Level Curve: Contour

Tuesday, 30 July 2024 1:30 pm

Topographical Maps

One of the most common examples of a contour diagram is a topographical map like that show Figure 12.32. It gives the elevation in the region and is a good way of getting an overall picture o terrain: where the mountains are, where the flat areas are. Such topographical maps are freque colored green at the lower elevations and brown, red, or white at the higher elevations.

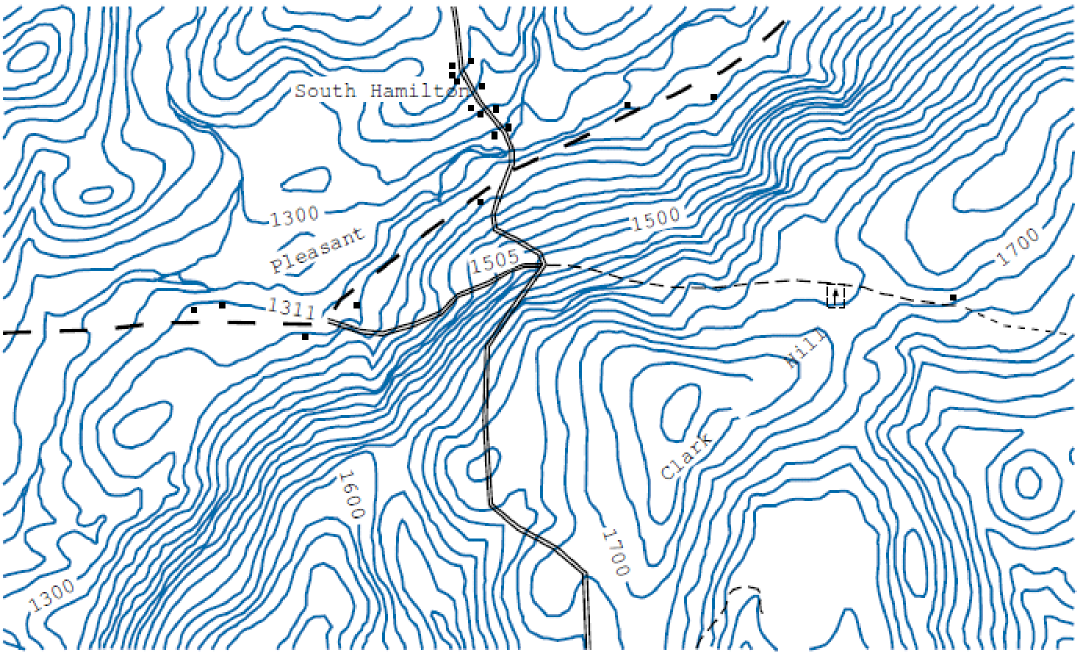

Figure 12.32: A topographical map showing the region around South Hamilton, NY

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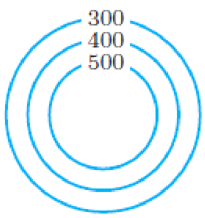

Figure 12.33: Mountain peak

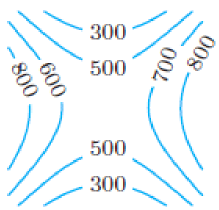

Figure 12.34: Pass between two mountains

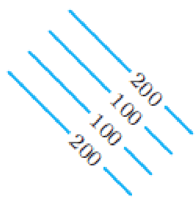

Figure 12.35: Long valley

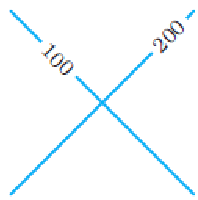

Figure 12.36: Impossible contour lines

Example 1 Use Figure 12.37 to estimate $f(18, 78)$ and $f(12, 76)$ and interpret in terms of corn production.

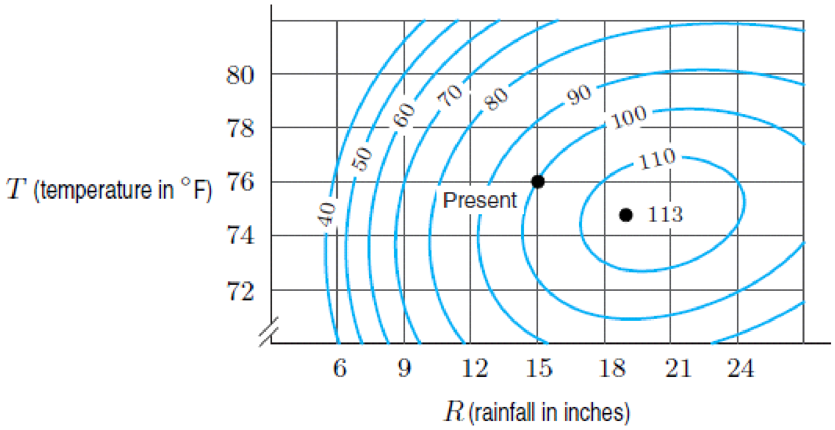

Figure 12.37: Corn production, C, as a function of rainfall and temperature

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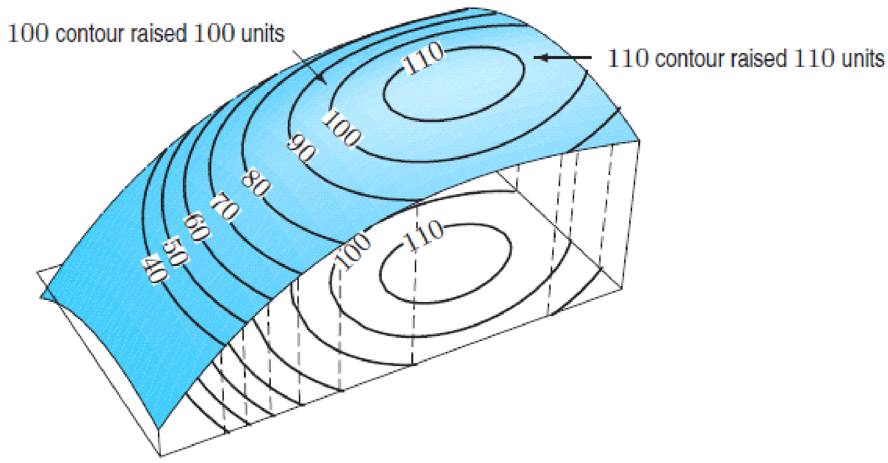

Figure 12.38: Getting the graph of the corn yield function from the contour diagram

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> Contour lines, or level curves, are obtained from a surface by slicing it with horizontal planes.
> A contour diagram is a collection of level curves labeled with function values.

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Finding Contours Algebraically

Algebraic equations for the contours of a function f are easy to find if we have a formula for $f(x, y)$. Suppose the surface has equation

$$z = f(x, y).$$

A contour is obtained by slicing the surface with a horizontal plane with equation $z = c$. Thus, the equation for the contour at height c is given by:

$$f(x, y) = c.$$

Example 3 Find equations for the contours of $f(x, y) = x^2 + y^2$ and draw a contour diagram for f. Relate the contour diagram to the graph of f.

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Solution The contour at height c is given by

$$f(x, y) = x^2 + y^2 = c.$$

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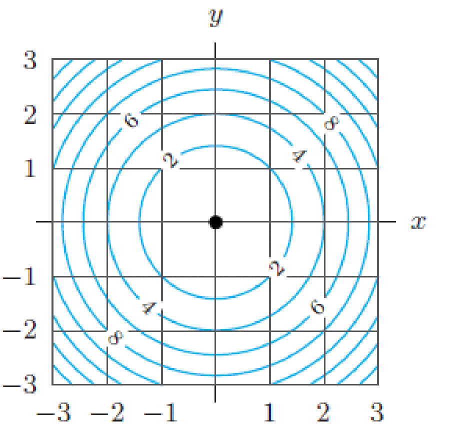

Figure 12.39: Contour diagram for $f(x, y) = x^2 + y^2$ (even values of c only)

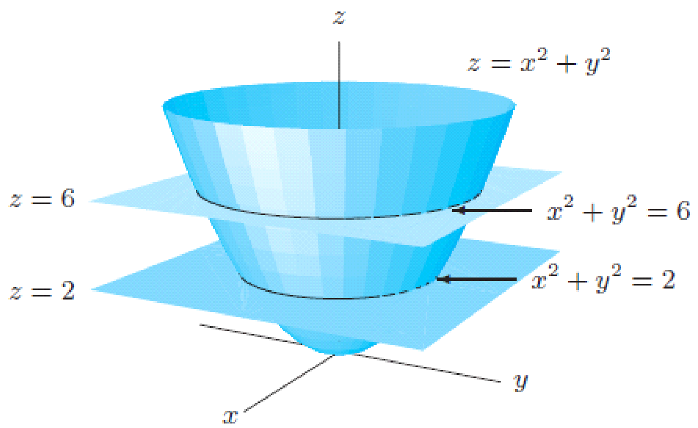

Figure 12.40: The graph of $f(x, y) = x^2 + y^2$

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Example 5 Draw a contour diagram for $f(x, y) = 2x + 3y + 1$.

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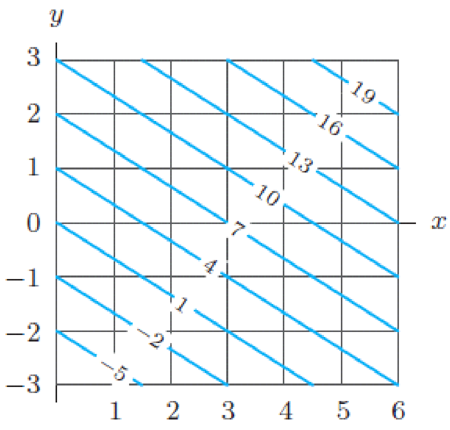


Figure 12.43: A contour diagram for $f(x, y) = 2x + 3y + 1$

EXAMPLE 5 Sketch the level curves for $k = -6, 0, 6, 12$

$f(x,y) = 6 - 3x - 2y$

SOLUTION: $f(x,y) = k$

So; $6 - 3x - 2y = k \Rightarrow 3x + 2y + (k-6) = 0$

or $y = -\frac{3}{2}x - \frac{(k-6)}{2}$

1) $k = -6$

$\boxed{y = -\frac{3}{2}x + 6}$

2) $k = 0$

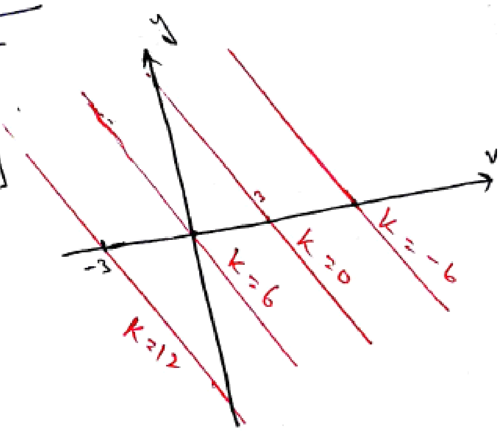
$\boxed{y = -\frac{3}{2}x + 3}$

3) $k = 6$

$\boxed{y = -\frac{3}{2}x}$

4) $k = 12$

$\boxed{y = -\frac{3}{2}x - 3}$



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EXAMPLE 6 | Sketch level curves for $g(x,y) = \sqrt{9-x^2-y^2}$ for $k = 0, 1, 2, 3$

SOLUTION :

$$\sqrt{9-x^2-y^2} = k$$

or $$9-x^2-y^2 = k^2$$

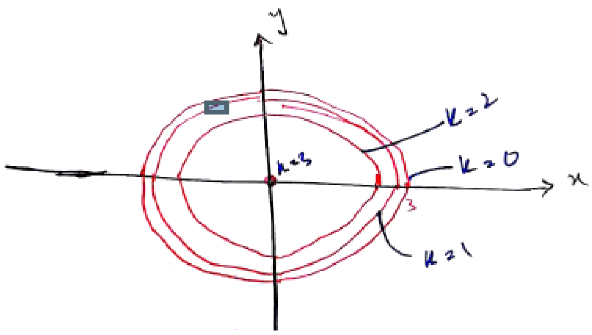
$$x^2+y^2 = 9-k^2$$

(1) $\underline{k=0}$ $x^2+y^2 = (3)^2 \rightarrow r = 3$

(2) $\underline{k=1}$ $x^2+y^2 = 8 \rightarrow r = 2.83$

(3) $\underline{k=2}$ $x^2+y^2 = 5 \rightarrow r = 2.24$

(4) $\underline{k=3}$ $x^2+y^2 = 0 \rightarrow r = 0$



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