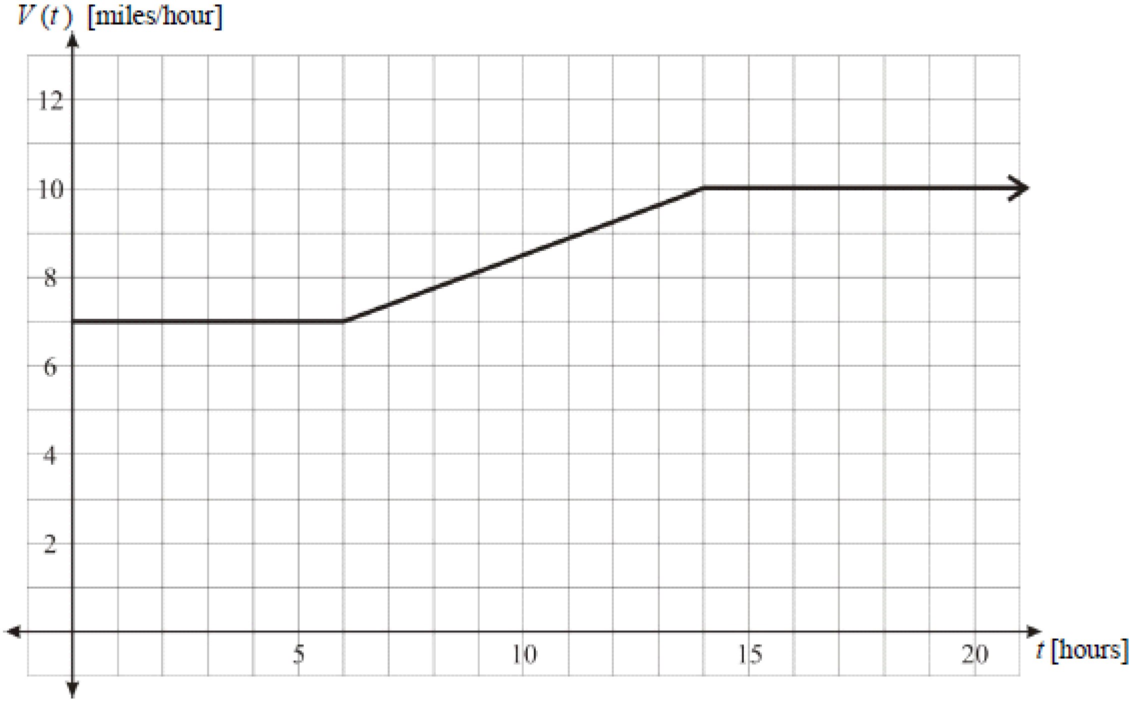
Integration: Concept & Area under a curve(Riemann Sum)

Monday, 28 April 2025 11:52 pm

Areas Bounded by Curves

Given a velocity function, we know that finding the distance traveled by the object, over an interval of time, involves finding the area bounded by the function's graph over that same interval.

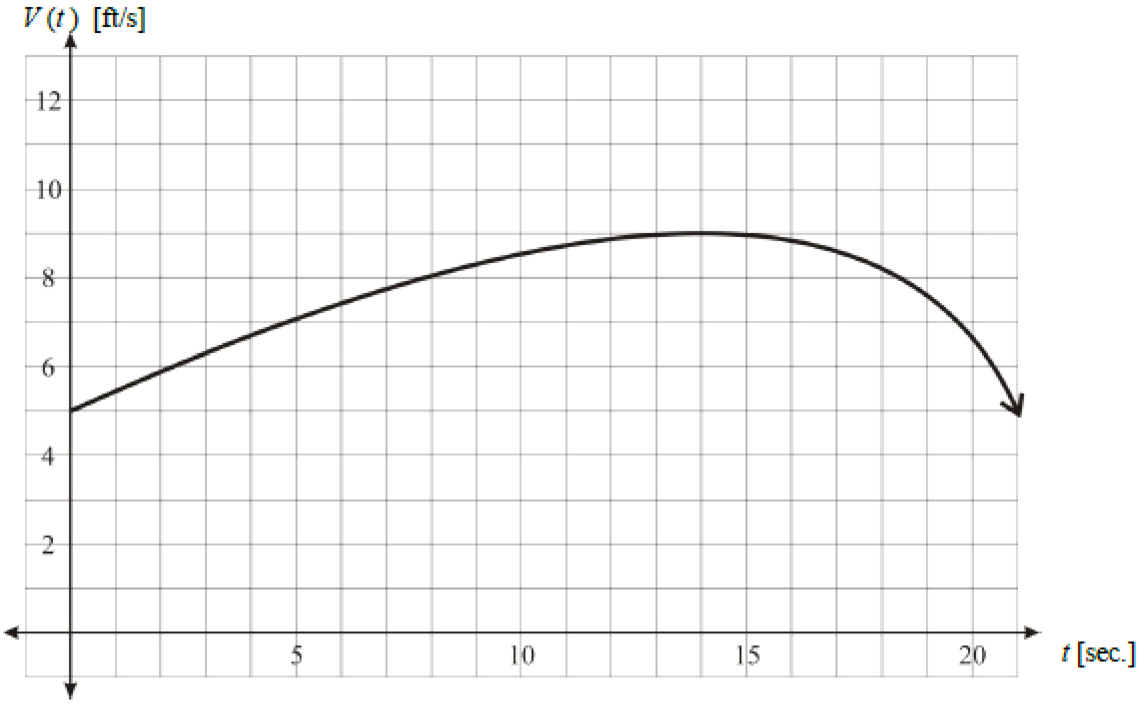
Below is a hypothetical graph of the velocity of a tractor traveling on a straight road. Find the distance traveled by the tractor between $t = 6$ and $t = 18$.



Area over the given interval = _______

Therefore, the distance traveled between the 6th and the 18th hour is _______________ miles.

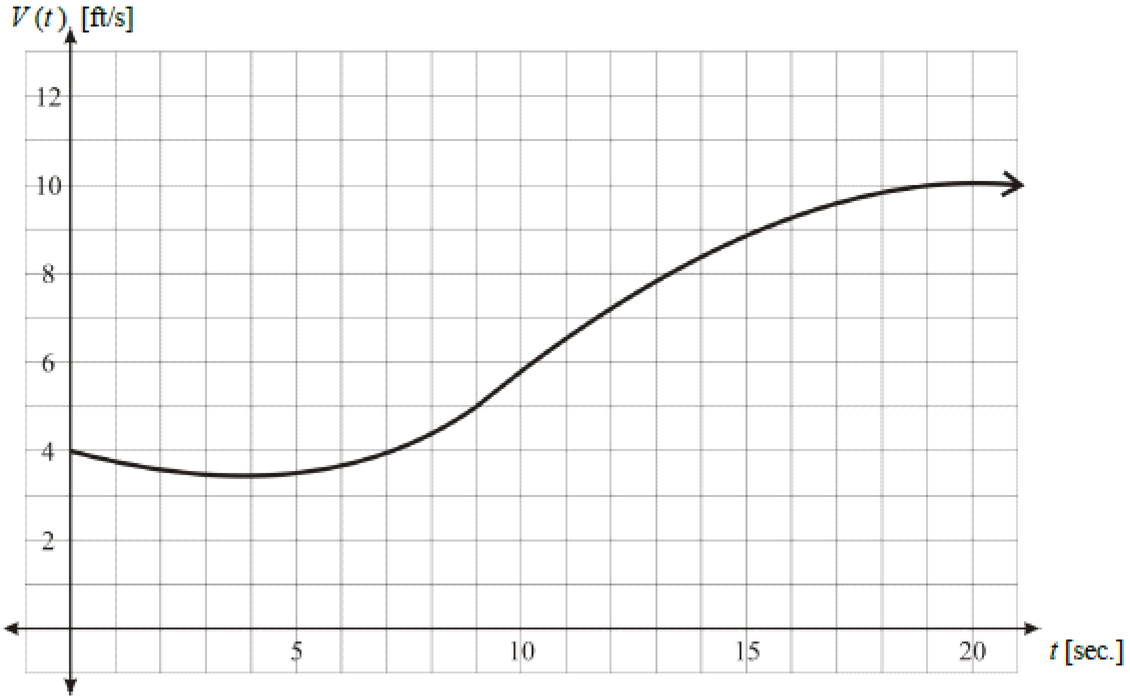
Next, let's consider the velocity function of a Victorian horse-carriage (graphed below). It is hard to envision being able to get an exact area, judging from the shape of the curve, but you can surely get an approximation. Use your creativity with various shapes and find the area under the curve over the interval [8, 20].



Area = ________

Therefore, the distance traveled between $t = 8$ and $t = 20$ is ________ feet.

Graphed below is the velocity of Mr. Kerai running after the unidentified student who stole his special TI 89 calculator! Find an approximation for the distance traveled over the interval [4, 19], but this time restricting yourselves to rectangles of width 1 second as your primary shape.



Area = ___________

Thus, the distance traveled between 4 and 19 seconds is _______________ feet.

Now answer the following questions based on your observations of the work you have done so far.

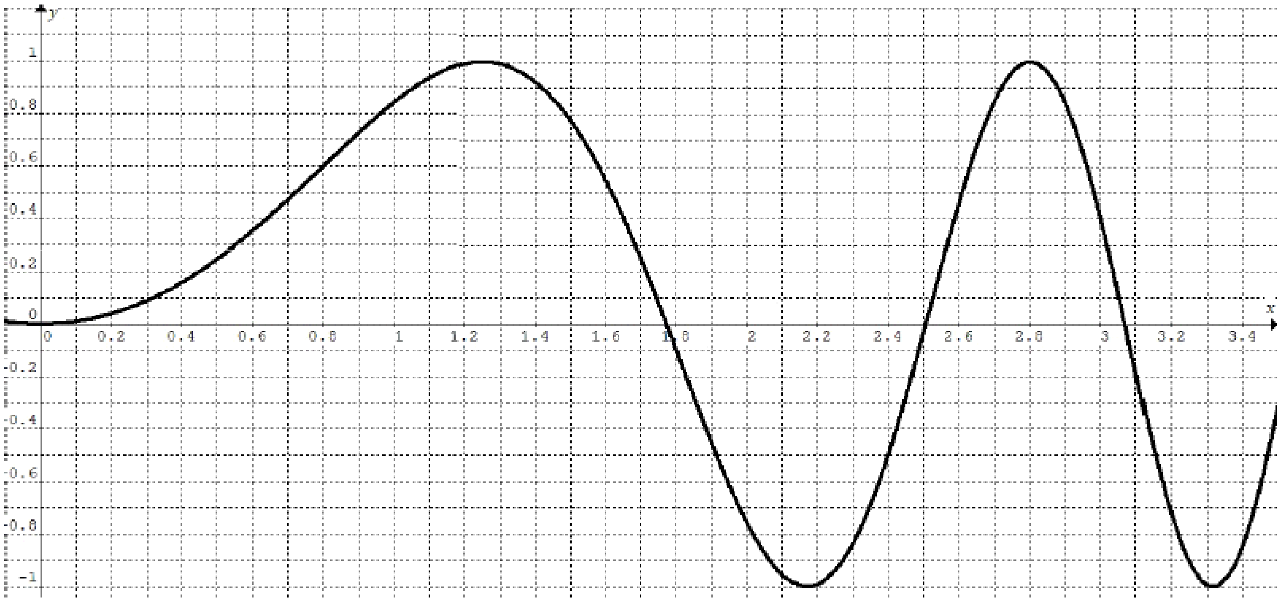
- What is the correspondence between the velocity function's output and the height of any given rectangle you used?

- If you were given a formula or equation for the function, in addition to the graph, how would that help your calculations for the area of the rectangles.

- If you were always restricted to using rectangles of uniform width to approximate areas bounded by curves, how could you improve the accuracy of your approximations?

More on Areas Bounded by Curves

The graph below is the graph of the function $f(x) = \sin(x^2)$.



Find the area bounded by the curve $f(x) = \sin(x^2)$ for $0.4 \le x \le 2.6$. (*Use rectangles of width 0.1 units.)*

Area under the curve
for $0.4 \le x \le 2.6$ $\quad = \quad f(0.4)\times 0.1 + f(0.5)\times 0.1 +$

We call this method **"Riemann Sum**".