

4200 - Formal Languages

Midterm 2 Solution

Fall 2019

Nov 4, 2019 at 2:00 - 2:50pm

Student:

Directions: The test is open book and open notes, but NOT open electronic devices (e.g. phone, tablets or computers). For each problem, show your work completely. Your written reasons and explanations supporting your solutions will be graded.

There are **3 problems** for a total of **65 points**.

Problem 1

20 points

Define a context-free grammar that generates exactly each of the following languages. Both ambiguous and unambiguous grammars are acceptable (it does not matter in this question). For grading purposes, please use S as the starting variable. Alphabet $\Sigma = \{0, 1\}$.

1. $A = \emptyset \cup \{\epsilon\} \cup \{0\}$

Answer:

We realize that

$$\begin{aligned} A &= \emptyset \cup \{\epsilon\} \cup \{0\} \\ &= \{\epsilon\} \cup \{0\} \\ &= \{\epsilon, 0\} \end{aligned}$$

Therefore, the CFG that generates A is:

$$S \rightarrow \epsilon \mid 0 \tag{1}$$

2. $B = \{ 1^i 1^j 0^k \mid i + j = k \text{ or } i + j = 2k \text{ with } i, j, k \geq 0 \}$

Answer:

We realize that

$$\begin{aligned} B &= \{ 1^i 1^j 0^k \mid i + j = k \text{ or } i + j = 2k \text{ with } i, j, k \geq 0 \} \\ &= \{ 1^k 0^k \mid k \geq 0 \} \cup \{ 1^{2k} 0^k \mid k \geq 0 \} \end{aligned} \tag{2}$$

Therefore, the CFG that generates B is:

$$S \rightarrow A \mid Z \tag{3}$$

$$A \rightarrow 1A0 \mid \epsilon \tag{4}$$

$$Z \rightarrow 11Z0 \mid \epsilon \tag{5}$$

Problem 2

30 points

For each of the languages below, please answer the following two questions:

- (a) Is the language context-free or not?
- (b) If context-free, please construct a corresponding pushdown automaton or a context-free grammar.

If not context-free, please show a proof using Pumping Lemma for context-free languages. For full credits, your proof need to include all the steps (e.g. the assumption, the choice of a string s , and etc. See examples in the book).

$$1. A = \{ (0^i 1^i 1^m 0^m)^k \mid i, m, k \geq 0 \}$$

Answer:

We realize that

$$A = \{ (0^i 1^i 1^m 0^m)^k \mid i, m, k \geq 0 \} \tag{6}$$

$$= \{ (0^i 1^i 1^m 0^m)^0 \mid i, m \geq 0 \} \cup \{ (0^i 1^i 1^m 0^m)^1 \mid i, m \geq 0 \} \cup \{ (0^i 1^i 1^m 0^m)^2 \mid i, m \geq 0 \} \cup \dots \tag{7}$$

$$= \{ \epsilon \} \cup \{ (0^i 1^i 1^m 0^m)^1 \mid i, m \geq 0 \} \cup \{ 0^i 1^i 1^m 0^m 0^i 1^i 1^m 0^m \mid i, m \geq 0 \} \cup \dots \tag{8}$$

Idea: We further recognize that the third term in Eq. 8 is not a context-free language because of 4 i 's and 4 m 's. That is, intuitively, with one stack of the PDA, it would be impossible to make sure four substrings to be of the same length i (the same constraint for 4 other substrings to be of the same length m).

Proof: Use pumping lemma for context-free languages.

2. $B = \{w^R w \mid w \in \{0, 1\}^*\}$ Here, w^R is the reverse of the string w .

Answer:

B is a context-free language and can be generated by the following CFG:

$$S \rightarrow 0S0 \mid 1S1 \mid \epsilon \tag{9}$$

Problem 3

15 points

Given a language $A = \{ 0^{2n}1^{n+1} \mid n \geq 0 \}$. Please answer the following questions:

1. Is the above language regular or non-regular?

Answer: Non-regular.

2. Is the above language context-free or not?

Answer: Context-free.

3. Please provide a model that recognizes or generates the language, if that is possible (choose one among the following five types: NFA, DFA, Regular expressions, Context-free Grammars, or Pushdown Automata).

Otherwise, provide a proof by contradiction using one Pumping Lemma (choose either a Pumping Lemma for Regular languages or Pumping Lemma for Context-Free languages).

Answer: First, we recognize that A can be re-written as:

$$A = \{ 0^{2n}1^{n+1} \mid n \geq 0 \} \tag{10}$$

$$= \{ 0^{2n}1^n1 \mid n \geq 0 \} \tag{11}$$

Therefore, A can be generated by the following CFG:

$$S \rightarrow B1 \tag{12}$$

$$B \rightarrow 00B1 \mid \epsilon \tag{13}$$

The end.