Integration Application Questions

Wednesday, 19 June 2024 8:11 pm

61. Water is pumped into a cylindrical tank, standing vertically, at a decreasing rate given at time t minutes by

$$r(t) = 120 - 6t \text{ ft}^3/\text{min} \quad \text{for } 0 \leq t \leq 10.$$

The tank has radius 5 ft and is empty when $t = 0$. Find the depth of water in the tank at $t = 4$.

62. A car moves along a straight line with velocity, in feet/second, given by

$$v(t) = 6 - 2t \quad \text{for } t \geq 0.$$

(a) Describe the car's motion in words. (When is it moving forward, backward, and so on?)

(b) The car's position is measured from its starting point. When is it farthest forward? Backward?

(c) Find s, the car's position measured from its starting point, as a function of time.

64. In drilling an oil well, the total cost, C, consists of fixed costs (independent of the depth of the well) and marginal costs, which depend on depth; drilling becomes more expensive, per meter, deeper into the earth. Suppose the fixed costs are 1,000,000 riyals (the riyal is the unit of currency of Saudi Arabia), and the marginal costs are

$$C'(x) = 4000 + 10x \text{ riyals/meter},$$

where x is the depth in meters. Find the total cost of drilling a well x meters deep.

65. Use the Fundamental Theorem to find the area under $f(x) = x^2$ between $x = 0$ and $x = 3$.

67. Calculate the exact area above the graph of $y = \frac{1}{2}\left(\frac{3}{\pi}x\right)^2$ and below the graph of $y = \cos x$. The curves intersect at $x = \pm\pi/3$.

68. Find the exact area of the shaded region in Figure 6.23 between $y = 3x^2 - 3$ and the x-axis.

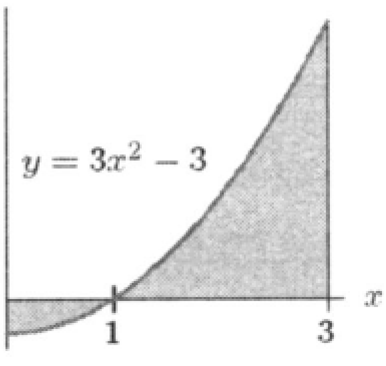

Figure 6.23

69. (a) Find the exact area between $f(x) = x^3 - 7x^2 + 10x$, the x-axis, $x = 0$, and $x = 5$.
(b) Find $\int_0^5 (x^3 - 7x^2 + 10x)\,dx$ exactly and interpret this integral in terms of areas.

70. Find the exact area between the curve $y = e^x - 2$ and the x-axis for $0 \le x \le 2$.

74. The area under $1/\sqrt{x}$ on the interval $1 \le x \le b$ is equal to 6. Find the value of b using the Fundamental Theorem.