Self-Assessment Quiz: Power Series, Taylor & Maclaurin Series

Ungraded — Multiple Choice Questions (choose the best answer)

Q1. A power series centered at a has the general form:

$$\sum_{n=0}^{\infty} c_n (x-a)^n.$$

Which statement is **true** about such a series?

(A) It always converges for all real x.

(B) It converges only at $x = a$.

(C) It converges for $|x-a| < R$ for some nonnegative R (the radius of convergence).

(D) It must represent an analytic function for all x.

Q2. The radius of convergence R of a power series can be found (when applicable) by using:

(A) The Ratio Test or Root Test.

(B) The Alternating Series Test.

(C) Integration by parts.

(D) L'Hôpital's rule.

Q3. The Maclaurin series of e^x is:

(A) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

(B) $\displaystyle\sum_{n=0}^{\infty} n!\, x^n$.

(C) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$.

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n}$.

Q4. The Maclaurin series for $\ln(1+x)$ (for $-1 < x \le 1$) begins as:

(A) $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots$.

(B) $1 + x + x^2 + x^3 + \cdots$.

(C) $x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$.

(D) $\frac{1}{1-x}$.

Q5. Which of the following Taylor series equals $\dfrac{1}{1-x}$ for $|x| < 1$?

(A) $\sum_{n=0}^{\infty} x^{2n}$

(B) $\sum_{n=0}^{\infty} x^{n}$

(C) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

(D) $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

Q6. If the Taylor series of a function f at a converges to $f(x)$ for $|x-a| < R$, then f is called:

(A) Continuous

(B) Differentiable infinitely often (smooth) but not necessarily analytic

(C) Analytic (real-analytic) on that interval

(D) Integrable only

Q7. The Lagrange form of the remainder $R_n(x)$ for the Taylor polynomial of degree n about a is:

(A) $R_n(x) = \frac{f^{(n+1)}(a)}{(n+1)!}(x-a)^{n+1}$ exactly.

(B) $R_n(x) = \frac{f^{(n+1)}(c)}{(n+1)!}(x-a)^{n+1}$ for some c between a and x.

(C) $R_n(x) = 0$ whenever f is continuous.

(D) $R_n(x)$ cannot be estimated.

Q8. Which expansion is the Maclaurin series for $\sin x$?

(A) $\sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!}$

(B) $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$

(C) $\sum_{n=0}^{\infty} (-1)^n x^n$

(D) $\sum_{n=1}^{\infty} \frac{x^n}{n}$

Q9. Consider the power series $\sum_{n=0}^{\infty} \frac{x^n}{n!}$. Its radius of convergence is:

(A) 0

(B) 1

(C) ∞ (i.e., converges for all real x)

(D) Depends on x

Q10. If $f(x) = \sum_{n=0}^{\infty} c_n (x-a)^n$ on its interval of convergence, then the derivative $f'(x)$ is given by:

(A) $\sum_{n=0}^{\infty} c_n (x-a)^n$ (same series)

(B) $\sum_{n=1}^{\infty} n c_n (x-a)^{n-1}$ (termwise differentiation)

(C) $\sum_{n=0}^{\infty} \frac{c_n}{n+1} (x-a)^{n+1}$

(D) Differentiation cannot be performed termwise

Q11. The Maclaurin series for $\arctan x$ (for $|x| \leq 1$, $x \neq \pm 1$ endpoints considered separately) is:

(A) $x - \dfrac{x^3}{3} + \dfrac{x^5}{5} - \cdots$

(B) $x + \dfrac{x^2}{2} + \dfrac{x^3}{3} + \cdots$

(C) $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$

(D) $\dfrac{1}{1+x^2}$

Q12. To find the Taylor series of $f(x) = \sqrt{1+x}$ about $x = 0$, which technique is easiest?

(A) Repeated differentiation only

(B) Use the binomial series expansion $(1+x)^\alpha = \sum \binom{\alpha}{n} x^n$

(C) Expand $\ln(1+x)$ and exponentiate

(D) It has no power series expansion about 0

Q13. Suppose a power series has radius of convergence $R = 2$. Which of the following is true?

(A) The series converges for all x with $|x| < 2$ and diverges for all $|x| > 2$; endpoints $x = \pm 2$ must be tested separately.

(B) The series converges for $|x| < 2$ and also automatically at $x = \pm 2$.

(C) The series diverges for all x with $|x| > 2$ and at $x = 0$.

(D) The radius means the series converges exactly at $x = 2$ only.

Q14. Which of the following is a correct Maclaurin series for $\dfrac{1}{1+x^2}$ valid for $|x| < 1$?

(A) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^{2n}$

(B) $\displaystyle\sum_{n=0}^{\infty} (-1)^n x^n$

(C) $\displaystyle\sum_{n=0}^{\infty} x^{2n+1}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{n}$

Q15. Which statement about Taylor series and uniform convergence is true?

(A) Every Taylor series converges uniformly on its entire radius interval $|x-a| < R$.

(B) A Taylor series always converges uniformly on every closed subinterval $[a-\rho, a+\rho]$ with $0 < \rho < R$.

(C) Taylor series never converge uniformly on any interval.

(D) Uniform convergence is irrelevant for termwise integration/differentiation.

Answer Key

Q1. (C)

Q2. (A)

Q3. (A)

Q4. (A)

Q5. (B)

Q6. (C)

Q7. (B)

Q8. (B)

Q9. (C)

Q10. (B)

Q11. (A)

Q12. (B)

Q13. (B)

Q14. (A)

Q15. (B)