Notes of Ch7: Estimation I, "introductory Statistics" by Wonnacott

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1 Introduction

because population is fixed so the population parameters like mean μ and variance σ^2 are fixed (though generally unknown).
the sample mean $\bar{X}$ and sample variance s^2 are random variables, varying from sample to sample, with certain probability distribution. the random variables calculated from the observations in a sample are called sample statistic.
the point estimate of μ is the an estimation by computing one sample mean, but it is distributed around μ. so we must estimate that μ is bracketed by some interval –known as confidence interval– of the following form:

$$\mu = \bar{X} \pm \text{ an error allowance} \tag{1}$$

as we can be specific about distribution of $\bar{X}$, we can be specific about this error allowance.
How confident do we wish to be that our estimate is right about the μ? it is common to choose 95%100 confidence. to get this confidence level the smallest range under normal distribution of $\bar{X}$ that we just enclose a 95%100 probability. since the distribution is symmetric around the middle chunk, we can leave 2.5%100 probability in each side. we note that this involves going above(or below) the mean by 1.96 standard deviation of $\bar{X}$. we there for write:

$$p(\mu - 1.96\frac{\sigma}{\sqrt{n}} < \bar{X} < \mu + 1.96\frac{\sigma}{\sqrt{n}}) = 95\%$$
$$\Rightarrow p(-1.96 < \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} < 1.96) = 95\%$$

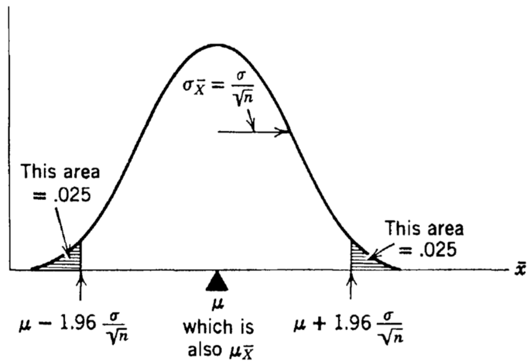

FIG. 7-1 Distribution of sample mean $\bar{X} \sim N[\mu, (\sigma^2/n)]$. (*Note.* μ is an *unknown constant;* we don't know what its value is; all we know is that, whatever μ may be, the *variable* $\bar{X}$ is distributed around it as shown in this diagram.)

as long as $\bar{X}$ is a random variable, it is referred to as an "estimator" of μ. but once a sample has been observed and $\bar{X}$ takes on a specific value, it is called then "estimate" of μ and denoted by $\bar{x}$. we might call $\bar{x}$ the realized value, and $\bar{X}$ the potential value. the last equation sometimes abbreviated to:

$$\text{95\% confidence interval :}$$
$$\mu = \bar{x} \pm z_{0.025} \frac{\sigma}{\sqrt{n}} \tag{2}$$

where $z_{0.025}$ is the critical value leaving 2.5% probability in the upper tail of the standard normal distribution.

2 Desirable properties of estimators