Friday, 28 June 2024 11:07 pm

Vector Field

Gravitational and electric forces have both a direction and a magnitude. They are represented by a vector at each point in their domain, producing a vector field.

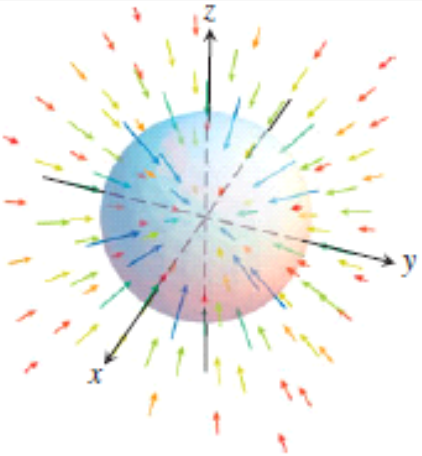

FIGURE 16.8 Vectors in a gravitational field point toward the center of mass that gives the source of the field.

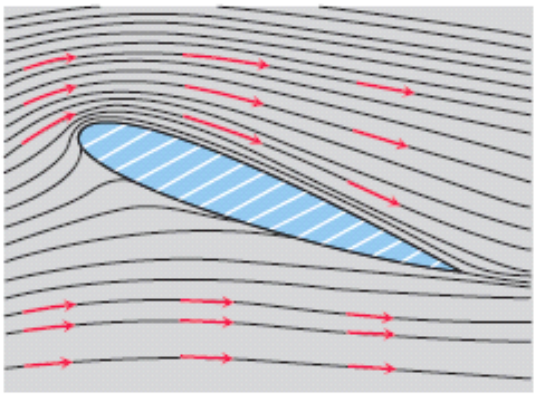

FIGURE 16.6 Velocity vectors of a flow around an airfoil in a wind tunnel.

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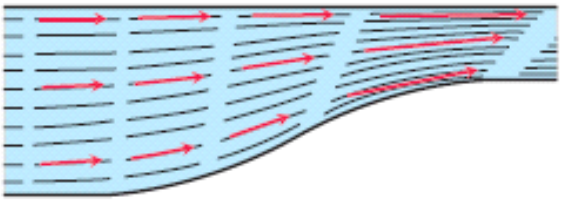


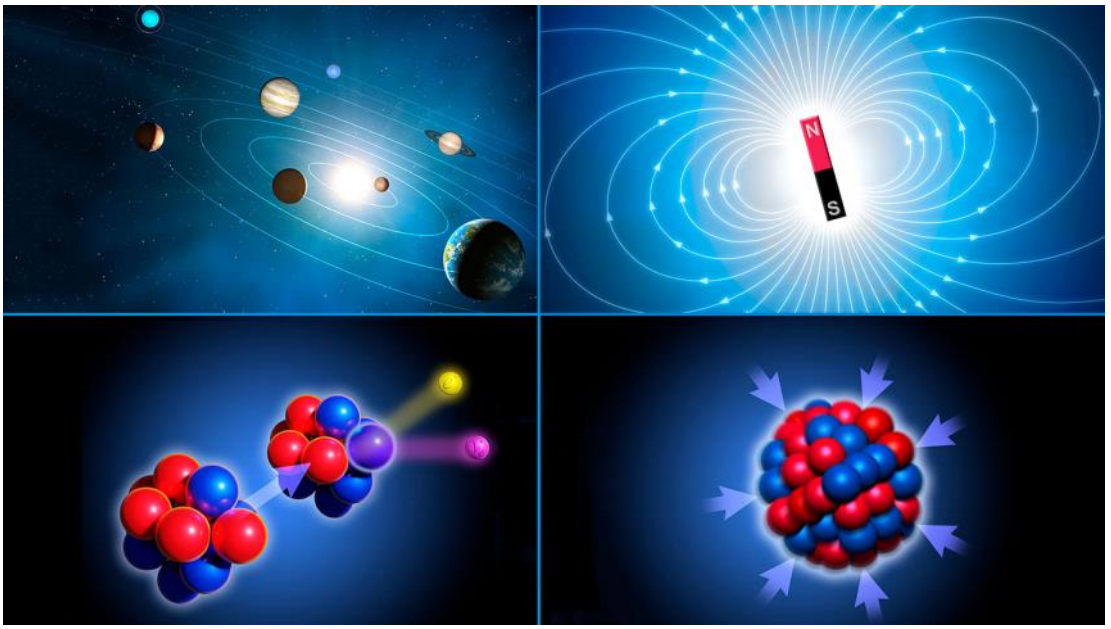
FIGURE 16.7 Streamlines in a contracting channel. The water speeds up as the channel narrows and the velocity vectors increase in length.

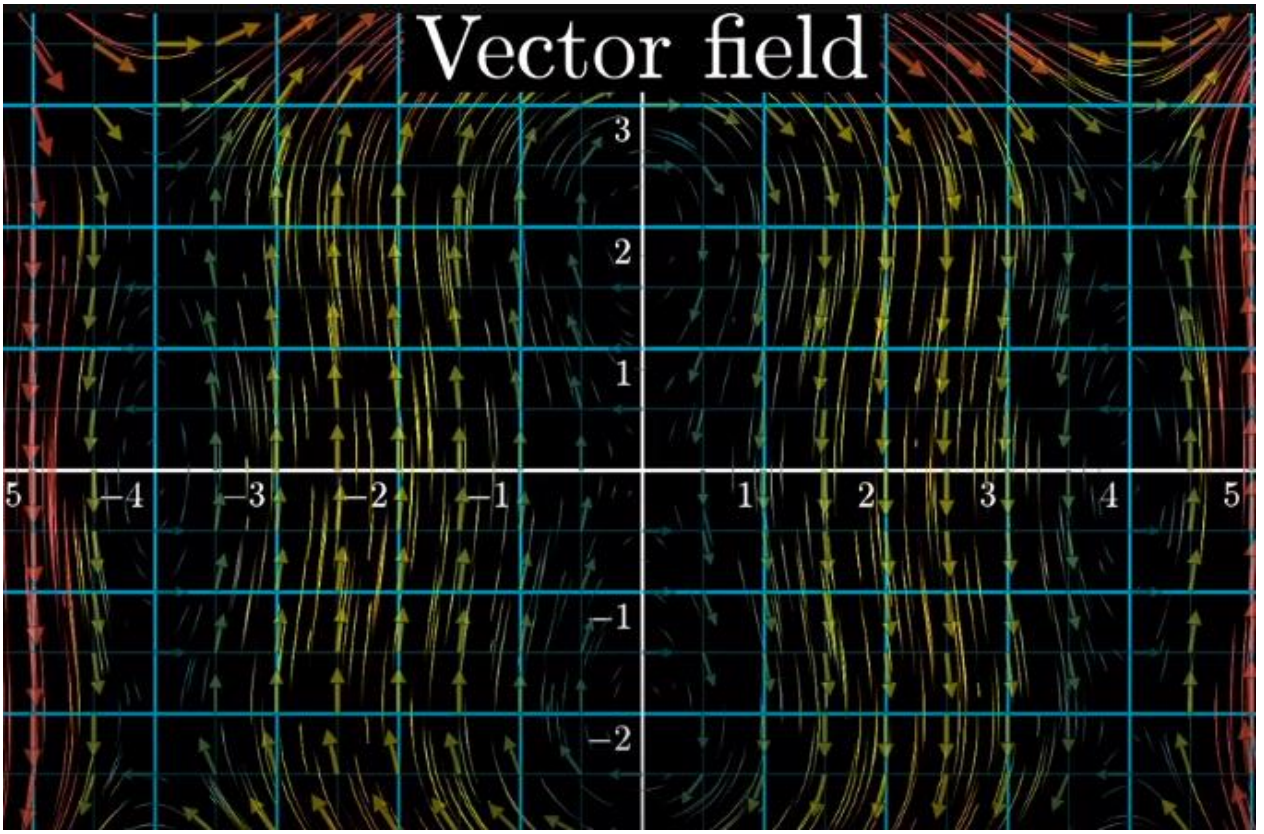
Generally, a **vector field** is a function that assigns a vector to each point in its domain. A vector field on a three-dimensional domain in space might have a formula like

$$\mathbf{F}(x, y, z) = M(x, y, z)\mathbf{i} + N(x, y, z)\mathbf{j} + P(x, y, z)\mathbf{k}.$$

The field is **continuous** if the **component functions** M, N, and P are continuous; it is **differentiable** if each of the component functions is differentiable. The formula for a field of two-dimensional vectors could look like

$$\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}.$$





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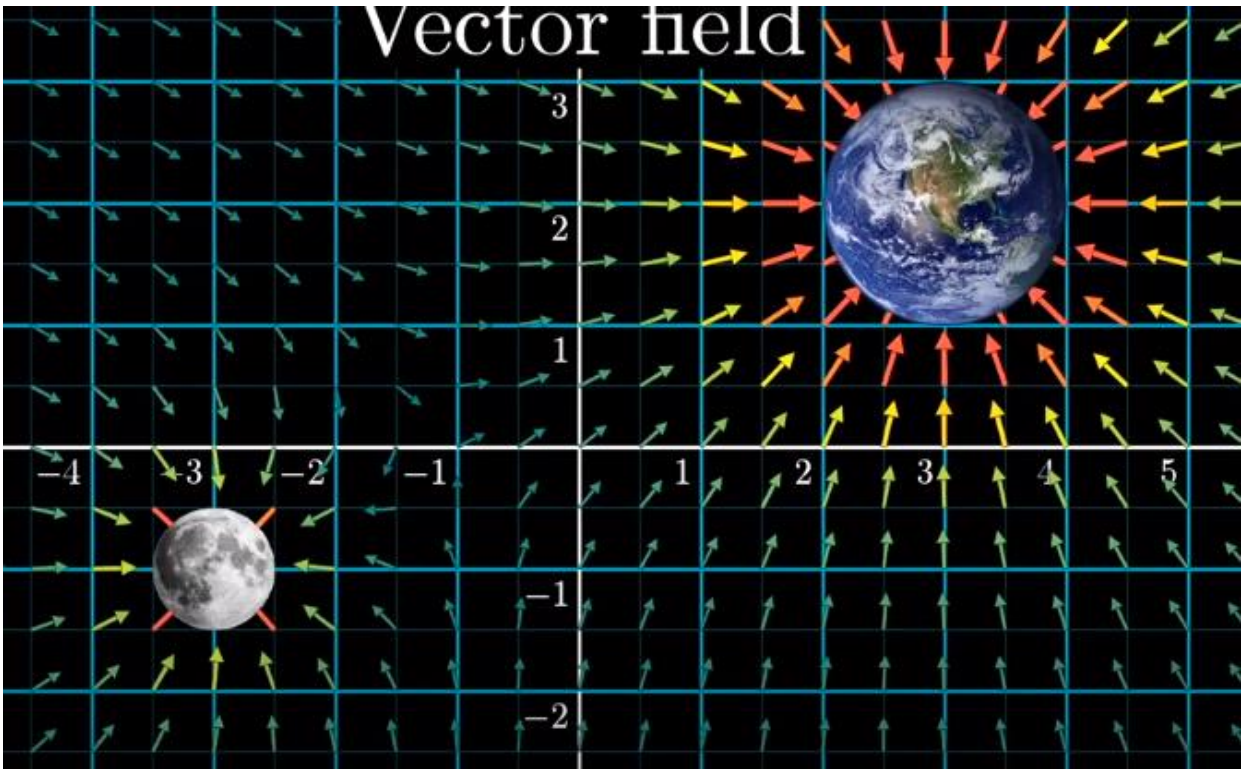

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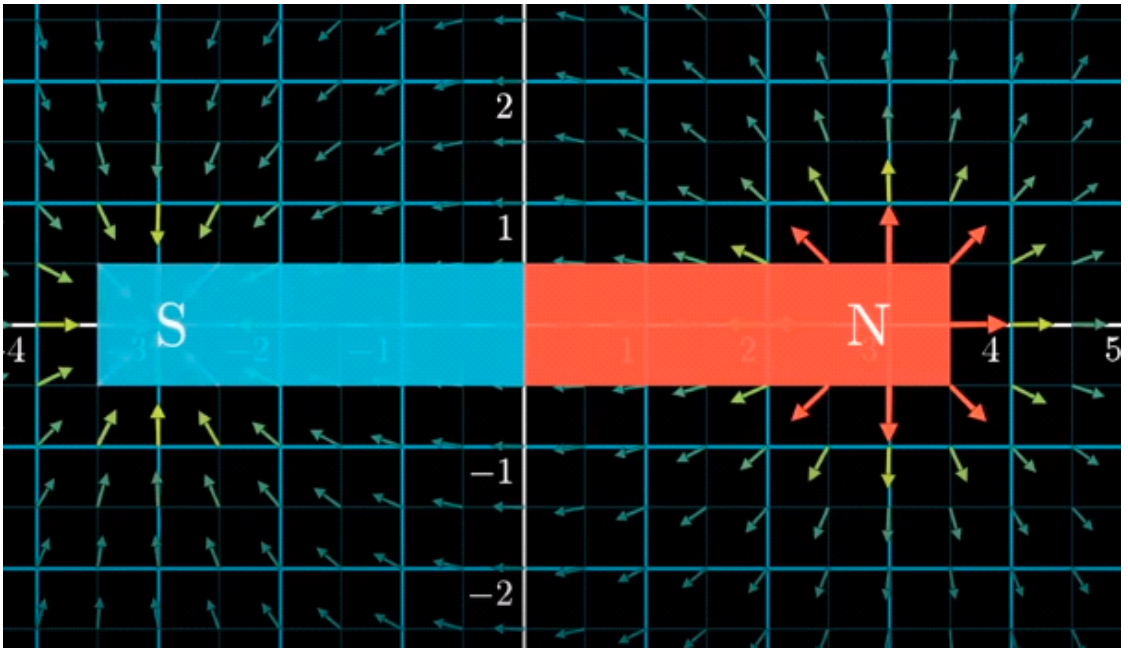

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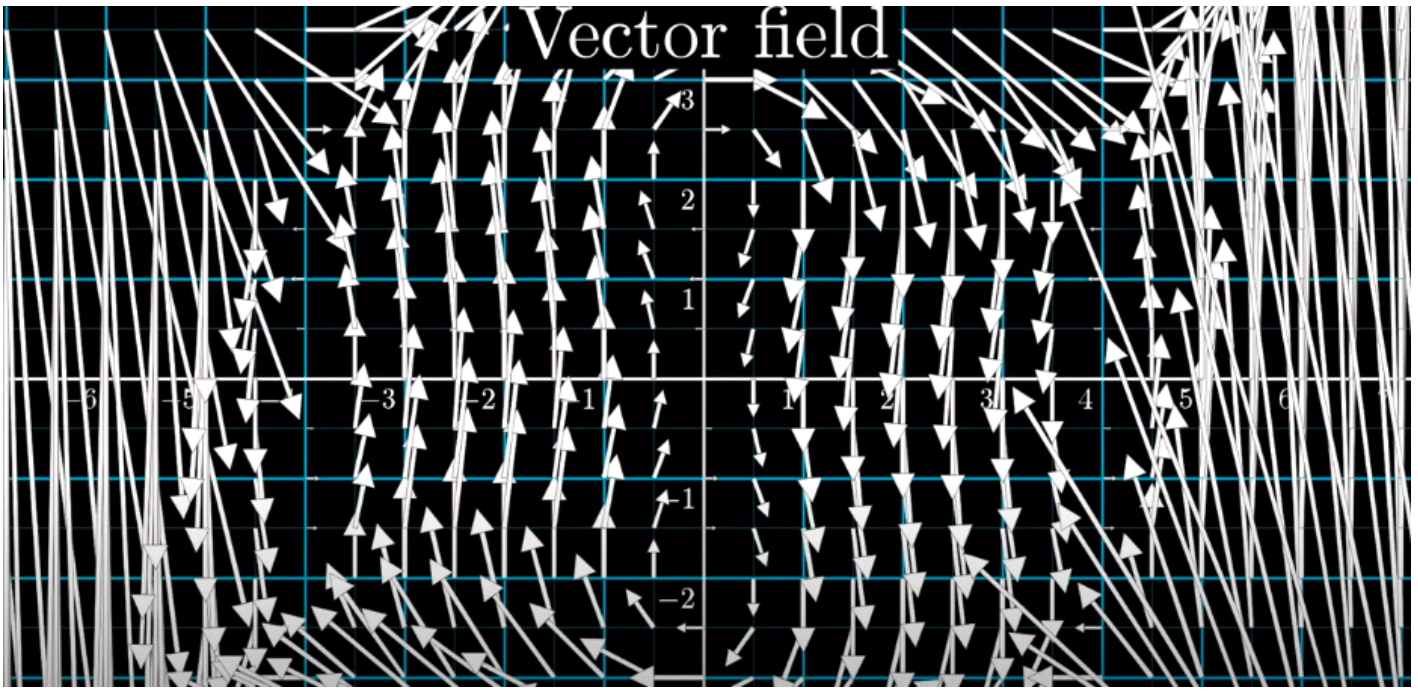

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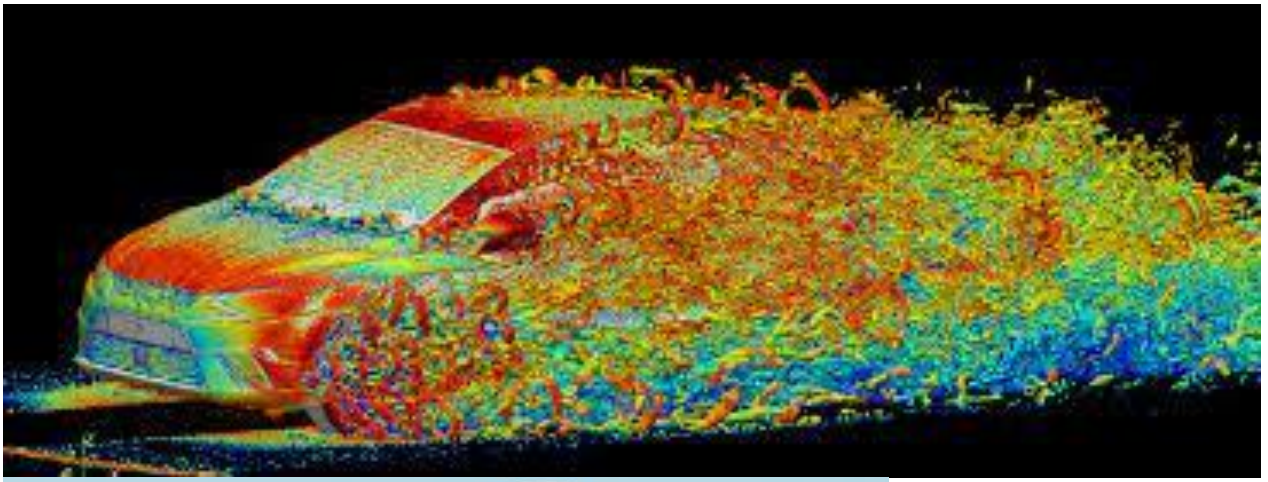
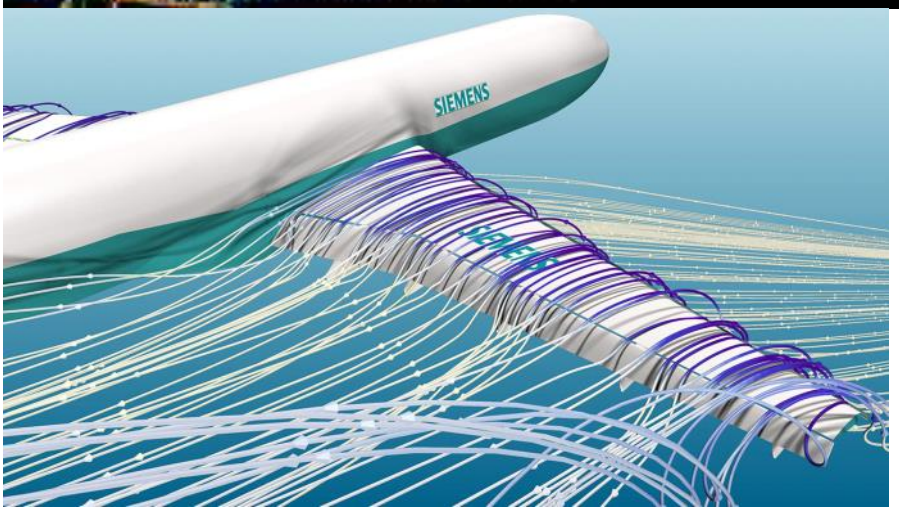
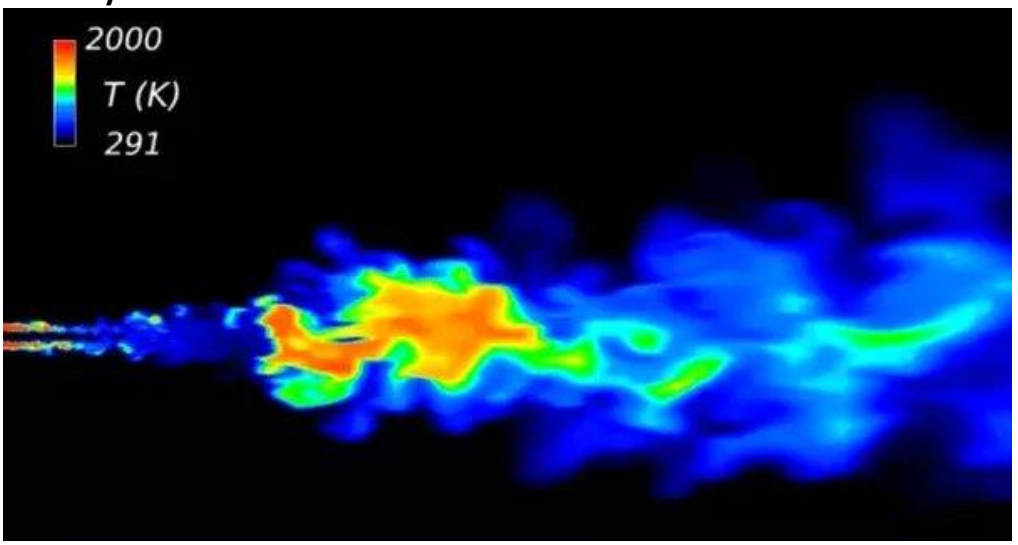
Aero Dynamics



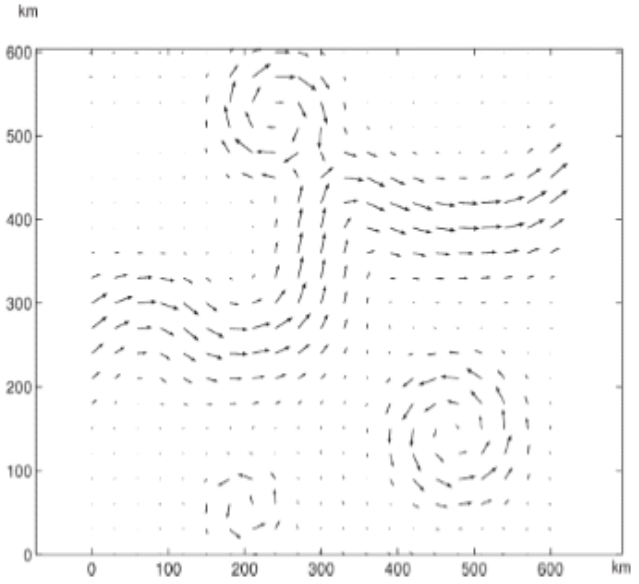


Fluid Dynamics



Section 17.3: Vector Fields

A *vector field* is a function which assigns a unique vector to each point in $\mathbb{R}^2$ or $\mathbb{R}^3$. An obvious example of a vector field would be the gradient field of a function $f(x, y)$ or $f(x, y, z)$.



The image above shows the velocity vector field of the gulf stream.

Force Fields

A very important vector quantity that will give rise to many of our vector fields is force. For example, the earth exerts a gravitational force at every point in space. Such a vector field would look like the following:

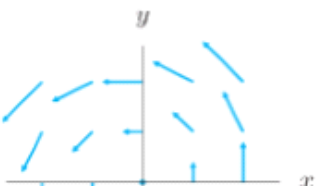
Definition of a Vector Field

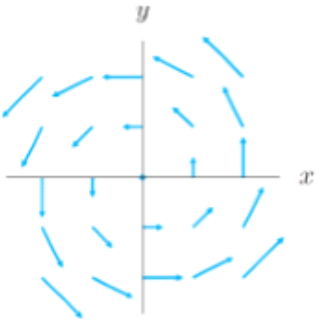
A *vector field* in $\mathbb{R}^2$ is a function $\vec{F}(x, y)$ whose value at a point (x, y) is a two-dimensional vector. Similarly, a vector field in $\mathbb{R}^3$ is a function $\vec{F}(x, y, z)$ whose value at a point (x, y, z) is a three-dimensional vector.

Often we will identify a point (x, y) or a point (x, y, z) with its position vector $\vec{r}$ and write our vector field as $\vec{F}(\vec{r})$.

Examples:

1. Sketch the vector field in $\mathbb{R}^2$ given by $\vec{F}(x, y) = -y\vec{i} + x\vec{j}$.





2. Describe the vector field in $\mathbb{R}^3$ given by $\vec{F}(\vec{r}) = \vec{r}$.

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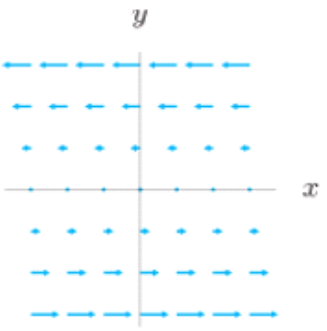
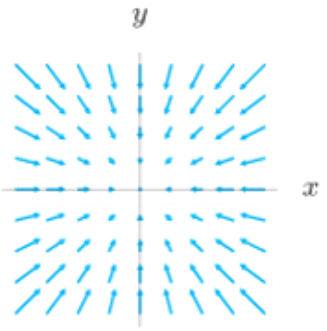
3. Find a possible formula for the given vector fields.



(a)



(b)