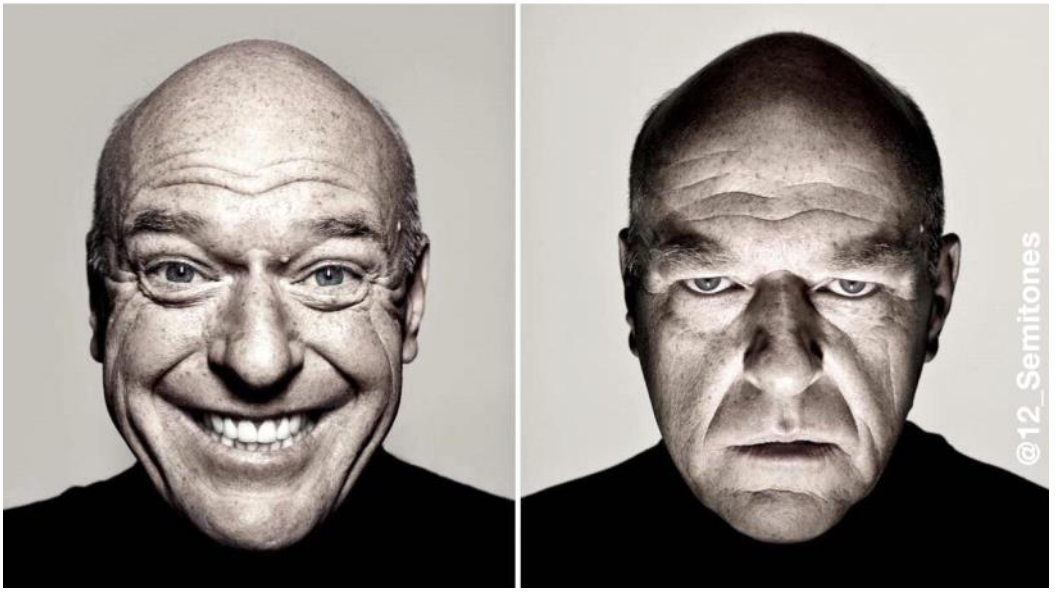
Integration: Substitution in Trigonometric

Thursday, 29 August 2024 11:54 pm

$$\int \frac{1}{x^5} dx \qquad \int \frac{1}{x^5+1} dx$$



Trigonometric Substitution

Table of Trigonometric Substitutions

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin\theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a \tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}$ or $\pi \leqslant \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

V EXAMPLE 1 Evaluate $\int \frac{\sqrt{9 - x^2}}{x^2} dx$.

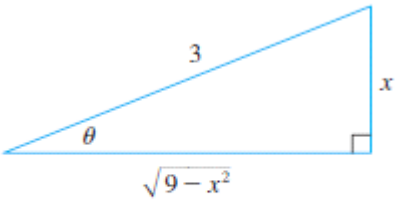


FIGURE 1

$\sin\theta = \frac{x}{3}$

V EXAMPLE 2 Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

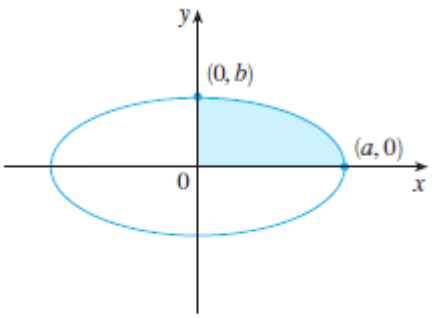


FIGURE 2

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

V EXAMPLE 3 Find $\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$.

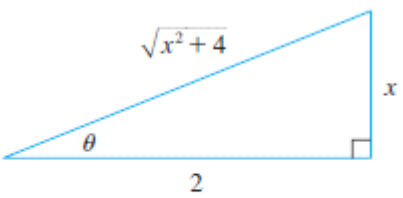


FIGURE 3

$\tan\theta = \frac{x}{2}$

EXAMPLE 4 Find $\int \frac{x}{\sqrt{x^2+4}}\,dx$.

EXAMPLE 5 Evaluate $\int \frac{dx}{\sqrt{x^2 - a^2}}$, where $a > 0$.

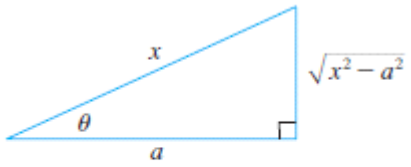


FIGURE 4

$\sec\theta = \frac{x}{a}$

EXAMPLE 6 Find $\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2 + 9)^{3/2}}\, dx$.

EXAMPLE 7 Evaluate $\int \frac{x}{\sqrt{3 - 2x - x^2}}\, dx$.