Average value & Area bounded by curves

Wednesday, 19 June 2024 8:46 pm

Average Function Value

The average value of a continuous function $f(x)$ over the interval $[a, b]$ is given by,

$$f_{avg} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

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Example 1 Determine the average value of each of the following functions on the given interval.

(a) $f(t) = t^2 - 5t + 6\cos(\pi t)$ on $\left[-1, \frac{5}{2}\right]$

(b) $R(z) = \sin(2z)\mathbf{e}^{1-\cos(2z)}$ on $[-\pi, \pi]$

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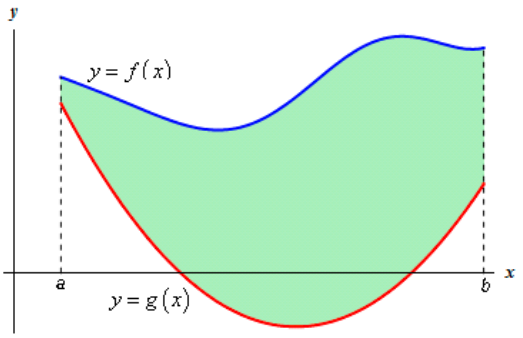
Section 6.2 : Area Between Curves

In this section we are going to look at finding the area between two curves. There are actually two cases that we are going to be looking at.

In the first case we want to determine the area between $y = f(x)$ and $y = g(x)$ on the interval $[a, b]$. We are also going to assume that $f(x) \geq g(x)$. Take a look at the following sketch to get an idea of what we're initially going to look at.



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In the **Area and Volume Formulas** section of the Extras chapter we derived the following formula for the area in this case.

$$A = \int_a^b f(x) - g(x)\,dx \tag{1}$$

The second case is almost identical to the first case. Here we are going to determine the area between $x = f(y)$ and $x = g(y)$ on the interval $[c, d]$ with $f(y) \geq g(y)$.

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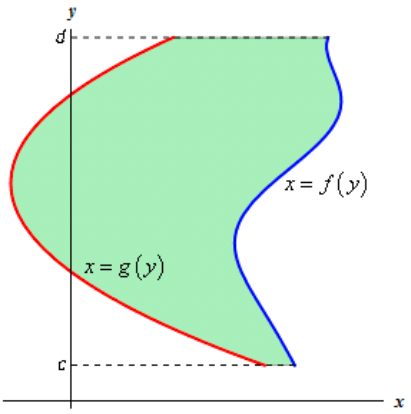

$$A = \int_c^d f(y) - g(y)\ dy$$

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$$A = \int_a^b \left(\begin{array}{c}\text{upper}\\ \text{function}\end{array}\right) - \left(\begin{array}{c}\text{lower}\\ \text{function}\end{array}\right) dx, \qquad a \le x \le b$$

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$$A = \int_c^d \left(\begin{array}{c}\text{right}\\ \text{function}\end{array}\right) - \left(\begin{array}{c}\text{left}\\ \text{function}\end{array}\right) dy, \qquad c \le y \le d$$

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Example 1 Determine the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$.

Example 2 Determine the area of the region bounded by $y = x\mathbf{e}^{-x^2}$, $y = x + 1$, $x = 2$, and the y-axis.

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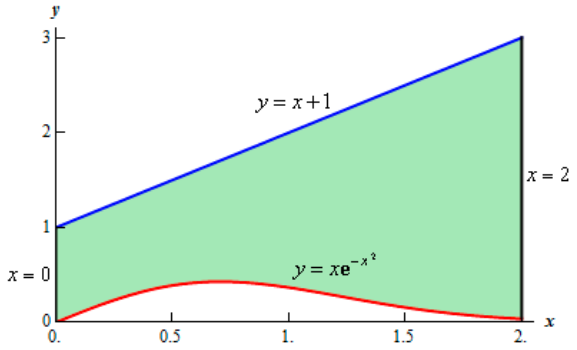

Example 3 Determine the area of the region bounded by $y = 2x^2 + 10$and $y = 4x + 16$.

Example 5 Determine the area of the region enclosed by $y = \sin x$, $y = \cos x$, $x = \frac{\pi}{2}$, and the y-axis.

Example 6 Determine the area of the region enclosed by $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$.

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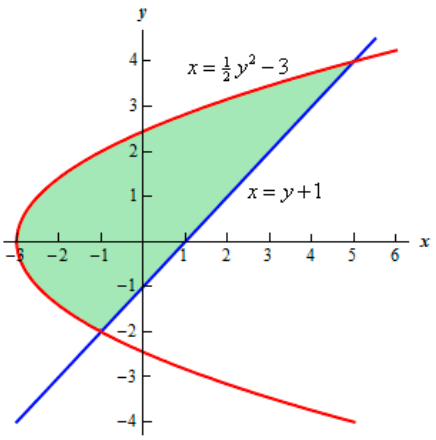

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Example 7 Determine the area of the region bounded by $x = -y^2 + 10$ and $x = (y-2)^2$.

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