4200 - Formal Languages
Midterm 1 - Solution
Fall 2021

Student:

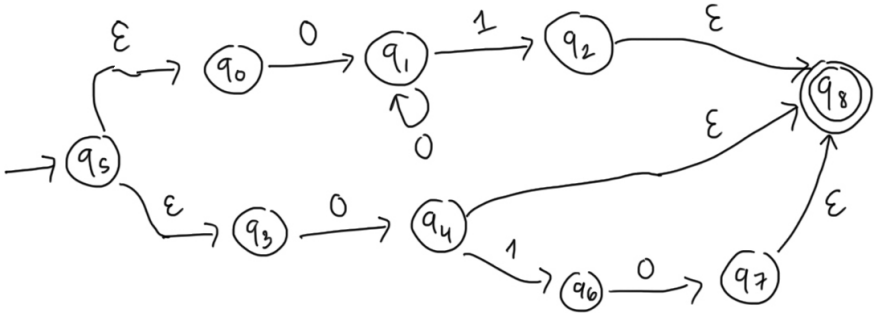
Directions: The test is open book and open notes, but NOT open electronic devices (e.g. phone, tablets or computers). For each problem, show your work completely. Give reasons for all answers. You will not only be graded on your mathematics, but also on your presentation, organization, proper use of English, spelling, punctuation, and logic.
There are **5 problems** for a total of **80 points**.

Problem 1

10 points
For the following regular expression, draw a finite-state automaton (may be deterministic or non-deterministic). Please do not simplify the given regular expression.

$$(00^*)1 \cup 0(10 \cup \epsilon) \tag{1}$$

Answer:



Problem 2

20 points
Convert the NFA M in Figure 1 into a regular expression α such that $L(\alpha) = L(M)$.
Notes: Please write the final expression below (on the "**Final expression:**"). In page 3, show step-by-step how you derive the final expression. For each step, (1) explain which state is being removed and (2) show the resultant intermediate FA.

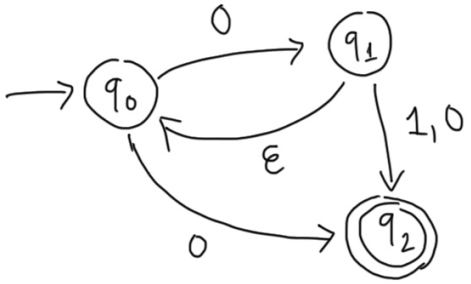
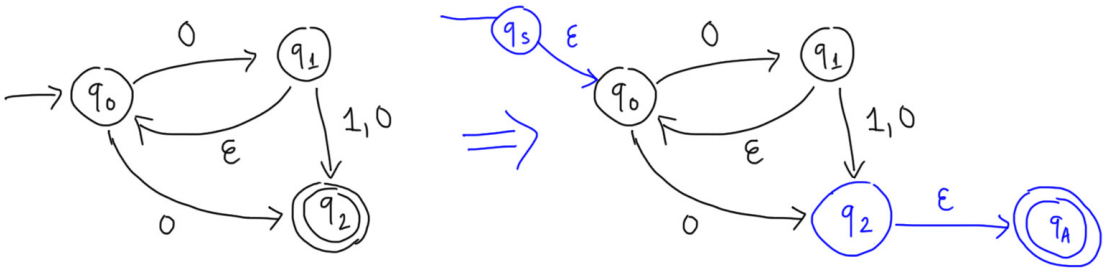
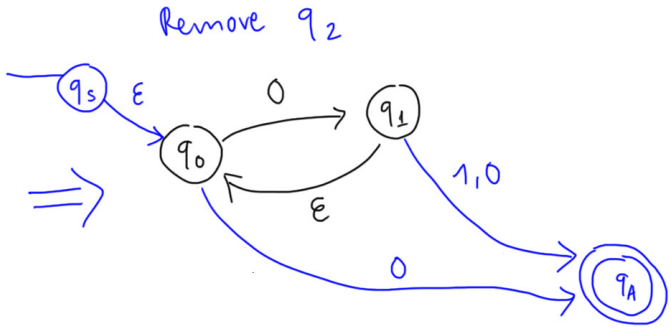
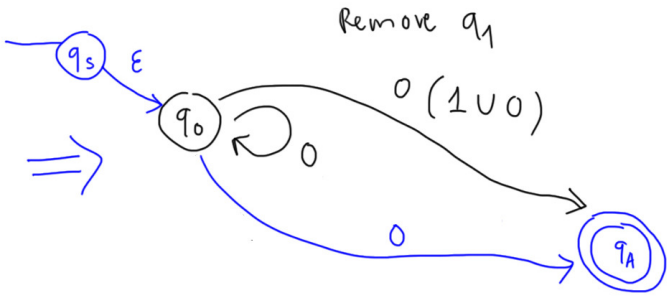


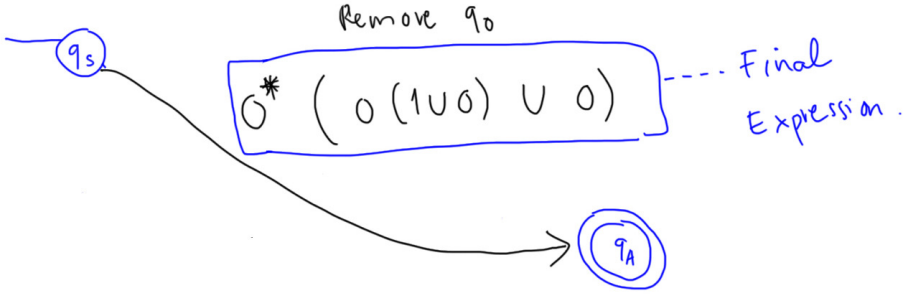
Figure 1: Non-deterministic finite-state automaton M.

Final expression: $0^*\big(0(1 \cup 0) \cup 0\big)$
Step-by-step derivations:









Problem 3

20 points

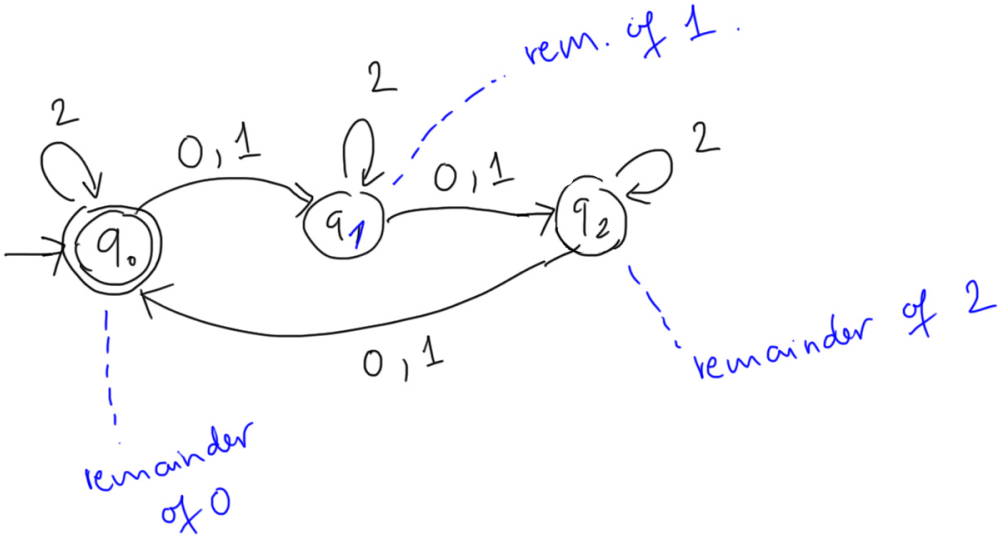
For the following set of strings A over alphabet $\Sigma = \{0, 1, 2\}$, demonstrate that A is regular by constructing an NFA, DFA or a regular expression.

$A = \{w \in \Sigma^* \mid (\#0(w) + \#1(w)\%3 = 0\}$
$= \{w \in \Sigma^* \mid$ dividing the total number of 0's and 1's combined by 3 would produce a remainder of 0 $\}$

An example string $\in A$ is: 0210 because there are two 0's and one 1's. Therefore, the total of 0's and 1's is three, which is divisible by 3 (i.e. giving a remainder of 0).

1. List out 5 strings in A:
Answer: $\epsilon, 001, 1012, 2222222, 21020$

2. Provide an NFA, DFA or a regular expression below.
Answer:



Problem 4

20 points

Alphabet $\Sigma = \{0, 1\}$.

For each provided language (X and Y), please answer these two questions:

(a) Is the language regular or non-regular?

(b) If your answer is regular, please provide either a DFA, NFA or regular expression (choose only one) that recognizes the language. If your answer is non-regular, please prove it by contradiction using Pumping Lemma. The presentation format of the proof is expected to follow those examples in the book.

1. $X = \{\ 0^m 0^n \mid m, n \geq 1 \ \text{ and } \ m = n\}$

   Answer:

   a. It is **regular** because $X = \{(00)^m \mid m \geq 1\}$

   b. Regular expression:

   $\alpha = 00(00)^*$

2. $Y = \{\ 0^m 10^n \mid m, n \geq 1 \ \text{ and } \ m = n + 1\}$

Answer:

a. It is **NOT regular**.

b. Proof using pumping lemma

0^p010^p

Follow examples in the textbook. Here we choose an counterexample $s = 0^p 1 0^p 0$ and use pumping-down technique ($i = 0$) to reach contradiction.

The end.