4200 - Formal Languages: Solution to Homework #2

Due on Wed, Sep 7, 2018 at 2:00pm

Instructor: Dr. Anh Nguyen

Note: Late assignments will not be graded. You will not only be graded on your mathematics, but also on your organization, proper use of English, spelling, punctuation, and logic.
There are in total 2 problems in this homework.

Problem 1

15 points
Draw the state diagram of DFAs recognizing the following languages.

a. A = $\{w|w$ starts with 0 and has odd length, or starts with 1 and has even length$\}$

b. B = $\{w|w$ is any string except 11 and 111$\}$

c. C = $\{\epsilon, 0\}$

Solution:

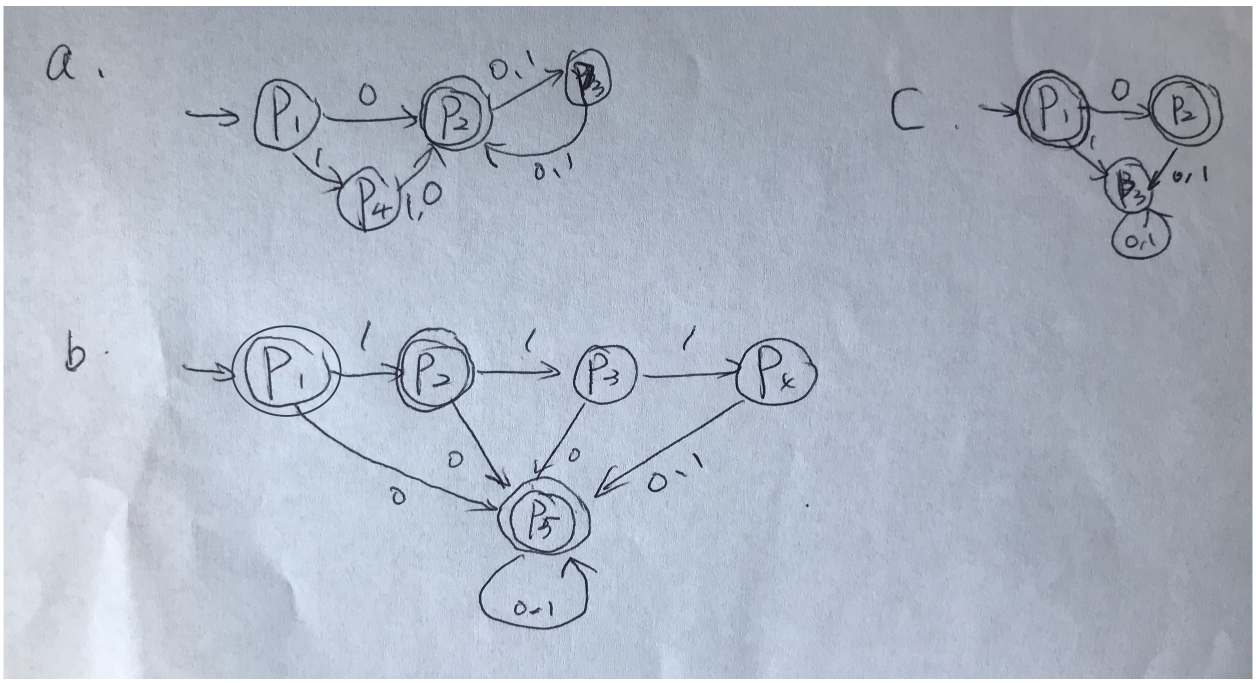

Figure 1:

Problem 2

10 points
Example of set difference: $A = \{0, 01\}$, and $B = \{0, 11\}$. Then, $A - B = \{01\}$.

Prove that regular languages are closed under the set *difference* operation. That is, if A and B are regular languages, then, $A - B$ is also a regular language.

Hint: One can prove the statement above by either (1) contradiction or (2) construction. For the proof, you may make use of the theorems that regular languages are closed under *union*, *intersection*, and *complement* (already discussed in class).

Solution 1 - Proof by contradiction
First, we know that $A - B = A \cap B^C$. And because B is regular, then $\Sigma^* - B = B^C$ is also regular via the closure of regular languages under the *complement* operation.
Assumption: We assume that $A \cap B^C$ is not regular.
Because regular languages are closed under intersection, therefore, the assumption implies that at least A or B^C must not be a regular language. However, this contradicts the known facts that both A and B^C are regular.
Therefore, $A \cap B^C$ is regular. $\square$

Solution 2 - Proof by construction
Similar to the proof in the **footnote** of page 46 in the textbook.