Limit & Continuity

Tuesday, 30 July 2024 1:30 pm

The function f has a **limit** L at the point (a, b), written

$$\lim_{(x,y)\to(a,b)} f(x, y) = L,$$

if $f(x, y)$ is as close to L as we please whenever the distance from the point (x, y) to the point (a, b) is sufficiently small, but not zero.

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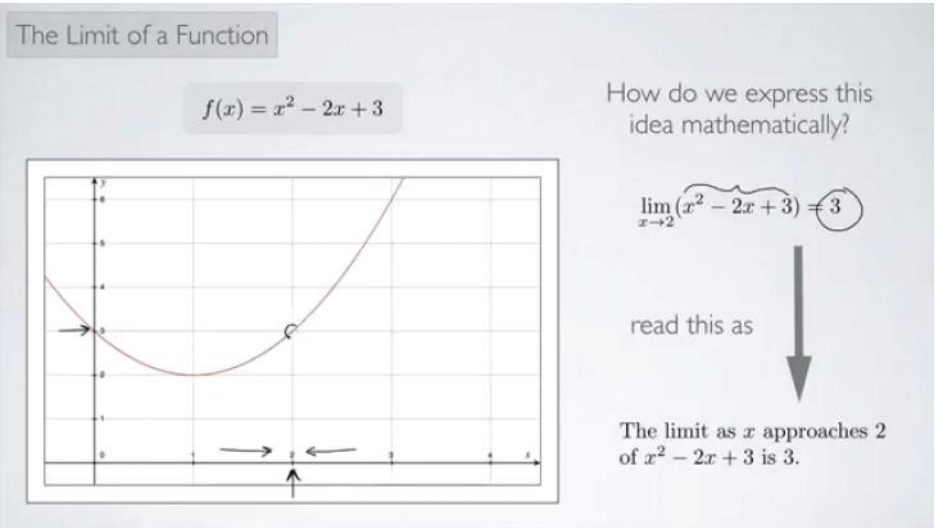
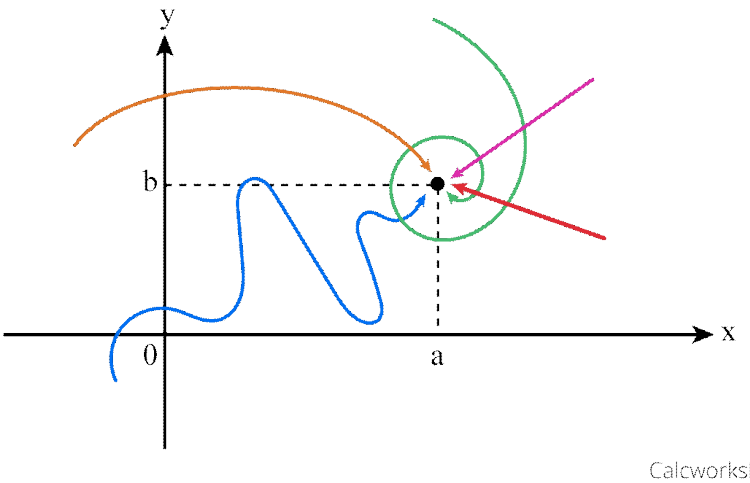
A function f is **continuous at the point** (a, b) if

$$\lim_{(x,y)\to(a,b)} f(x, y) = f(a, b).$$

A function is **continuous on a region** R in the xy-plane if it is continuous at each point in R.





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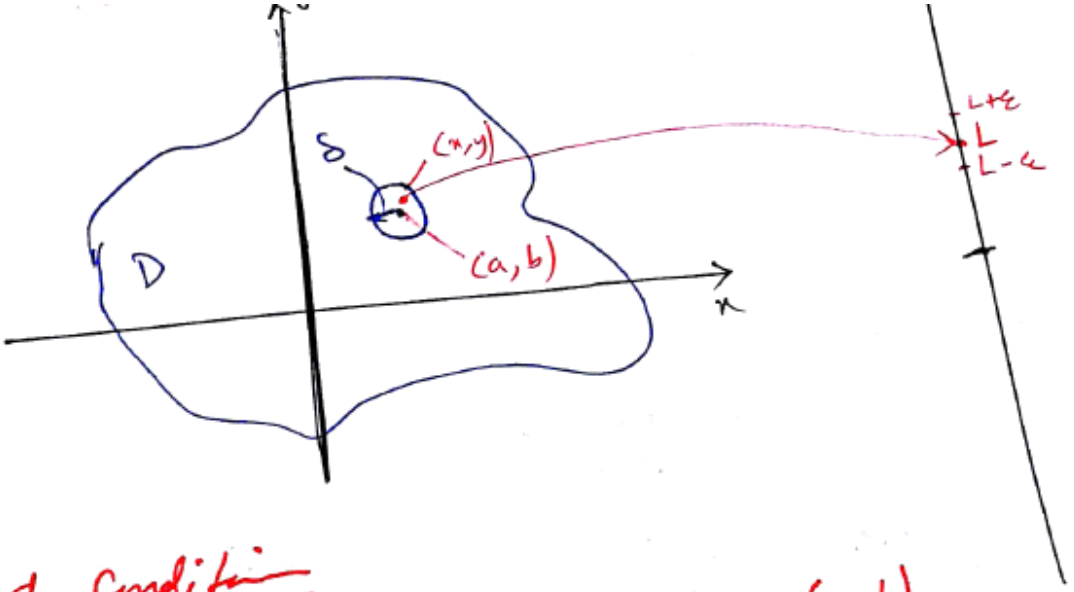

Important Condition

- If $f(x,y) \to L_1$ as $(x,y) \to (a,b)$ along a path C_1 and $f(x,y) \to L_2$ as $(x,y) \to (a,b)$ along a path C_2, where $L_1 \neq L_2$ - then the limit does not exist.

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EXAMPLE: Show that the limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{x^2+y^2}$$

SOLUTION:

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EXAMPLE: Find $\lim_{(x,y)\to(0,0)} f(x,y)$ where

$$f(x,y) = \frac{xy}{x^2+y^2}$$

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Q - $f(x,y) = \frac{xy^2}{x^2+y^4}$ as $(x,y) \to (0,0)$

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Q - $f(x,y) = \frac{3x^2y}{x^2+y^2}$ as $(x,y) \to (0,0)$ which shows that along x-axis

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CONTINUITY

DEFINITION: A function of 2 variables is continuous at (a,b) if

$$\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$$

$f(x,y)$ is continuous on Domain if 'f' is continuous on every point (a,b) in D.

Q $\lim_{(x,y)\to(1,2)} f(x,y) = \lim_{(x,y)\to(1,2)} [x^2y^3 - x^3y^2 + 3x + 2y]$

Sol: Polynomial functions are usually continuous so;

$$\lim_{(x,y)\to(1,2)} f(x,y) = (1)^2(2)^3 - (1)^3(2)^2 + 3(1) + 2(2)$$

$$= 8 - 4 + 3 + 4$$

$$\lim_{(x,y)\to(1,2)} f(x,y) = 11$$

Q Discuss continuity of

$$f(x,y) = \frac{x^2-y^2}{x^2+y^2}$$

Since function is discontinuous at $(x,y) = (0,0)$. This rational function is continuous on its Domain i.e.

$$D = \{(x,y) \mid (x,y) \neq (0,0)\}$$

(b) $\lim_{(x,y)\to(5,1)} \frac{xy}{x+y}$

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Example 2 Determine if the following limit exist or not. If they do exist give the value of the limit.

$$\lim_{(x,y)\to(1,1)} \frac{2x^2 - xy - y^2}{x^2 - y^2}$$

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Example 3 Determine if the following limits exist or not. If they do exist give the value of the limit.

(a) $\lim_{(x,y)\to(0,0)} \frac{x^2y^2}{x^4 + 3y^4}$

(b) $\lim_{(x,y)\to(0,0)} \frac{x^3y}{x^6 + y^2}$

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