $\hat{M}-1$ such that

$$\overline{\sigma}^{-1}(\sigma)<\frac{(\hat{M}-1)+\hat{M}}{2}.$$

By decreasing $\hat{M}$ to $\hat{M}-1$, the quality increases, as do the contribution benefits for each of the $\hat{M}$ contributors. Yet, this change may actually lead to a decrease in total welfare if the change in the quality, weighted by the number of consumers plus the change in the contribution benefits, weighted by the number of contributors, is less than the loss in the

[10] If $\sigma=0$, then the consumption benefit is a net loss on the new contributor, but by (3), the marginal impact of the loss is less than the marginal increase in quality leading to a positive change in utility.

contribution benefits by the $\hat{M}^{th}$ contributor:

$$\begin{aligned}(N-M)\big[&\max\{B(X(\hat{M}-1,\sigma))-C(X(\hat{M}-1,\sigma)),0\}\\&-\max\{B(X(\hat{M},\sigma))-C(X(\hat{M},\sigma)),0\}\big]\\&+M\big[B(X(\hat{M}-1,\sigma))-C(X(\hat{M}-1,\sigma))\\&-[B(X(\hat{M},\sigma))-C(X(\hat{M},\sigma))]\big]\\&+(\hat{M}-1)[\sigma b(x(\hat{M}-1,\sigma),X_{-i}(\hat{M}-1,\sigma))-c(x(\hat{M}-1,\sigma))\\&-[\sigma b(x(\hat{M},\sigma),X_{-i}(\hat{M},\sigma))-c(x(\hat{M},\sigma))]]\\&<\sigma b(x(\hat{M},\sigma),X_{-i}(\hat{M},\sigma))-c(x(\hat{M},\sigma)).\end{aligned} \tag{6}$$

Increasing quality by decreasing the number of contributors increases the utility for each contributor and each free-rider, but may decrease welfare due to the decrease in the utility of the contributor turned free-rider.

**Proposition 5.** *If* (6) *is satisfied, then the optimal strategy for a social planner interested in maximizing total welfare need not match the optimal strategy for a social planner interested in maximizing quality.*

Proposition 5 has bite due to the fact that maximizing quality is akin to concurrently maximizing both the quality and the contribution benefit for any given contributor. For the given set of contributors and consumers, each individual's utility is at its peak. A tension still remains on the extensive margin. Adding an additional contributor may decrease both the quality for all consumers and the contribution benefits for the original set of contributors, but increases the new contributor's payoff by becoming a contributor are large enough that aggregate welfare of the population at large is increased. Even if there is an integer $M$ such that $M = \overline{\sigma}^{-1}(\sigma)$, it may not be the welfare maximizer. Supply-side congestion exists on both the intensive and extensive margins.

Regardless of the type of contributor, the above shows that a social planner is willing to sacrifice the utility of a consumer to increase the utility of a contributor in order to maximize total welfare.

**Corollary 1.** *If the set of potential contributors is a strict subset of the population and $\sigma$ is small, then for a finite population, a social planner seeking to maximize total welfare will never choose the maximum number of contributors that maximizes quality.*

Corollary 1 provides a general extension of the results in Kearl (1983), Quandt (1983), Nitzan et al. (2013), and the other associated works on regulatory complexity. The economics underpinning regulatory capture are not unique to regulation, and can be applied to public goods more broadly. Under this welfare-maximization program, the social planner transfers surpluses from consumers to contributors. In order to obtain a more equitable distribution of surpluses between consumers and contributors, additionally measures need to be put in place that limit the incentives for over-provision, e.g. anonymizing contributions to lower $\sigma$.

# 5 Discussion and Concluding Remarks

This paper has outlined several novel and empirically relevant results. When extending the standard theory of public goods to incorporate consumption costs and contribution benefits, well known incentive problems change and new issues are introduced. While inefficiencies due to free-riding may still persist, over-provision due to supply-side congestion also presents a concern, but is not dealt with in the same manner as free-riding. Over-provision is made more severe by large populations. Moreover, policy prescriptions typically used to internalize the externalities driving these inefficiencies no longer necessarily have the same desirable properties. Without further consideration, maximizing welfare by controlling the number of contributors can lead to a deterioration of the quality to benefit the contributors. Policies need to be evaluated on a case-by-case basis depending on the presence of consumption costs and contribution benefits.

An interesting extension to consider is the dynamic aspects of public good provision. Regulations, statutes, and other legal guidelines such as tax codes tend to evolve over time with contributors (regulators) who have their own private and public incentives. The quality-enhancing properties of early contributions can induce a feedback loop that increases the contributions made in these regulations over time. The early contributions may lead to improvements in the quality of these regulations, but over time, these improvements diminish and eventually have deleterious effects, decaying the quality of these regulations. A similar story can be told with respect to open source software. Such a dynamic analysis focusing on the efficiency of provision over time presents an interesting avenue for future research.

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# Appendix

**Proof of Lemma 1**.

*Proof.* The (interior) first-order condition with respect to $i$ is

$$\frac{dB\left(X^*\right)}{dX} - \gamma\frac{dC\left(X^*\right)}{dX} + \sigma\frac{\partial b(x_i^*, X_{-i}^*)}{\partial x_i} - \frac{dc(x_i^*)}{dx_i} = 0.$$

Hence for an arbitrary $i$ and $j$,

$$\frac{dB\left(X^*\right)}{dX} - \gamma\frac{dC\left(X^*\right)}{dX} + \sigma\frac{\partial b(x_i^*, X_{-i}^*)}{\partial x_i} - \frac{dc(x_i^*)}{dx_i} \\ = \frac{dB\left(X^*\right)}{dX} - \gamma\frac{dC\left(X^*\right)}{dX} + \sigma\frac{\partial b(x_j^*, X_{-j}^*)}{\partial x_j} - \frac{dc(x_j^*)}{dx_i},$$

which simplifies to

$$\sigma\left(\frac{\partial b(x_i^*, X_{-i}^*)}{\partial x_i} - \frac{\partial b(x_j^*, X_{-j}^*)}{\partial x_j}\right) = \frac{dc(x_i^*)}{dx_i} - \frac{dc(x_j^*)}{dx_i}$$

If $x_i^* > x_j^*$, then the right-hand side is nonpositive while the left-hand side is strictly positive. Thus $x_i^* = x_j^*$. □

**Proof of Proposition 1**.

*Proof.* Set $\sigma = 0$ and $\gamma = 1$. For $x_i^* > 0$,

$$\frac{dB\left(X^*\right)}{dX} - \frac{dC\left(X^*\right)}{dX} - \frac{dc(x_i^*)}{dx_i} = 0, \tag{7}$$

By Lemma 1, $x_i^* = x_j^* = x^*$. Moreover,

$$N\left(\frac{dB\left(X\right)}{dX} - \frac{dC\left(X\right)}{dX}\right) > 0$$

for all $X$ and $N$ as $\frac{dc(x)}{dx_i} > 0$ when $x > 0$. Hence there is no equilibrium with $M < N$ as at least one non-contributing individual has a profitable deviation and there exists a symmetric Nash equilibrium with $N$ contributors and both $M^*(N) = \tilde{M}(N) = N$. The relevant first-order condition is now

$$\frac{dB\left(X(N,0)\right)}{dX} - \frac{dC\left(X(N,0)\right)}{dX} - \frac{dc\left(x_i(N,0)\right)}{dx_i} = 0, \quad \forall\, i. \tag{8}$$

To prove that the equilibrium level of contributions converges to the quality-maximizing level, suppose to the contrary that $x_i^* \to z > 0$ as $N \to \infty$. Convergence to a positive $z$ implies that $X^* \to \infty$ as $N \to \infty$. It follows that

$$\frac{dB\left(\infty\right)}{dX} - \frac{dC\left(\infty\right)}{dX} < 0.$$

As $\frac{dc(z)}{dx_i} < 0$, (8) cannot be satisfied. Thus $x_i^* \to 0$ as $N \to \infty$. As $x_i^*$ approaches zero and $\lim_{N\to\infty} \frac{dc\left(x_i^*\right)}{dx_i} = 0$, (8) holds only if

$$\lim_{N\to\infty}\left(\frac{dB\left(X^*\right)}{dX} - \frac{dC\left(X^*\right)}{dX}\right) = 0,$$

which is true when $\lim_{N\to\infty} X^* = \hat{X}$.

Now, consider the welfare maximizing case. The welfare maximizing contributions are determined by the solution to

$$\max_{\mathbf{x}} \sum_{j=1}^{N} u(X, x_j; \sigma).$$

The arbitrary first-order condition with respect to contribution $x_i > 0$ is given by

$$\frac{dB(\tilde{X})}{dX} - \frac{dC(\tilde{X})}{dX} - \frac{1}{N}\frac{dc(\tilde{x}_i)}{dx_i} = 0. \tag{9}$$

By inspection, $\tilde{x}_i = \tilde{x}_j$ for all $i, j$ and by an analogous argument to the above, $\tilde{M}(N) = N$. Taking the limit of (9) as $N \to \infty$ yields $\tilde{X} \to \hat{X}$.

Lastly to prove that $X^* < \tilde{X} < \hat{X}$ for all finite $N > 1$, note that $\frac{dc(x_i)}{dx_i} > 0$ for all $x_i > 0$. Evaluating the LHS of (8) at $x_i = x_i(N, 0)$ for all $i$ yields

$$\frac{dB\left(X(N,0)\right)}{dX} - \frac{dC\left(X(N,0)\right)}{dX} - \frac{1}{N}\frac{dc\left(x_i(N,0)\right)}{dx_i}.$$

As $\frac{dc(x_i(N,0))}{dx_i} > 0$,

$$-\frac{dc\left(x_i(N,0)\right)}{dx_i} < -\frac{1}{N}\frac{dc\left(x_i(N,0)\right)}{dx_i}.$$

From (8),

$$\begin{aligned}\frac{dB\left(X(N,0)\right)}{dX} - \frac{dC\left(X(N,0)\right)}{dX} - \frac{1}{N}\frac{dc\left(x_i(N,0)\right)}{dx_i}& \\ > \frac{dB\left(X(N,0)\right)}{dX} - \frac{dC\left(X(N,0)\right)}{dX} - \frac{dc\left(x_i(N,0)\right)}{dx_i} &= 0,\end{aligned}$$

so $\tilde{x}_i > x_i^*$ and $\tilde{X} > X^*$ for all $i$ and $j$ and all finite $N > 1$. The presence of $\frac{dc(x_i)}{dx_i} > 0$ for all $x_i > 0$ implies that both $X^*$ and $\tilde{X}$ are less than $\hat{X}$. □

## Proof of Proposition 2

*Proof.* At $x_i = 0$,

$$\begin{aligned}\frac{dB(X)}{dx} + \frac{\partial b(0, X_{-i})}{\partial x} &> 0 \\ N\frac{dB(X)}{dx} + \frac{\partial b(0, X_{-i})}{\partial x} &> 0,\end{aligned}$$

for all $X > 0$, so $M^*(N) = \tilde{M}(N) = N$. The remainder of the proof proceeds in two cases.

*Case 1*: $b(x_i, X_{-i}) = b(x_i)$. The first-order condition for the arbitrary contributor is given by

$$\frac{dB(X^*)}{dX} + \sigma \frac{db(x^*)}{dx_i} - \frac{dc(x^*)}{dx_i} = 0$$

As $B(\cdot)$ and $b(\cdot)$ are weakly concave while $c(\cdot)$ is strictly convex, the first two terms are nonincreasing while the third term is strictly in increasing. Therefore both $x^*$ and $X^*$ are strictly increasing in $\sigma$. The welfare-maximizing first-order condition for an arbitrary contributor is given by

$$N\frac{dB(X^*)}{dX} + \sigma \frac{db(x^*)}{dx_i} - \frac{dc(x^*)}{dx_i} = 0,$$

so by the same argument, $\tilde{x}$ and $\tilde{X}$ are strictly increasing in $\sigma$. Now suppose that $\tilde{x} = x^*$, so $X^* = \tilde{X}$. Then

$$\frac{dB(X^*)}{dX} + \sigma \frac{db(x^*)}{dx_i} - \frac{dc(x^*)}{dx_i} = N\frac{dB(\tilde{X})}{dX} + \sigma \frac{db(\tilde{x})}{dx_i} - \frac{dc(\tilde{x})}{dx_i} = 0,$$

which reduces to

$$\frac{dB(X^*)}{dX} = N\frac{dB(\tilde{X})}{dX},$$

a contradiction. As the right-hand side exceeds the left and $B(\cdot)$ is weakly concave, it follows that $\tilde{X} > X^*$.

*Case 2*: $b(x_i, X_{-i}) = b\left(\frac{x_i}{X}\right)$. The equilibrium and welfare-maximizing first order conditions for individual $i$ are respectively given by

$$\frac{dB(X^*)}{dX} + \sigma \frac{\partial b\left(\frac{x^*}{X^*}\right)}{\partial x_i} \frac{X^*_{-i}}{(X^*)^2} - \frac{dc(x^*)}{dx_i} = 0$$

$$N\frac{dB(\tilde{X})}{dX} + \sigma \frac{\partial b\left(\frac{\tilde{x}}{\tilde{X}}\right)}{\partial x_i} \frac{\tilde{X}_{-i}}{\tilde{X}^2} - \sigma \sum_{j \neq i} \frac{\partial b\left(\frac{\tilde{x}}{\tilde{X}}\right)}{\partial x_j} \frac{\tilde{x}}{\tilde{X}^2} - \frac{dc(\tilde{x})}{dx_i} = 0.$$

By Lemma 1, $\frac{x^*}{X^*} = \frac{\tilde{x}}{\tilde{X}} = \frac{1}{N}$, $\frac{X^*_{-i}}{(X^*)^2} = \frac{N-1}{N^2 x^*}$, and $\frac{\tilde{X}_{-i}}{\tilde{X}^2} = \frac{N-1}{N^2 \tilde{x}}$. Thus the above first-order conditions can be rewritten as

$$\frac{dB(X^*)}{dX} + \sigma \frac{\partial b\left(\frac{1}{N}\right)}{\partial x_i} \frac{N-1}{N^2 x^*} - \frac{dc(x^*)}{dx_i} = 0$$

$$N\frac{dB(\tilde{X})}{dX} + \underbrace{\sigma \frac{\partial b\left(\frac{1}{N}\right)}{\partial x_i} \frac{N-1}{N^2 \tilde{x}} - \sigma(N-1) \frac{\partial b\left(\frac{1}{N}\right)}{\partial x_j} \frac{1}{N^2 \tilde{x}}}_{=0} - \frac{dc(\tilde{x})}{dx_i} = 0.$$

It follows that $\tilde{x}$ and $\tilde{X}$ are independent of $\sigma$. Now, fix $N$. As $\sigma \frac{\partial b\left(\frac{1}{N}\right)}{\partial x_i} \frac{N-1}{N^2 x^*}$ is strictly increasing in $\sigma$, it must be that $x^*$ and thus $X^*$ are increasing in $\sigma$ and a cutoff value $\overline{\sigma}(N)$ exists where $X^* > \tilde{X}$ if and only if $\sigma > \overline{\sigma}(N)$. □

**Proof of Proposition 3**

*Proof.* Fix $M' < N$. By an analogous argument to that in Proposition 2, $M^*(N) = \tilde{M}(N) = M'$. The relevant first-order conditions for the equilibrium and welfare-maximizing program are respectively given by

$$\frac{dB(X^*)}{dX} + \sigma\frac{\partial b\left(\frac{1}{M'}\right)}{\partial x_i}\frac{M'-1}{(M')^2x^*} - \frac{dc(x^*)}{dx_i} = 0$$
$$N\frac{dB(\tilde{X})}{dX} - \frac{dc(\tilde{x})}{dx_i} = 0.$$

As in Proposition 2, $\sigma\frac{\partial b\left(\frac{1}{M'}\right)}{\partial x_i}\frac{M'-1}{(M')^2x^*}$ is strictly increasing in $\sigma$ and decreasing in $x^*$, so $\overline{\overline{\sigma}}(M', N)$ is defined analogously to $\overline{\sigma}(N)$. As $\tilde{X}$ is strictly increasing in $N$, $\overline{\overline{\sigma}}(M', N)$ is strictly increasing in $N$. □

**Proof of Proposition 4**.

*Proof.* I begin by first proving that the cutoff value $\overline{\sigma}(M)$ exists for any arbitrary number of contributors $M$. By lemma 1, $\frac{\partial x_i(M,\sigma)}{\partial\sigma} = \frac{\partial x_j(M,\sigma)}{\partial\sigma} = \frac{\partial x(M,\sigma)}{\partial\sigma}$. The first step is to show that $\frac{\partial x(M,\sigma)}{\partial\sigma} > 0$. Fix $M$ and recall that, for each contributor $i = 1, \ldots, M$, it must be that

$$\frac{dB\left(Mx^*\right)}{dX} - \frac{dC\left(Mx^*\right)}{dX} + \sigma\frac{\partial b\left(x^*, (M-1)x^*\right)}{\partial x_i} - \frac{dc(x^*)}{dx_i} = 0 \tag{10}$$

holds in equilibrium. Differentiating (10) with respect to $\sigma$ yields

$$M\underbrace{\left(\frac{d^2B(X(M,\sigma))}{dX^2} - \frac{d^2C(X(M,\sigma))}{dX^2}\right)}_{\Omega_1}\frac{\partial x(M,\sigma)}{\partial\sigma} + \sigma M\underbrace{\left(\frac{\partial^2 b(x(M,\sigma),(M-1)x(M,\sigma))}{\partial {x_i}^2}\right)}_{\Omega_2}\frac{\partial x(M,\sigma)}{\partial\sigma} \underbrace{-\frac{d^2c(x^*)}{dx_i^2}}_{\Omega_3}\frac{\partial x(M,\sigma)}{\partial\sigma} = -\frac{\partial b\left(x^*, (M-1)x^*\right)}{\partial x_i} \tag{11}$$

The right-hand side of (11) is strictly negative, so $\frac{\partial x(M,\sigma)}{\partial\sigma} \neq 0$ for all $\sigma \geq 0$. Therefore $x(M,\sigma)$ is monotonic in $\sigma$. $\Omega_1 < 0$, $\Omega_2 \leq 0$, and $\Omega_3 < 0$. Hence the left-hand side is negative if and only if $\frac{\partial x(M,\sigma)}{\partial\sigma} > 0$.

As $\frac{\partial x(M,\sigma)}{\partial \sigma} > 0$, $X(M,\sigma)$ is also increasing in $\sigma$. At $\sigma = 0$,

$$\frac{dB(X(M,0))}{dX} - \frac{dC(X(M,0))}{dX} > 0,$$

so there must exist a value $\overline{\sigma}(M) > 0$ for every $M \geq 1$ such that for all $\sigma < \overline{\sigma}(M)$, property $P1$ holds and for all $\sigma > \overline{\sigma}(M)$, property $P2$ holds.

Proving the monotonicity / non-monotonicity of $\overline{\sigma}(M)$ proceeds in two cases.

*Case 1*: $b(x_i, X_{-i}) = b(x_i)$. Proving that $\overline{\sigma}(M)$ is decreasing in $M$ as it was without consumption costs (Proposition 2) first requires showing that $X^*$ is increasing in $M$. Fix $\sigma$ and suppose to the contrary that $X^*$ is decreasing in $M$ on any subset of $\mathbb{N} \setminus \{0, 1\}$. Then,

$$\begin{aligned}\frac{dB\left(X(M,\sigma)\right)}{dX} - \frac{dC\left(X(M,\sigma)\right)}{dX} + \sigma\frac{\partial b\left(x(M,\sigma)\right)}{\partial x_i} &= \frac{dc(x(M,\sigma))}{dx_i} \\ > \frac{dc(x(M+1,\sigma))}{dx_i} &= \frac{dB\left(X(M+1,\sigma)\right)}{dX} - \frac{dC\left(X(M+1,\sigma)\right)}{dX} + \sigma\frac{\partial b\left(x(M+1)\right)}{\partial x_i},\end{aligned}$$

a contradiction. Thus $X^*$ is increasing in $M$.

Now to show that $\overline{\sigma}(M)$ is decreasing in $M$, suppose that $X(M,\sigma)$ is such that

$$\frac{dB\left(X(M,\sigma)\right)}{dX} - \frac{dC\left(X(M,\sigma)\right)}{dX} \leq 0$$

and

$$\frac{dB\left(X(M+1,\sigma)\right)}{dX} - \frac{dC\left(X(M+1,\sigma)\right)}{dX} < 0.$$

It follows that $\sigma \leq \overline{\sigma}(M)$ but $\sigma > \overline{\sigma}(M+1)$, which implies that $\overline{\sigma}(M)$ is decreasing in $M$. To show that as $M \to \infty$, $\overline{\sigma}(M) \to 0$ recall that by Proposition 1, $x^* \to 0$ as $M \to \infty$, while $\lim_{M\to\infty} \frac{\partial b(x(M,\sigma),(M-1)x(M,\sigma))}{\partial x_i} - \frac{dc(x(M,\sigma))}{dx_i} > 0$ for all $\sigma > 0$. The result follows from $P2$.

*Case 2*: $b(x_i, X_{-i}) = b\left(\frac{x_i}{X}\right)$. This case is proven by a counter-example to monotonicity provided in the text.

To prove statement (i), suppose that $\sigma < \overline{\sigma}(N)$. By the above argument, $P1$ applies and the result follows immediately from an identical argument to that of Proposition 1.

To prove statement (ii), suppose that $\sigma > \overline{\sigma}(N)$, so $P2$ applies. Otherwise, there would exist an equilibrium with $M^*(N) < N$ under $P1$, a contradiction. The first-order conditions for a maximum are given by

$$\frac{dB\left(X^*\right)}{dX} - \frac{dC\left(X^*\right)}{dX} + \frac{\partial b\left(x^*, (M-1)x^*\right)}{\partial x_i} - \frac{dc(x^*)}{dx_i} = 0, \quad \forall\, i = 1, \ldots, M \tag{12}$$

while the welfare maximizing conditions with $M$ contributors is given by

$$\frac{dB(\tilde{X})}{dX} - \frac{dC(\tilde{X})}{dX} + \frac{1}{N}\left(\frac{\partial b(\tilde{x},(M-1)\tilde{x})}{\partial x_i} - \frac{dc(\tilde{x})}{dx_i}\right) = 0, \quad \forall\, i \leq M. \tag{13}$$

Given (12),

$$\frac{dB\left(X^*\right)}{dX} - \frac{dC\left(X^*\right)}{dX} + \frac{1}{N}\left(\frac{\partial b\left(x^*,(M-1)x^*\right)}{\partial x_i} - \frac{dc(x^*)}{dx_i}\right) < 0.$$

Thus $x^* > \tilde{x}$ and by extension, $X^* > \tilde{X}$. Both being greater than $\hat{X}$ follows immediately.

To prove statement (iii), note that $\tilde{X} \to \hat{X}$ as $N \to \infty$ follows from taking the limit of (13) as $N \to \infty$. By Proposition 1, $x^* \to 0$ as $N \to \infty$. As $\lim_{x\to 0^+} \frac{\partial b(x,X_{-i})}{\partial x} - \frac{dc(x)}{dx_i} > 0$ for $\sigma > 0$, $\frac{dB(X(\infty,\sigma))}{dX} - \frac{dC(X(\infty,\sigma))}{dX} < 0$, which implies that $X^* > \hat{X}$ for all $\sigma > 0$. □

**Proof of Proposition 5**.

*Proof.* Suppose that $\hat{M} > \overline{\sigma}^{-1}(\sigma)$ and there exists an integer $\hat{M}-1$ such that

$$\overline{\sigma}^{-1}(\sigma) < \frac{(\hat{M}-1)+\hat{M}}{2},$$

so producing the good with $\hat{M}-1$ contributors leads to a good with a higher quality than having $\hat{M}$ contributors. It immediately follows that if (6) holds, then $\hat{M}$ leads to greater aggregate welfare than $\hat{M}-1$. □

**Proof of Corollary 1**.

*Proof.* The result follows from Proposition 5. □