

The Newton interpolation formula calculates a polynomial which interpolates a set of coordinates  $\{(x_i, y_i)\}$  and is of the form:

$$p_n(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1) + \dots$$

$$\dots + a_n \prod_{i=0}^{n-1} (x - x_i)$$

more specifically

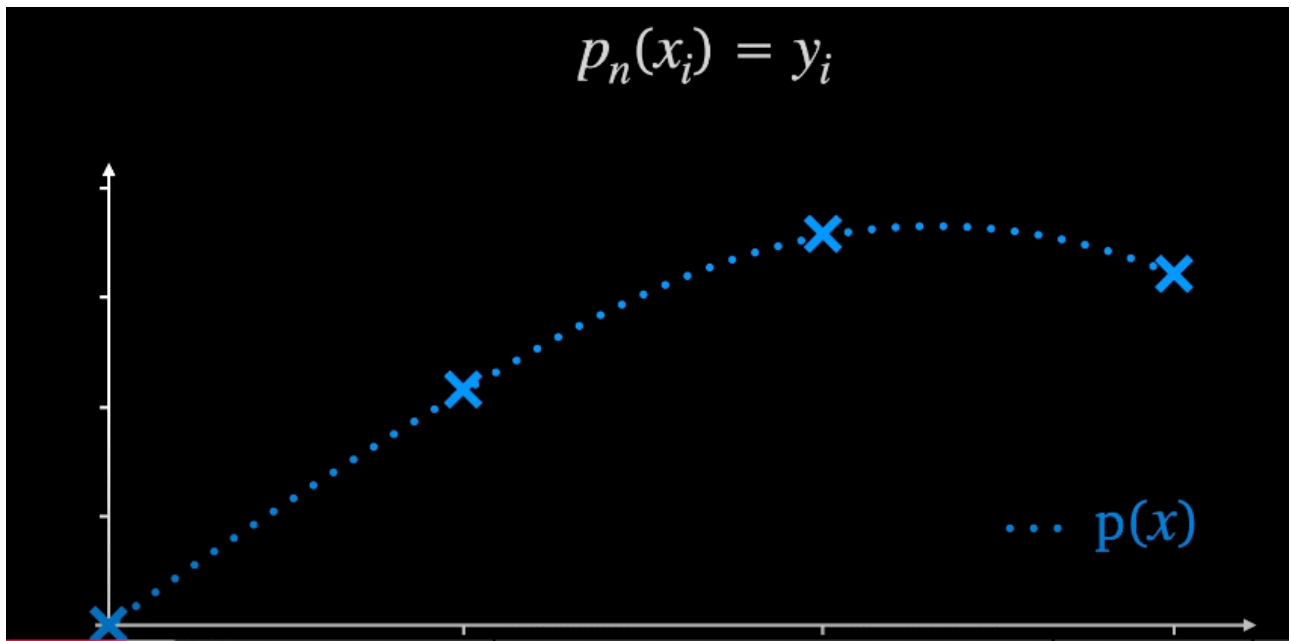
$$a_0 = y_0 \quad a_1 = f[x_0, x_1] \quad a_2 = f[x_0, x_1, x_2]$$

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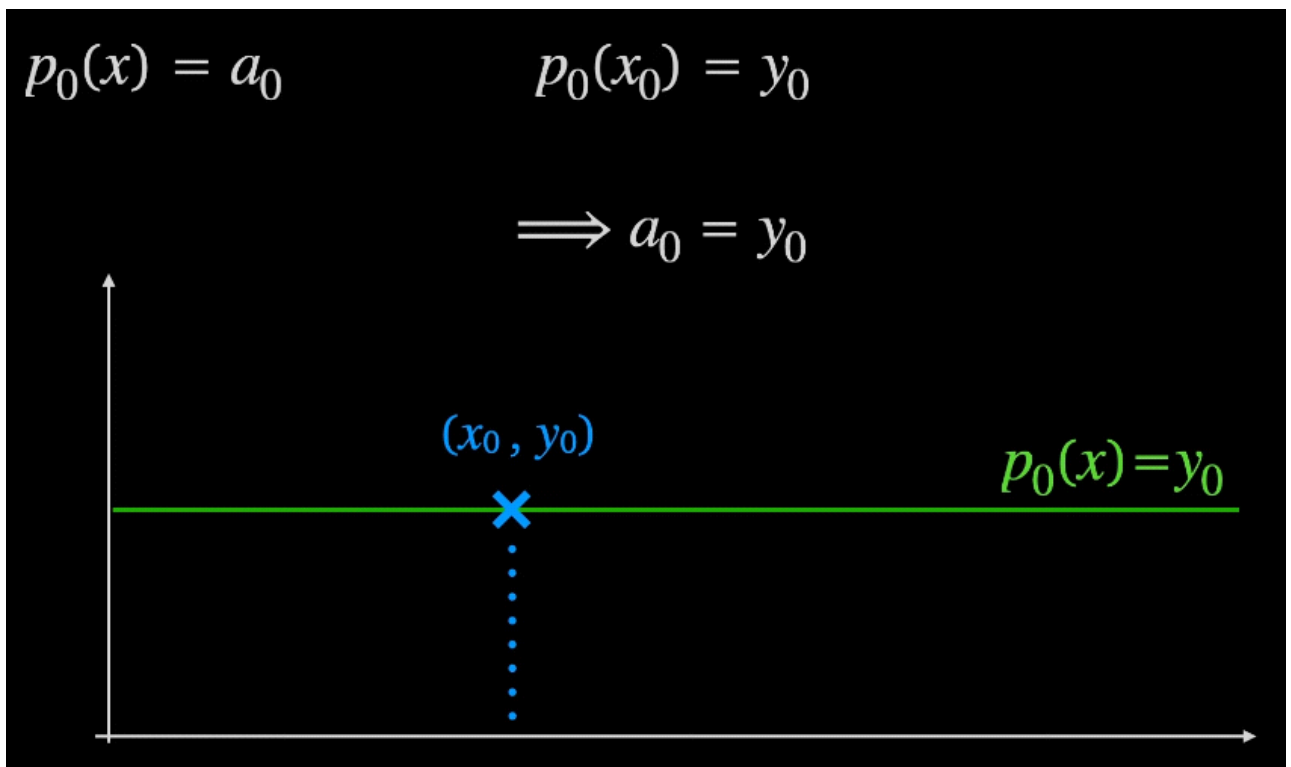
## Summary of the approach

No. Nodes	formula
1	$p_0(x) = a_0$
2	$p_1(x) = a_0 + a_1(x - x_0)$
3	$p_2(x) = a_0 + a_1(x - x_0) + a_2(x - x_0)(x - x_1)$

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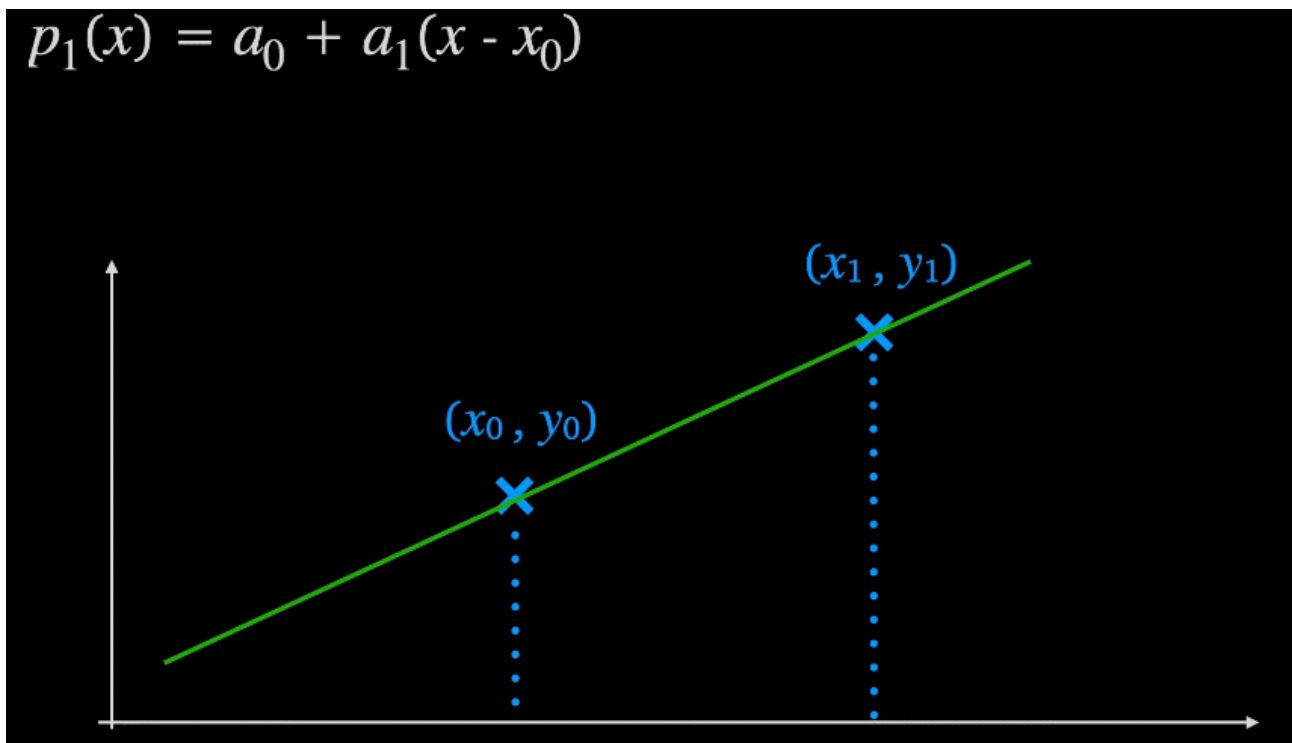


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$$p_1(x) = a_0 + a_1(x - x_0)$$



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$$p_1(x) = y_0 + a_1(x - x_0)$$

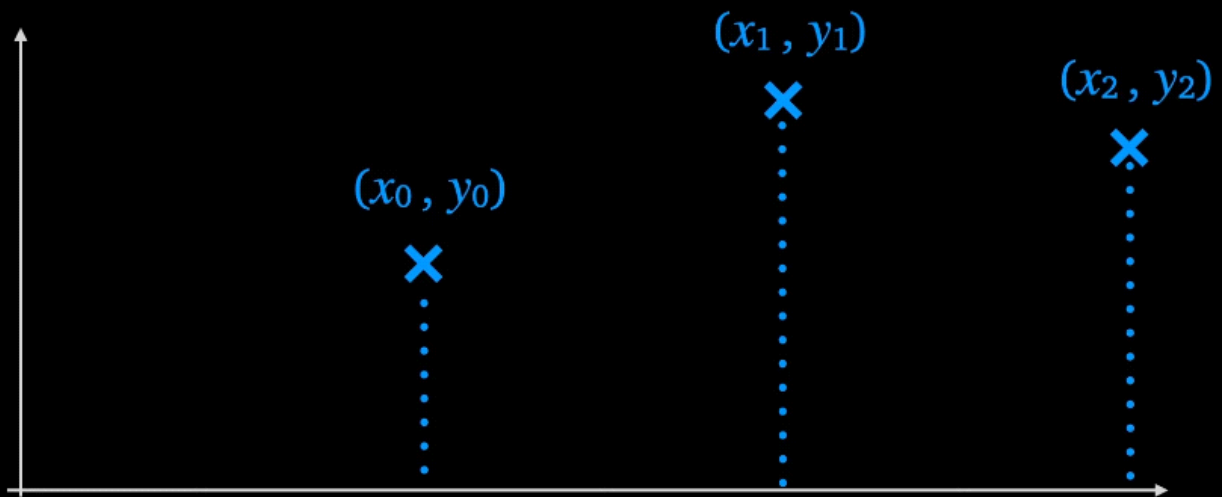
$$p_1(x_1) = \underline{y_0 + a_1(x_1 - x_0)} = y_1$$

$$a_1 = \frac{y_1 - y_0}{x_1 - x_0}$$

$$\frac{y_1 - y_0}{x_1 - x_0} = f[x_0, x_1]$$

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$$p_2(x) = y_0 + f[x_0, x_1](x - x_0) + a_2(x - x_0)(x - x_1)$$



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$$p_2(x) = y_0 + f[x_0, x_1](x - x_0) + a_2(x - x_0)(x - x_1)$$

$$p_2(x_2) = y_0 + f[x_0, x_1](x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1) \\ = y_2$$

$$y_2 - y_0 = f[x_0, x_1](x_2 - x_0) + a_2(x_2 - x_0)(x_2 - x_1)$$

$$f[x_0, x_1] + a_2(x_2 - x_1) = \frac{y_2 - y_0}{x_2 - x_0} = f[x_0, x_2]$$

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$$f[x_0, x_2] = f[x_0, x_1] + a_2(x_2 - x_1)$$

$$a_2 = \frac{f[x_0, x_2] - f[x_0, x_1]}{x_2 - x_1}$$

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$$a_2 = \frac{f[x_0, x_2] - f[x_0, x_1]}{x_2 - x_1}$$

But in standard form, the second divided difference is

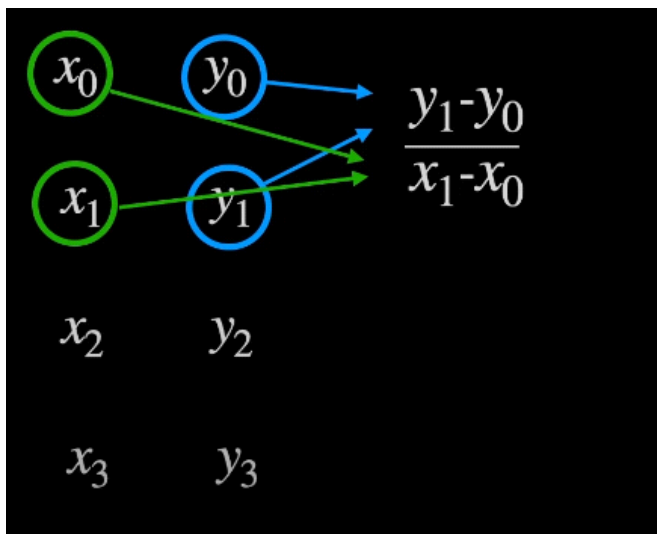
$$a_2 = \frac{f[x_1, x_2] - f[x_0, x_1]}{x_2 - x_0} = f[x_0, x_1, x_2]$$

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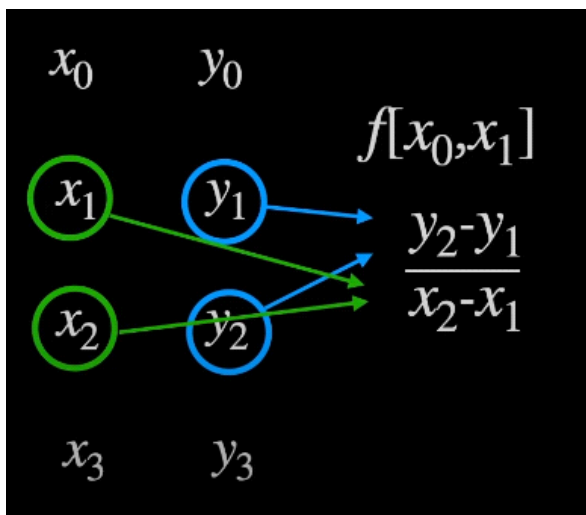
# The divided difference table

$x_0$	$y_0$			
		$f[x_0, x_1]$		
$x_1$	$y_1$		$f[x_0, x_1, x_2]$	
		$f[x_1, x_2]$		$f[x_0, x_1, x_2, x_3]$
$x_2$	$y_2$		$f[x_1, x_2, x_3]$	
		$f[x_2, x_3]$		
$x_3$	$y_3$			

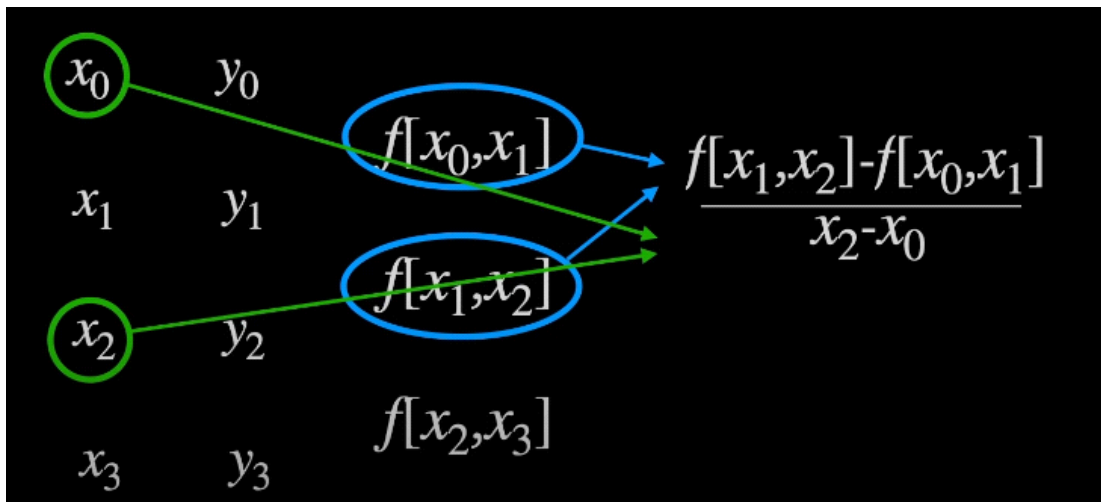
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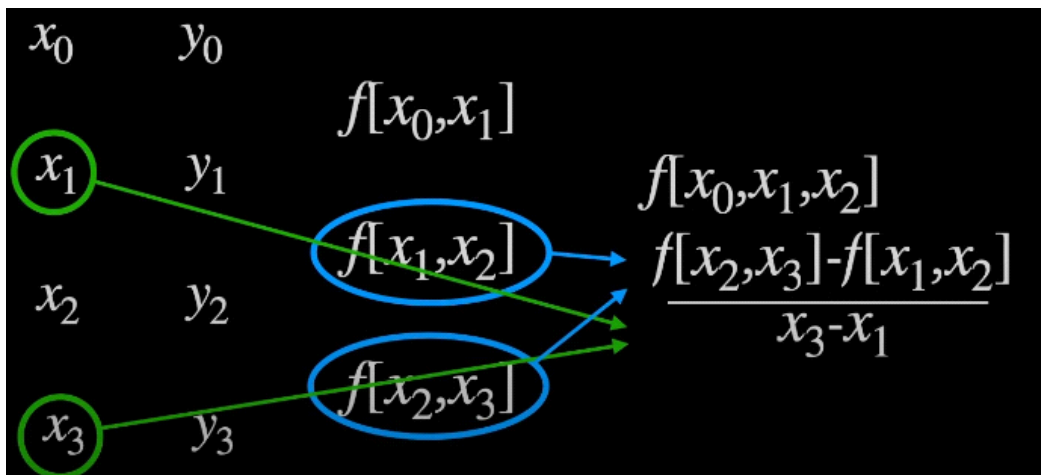
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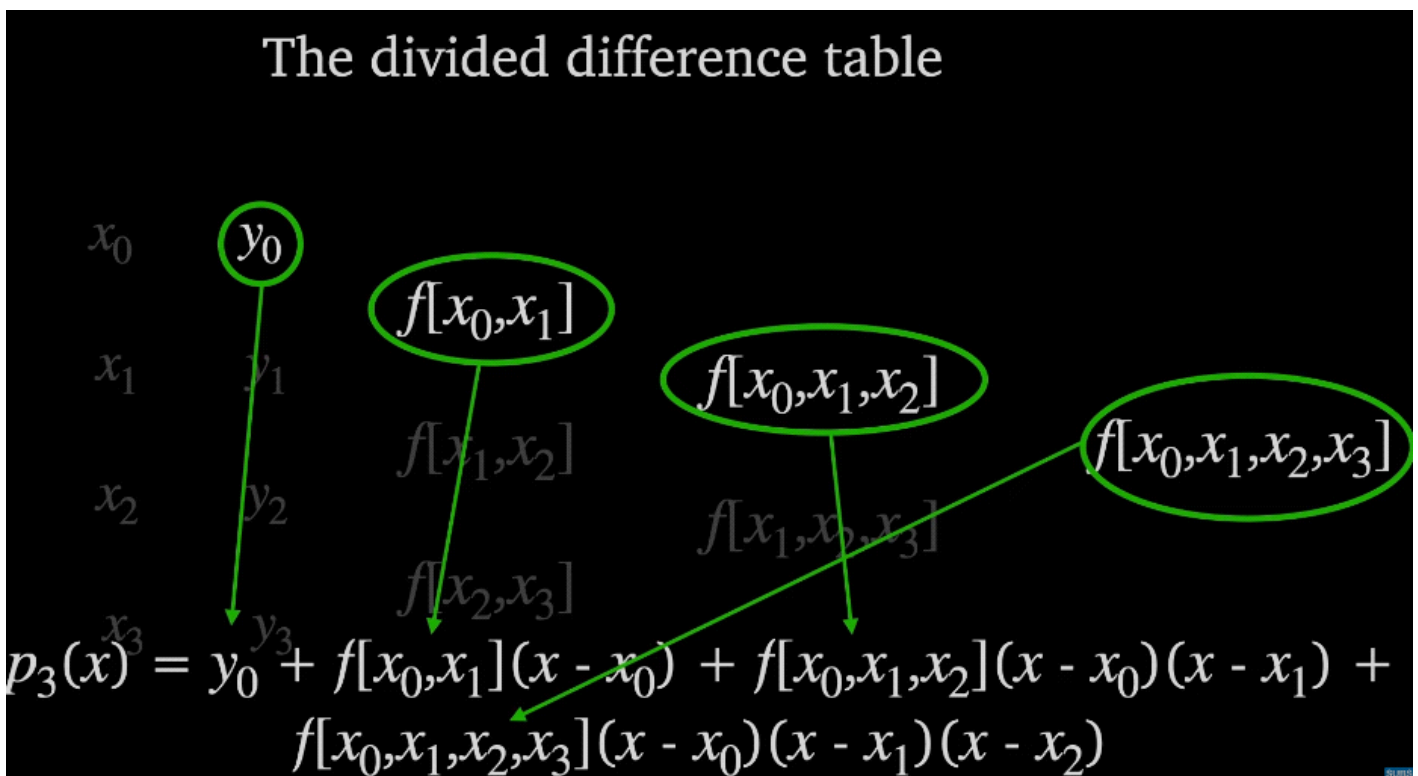
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$x$	$\sin(x)$	1 <sup>st</sup> order	2 <sup>nd</sup> order	3 <sup>rd</sup> order
0	0	0.901		
0.785	0.707		-0.336	
		0.373		$\frac{-0.475 + 0.336}{2.356 - 0}$
1.571	1		-0.475	
		-0.373		
2.356	0.707			

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