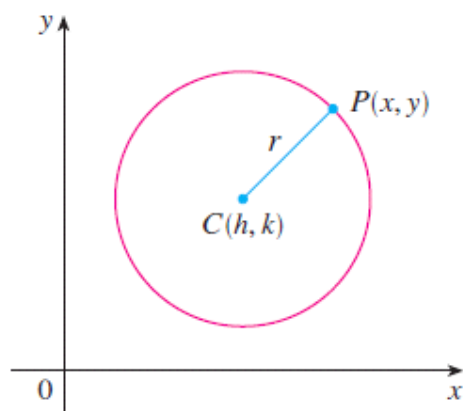


Conic Section

Saturday, 12 October 2024

5:29 pm

Circles



1 Equation of a Circle An equation of the circle with center (h, k) and radius r is

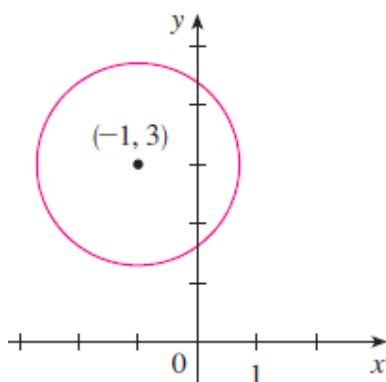
$$(x - h)^2 + (y - k)^2 = r^2$$

In particular, if the center is the origin $(0, 0)$, the equation is

$$x^2 + y^2 = r^2$$

EXAMPLE 1 Find an equation of the circle with radius 3 and center $(2, -5)$.

EXAMPLE 2 Sketch the graph of the equation $x^2 + y^2 + 2x - 6y + 7 = 0$ by first showing that it represents a circle and then finding its center and radius.

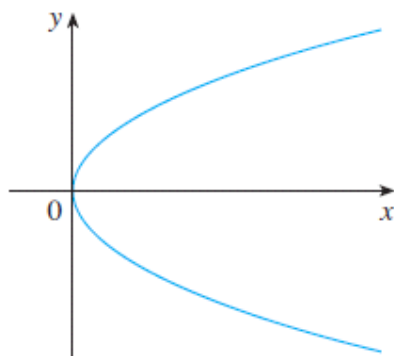
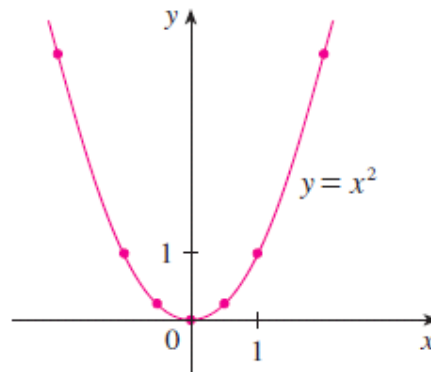


Parabolas

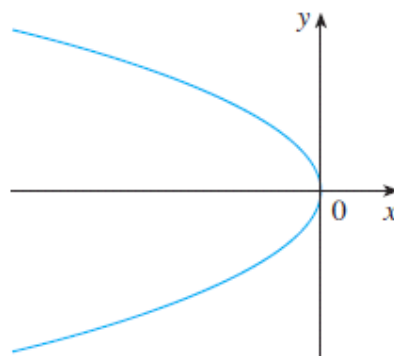
The geometric properties of parabolas are reviewed in Section 10.5. Here we regard a parabola as a graph of an equation of the form $y = ax^2 + bx + c$.

EXAMPLE 3 Draw the graph of the parabola $y = x^2$.

x	$y = x^2$
0	0
$\pm\frac{1}{2}$	$\frac{1}{4}$
± 1	1
± 2	4
± 3	9



(a) $x = ay^2$, $a > 0$



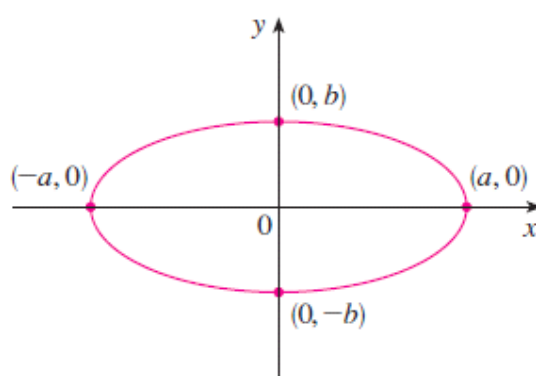
(b) $x = ay^2$, $a < 0$

Ellipses

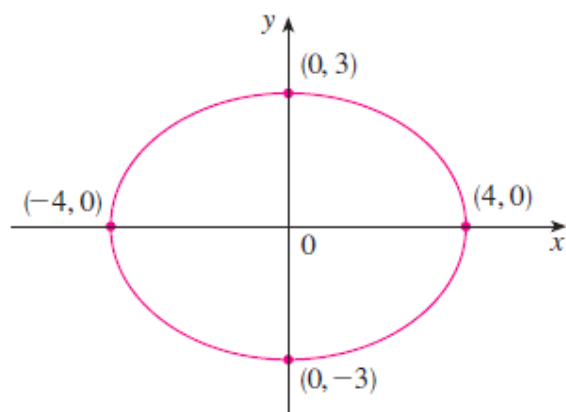
The curve with equation

2

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



EXAMPLE 5 Sketch the graph of $9x^2 + 16y^2 = 144$.



Hyperbolas

The curve with equation

3

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

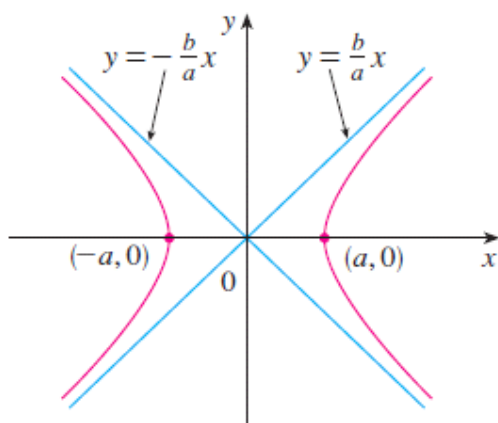


FIGURE 10

The hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

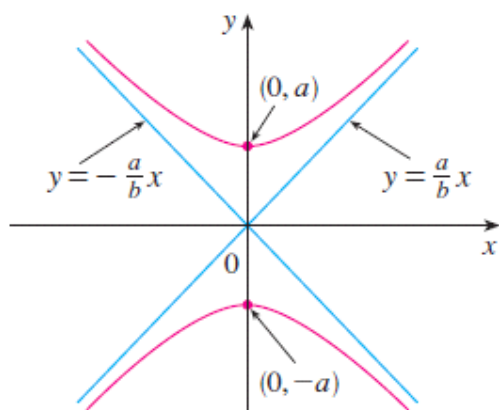
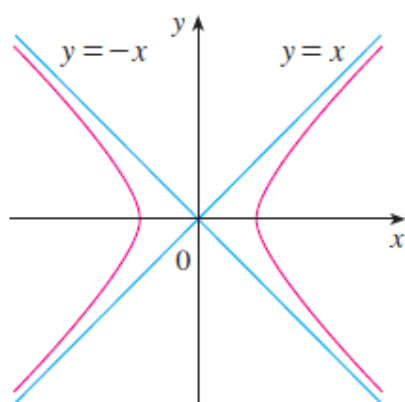
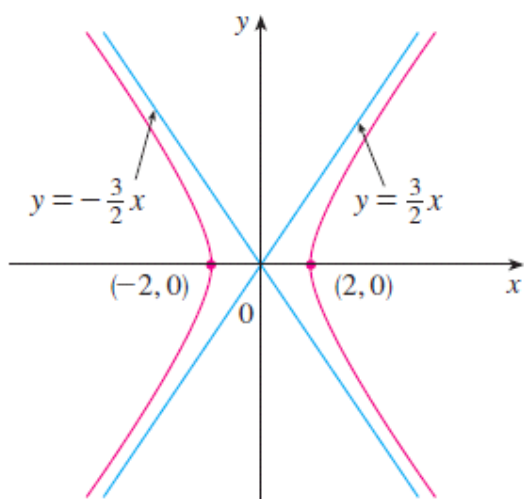


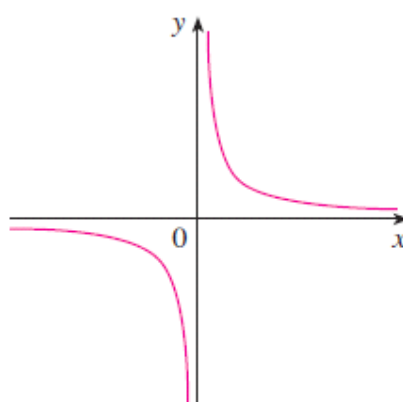
FIGURE 11

The hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

EXAMPLE 6 Sketch the curve $9x^2 - 4y^2 = 36$.



(a) $x^2 - y^2 = a^2$



(b) $xy = k \ (k > 0)$

Shifted Conics

Recall that an equation of the circle with center the origin and radius r is $x^2 + y^2 = r^2$, but if the center is the point (h, k) , then the equation of the circle becomes

$$(x - h)^2 + (y - k)^2 = r^2$$

Similarly, if we take the ellipse with equation

4

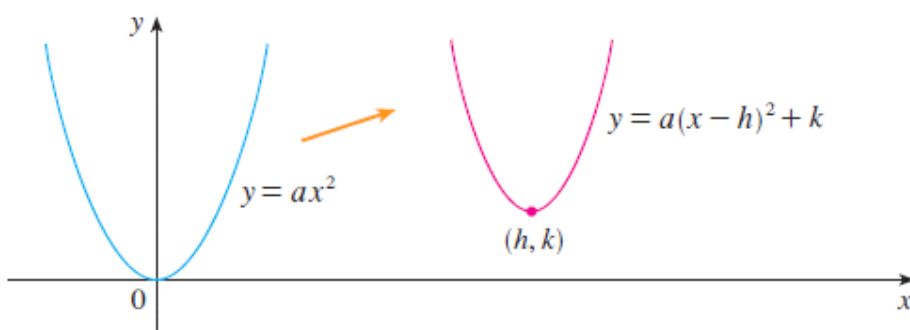
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

and translate it (shift it) so that its center is the point (h, k) , then its equation becomes

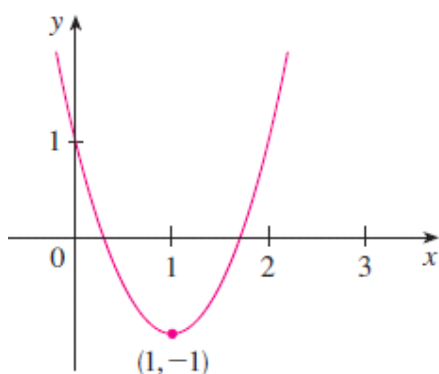
5

$$\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1$$

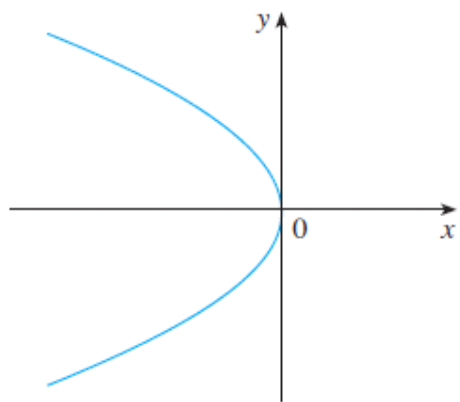
$$y - k = a(x - h)^2 \quad \text{or} \quad y = a(x - h)^2 + k$$



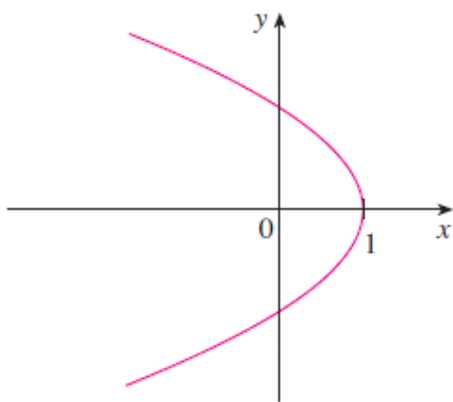
EXAMPLE 7 Sketch the graph of the equation $y = 2x^2 - 4x + 1$.



EXAMPLE 8 Sketch the curve $x = 1 - y^2$.



(a) $x = -y^2$



(b) $x = 1 - y^2$