4200 - Formal Languages: Homework #5

Due on Oct 17, 2019 at 2:00pm

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Problem 1

30 points

Exercise 2.6. Give context-free grammars (CFGs) generating the following languages.

1. The set of strings over the alphabet $\Sigma = \{a, b\}$ with more a's than b's

   Answer:

$$S \to Aa \mid MS \mid SMA$$
$$A \to Aa \mid \epsilon$$
$$M \to \epsilon \mid MM \mid bMA \mid aMb$$

2. The complement of the language $\{a^n b^n \mid n \geq 0\}$

   Answer:

   A is the complement of the language $\{a^n b^n \mid n \geq 0\}$.

   That is, A is the language: $A = \{a^n b^m \mid n \neq m\} \cup \{ (a \cup b)^* ba (a \cup b)^* \}$

   Let's call the leftmost language A_1 and rightmost language A_2 i.e. $A = A_1 \cup A_2$.

   The CFG that generates A_1 is

$$S_1 \to aS_1b \mid T \mid U$$
$$T \to aT \mid a$$
$$U \to Ub \mid b$$

   The CFG that generates L_2 is:

$$S_2 \to RbaR$$
$$R \to RR \mid a \mid b \mid \epsilon$$

   Therefore, the CFG that generates language A when $A = A_1 \cup A_2$ is:

$$S \to S_1 \mid S_2$$
$$S_1 \to aS_1b \mid T \mid U$$
$$S_2 \to RbaR$$
$$T \to aT \mid a$$
$$U \to Ub \mid b$$
$$R \to RR \mid a \mid b \mid \epsilon$$

Problem 2

15 points
Exercise 2.9. Give context-free grammars (CFGs) generating the following language:

$A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$

Is your grammar ambiguous? Why or why not?
Answer
Note that $A = A_1 \cup A_2$ where $A_1 = \{a^i b^j c^k \mid i = j\}$ and $A_2 = \{a^i b^j c^k \mid j = k\}$.
The CFG that generates A_1 is:

$$S_1 \to aS_2bS_3 \mid \epsilon$$
$$S_2 \to aS_2b \mid \epsilon$$
$$S_3 \to cS_3 \mid \epsilon$$

The CFG that generates A_2 is:

$$S_4 \to S_5bS_6c \mid \epsilon$$
$$S_5 \to aS_5 \mid \epsilon$$
$$S_6 \to bS_6c \mid \epsilon$$

The CFG that generates A would contain all the above 6 rules and starts with the following rule:

$$S \to S_1 \mid S_4$$

The above CFG is ambiguous because you can derive two leftmost derivations each has a different parse tree.

Problem 3

15 points
Exercise 2.14. Convert the following CFG into an equivalent CFG in Chomsky normal form, using the procedure given in Theorem 2.9.

$$A \to BAB \mid B \mid \epsilon$$
$$B \to 00 \mid \epsilon$$

Answer: Below is the final result after following the procedure in the book. *Note*: For the Midterm 2, for this type of question, you will be asked to provide all intermediate steps, not just the final one.

$$S_0 \to BA_1 \mid BS \mid SB \mid UU \mid BB \mid \epsilon$$
$$S \to BA_2 \mid BS \mid SB \mid UU \mid BB$$
$$B \to UU$$
$$U \to 0$$
$$A_1 \to SB$$
$$A_2 \to SB$$