Polar Integration

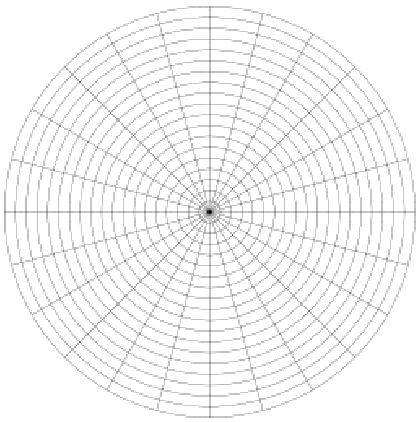
Monday, 10 June 2024 4:58 pm

Section 16.4: Double Integrals in Polar Coordinates

Integration in Polar Coordinates

It is often convenient to view $\mathbb{R}^2$ as a *polar grid* instead of a rectangular grid when setting up and computing double integrals. In this case the relationship between the Cartesian coordinates (x, y) and the polar coordinates (is given by

$$x = \quad ,$$
$$y =$$
$$x^2 + y^2 =$$



Now, in order to be able to utilize this coordinate system to integrate effectively, it will be important to determine what form the area element $dA =$ kes when written in polar coordinates.

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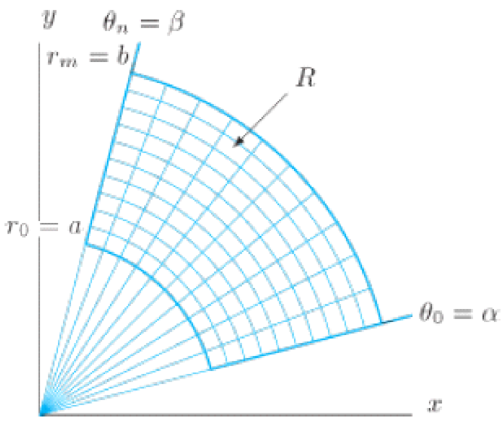
What is dA in Polar Coordinates?

We first note that $r =$ const gives a circle of constant radius in polar coordinates, while $\theta =$ const gives a ray emanating from the origing and making an angle of θ with the positive x-axis. A polar grid (such as the one shown above) is built out of such circles and rays. Now, suppose we want to integrate a function $f(x, y) = f(\quad , \quad)$ over the region R shown below:



It would be natural to form a Riemann sum where we sum over each of the "bent" rectangles depicted in the figure. Such a Riemann sum would look something like

$$\sum_{i,j} \quad .$$

In order to determine what ΔA looks like in the Riemann sum, let us look closer at one of the small bent rectangles.

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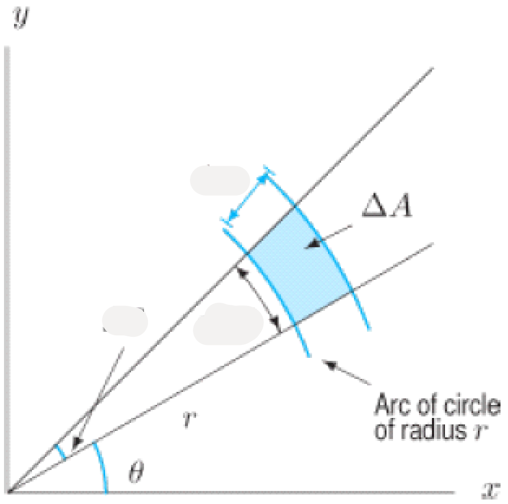

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Analyzing the figure on the previous page, we can see that

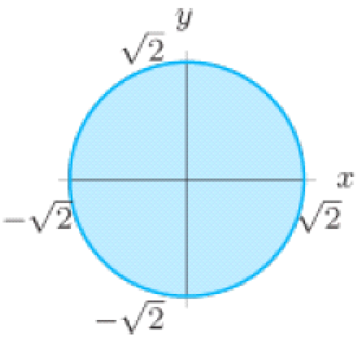
$$\Delta A \approx$$

Thus we develop the following scheme for integrating in polar coordinates:

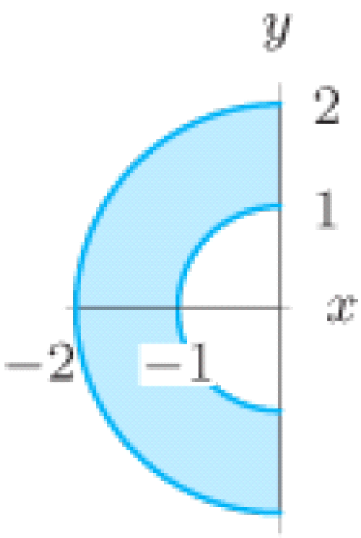
> When computing integrals in polar coordinates, we use $x = r\cos\theta$, $y = r\sin\theta$, $x^2 + y^2 = r^2$. Put $dA =$.

Examples:

1. For the following regions R, write $\int_R f\, dA$ as an iterated integral using polar coordinates.



(a)



(b)

2. Sketch the region of integration.

(a) $\displaystyle\int_{\pi/2}^{\pi}\int_0^1 f(r,\theta)\, r\, dr\, d\theta$

(b) $\displaystyle\int_{\pi/6}^{\pi/3} \int_0^1 f(r,\theta)\, r\, dr\, d\theta.$

3. Evaluate the integral

   (a) $\displaystyle\int_R \sqrt{x^2+y^2}\, dA$, where R is $4 \leq x^2+y^2 \leq 9$.

Compute the integral of $f(x, y) = 1/(x^2 + y^2)^{3/2}$ over the region R shown in Figure 16.34.

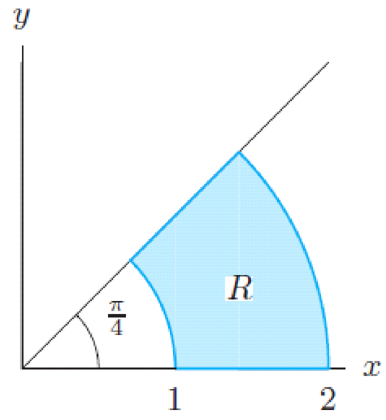


Figure 16.34: Integrate f over the polar region