4200 - Formal Languages
Midterm 2

Fall 2021

Oct 21, 2021 at 2:00 - 2:50pm

Student:

Directions: The test is open book and open notes, but NOT open electronic devices (e.g. phone, tablets or computers). For each problem, show your work completely. Give reasons for all answers. You will not only be graded on your mathematics, but also on your presentation, organization, proper use of English, spelling, punctuation, and logic.
There are **3 problems** for a total of **65 points**.

Problem 1

30 points
For each of the languages below, please answer the following two questions:
(a) Is the language context-free or not?
(b) If context-free, please construct a context-free grammar (CFG) that generates exactly the language. Both ambiguous and unambiguous grammars are acceptable (it does not matter in this question). For grading purposes, please use S as the starting variable. Alphabet $\Sigma = \{0, 1\}$.

1. $A = \{1, \epsilon\} \cup \{1\}^*$

a. Yes $A = \{1, \varepsilon\} \cup \{1\}^* = \{1\}^*$

b. $S \rightarrow \varepsilon \mid 1S$

2. $B = \{\ 1^n 0^{2n} 0^{n+1} \mid n \geq 0\ \}$

a. Yes $B = \{1^n 0^{3n} 0 \mid n \geq 0\}$

b. $S \rightarrow A0$
$A \rightarrow 1A000 \mid \varepsilon$

3. $C = \{\ 0^n 1^{2n} 0^{3n} \mid n \geq 0\ \}$

a. NO, C is not context free.

Problem 2

30 points

For each of the languages below, please answer the following ~~two~~ three questions:

(a) Is the language regular or not?

(b) Is the language context-free or not?

(c) If context-free, please construct a pushdown automaton (PDA) that recognizes the language.

1. $D = \{ww^R w^R \mid w \in \{0,1\}^*\}$
   Here, w^R is the reverse of the string w (e.g., 001 is the reverse of 100).

a. NO

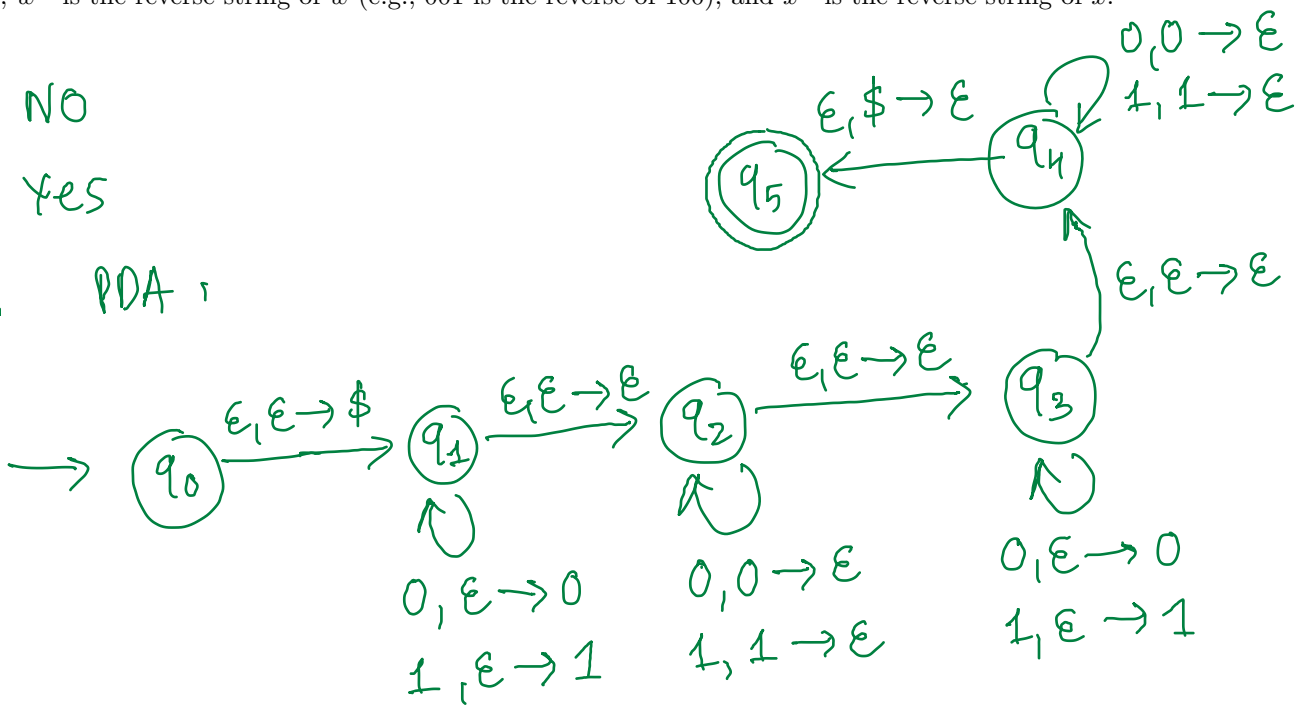
b. NO

2. $E = \{ww^R x^R x \mid w \in \{0,1\}^* \text{ and } x \in \{0,1\}^*\}$
   Here, w^R is the reverse string of w (e.g., 001 is the reverse of 100), and x^R is the reverse string of x.

a. NO

b. Yes

c. PDA:



The end.