

Continuity Questions

Friday, 2 August 2024

9:59 am

In Exercises 1–6, is the function continuous at all points in the given region?

1. $\frac{1}{x^2 + y^2}$ on the square $-1 \leq x \leq 1, -1 \leq y \leq 1$

2. $\frac{1}{x^2 + y^2}$ on the square $1 \leq x \leq 2, 1 \leq y \leq 2$

3. $\frac{y}{x^2 + 2}$ on the disk $x^2 + y^2 \leq 1$

4. $\frac{e^{\sin x}}{\cos y}$ on the rectangle $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, 0 \leq y \leq \frac{\pi}{4}$

5. $\tan(xy)$ on the square $-2 \leq x \leq 2, -2 \leq y \leq 2$

6. $\sqrt{2x - y}$ on the disk $x^2 + y^2 \leq 4$

17. Let $f(x, y) = \begin{cases} \frac{|x|}{x}y & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$

Is $f(x, y)$ continuous

(a) On the x -axis?

(b) On the y -axis?

(c) At $(0, 0)$?

23. The function f , whose graph and contour diagram are in Figures 12.89 and 12.90, is given by

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0), \\ 0, & (x, y) = (0, 0). \end{cases}$$

(a) Show that $f(0, y)$ and $f(x, 0)$ are each continuous functions of one variable.

(b) Show that rays emanating from the origin are contained in contours of f .

(c) Is f continuous at $(0, 0)$?

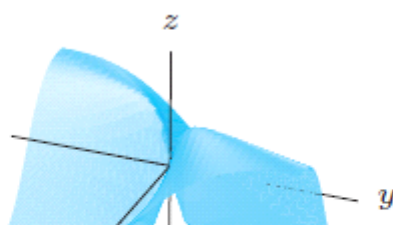




Figure 12.89: Graph of $z = xy/(x^2 + y^2)$

20. Is the following function continuous at $(0, 0)$?

$$f(x, y) = \begin{cases} x^2 + y^2 & \text{if } (x, y) \neq (0, 0) \\ 2 & \text{if } (x, y) = (0, 0) \end{cases}$$

21. What value of c makes the following function continuous at $(0, 0)$?

$$f(x, y) = \begin{cases} x^2 + y^2 + 1 & \text{if } (x, y) \neq (0, 0) \\ c & \text{if } (x, y) = (0, 0) \end{cases}$$