Homework 1

1. This will ultimately result in an empty set. The set could go up to infinity but there is no integer n such as $n = n+1$.

2. This set only contains Palindromes. It would have an infinite amount of Palindrome strings.

3. $D = \{(1,6)(1,7)(1,8)(1,9)(1,10)(2,6)(2,7)(2,8)(2,9)(2,10)(3,6)(3,7)(3,8)(3,9)(3,10)(4,6)(4,7)(4,8)(4,9)(4,10)(5,6)(5,7)(5,8)(5,9)(5,10)\}$

   $R = \{10, 10, 10, 10, 10, 7, 8, 9, 10, 6, 7, 7, 8, 8, 9, 9, 8, 7, 6, 10, 6, 6, 6, 6, 6\}$

4. $f(4) = 7$, $g(4,7)$ is equal to 8 from the table.

① Base:

1. If $n=1$ $\sum_{i=1}^{1} \frac{1}{i(i+1)} = \frac{1}{2} = \frac{n}{n+1} = \frac{1}{1+1} = \frac{1}{2}$

② Inductive:

The assumption that $\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1}$

③ Inductive step:

$\sum_{i=1}^{k+1} \frac{1}{i(i+1)} = \sum_{i=1}^{k} \frac{1}{i(i+1)} + \frac{1}{k^2+3k+2} = \frac{k}{k+1} + \frac{1}{(k+2)(k+1)}$

$= \frac{k^2+2k+1}{(k+1)(k+2)} = \frac{(k+1)(k+1)}{(k+2)(k+1)} = \frac{(k+1)}{(k+2)}$

This means the statement holds at $i=k+1$ and $n \leq k+1$ and $n+1 \leq k+2$ so at all ingers n.

2. ① Base: If $n=1$, $1^3 = 1 = [\frac{n(n+1)}{2}]^2 = [\frac{2}{2}]^2 = \underline{1}$

② Inductive Hypothesis:

The assumption that $\sum_{i=1}^{k} i^3 = (\frac{k(k+1)}{2})^2$

③ Inductive step:

$\sum_{i=1}^{k+1} i^3 = \sum_{i=1}^{k} i^3 = (k+1)^3 = (\frac{k(k+1)}{2})^2 + (k+1)^3 =$

$\frac{k^4+2k^3+k^2}{2+k^3+3k^2+3k+1} = \frac{k^4}{2} + 2k^3 + \frac{7k^2}{2} + 3k + 1 = (\frac{(k+1)(k+2)}{2})^2$

This means the statment holds at $i=k+1$ and $n \leq k+1$ and $n+1 \leq k+2$ so at all integers n.