1 Introduction

Recommended Problems

P1.1

Evaluate each of the following expressions for the complex number $z = \frac{1}{2}e^{j\pi/4}$.

(a) $Re\{z\}$

(b) $Im\{z\}$

(c) $|z|$

(d) $\sphericalangle z$

(e) z^* (* denotes complex conjugation)

(f) $z + z^*$

P1.2

Let z be an arbitrary complex number.

(a) Show that

$$Re\{z\} = \frac{z + z^*}{2}$$

(b) Show that

$$jIm\{z\} = \frac{z - z^*}{2}$$

P1.3

Using Euler's formula, $e^{j\theta} = \cos\theta + j\sin\theta$, derive the following relations:

(a) $\cos\theta = \dfrac{e^{j\theta} + e^{-j\theta}}{2}$

(b) $\sin\theta = \dfrac{e^{j\theta} - e^{-j\theta}}{2j}$

P1.4

(a) Let $z = re^{j\theta}$. Express in polar form (i.e., determine the magnitude and angle for) the following functions of z:

(i) z^*

(ii) z^2

(iii) jz

(iv) zz^*

(v) $\dfrac{z}{z^*}$

(vi) $\dfrac{1}{z}$

(b) Plot in the complex plane the vectors corresponding to your answers to Problem P1.4a(i)–(vi) for $r = \frac{2}{3}$, $\theta = \pi/6$.

P1.5

Show that

$$(1 - e^{j\alpha}) = 2 \sin\left(\frac{\alpha}{2}\right) e^{j[(\alpha-\pi)/2]}$$

P1.6

For $x(t)$ indicated in Figure P1.6, sketch the following:

(a) $x(-t)$

(b) $x(t + 2)$

(c) $x(2t + 2)$

(d) $x(1 - 3t)$

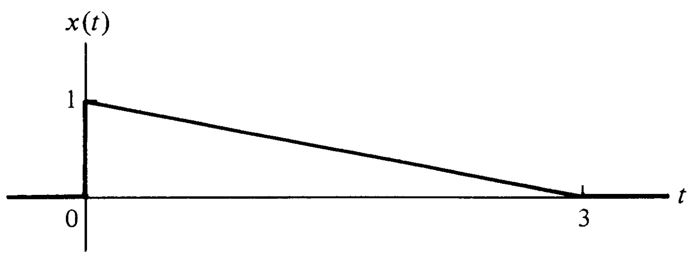

Figure P1.6

P1.7

Evaluate the following definite integrals:

(a) $\int_0^a e^{-2t}\, dt$

(b) $\int_2^\infty e^{-3t}\, dt$

2 Signals and Systems: Part I

Recommended Problems

P2.1

Let $x(t) = \cos(\omega_x(t + \tau_x) + \theta_x)$.

(a) Determine the frequency in hertz and the period of $x(t)$ for each of the following three cases:

	ω_x	τ_x	θ_x
(i)	$\pi/3$	0	2π
(ii)	$3\pi/4$	1/2	$\pi/4$
(iii)	3/4	1/2	1/4

(b) With $x(t) = \cos(\omega_x(t + \tau_x) + \theta_x)$ and $y(t) = \sin(\omega_y(t + \tau_y) + \theta_y)$, determine for which of the following combinations $x(t)$ and $y(t)$ are identically equal for all t.

	ω_x	τ_x	θ_x	ω_y	τ_y	θ_y
(i)	$\pi/3$	0	2π	$\pi/3$	1	$-\pi/3$
(ii)	$3\pi/4$	1/2	$\pi/4$	$11\pi/4$	1	$3\pi/8$
(iii)	3/4	1/2	1/4	3/4	1	3/8

P2.2

Let $x[n] = \cos(\Omega_x(n + P_x) + \theta_x)$.

(a) Determine the period of $x[n]$ for each of the following three cases:

	Ω_x	P_x	θ_x
(i)	$\pi/3$	0	2π
(ii)	$3\pi/4$	2	$\pi/4$
(iii)	3/4	1	1/4

(b) With $x[n] = \cos(\Omega_x(n + P_x) + \theta_x)$ and $y[n] = \cos(\Omega_y(n + P_y) + \theta_y)$, determine for which of the following combinations $x[n]$ and $y[n]$ are identically equal for all n.

	Ω_x	P_x	θ_x	Ω_y	P_y	θ_y
(i)	$\pi/3$	0	2π	$8\pi/3$	0	0
(ii)	$3\pi/4$	2	$\pi/4$	$3\pi/4$	1	$-\pi$
(iii)	3/4	1	1/4	3/4	0	1

P2.3

(a) A discrete-time signal $x[n]$ is shown in Figure P2.3.

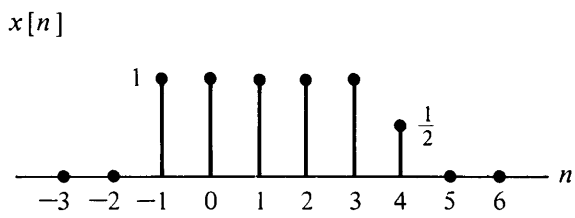

Figure P2.3

Sketch and carefully label each of the following signals:

(i) $x[n-2]$

(ii) $x[4-n]$

(iii) $x[2n]$

(b) What difficulty arises when we try to define a signal as $x[n/2]$?

P2.4

For each of the following signals, determine whether it is even, odd, or neither.

(a)

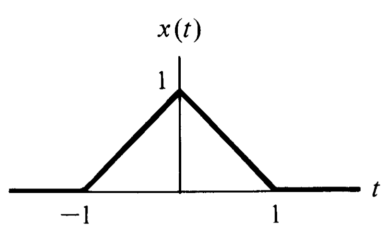

Figure P2.4-1

(b)

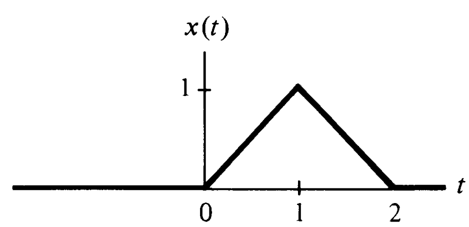

Figure P2.4-2

(c)

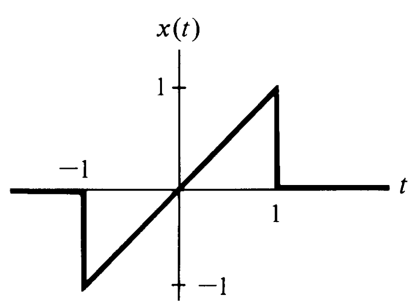

Figure P2.4-3

(d)

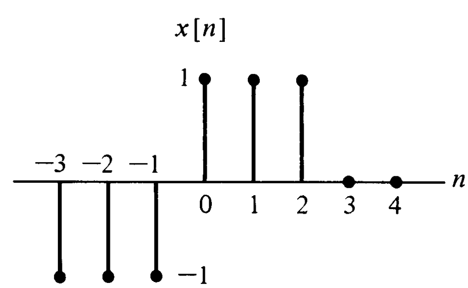

Figure P2.4-4

(e)

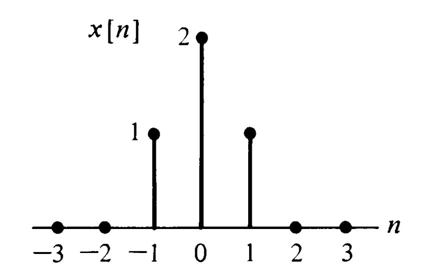

Figure P2.4-5

(f)

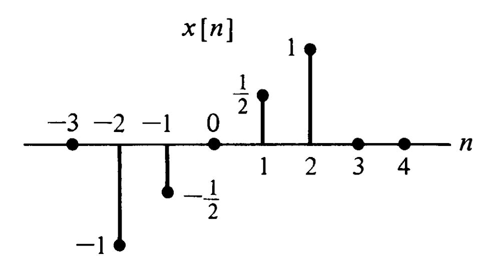

Figure P2.4-6

P2.5

Consider the signal $y[n]$ in Figure P2.5.

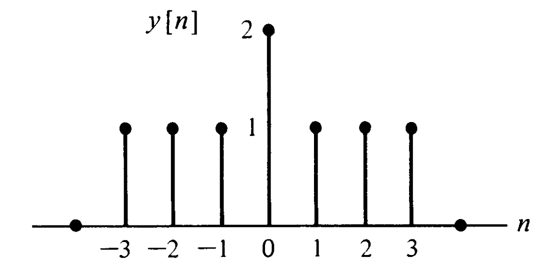

Figure P2.5

(a) Find the signal $x[n]$ such that $Ev\{x[n]\} = y[n]$ for $n \geq 0$, and $Od\{x[n]\} = y[n]$ for $n < 0$.

(b) Suppose that $Ev\{w[n]\} = y[n]$ for all n. Also assume that $w[n] = 0$ for $n < 0$. Find $w[n]$.

P2.6

(a) Sketch $x[n] = \alpha^n$ for a typical α in the range $-1 < \alpha < 0$.

(b) Assume that $\alpha = -e^{-1}$ and define $y(t)$ as $y(t) = e^{\beta t}$. Find a complex number β such that $y(t)$, when evaluated at t equal to an integer n, is described by $(-e^{-1})^n$.

(c) For $y(t)$ found in part (b), find an expression for $Re\{y(t)\}$ and $Im\{y(t)\}$. Plot $Re\{y(t)\}$ and $Im\{y(t)\}$ for t equal to an integer.

P2.7

Let $x(t) = \sqrt{2}(1 + j)e^{j\pi/4}e^{(-1+j2\pi)t}$. Sketch and label the following:

(a) $Re\{x(t)\}$

(b) $Im\{x(t)\}$

(c) $x(t + 2) + x^*(t + 2)$

P2.8

Evaluate the following sums:

(a) $\displaystyle\sum_{n=0}^{5} 2\left(\frac{3}{a}\right)^n$

(b) $\displaystyle\sum_{n=2}^{6} b^n$

(c) $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2}{3}\right)^{2n}$

Hint: Convert each sum to the form

$$C\sum_{n=0}^{N-1} \alpha^n = S_N \qquad \text{or} \qquad C\sum_{n=0}^{\infty} \alpha^n = S_\infty$$

and use the formulas

$$S_N = C\left(\frac{1-\alpha^N}{1-\alpha}\right), \qquad S_\infty = \frac{C}{1-\alpha} \qquad \text{for } |\alpha| < 1$$

P2.9

(a) Let $x(t)$ and $y(t)$ be periodic signals with fundamental periods T_1 and T_2, respectively. Under what conditions is the sum $x(t) + y(t)$ periodic, and what is the fundamental period of this signal if it is periodic?

(b) Let $x[n]$ and $y[n]$ be periodic signals with fundamental periods N_1 and N_2, respectively. Under what conditions is the sum $x[n] + y[n]$ periodic, and what is the fundamental period of this signal if it is periodic?

(c) Consider the signals

$$x(t) = \cos\frac{2\pi t}{3} + 2\sin\frac{16\pi t}{3},$$
$$y(t) = \sin \pi t$$

Show that $z(t) = x(t)y(t)$ is periodic, and write $z(t)$ as a linear combination of harmonically related complex exponentials. That is, find a number T and complex numbers c_k such that

$$z(t) = \sum_k c_k e^{jk(2\pi/T)t}$$

P2.10

In this problem we explore several of the properties of even and odd signals.

(a) Show that if $x[n]$ is an odd signal, then

$$\sum_{n=-\infty}^{+\infty} x[n] = 0$$

(b) Show that if $x_1[n]$ is an odd signal and $x_2[n]$ is an even signal, then $x_1[n]x_2[n]$ is an odd signal.

(c) Let $x[n]$ be an arbitrary signal with even and odd parts denoted by

$$x_e[n] = Ev\{x[n]\}, \qquad x_o[n] = Od\{x[n]\}$$

Show that

$$\sum_{n=-\infty}^{+\infty} x^2[n] = \sum_{n=-\infty}^{+\infty} x_e^2[n] + \sum_{n=-\infty}^{+\infty} x_o^2[n]$$

(d) Although parts (a)–(c) have been stated in terms of discrete-time signals, the analogous properties are also valid in continuous time. To demonstrate this, show that

$$\int_{-\infty}^{+\infty} x^2(t)\, dt = \int_{-\infty}^{+\infty} x_e^2(t)\, dt + \int_{-\infty}^{+\infty} x_o^2(t)\, dt,$$

where $x_e(t)$ and $x_o(t)$ are, respectively, the even and odd parts of $x(t)$.

P2.11

Let $x(t)$ be the continuous-time complex exponential signal $x(t) = e^{j\omega_0 t}$ with fundamental frequency ω_0 and fundamental period $T_0 = 2\pi/\omega_0$. Consider the discrete-time signal obtained by taking equally spaced samples of $x(t)$. That is, $x[n] = x(nT) = e^{j\omega_0 nT}$.

(a) Show that $x[n]$ is periodic if and only if T/T_0 is a rational number, that is, if and only if some multiple of the sampling interval *exactly equals* a multiple of the period $x(t)$.

(b) Suppose that $x[n]$ is periodic, that is, that

$$\frac{T}{T_0} = \frac{p}{q}, \qquad \text{(P2.11-1)}$$

where p and q are integers. What are the fundamental period and fundamental frequency of $x[n]$? Express the fundamental frequency as a fraction of $\omega_0 T$.

(c) Again assuming that T/T_0 satisfies eq. (P2.11-1), determine precisely how many periods of $x(t)$ are needed to obtain the samples that form a single period of $x[n]$.

3 Signals and Systems: Part II

Recommended Problems

P3.1

Sketch each of the following signals.

(a) $x[n] = \delta[n] + \delta[n-3]$

(b) $x[n] = u[n] - u[n-5]$

(c) $x[n] = \delta[n] + \frac{1}{2}\delta[n-1] + (\frac{1}{2})^2\delta[n-2] + (\frac{1}{2})^3\delta[n-3]$

(d) $x(t) = u(t+3) - u(t-3)$

(e) $x(t) = \delta(t+2)$

(f) $x(t) = e^{-t}u(t)$

P3.2

Below are two columns of signals expressed analytically. For each signal in column A, find the signal or signals in column B that are identical.

A	B
(1) $\delta[n+1]$	**(a)** $\sum_{k=-\infty}^{n} \delta[k]$
(2) $(\frac{1}{2})^n u[n]$	**(b)** $\frac{du(t)}{dt}$
(3) $\delta(t)$	**(c)** $\sum_{k=0}^{n} \delta[k]$
(4) $u(t)$	**(d)** $\sum_{k=0}^{\infty} (\frac{1}{2})^k\delta[n-k]$
(5) $u[n]$	**(e)** $\int_{-\infty}^{t} \delta(\tau)\, d\tau$
(6) $\delta[n+1]u[n]$	**(f)** $u[n]$
	(g) $\sum_{k=-\infty}^{\infty} (\frac{1}{2})^k\delta[n-k]$
	(h) $\delta[n+1]$
	(i) ϕ

P3.3

(a) Express the following as sums of weighted delayed impulses, i.e., in the form

$$x[n] = \sum_{k=-\infty}^{\infty} a_k\delta[n-k]$$

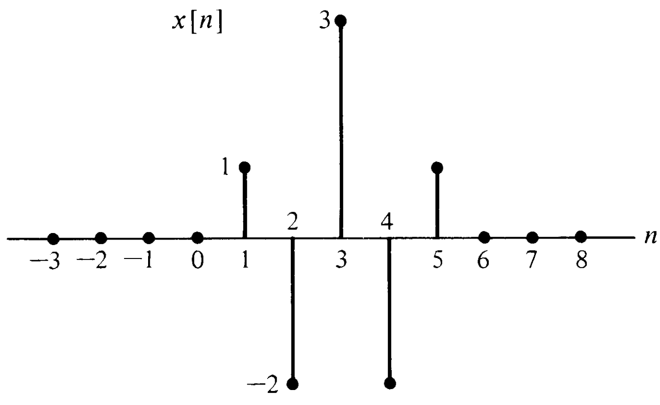

Figure P3.3-1

(b) Express the following sequence as a sum of step functions, i.e., in the form

$$s[n] = \sum_{k=-\infty}^{\infty} a_k u[n-k]$$

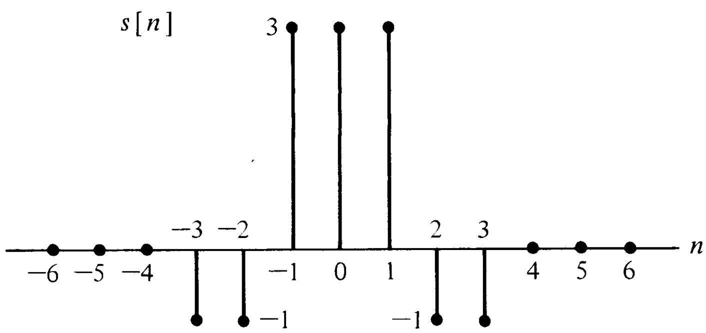

Figure P3.3-2

P3.4

For $x(t)$ indicated in Figure P3.4, sketch the following:

(a) $x(1-t)[u(t+1) - u(t-2)]$

(b) $x(1-t)[u(t+1) - u(2-3t)]$

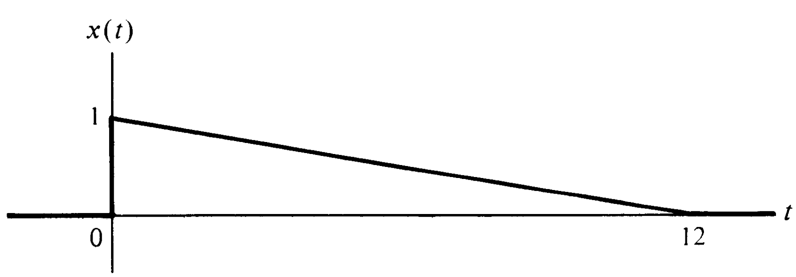

Figure P3.4

P3.5

Consider the three systems H, G, and F defined in Figure P3.5-1.

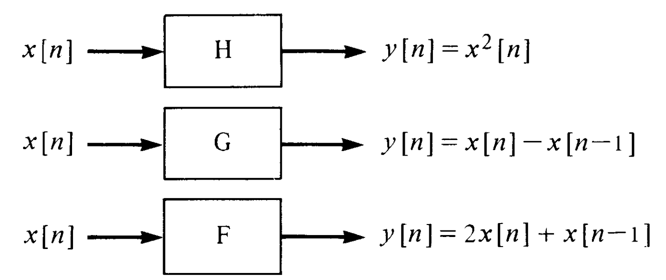

Figure P3.5-1

The systems in Figures P3.5-2 to P3.5-7 are formed by parallel and cascade combination of H, G, and F. By expressing the output $y[n]$ in terms of the input $x[n]$, determine which of the systems are equivalent.

(a)

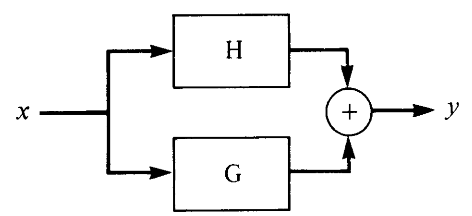

Figure P3.5-2

(b)

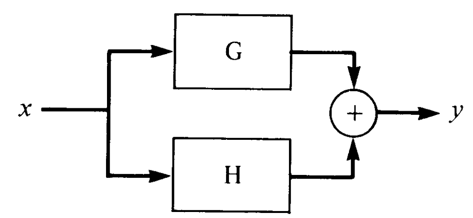

Figure P3.5-3

(c)

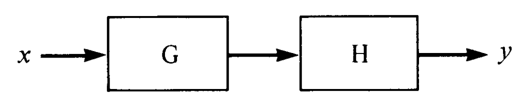

Figure P3.5-4

(d)

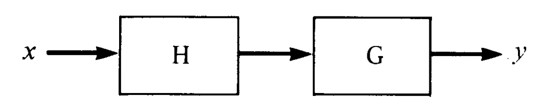

Figure P3.5-5

(e)

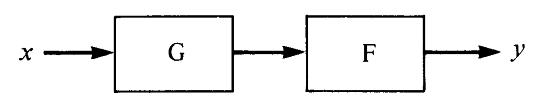

Figure P3.5-6

(f)

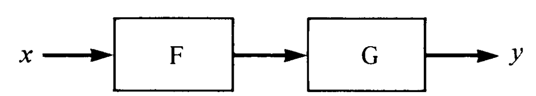

Figure P3.5-7

P3.6

Table P3.6 contains the input-output relations for several continuous-time and discrete-time systems, where $x(t)$ or $x[n]$ is the input. Indicate whether the property along the top row applies to each system by answering yes or no in the appropriate boxes. Do not mark the shaded boxes.

$y(t)$, $y[n]$	Memoryless	Linear	Time-Invariant	Causal	Invertible	Stable
(a) $(2 + \sin t)x(t)$						
(b) $x(2t)$						
(c) $\sum_{k=-\infty}^{\infty} x[k]$						
(d) $\sum_{k=-\infty}^{n} x[k]$						
(e) $\frac{dx(t)}{dt}$						
(f) $\max\{x[n], x[n-1], \ldots, x[-\infty]\}$						

Table P3.6

P3.7

Consider the following systems

$$\text{H:} \quad y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau \qquad \text{(an integrator)},$$

$$\text{G:} \quad y(t) = x(2t),$$

where the input is $x(t)$ and the output is $y(t)$.

(a) What is H^{-1}, the inverse of H? What is G^{-1}?

(b) Consider the system in Figure P3.7. Find the inverse F^{-1} and draw it in block diagram form in terms of H^{-1} and G^{-1}.

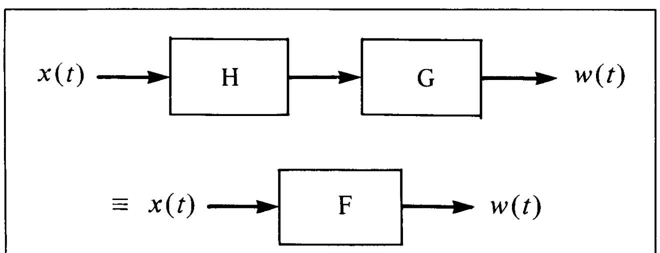

Figure P3.7