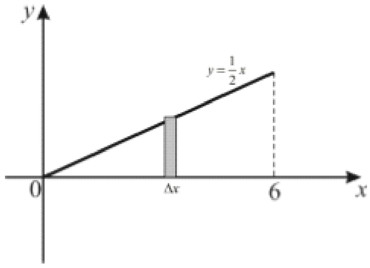
Cal 1 Volume of Revolution

Thursday, 27 June 2024 12:23 am

Slice and Dice!

Consider the function $y = \frac{1}{2}x$, whose graph is shown below. If you were asked to find the area bounded by the function and the x-axis for the interval [0, 6], then you would simply calculate the area of a triangle, without needing any *definite integral*.

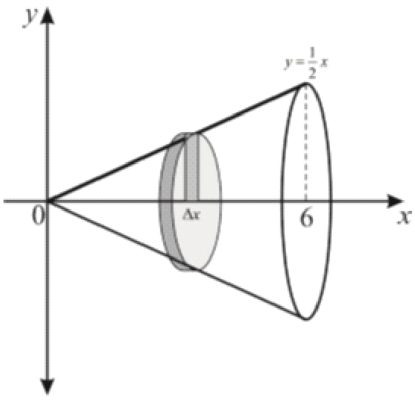


However, if you were to make a Riemann Sum argument then you could use rectangles of width Δx and height equaling function values and get an expression such as

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} \frac{1}{2} x_i \cdot \Delta x = \int_0^6 \frac{1}{2} x \cdot dx .$$

If you calculated this integral you would get an answer of 9 sq. units, which can be confirmed by the area of a triangle formula.

Next, if you were asked to rotate this line with the x-axis as the axis of rotation then you should get a cone as the resulting shape as shown below.



Notice, that as a result of the rotation, each rectangle that was being used in the integration process earlier, turns into a circular disc (a very thin cylinder).

Write an expression for the volume of any one of these discs.

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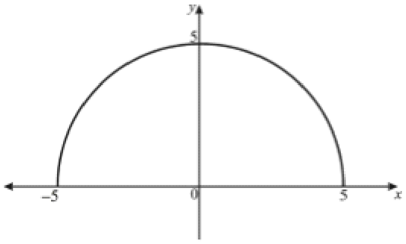
Next, finish the Riemann Sum statement, $\lim_{\Delta x \to 0} \sum_{i=1}^{n}$ _______________, that expresses the volume of this cone in terms of the sums of these discs.

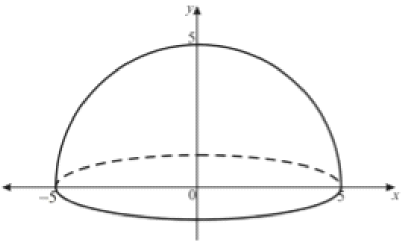
Finally, your expression for the volume is $\int_0^6$ ______________________.

Next, evaluate the definite integral and find the volume of the cone of height 6 and radius 3 units.

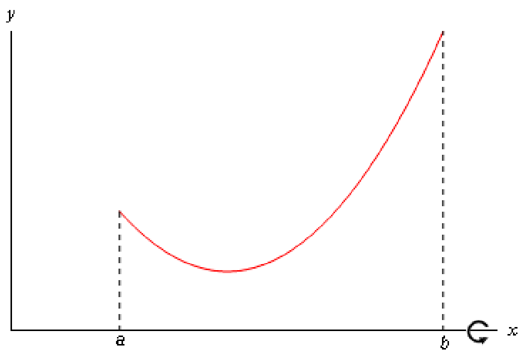
Then, verify your result by using the formula $V = \frac{1}{3}\pi r^2 h$ for the volume of a cone.

Now use a similar process as above to find the volume of a hemisphere by rotating the equation of a semicircle $y = \sqrt{25 - x^2}$ about the y-axis as the axis of rotation. Make sure to first determine what kind of slice you will get for the Riemann Sum argument.





Example 1 Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 4x + 5$, $x = 1$, $x = 4$, and the x-axis about the x-axis.



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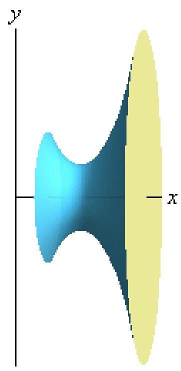
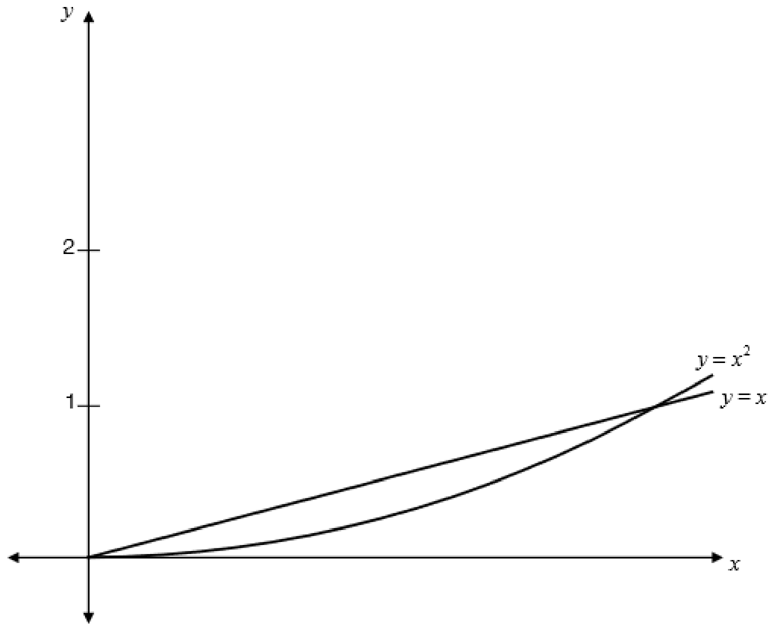


$$A = \pi(\text{radius})^2$$

Example 2 Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \dfrac{x}{4}$ that lies in the first quadrant about the y-axis.

Consider the region bounded by the curves $y = x$ and $y = x^2$ shown in the graph below. If you were to rotate this region about the horizontal line, $y = 1.5$, what kind of shape would you get? Try making some sketches on the diagram below by first marking the axis of rotation.



Next , use the Reimann Sum argument and create an expression for the volume of one slice of this 3D shape.

$$\lim_{\Delta x \to 0} \sum_{i=1}^{n} \underline{\qquad\qquad\qquad\qquad\qquad\qquad}$$

Finally, use a definite integral to find the volume of the solid formed.

$$A = \pi\left(\begin{pmatrix}\text{outer}\\ \text{radius}\end{pmatrix}^2 - \begin{pmatrix}\text{inner}\\ \text{radius}\end{pmatrix}^2\right)$$