Concept of Integration

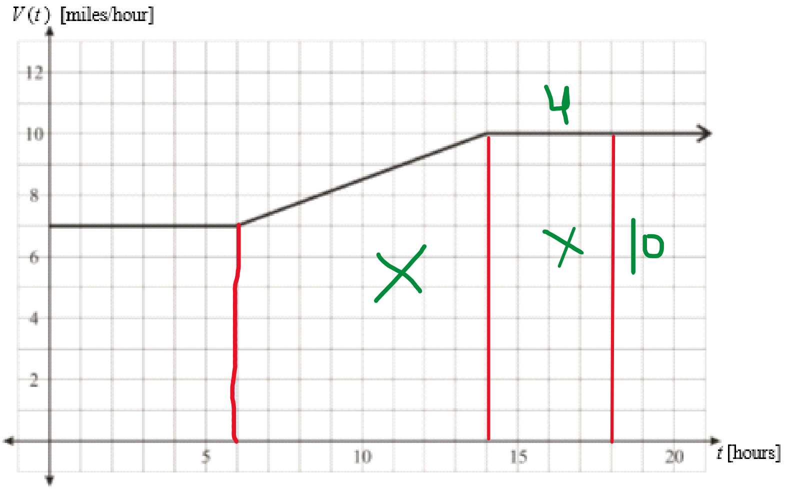
Monday, 9 December 2024 1:02 pm

Calculus 1 *Handout 18*

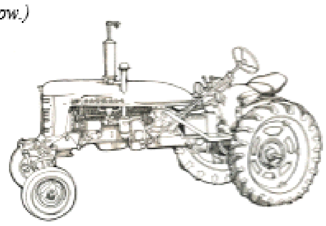
Areas Bounded by Curves

Given a velocity function, we know that finding the distance traveled by the object, over an interval of time, involves finding the area bounded by the function's graph over that same interval.

Below is a hypothetical graph of the velocity of a tractor traveling on a straight road. Find the distance traveled by the tractor between $t = 6$ and $t = 18$.



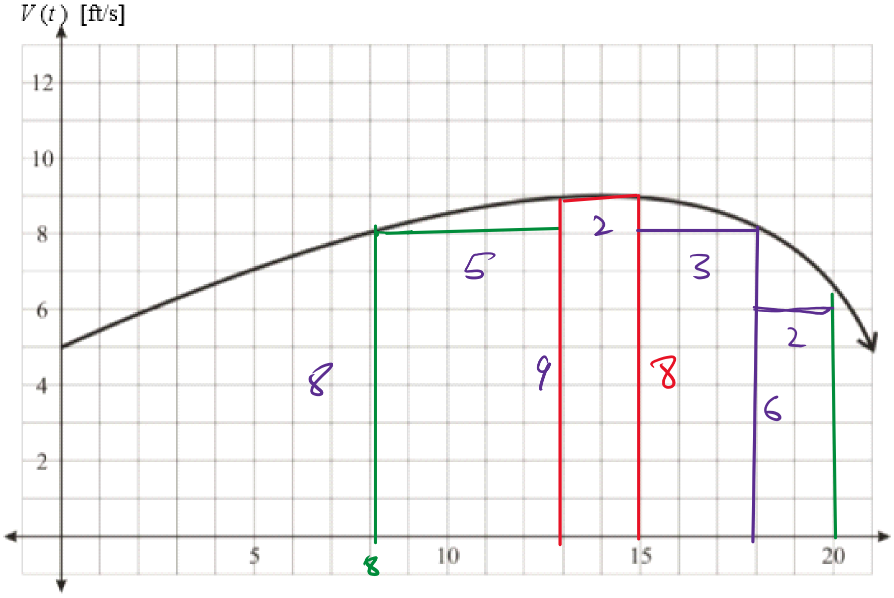
(Document your method of approximating the area in the space below.)



Area over the given interval = _______

Therefore, the distance traveled between the 6th and the 18th hour is ______________ miles.

Next, let's consider the velocity function of a Victorian horse-carriage (graphed below). It is hard to envision being able to get an exact area, judging from the shape of the curve, but you can surely get an approximation. Use your creativity with various shapes and find the area under the curve over the interval [8, 20].



(Document your method of approximating the area in the space below.)

Area = ________

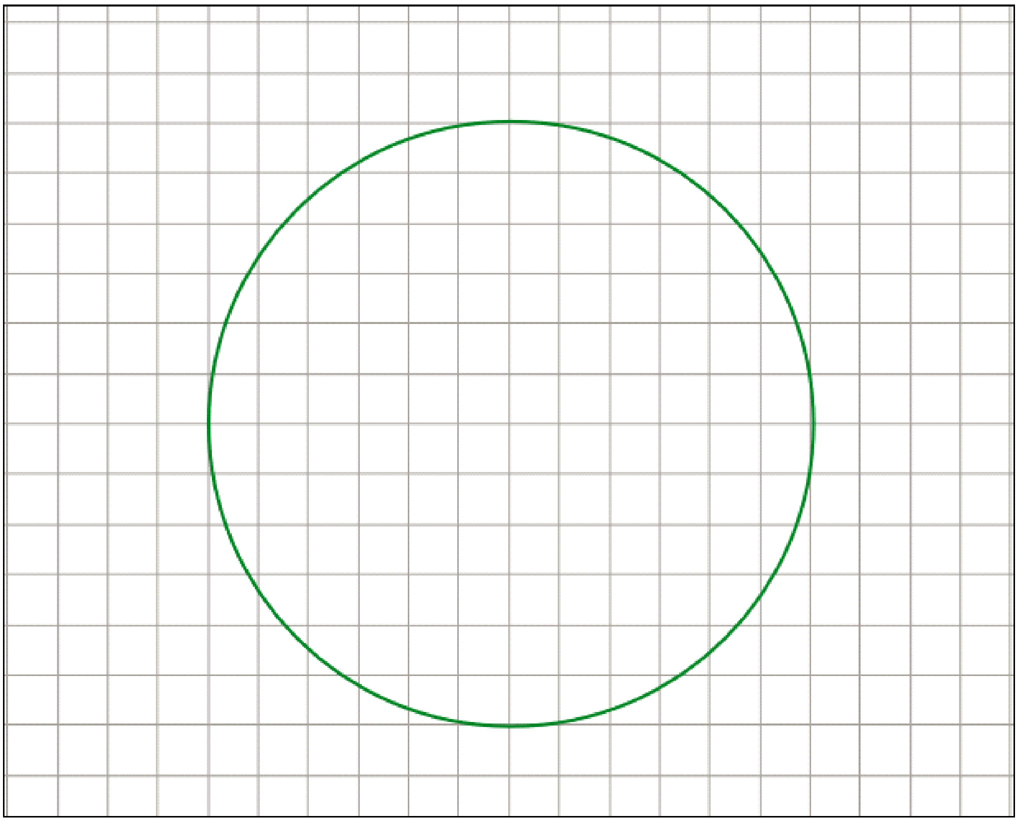
Therefore, the distance traveled between $t = 8$ and $t = 20$ is _________ feet.

Now look up at the board as I point something out to you.....

Area of a Circle:

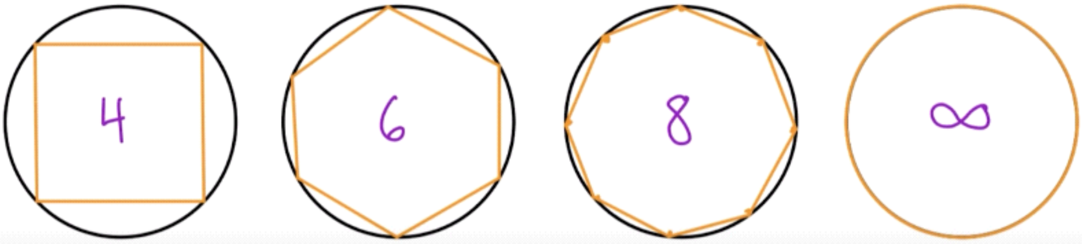
A personal Approach

Find a way to approximate the area of the circle below without using any prior knowledge of a formula for the area. You may use your knowledge of areas of rectangles, triangles and other polygons. Document your process clearly and come up with a good approximation. You can assume one of the squares on the grid to be 1 sq. unit.



The Method of Exhaustion

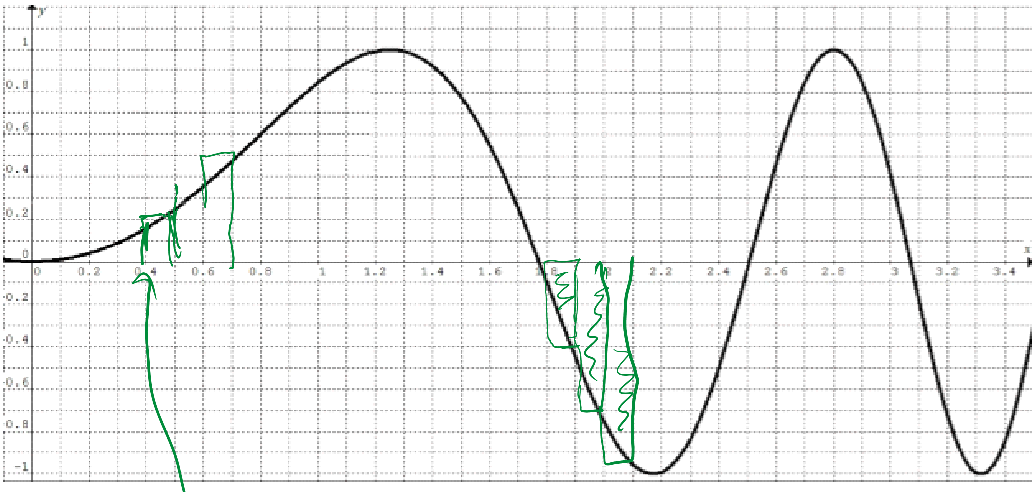
...the **Method of Exhaustion** builds on this to find area of curved shapes:



- Integral calculus is directly based on this method.
- Calculus in general relies on the idea of approaching infinity.
  - Leibniz gave field the name "calculus of the infinitesimals"

More on Areas Bounded by Curves

The graph below is the graph of the function $f(x) = \sin(x^2)$.



Find the area bounded by the curve $f(x) = \sin(x^2)$ for $0.4 \le x \le 2.6$. (*Use rectangles of width 0.1 units.*)

Show all your work, i.e. write down all your individual calculations below. I have started you off for the first and second rectangles using left-hand sums, i.e. the top left corner of each rectangle touches the curve.

Area under the curve
for $0.4 \le x \le 2.6$ $\quad = \boxed{f(0.4)\times 0.1} + \boxed{f(0.5)\times 0.1} +$ $f(0.6)\times 0.1 + \ldots + f(2.5)\times 0.1$