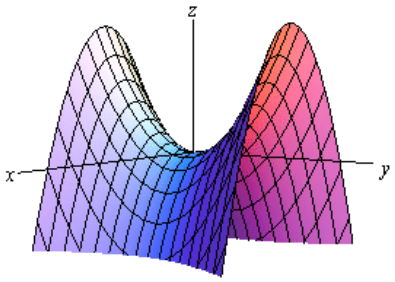
# Partial Derivatives II: Higher Derivatives, Implicit & Explicit

Friday, 2 August 2024 9:52 am

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***Example 1*** Determine if $f(x,y) = \dfrac{x^2}{y^3}$ is increasing or decreasing at $(2,5)$,

**(a)** if we allow $x$ to vary and hold $y$ fixed.

**(b)** if we allow $y$ to vary and hold $x$ fixed.

Determine if $f(x,y) = x\ln(4y) + \sqrt{x+y}$ is increasing or decreasing at $(-3,6)$ if

**(a)** we allow $x$ to vary and hold $y$ fixed.

**(b)** we allow $y$ to vary and hold $x$ fixed.

## Higher Order Partial Derivatives

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$$(f_x)_x = f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$
$$(f_x)_y = f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$
$$(f_y)_x = f_{yx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y}$$
$$(f_y)_y = f_{yy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2}$$

Find all the second order derivatives for $f(x,y) = \cos(2x) - x^2\mathbf{e}^{5y} + 3y^2$.

### Clairaut's Theorem

Suppose that $f$ is defined on a disk $D$ that contains the point $(a,b)$. If the functions $f_{xy}$ and $f_{yx}$ are continuous on this disk then,

$$f_{xy}(a,b) = f_{yx}(a,b)$$

***Example 2*** Verify Clairaut's Theorem for $f(x,y) = x\mathbf{e}^{-x^2y^2}$.

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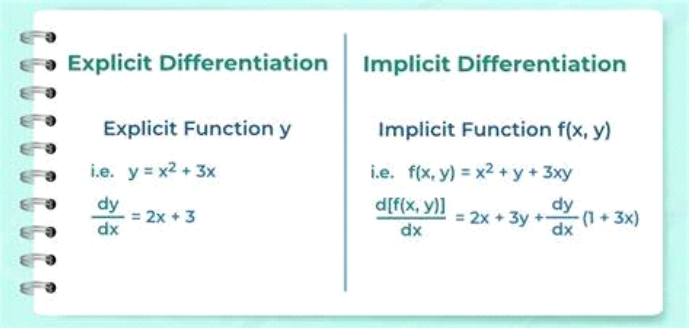
***Example 3*** Find the indicated derivative for each of the following functions.

**(a)** Find $f_{xxyzz}$ for $f(x,y,z) = z^3y^2\ln(x)$

**(b)** Find $\dfrac{\partial^3 f}{\partial y \partial x^2}$ for $f(x,y) = \mathbf{e}^{xy}$

| Explicit Function Examples | Implicit Function Examples |
|---|---|
| $y = 3x + 5$ | $x^2 + y^2 = 25$ |
| $y = x^2 + 6x - 8$ | $3x^2 - 5y^2 + 9x = 25 - 15y$ |
| $x = a\cos(n\theta)$ | $m^2n + mn^2 - mn - 2$ |
| $d = rt$ | $a(r\cos\theta - a) = k\cos^2\theta$ |




# Practice Questions

***Example 1*** Find $y'$ for $xy = 1$.

***Example 4*** Find the equation of the tangent line to

$$x^2 + y^2 = 9$$

at the point $\left(2,\ \sqrt{5}\right)$.

***Example 5*** Find $y'$ for each of the following.

**(a)** $x^3y^5 + 3x = 8y^3 + 1$

Given $x^2 + y^2 + z^2 = \sin(yz)$
find $dz/dx$

EXAMPLE: For $f(x,y) = e^{xy} + x/y$
Find $f_x(x,y)$ & $f_y(x,y)$

EXAMPLE: $f(x,y) = \sin\left(\frac{x}{1+y}\right)$

Q Find $\partial z/\partial x$ and $\partial z/\partial y$ for the implicit equation given as

$$x^3 + y^3 + z^3 + 6xyz = 1$$

Sol

EXAMPLE: (Given) For a function of 3 variables
$f(x,y,z) = e^{xy}\ln(z)$
Find $f_x$, $f_y$, $f_z$

EXAMPLE: Find $f_{xxyz}$ if $f(x,y,z) = \sin(3x+yz)$

Q Find $f_{xyyy}$ for $f(x,y) = \cos(xy) - x^3 y^4$