Integration: Trigonometric Functions

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$$\int \frac{\ln x}{x^2}\, dx$$

$$\int (\ln x)^2\, dx$$

Trigonometric Integrals

V EXAMPLE 3 Evaluate $\int_0^\pi \sin^2 x\, dx$.

EXAMPLE 4 Find $\int \sin^4 x\, dx$.

EXAMPLE 1 Evaluate $\int \cos^3 x\, dx$.

V EXAMPLE 2 Find $\int \sin^5 x \cos^2 x\, dx$.

Strategy for Evaluating $\int \sin^m x \cos^n x\, dx$

(a) If the power of cosine is odd ($n = 2k + 1$), save one cosine factor and use $\cos^2 x = 1 - \sin^2 x$ to express the remaining factors in terms of sine:

$$\int \sin^m x \cos^{2k+1} x\, dx = \int \sin^m x\, (\cos^2 x)^k \cos x\, dx$$

$$= \int \sin^m x\, (1 - \sin^2 x)^k \cos x\, dx$$

Then substitute $u = \sin x$.

(b) If the power of sine is odd ($m = 2k + 1$), save one sine factor and use $\sin^2 x = 1 - \cos^2 x$ to express the remaining factors in terms of cosine:

$$\int \sin^{2k+1} x \cos^n x\, dx = \int (\sin^2 x)^k \cos^n x \sin x\, dx$$

$$= \int (1 - \cos^2 x)^k \cos^n x \sin x\, dx$$

Then substitute $u = \cos x$. [Note that if the powers of both sine and cosine are odd, either (a) or (b) can be used.]

(c) If the powers of both sine and cosine are even, use the half-angle identities

$$\sin^2 x = \tfrac{1}{2}(1 - \cos 2x) \qquad \cos^2 x = \tfrac{1}{2}(1 + \cos 2x)$$

It is sometimes helpful to use the identity

$$\sin x \cos x = \tfrac{1}{2} \sin 2x$$

Strategy for Evaluating $\int \tan^m x \sec^n x\, dx$

(a) If the power of secant is even ($n = 2k$, $k \geqslant 2$), save a factor of $\sec^2 x$ and use $\sec^2 x = 1 + \tan^2 x$ to express the remaining factors in terms of $\tan x$:

$$\int \tan^m x \sec^{2k} x\, dx = \int \tan^m x\, (\sec^2 x)^{k-1} \sec^2 x\, dx$$

$$= \int \tan^m x\, (1 + \tan^2 x)^{k-1} \sec^2 x\, dx$$

Then substitute $u = \tan x$.

(b) If the power of tangent is odd ($m = 2k + 1$), save a factor of $\sec x \tan x$ and use $\tan^2 x = \sec^2 x - 1$ to express the remaining factors in terms of $\sec x$:

$$\int \tan^{2k+1} x \sec^n x\, dx = \int (\tan^2 x)^k \sec^{n-1} x \sec x \tan x\, dx$$

$$= \int (\sec^2 x - 1)^k \sec^{n-1} x \sec x \tan x\, dx$$

Then substitute $u = \sec x$.

V EXAMPLE 5 Evaluate $\int \tan^6 x \sec^4 x\,dx$.

EXAMPLE 6 Find $\int \tan^5\theta \sec^7\theta\,d\theta$.

EXAMPLE 7 Find $\int \tan^3 x\,dx$.

EXAMPLE 8 Find $\int \sec^3 x\,dx$.

EXAMPLE 9 Evaluate $\int \sin 4x \cos 5x\,dx$.

2 To evaluate the integrals (a) $\int \sin mx \cos nx\,dx$, (b) $\int \sin mx \sin nx\,dx$, or (c) $\int \cos mx \cos nx\,dx$, use the corresponding identity:

(a) $\sin A \cos B = \frac{1}{2}[\sin(A - B) + \sin(A + B)]$

(b) $\sin A \sin B = \frac{1}{2}[\cos(A - B) - \cos(A + B)]$

(c) $\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$