

Homework 6

1. By pumping lemma, there exists an integer $p \geq 1$, such that every string s in A with $|s| \geq p$ can be written as $s = uvwxy$

1. $|vwx| \geq 1$

2. $|vwx| \leq p$

3. $uv^nwx^ny \in A$ for all $n \geq 0$

Now string $s = 0^p \# 0^{2p} \# 0^{3p}$. It is in A . So

to satisfy the 3 conditions we write $s = uvwxy$

Now, since $|vwx| \leq p$ therefore the following 2 cases are only possible

1) substring vwx spans completely over $0^p \# 0^{2p} \# 0^{3p}$ but pumping lemma fails in this case

2) substring vwx spans completely over suffix $0^{2p} \# 0^{3p}$ of string $s = 0^p \# 0^{2p} \# 0^{3p}$

and this also fails

So it is Not a context-free language.

2. $\{w \mid \text{the number of 0's on the left and right is equal}\}$
 $\{s \rightarrow ASA \mid \# \}$ because $B = \{0^{3n} \# 0^{3n} \mid n \geq 0\}$
 $\{A \rightarrow 000\}$

Here, only A is equal to 3 zeroes, and s is a recursive segment, which calls on both sides equally and will add 3 zeroes to both sides.

S will stop accepting $\#$ at the center position.

3. Using Pumping lemma to show that b is not context free. we use 111000000111

$$p=3, S=1^{3n}0^{3n}1^{3n}$$

$u=111, vxy=000, z=000111$, now substitute uv^nxy^nz , $n=2, uv^2xy^2z$

The result is $u=111, v^2=00, x=0, y^2=00, z=000111$
 $\Rightarrow 111000000111$

It's a palindrome but not equal number of 0's and 1's
So they are not equal.

It fails the first pumping lemma rule.

So it is NOT a context-free language.

4. Using contradiction we assume C has pumping length p . So, $S=1^p3^p2^p4^p2^pC > p$ so we can have 4 cases to show a contradiction.

1) vxy contains a 1. $uv^2xy^2z=C$, it will not have the same amount of 1's and 2's.

2) vxy contains a 2. $uv^2xy^2z=C$, it will not have the same amount of 1's and 2's.

3) vxy contains a 3. $uv^2xy^2z=C$, it will not have the same amount of 3's and 4's.

4) vxy contains a 4. $uv^2xy^2z=C$, it will not have the same amount of 3's and 4's.

Therefore it means it contradicts at least 1 from the requirement

So it is NOT a context-free language.