4200 - Formal Languages: Final Exam

Fall 2020

May 1, 2020 at 8:00 - 10:30am

Student:

Directions: The test is open-book and open-note, but using electronic devices (e.g. phone, tablets or computers) during the exam is NOT allowed. For each problem, show your work completely. Give reasons for all answers.
There are **7 problems** for a total of **85 points**.

Problem 1

10 points

$$A = \{0, 1, 2\}^*$$

$$B = \{0, 1\}^*$$

$$C = A - B$$

For each of the languages (A, B, and C), write down five example strings in it. Define a deterministic finite automaton (DFA) that recognizes the language C.

Problem 2

10 points

Define a regular expression that recognizes the set A that passwords that satisfy the following criteria:

- Minimum password length: 2
- Maximum password length: 4
- Each character in the password must be one of the following LaTeX: { a, b, c, A, B, C, 1, 2, 3 }
- Password must contain at least one upper-case letter
- Password must contain at least one number

Answer:

$$\Sigma = \{a, b, c, A, B, C, 1, 2, 3\} \tag{1}$$
$$U = \{A, B, C\} \tag{2}$$
$$N = \{1, 2, 3\} \tag{3}$$
$$\alpha = \tag{4}$$
$$NU \cup UN \tag{5}$$
$$\cup\ \Sigma NU \cup \Sigma UN \cup U\Sigma N \cup UN\Sigma \cup N\Sigma U \cup NU\Sigma \tag{6}$$

Problem 3

15 points

Given a language $E = \{1^n 0^m 0^{2n+1} \mid n, m \geq 0 \}$. Please answer the following questions:

1. Is the above language regular or non-regular?

   Answer: This is NOT regular. To prove it using Pumping Lemma, one can use an counterexample string: $s = 1^p 0^{2p} 0$ and reach contradiction by using $i = 0$.
   That is, because $|xy| \leq p$, y must contain only 1's. Therefore, when we pump down (i.e., set $i = 0$), $xy^0z = xz \notin E$ because the number of 1's is less than half of the number of 0's.

2. Is the above language context-free or not?

3. Is the above language Turing-recognizable or not?

4. Based on your answers to the above questions, please answer one of the following:

   a. If you answer E is *regular* and *context-free*, please provide an NFA or a regular expression that recognizes E.

   b. If you answer E is *regular* and NOT *context-free*, please provide a proof (showing E is not context-free) using Pumping Lemma for Context-free Languages.

   c. If you answer E is NOT *regular* and *context-free*, please provide a proof (showing E is not regular) using Pumping Lemma for Regular Languages.

   d. If you answer E is NOT *regular* and NOT *context-free*, please provide a proof (showing E is not context-free) using Pumping Lemma for Context-free Languages.

Problem 4

15 points

Given a language $G = \{1^x0^y1^x0^y \mid x, y \geq 0\} \cup \{1^k0^n1^k0^v \mid k, v, n \geq 0\}$.
Please answer the following questions:

1. Is the above language regular or non-regular?

   Answer:

   $$\begin{aligned} G &= \{1^x0^y1^x0^y \mid x, y \geq 0\} \cup \{1^k0^n1^k0^v \mid k, v, n \geq 0\} \\ &= \{1^x0^n1^x0^v \mid x, v, n \geq 0\} \end{aligned}$$

   This is NOT regular. To prove it using Pumping Lemma, one can use an counterexample string: $s = 1^p01^p$ and reach contradiction by using $i = 0$ (similar to the proof in Problem 3.1 in the previous page).

2. Is the above language context-free or not?

3. Is the above language Turing-recognizable or not?

4. Based on your answers to the above two questions, please answer one of the following:

   a. If you answer G is *regular* and *context-free*, please provide an NFA or a regular expression that recognizes G.

   b. If you answer G is *regular* and NOT *context-free*, please provide a proof (showing G is not context-free) using Pumping Lemma for Context-free Languages.

   c. If you answer G is NOT *regular* and *context-free*, please provide a proof (showing G is not regular) using Pumping Lemma for Regular Languages.

   d. If you answer G is NOT *regular* and NOT *context-free*, please provide a proof (showing G is not context-free) using Pumping Lemma for Context-free Languages.

Problem 5

15 points

Given a language $H = \{1^x0^y1^x0^y \mid x, y \geq 0\} \cap \{1^k0^n1^k0^v \mid k, v, n \geq 0\}$.
Please answer the following questions:

1. Is the above language regular or non-regular?

   Answer:

   $$\begin{aligned} H &= \{1^x0^y1^x0^y \mid x, y \geq 0\} \cap \{1^k0^n1^k0^v \mid k, v, n \geq 0\} \\ &= \{1^x0^y1^x0^y \mid x, y \geq 0\} \end{aligned}$$

   This is NOT regular. To prove it using Pumping Lemma, one can use an counterexample string: $s = 1^p01^p$ and reach contradiction by using $i = 0$ (similar to the proof in Problem 3.1 in the previous page).

2. Is the above language context-free or not?

3. Is the above language Turing-recognizable or not?

4. Based on your answer to question 1 above, please answer one of the following:

   a. If you answer H is *regular*, please provide an NFA or a regular expression that recognizes H.

   b. If you answer H is NOT *regular*, please provide a proof (showing H is not regular) using Pumping Lemma for Regular Languages.