Fundamental Theorems of Calculus

Sunday, February 5, 2023 12:35 PM

The Fundamental Theorem of Calculus, Part 1 If f is continuous on $[a, b]$, then the function g defined by

$$g(x) = \int_a^x f(t)\, dt \qquad a \leqslant x \leqslant b$$

is continuous on $[a, b]$ and differentiable on (a, b), and $g'(x) = f(x)$.

The Fundamental Theorem of Calculus, Part 2 If f is continuous on $[a, b]$, then

$$\int_a^b f(x)\, dx = F(b) - F(a)$$

where F is any antiderivative of f, that is, a function such that $F' = f$.

The Fundamental Theorem of Calculus is appropriately named because it establishes a connection between the two branches of calculus: differential calculus and integral calculus. Differential calculus arose from the tangent problem, whereas integral calculus arose from a seemingly unrelated problem, the area problem. Newton's mentor at Cambridge, Isaac Barrow (1630–1677), discovered that these two problems are actually closely related. In fact, he realized that differentiation and integration are inverse processes. The Fundamental Theorem of Calculus gives the precise inverse relationship between the derivative and the integral. It was Newton and Leibniz who exploited this relationship and used it to develop calculus into a systematic mathematical method. In particular, they saw that the Fundamental Theorem enabled them to compute areas and integrals very easily without having to compute them as limits of sums as we did in Sections 5.1 and 5.2.

$$g(x) = \int_a^x f(t)\, dt$$

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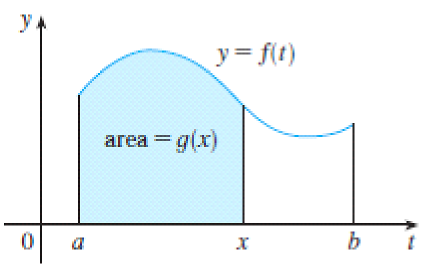


V **EXAMPLE 2** Find the derivative of the function $g(x) = \int_0^x \sqrt{1 + t^2}\, dt$.

EXAMPLE 4 Find $\dfrac{d}{dx} \int_1^{x^4} \sec t\, dt$.

SOLUTION Here we have to be careful to use the Chain Rule in conjunction with FTC1. Let $u = x^4$. Then

$$\frac{d}{dx} \int_1^{x^4} \sec t\, dt = \frac{d}{dx} \int_1^{u} \sec t\, dt$$

$$= \frac{d}{du}\left[\int_1^{u} \sec t\, dt\right] \frac{du}{dx} \qquad \text{(by the Chain Rule)}$$

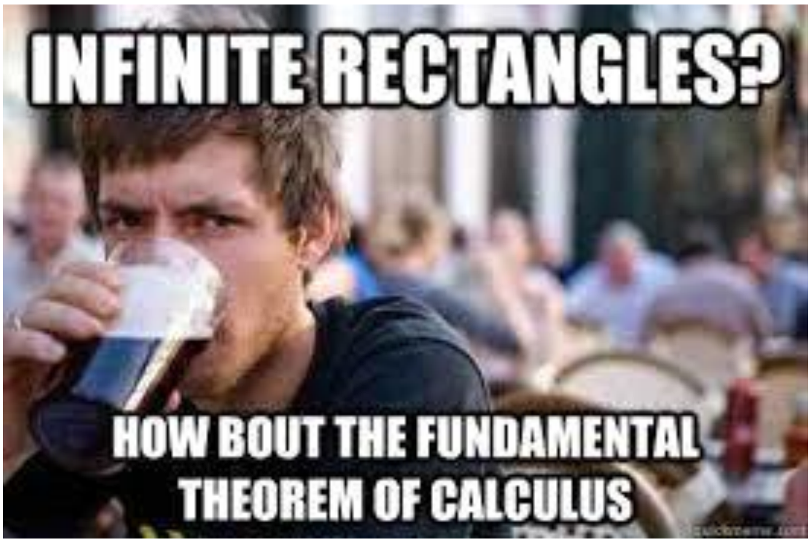
$$= \sec u \frac{du}{dx} \qquad \text{(by FTC1)}$$

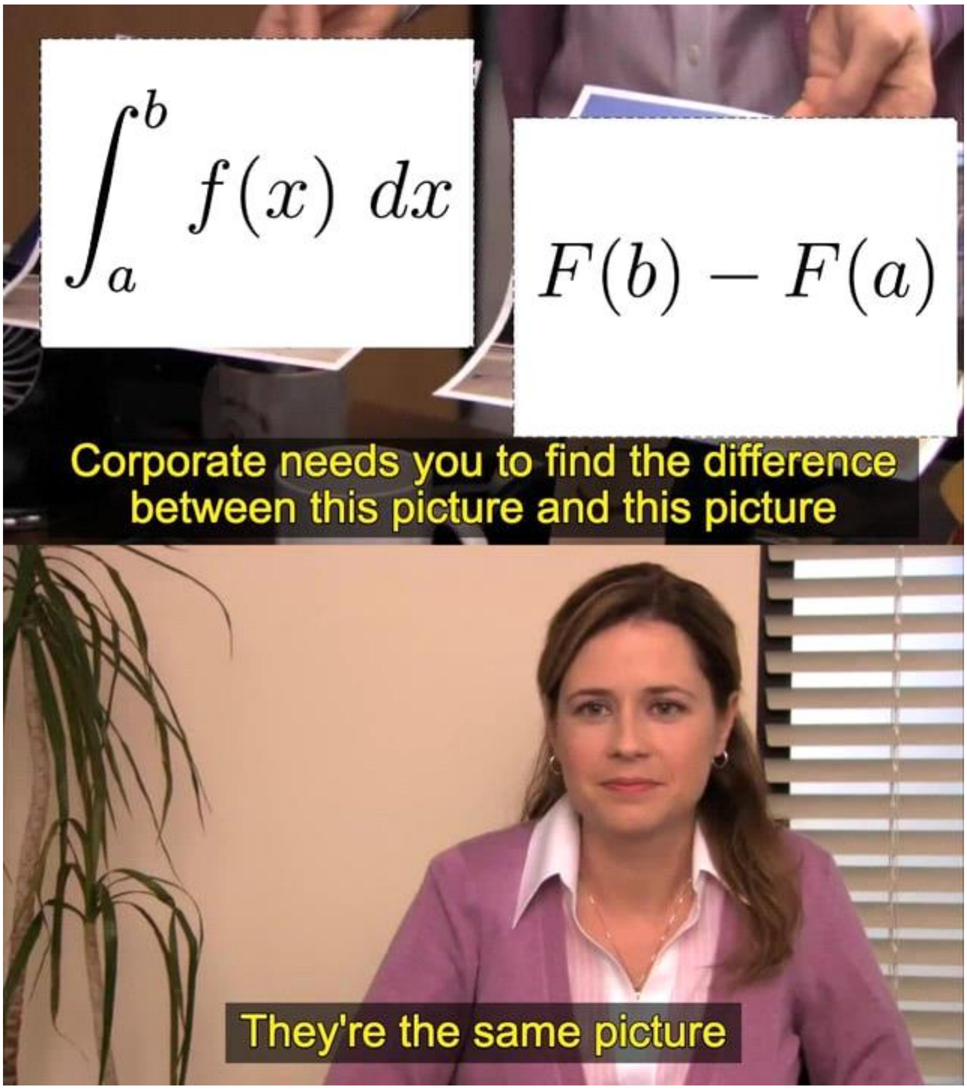
$$= \sec(x^4) \cdot 4x^3$$

V **EXAMPLE 5** Evaluate the integral $\int_1^3 e^x\, dx$.

EXAMPLE 6 Find the area under the parabola $y = x^2$ from 0 to 1.

EXAMPLE 8 Find the area under the cosine curve from 0 to b, where $0 \leqslant b \leqslant \pi/2$.


INFINITE RECTANGLES?
HOW BOUT THE FUNDAMENTAL THEOREM OF CALCULUS


$\int_a^b f(x)\, dx$
$F(b) - F(a)$
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They're the same picture