Practice Questions

Monday, 26 January 2026 12:11 pm

Optimization: Maxima Minima

Example 2 Find and classify all the critical points for $f(x,y) = 3x^2y + y^3 - 3x^2 - 3y^2 + 2$

$$f(x,y) = \left(3x + 4x^3\right)\left(y^2 + 2y\right)$$

3–14 Use Lagrange multipliers to find the maximum and minimum values of the function subject to the given constraint.

3. $f(x, y) = x^2 + y^2; \quad xy = 1$

20. $f(x, y) = 2x^2 + 3y^2 - 4x - 5, \quad x^2 + y^2 \leq 16$

Find the maximum and minimum values of $f(x,y) = 81x^2 + y^2$ subject to the constraint $4x^2 + y^2 = 9$.

Find the maximum and minimum values of $f(x,y) = 8x^2 - 2y$ subject to the constraint $x^2 + y^2 = 1$.

Double Integration

Example 1 Evaluate each of the following integrals over the given region D.

(a) $\iint_D \mathbf{e}^{\frac{x}{y}}\, dA,\ D = \{(x,y) \mid 1 \le y \le 2,\ y \le x \le y^3\}$

(b) $\iint_D 4xy - y^3\, dA$, D is the region bounded by $y = \sqrt{x}$ and $y = x^3$.

Example 2 Evaluate the following integrals by first reversing the order of integration.

(a) $\int_0^3 \int_{x^2}^9 x^3 \mathbf{e}^{y^3}\, dy\, dx$

(b) $\int_0^8 \int_{\sqrt[3]{y}}^2 \sqrt{x^4 + 1}\, dx\, dy$

Polar Integration

Example 1 Evaluate the following integrals by converting them into polar coordinates.

(a) $\iint_D 2x\, y\, dA$, D is the portion of the region between the circles of radius 2 and radius 5 centered at the origin that lies in the first quadrant.

(b) $\iint_D \mathbf{e}^{x^2+y^2}\, dA$, D is the unit disk centered at the origin.

Example 5 Evaluate the following integral by first converting to polar coordinates.

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{0}\cos\left(x^2+y^2\right)\,dy\,dx$$

Evaluate $\iint_D y^2+3x\,dA$ where D is the region in the 3rd quadrant between $x^2+y^2=1$ and $x^2+y^2=9$.

Evaluate the following integral by first converting to an integral in polar coordinates.

$$\int_{0}^{3}\int_{-\sqrt{9-x^2}}^{0}\mathbf{e}^{x^2+y^2}\,dy\,dx$$

Triple Integration: Cartesian

Evaluate the following integral.

$$\iiint_B 8xyz\,dV \qquad B=[2,3]\times[1,2]\times[0,1]$$

Evaluate $\int_2^3\int_{-1}^4\int_1^0 4x^2y-z^3\,dz\,dy\,dx$

Evaluate $\int_0^1\int_0^{z^2}\int_0^3 y\cos\left(z^5\right)\,dx\,dy\,dz$

Triple Integration: Cylindrical

Example 1 Evaluate $\iiint_E y\,dV$ where E is the region that lies below the plane $z=x+2$ above the xy-plane and between the cylinders $x^2+y^2=1$ and $x^2+y^2=4$.

Example 2 Convert $\int_{-1}^{1}\int_{0}^{\sqrt{1-y^2}}\int_{x^2+y^2}^{\sqrt{x^2+y^2}} xyz\,dz\,dx\,dy$ into an integral in cylindrical coordinates.

Evaluate $\iiint_E 4xy\,dV$ where E is the region bounded by $z=2x^2+2y^2-7$ and $z=1$.

Evaluate $\iiint_E \mathbf{e}^{-x^2-z^2}\,dV$ where E is the region between the two cylinders $x^2+z^2=4$ and $x^2+z^2=9$ with $1\le y\le 5$ and $z\le 0$.

Triple Integration: Spherical

Example 1 Evaluate $\iiint_E 16z\,dV$ where E is the upper half of the sphere $x^2+y^2+z^2=1$.

Evaluate $\iiint_E 10xz + 3\,dV$ where E is the region portion of $x^2 + y^2 + z^2 = 16$ with $z \geq 0$.

Evaluate $\iiint_E x^2 + y^2\,dV$ where E is the region portion of $x^2 + y^2 + z^2 = 4$ with $y \geq 0$.