Cal 1 Partial Fraction

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Method of Partial Fractions

The integral of some rational functions can be obtained by splitting the integrand into *partial fractions*. For example, to find

$$\int \frac{1}{(x-2)(x-5)}\,dx,$$

Partial fraction decomposition table

Type	*Factor example*	*Decomposition*
Linear factor	$(x-4)$	$\frac{A}{x-4}$
Repeated linear factor	$(x-4)^2$	$\frac{A}{(x-4)} + \frac{B}{(x-4)^2}$
Quadratic irreducible factor	(x^2+4)	$\frac{Ax+B}{(x^2+4)}$
Repeated quadratic irreducible factor	$(x^2+4)^2$	$\frac{Ax+B}{(x^2+4)} + \frac{Cx+D}{(x^2+4)^2}$

the integrand is split into partial fractions with denominators $(x-2)$ and $(x-5)$. We write

$$\frac{1}{(x-2)(x-5)} = \frac{A}{x-2} + \frac{B}{x-5},$$

where A and B are constants that need to be found. Multiplying by $(x-2)(x-5)$ gives the identity

$$1 = A(x-5) + B(x-2)$$

so

$$1 = (A+B)x - 5A - 2B.$$

Since this equation holds for all x, the constant terms on both sides must be equal.[6] Similarly, the coefficients of x on both sides must be equal. So

$$-5A - 2B = 1$$
$$A + B = 0.$$

Solving these equations gives $A = -1/3$, $B = 1/3$. Thus,

$$\frac{1}{(x-2)(x-5)} = \frac{-1/3}{x-2} + \frac{1/3}{x-5}.$$

(Check the answer by writing the right-hand side over the common denominator $(x-2)(x-5)$.)

Example 2 Find $\displaystyle\int \frac{x+2}{x^2+x}\,dx.$

Calculate $\displaystyle\int \frac{x^3-7x^2+10x+1}{x^2-7x+10}\,dx$ using long division before integrating.