# 4200 - Formal Languages
# Midterm 1

Fall 2022

Sep 28, 2022 at 2:00 - 2:50pm

Student:

AU username:

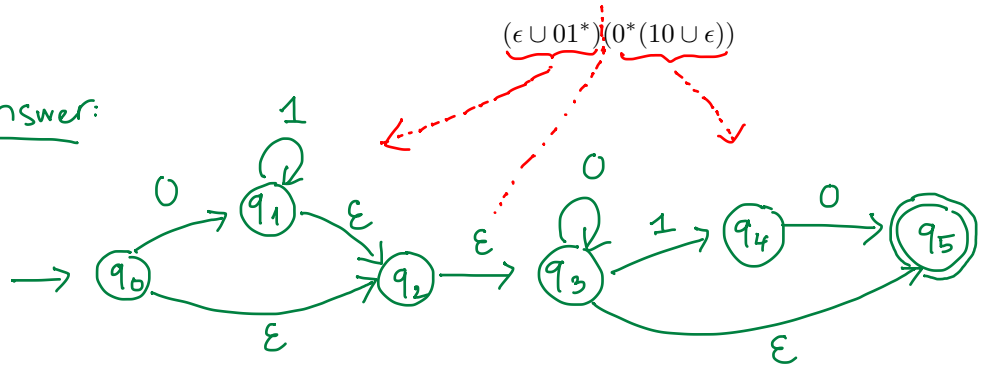
**Directions:** The test is open book and open notes, but NOT open electronic devices (e.g. phone, tablets or computers). For each problem, show your work completely. Give reasons for all answers. You will not only be graded on your mathematics, but also on your presentation, organization, proper use of English, spelling, punctuation, and logic.
There are **4 problems** for a total of **70 points**.

# Problem 1

**10 points**
For the following regular expression, draw a finite-state automaton (may be deterministic or non-deterministic). Please do not simplify the given regular expression.

$$(\epsilon \cup 01^*)(0^*(10 \cup \epsilon)) \tag{1}$$

Answer:




# Problem 2

**20 points**
Convert the NFA $M$ in Figure 1 into a regular expression $\alpha$ such that $L(\alpha) = L(M)$.
**Notes:** Please write the final expression below (on the "**Final expression:**"). In page 3, show step-by-step how you derive the final expression. For each step, (1) explain which state is being removed and (2) show the resultant intermediate FA.

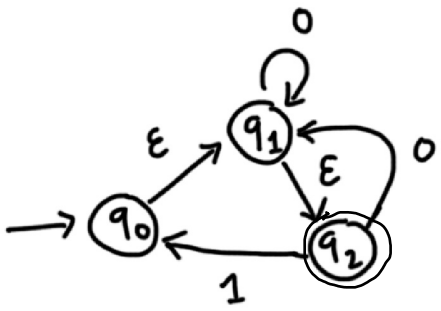


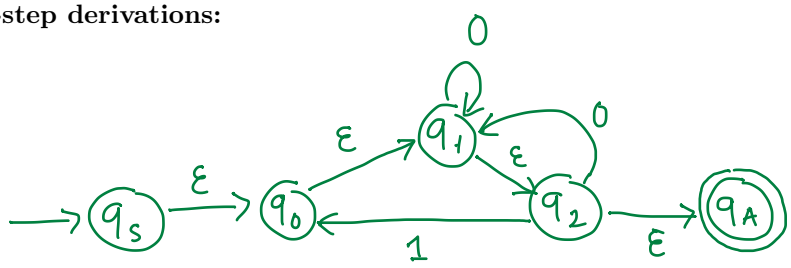

Figure 1: Non-deterministic finite-state automaton $M$.

$(0 \cup 1)^*$

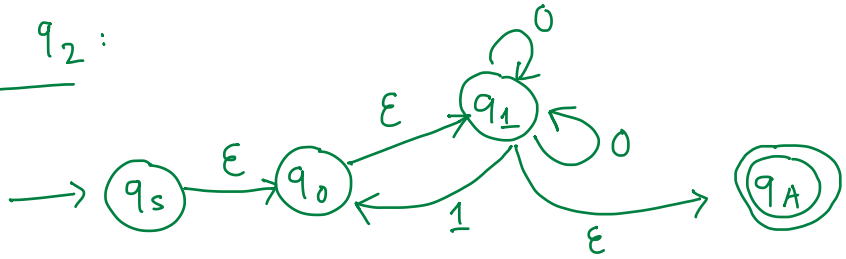
**Final expression:**

**Step-by-step derivations:**

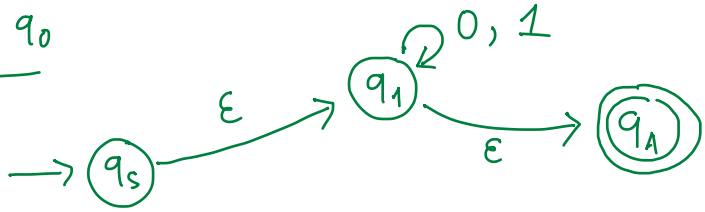
gNFA :



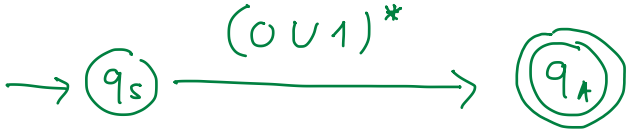

Remove $q_2$ :




Remove $q_0$




Remove $q_1$



# Problem 3

**20 points**

For the following set of strings $A$ over alphabet $\Sigma = \{0, 1\}$, demonstrate that $A$ is regular by constructing an NFA, DFA or a regular expression.

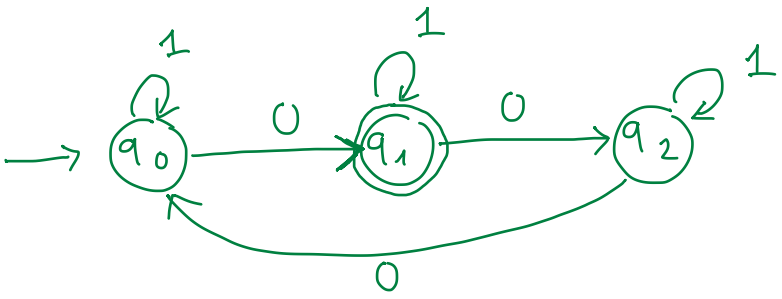
$$A = \{w \in \Sigma^* \mid (\#0(w)\%3 = 1\}$$
$$= \{w \in \Sigma^* \mid \text{dividing the number of 0's in the string by 3 yields a remainder of 1}\}$$

An example string $\in A$ is: 001010 because the total number of 0's is four, which yields a remainder of 1 when divided by 3.

1. List out 5 strings in $A$:

011, 01, 0000, 010010 11, 0, ...

2. Provide an NFA, DFA or a regular expression below.



# Problem 4

**20 points**

Alphabet $\Sigma = \{0, 1\}$.

For each provided language ($X$ and $Y$), please answer these two questions:

(a) Is the language regular or non-regular?

(b) If your answer is regular, please provide either a DFA, NFA or regular expression (choose only one) that recognizes the language. If your answer is non-regular, please prove it by contradiction using Pumping Lemma. The presentation format of the proof is expected to follow those examples in the book.

1. $X = \{\ 0^m 0^n \mid m, n \geq 0 \ \text{ and } \ m > n + 1\}$

Regular. Because X can be written as:

$X = \{ 0^{m+n} \mid m, n \geq 0 \text{ and } m + n > 2n + 1 \geq 1 \}$

Regular Expression:

$\alpha = 000^*$

$\alpha = 000^*$ ✓

2. $Y = \{\ 0^m 1^n \mid m, n \geq 1 \text{ and } m < n + 1\}$

Answer : not regular.

Follow the prove for $\{ 0^n 1^n \mid n \geq 0 \}$ in the book.

Here choose counterexample string $s = 0^p 1^p$

and when you pump up (not down!)

$xy^2z$ would be a string where # of 0's is > the # of 1's and not satisfying $m < n+1$.

**The end.**