4200 - Formal Languages: Final Exam

Spring 2021

May 1, 2020 at 8:00 - 10:30am

Student:

Directions: The test is open-book and open-note, but using electronic devices (e.g. phone, tablets or computers) during the exam is NOT allowed. For each problem, show your work completely. Give reasons for all answers.
There are **7 problems** for a total of **50 points**.

Problem 1

20 points
Provide a regular expression α such that $L(\alpha) = A$ where A is the following language:

$$A = \Sigma^* - \{01\}$$

where the alphabet $\Sigma = \{0, 1\}$.

Hint: One approach is to write down a DFA or an NFA that recognizes the language A first (in your scratch paper). And then, construct a regular expression that is equivalent to that DFA/NFA.

Answer: The steps (in scratch paper) are:

1. Write down a DFA that recognizes $\{01\}$ language.
2. Swap the accept and reject states to create a DFA that recognizes $A = \Sigma^* - \{01\}$.
3. Look at transitions in the above DFA, and write down the regular expression accordingly.

Final answer:

$$\alpha = \epsilon \ \cup \ (0 \cup 11)(0 \cup 1)^* \ \cup \ 10(0 \cup 1)(0 \cup 1)^* \tag{1}$$

Problem 2

10 points

Write down a context-free grammar for the following language E. Please use S as the start variable.

$$B = \{\ 1^x 0^{x-y} \mid x \geq y \geq 0\ \}$$

Answer:

$$S \to 1S \mid 1S0 \mid \epsilon \tag{2}$$

Problem 3

20 points

Given a language $C = \{1^x 1^y 0^{2z+1} \mid x, y, z \geq 0 \text{ and } x = y\}$. Please answer the following questions:

1. Is the above language regular or non-regular? If Yes, provide a regular expression that recognizes C.
   Answer: Regular. Regular expression: $(11)^*(00)^*0$.

2. Is the above language context-free or not? If Yes, provide a context-free grammar that generates C.
   Answer: Context-free.

$$S \to AB0$$
$$A \to 11A \mid \epsilon$$
$$B \to 00B \mid \epsilon$$

3. Is the above language Turing-decidable or not?
   Answer: Yes. Because any context-free language is also a Turing-decidable language.

4. Is the above language Turing-recognizable or not?
   Answer: Yes. Because any context-free language is also Turing-recognizable language.

Problem 4

5 points
A and B are both regular languages, would $A \cup B$ always be **regular**? Provide 1-3 sentences explaining your answer (i.e. why or why not).

Answer: Yes, $A \cup B$ would always be regular because regular languages are closed under the Union operation.

Problem 5

5 points
A is a regular language and B is a non-regular language, would $A \cup B$ always be **non-regular**? Please provide an example (or counterexample) to support your answer.

Answer: No. For example, if $A = \{0, 1\}^*$, i.e., the set of all possible strings, and B is any non-regular language, then $A \cup B = A$. And because $\{0, 1\}^*$ is regular, then in this case, $A \cup B$ is regular.

Problem 6

5 points
A is regular language and B is a non-context-free language, would $A \cap B$ always be **non-context-free**? Please provide an example (or counterexample) to support your answer.

Answer: No. For example, if $A = \emptyset$, i.e., the empty set, and B is a context-free language, then $A \cap B = A = \emptyset$. Because $\emptyset$ is a regular language, then in this case, $A \cap B$ is a regular language, which by definition is also context-free.