4200 - Formal Languages: Homework #7 Solutions

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Problem 1

Draw the state-transition diagram for a Turing Machine M that *decides* each of the following languages.That is, M is supposed to accept all strings in the language and reject all strings not in the language (it *cannot* loop forever on any input).

1. $A = \emptyset$
   $q_0\ \square \longrightarrow q_{reject}$
   $q_0\ 0 \longrightarrow q_{reject}$
   $q_0\ 1 \longrightarrow q_{reject}$
2. $B = \{\epsilon\}$
   $q_0\ \square \longrightarrow q_{accept}$
   $q_0\ 0 \longrightarrow q_{reject}$
   $q_0\ 1 \longrightarrow q_{reject}$
3. $C = \{0\}$
   $q_0\ 0 \longrightarrow q_1\ 0\ R$
   $q_0\ 1 \longrightarrow q_{reject}$
   $q_1\ 0 \longrightarrow q_{reject}$
   $q_1\ 1 \longrightarrow q_{reject}$
   $q_1\ \square \longrightarrow q_{accept}$
4. $D = \{1\}^*$
   $q_0\ 1 \longrightarrow q_0\ 1\ R$
   $q_0\ 0 \longrightarrow q_{reject}$
   $q_0\ \square \longrightarrow q_{accept}$

Problem 2

Show a Turing Machine M that decides the following language.

$$E = \{\ yy \mid y \in \{0,1\}^*\ \}$$

Idea

We will need following additional characters - a, b, α, β and $*$.

First step will be to divide the string into two halves. We will do that with following steps:

1. At the start, we are on the leftmost 0 or 1. Replace it by a or b respectively and go to rightmost 0 or 1.
2. Now, we are at the rightmost 0 or 1. Replace it by α or β respectively and go to rightmost 0 or 1. Perform step 1.

So now we easily distinguish between halves of the string. During this two steps, we can also check if the length of string is divisible by 2.

Next step will be to actually compare this two halves. We will do it by following steps:

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1. Suppose we are at the leftmost α or β. Replace it with $*$, save it with state and go to leftmost a or b, compare with value saved and delete the leftmost a or b.
2. Return to the leftmost α or β, perform step 1.

If we don't encounter any mismatches, then string will be accepted.

Transitions

Phase 1 - Replace occurrences 0 *and* 1 *with* a, b, α *and* β

$q_0^1\ 0 \longrightarrow q_R^1\ a\ R$
$q_0^1\ 1 \longrightarrow q_R^1\ b\ R$
$q_0^1\ \alpha \longrightarrow q_0^2\ \alpha\ L$
$q_0^1\ \beta \longrightarrow q_0^2\ \beta\ L$
$q_0^1\ \Delta \longrightarrow q_{accept}$

$q_1^1\ 0 \longrightarrow q_L^1\ \alpha\ R$
$q_1^1\ 1 \longrightarrow q_L^1\ \beta\ R$
$q_1^1\ a \longrightarrow q_{reject}$
$q_1^1\ b \longrightarrow q_{reject}$

$q_R^1\ 0 \longrightarrow q_R^1\ 0\ R$
$q_R^1\ 1 \longrightarrow q_R^1\ 1\ R$
$q_R^1\ \alpha \longrightarrow q_1^1\ \alpha\ R$
$q_R^1\ \beta \longrightarrow q_1^1\ \beta\ R$
$q_R^1\ \square \longrightarrow q_1^1\ \square\ L$

$q_L^1\ 0 \longrightarrow q_L^1\ 0\ L$
$q_L^1\ 1 \longrightarrow q_L^1\ 1\ L$
$q_L^1\ a \longrightarrow q_0^1\ a\ L$
$q_L^1\ b \longrightarrow q_0^1\ b\ L$
$q_L^1\ \square \longrightarrow q_0^1\ \square\ R$

Phase 2 - The head is pointing to the leftmost α *or* β*. We will start comparing the two halves*

$q_0^2\ \alpha \longrightarrow q_{L0}^2\ *\ L$
$q_0^2\ \beta \longrightarrow q_{L1}^2\ *\ L$
$q_0^2\ a \longrightarrow q_0^2\ a\ R$
$q_0^2\ b \longrightarrow q_0^2\ b\ R$
$q_0^2\ * \longrightarrow q_0^2\ *\ R$
$q_0^2\ \square \longrightarrow q_{accept}$

$q_{L0}^2\ a \longrightarrow q_{L0}^2\ a\ L$
$q_{L0}^2\ b \longrightarrow q_{L0}^2\ b\ L$
$q_{L0}^2\ \square \longrightarrow q_{C0}^2\ \square\ R$

$q_{L1}^2\ a \longrightarrow q_{L1}^2\ a\ L$
$q_{L1}^2\ b \longrightarrow q_{L1}^2\ b\ L$
$q_{L1}^2\ \square \longrightarrow q_{C1}^2\ \square\ R$

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$q^2_{C0}\ a \longrightarrow q^2_0\ \square\ R$
$q^2_{C0}\ b \longrightarrow q_{reject}$

$q^2_{C1}\ a \longrightarrow q_{reject}$
$q^2_{C1}\ b \longrightarrow q^2_0\ \square\ R$