Question 1 [10 Marks]

a) **Explain** the domain of the function $f(x) = \sqrt{5-2x} + \frac{x+3}{x^2-9}$. **(2)**

$5-2x \geq 0 \Rightarrow x \leq \frac{5}{2}$ & $x^2-9=0$

$x = \pm 3$

$\{x \mid \leq \frac{5}{2} \;\&\; x \neq -3\}$

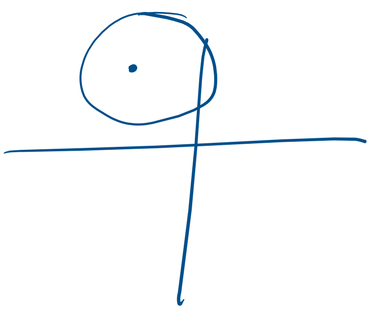
b) **Picture graphically** the shape of the following equations. **(3)**

i. $x^2 + y^2 + 2x - 6y + 7 = 0,$

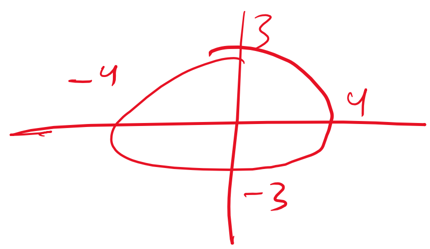
ii. $16\,y^2 + 9\,x^2 = 144,$

$f = -\frac{2}{2} = -1$ & $g = -\left(-\frac{6}{2}\right) = 3$

$(f, g) = (-1, 3)$ & $r = \sqrt{(-1)^2+(3)^2-7} = \sqrt{3}$



& $\frac{y^2}{9} + \frac{x^2}{16} = 1$



c) **Compute** the limit of the following function **(2)**

$$f(x) = \lim_{x\to 1}\left(\frac{1-\sqrt{x}}{1-x}\right)$$

Simplify first $\frac{1-\sqrt{x}}{1-x} \times \frac{1+\sqrt{x}}{1+\sqrt{x}} = \frac{\cancel{1-x}}{\cancel{(1-x)}(1+\sqrt{x})}$

$\lim_{x\to 1}\left(\frac{1}{1+\sqrt{x}}\right) = \frac{1}{2}$

d) **Discuss** the continuity of the following function at the given values of x. If the function is discontinuous at any point, determine the type of discontinuity.

(i) $x = -2$, **(ii)** $x = 0$ and **(iii)** $x = 3$? **(3)**

$$f(x) = \begin{cases} 2x + 34, & x < -2 \\ 30\,, & x = -2 \\ 29 - \dfrac{x}{2}, & -2 < x < 0 \\ 2x^2 + 9x + 29, & 0 < x \leq 3 \\ -4, & x > 3 \end{cases}$$

[CLO 1]

$\lim_{x \to -2^-} 2(-2)+34 = 30$ & $\lim_{x \to -2^+} 29 - \left(\frac{-2}{2}\right) = 30$

$f(-2) = 30$ continuous

$f(0)$ is not defined, not continuous
removable discontinuity

$\lim_{x \to 3^-} f(x) = 2(3)^2 + 9(3) + 29 = 74$ & $\lim_{x \to 3^+} f(x) = -4$

Jump discontinuity

Question 2 **[15 Marks]**

a) Explain why this function $f(x) = |x|$ is not differentiable by **applying** the definition of derivative. **(3)**

Let $a = 0$

$\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 0} \frac{|x| - |0|}{x - 0} = \frac{|x|}{x}$

$f'(x) \begin{cases} 1 & x \geq 0 \\ -1 & x < 0 \end{cases}$

Hence it is not diff.

b) If A is the area of a circle with radius and the circle expands as time passes, find dA/dt in terms of dr/dt and **project** that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of $2\ ft/s$. How fast is the area of the spill increasing when the radius of the spill is $60\ ft$? **(3)**

$$A = \pi r^2 \Rightarrow \frac{dA}{dt} = \pi 2r \frac{dr}{dt}$$

$$\frac{dA}{dt} = \pi 2 (60)(2) = 240\pi$$

c) **Determine** the derivative of the function $f(t) = \left(\frac{e^t - 2}{3\cos(t) + \sin(2t)}\right)^9$. **(2)**

$$f'(t) = 9\left(\frac{e^t - 2}{(3\cos(t) + \sin(2t))}\right)^8 \frac{(3\cos t + \sin(2t))(e^t) - (e^t - 2) *}{(3\cos t + \sin(2t))^2}$$

$$* = 3\sin t + 2\cos(2t)$$

d) **Use** implicit differentiation to find $\frac{dy}{dx}$ if $5y^2 + \sqrt{2x+1}\, sinx = x^2$. **(3)**

$$5\left(2y\frac{dy}{dx}\right) + \frac{1}{\cancel{2}}(2x+1)^{-1/2}(\cancel{2})\sin x + \sqrt{2x+1}\cos x = 2x$$

$$10y\frac{dy}{dx} = 2x - \frac{\sin x}{\sqrt{2x+1}} - \sqrt{2x+1}\cos x$$

$$\frac{dy}{dx} = \frac{1}{10y}\left(2x - \frac{\sin x}{\sqrt{2x+1}} - \sqrt{2x+1}\cos x\right.$$

e) **Solve** the limits using L'Hôpital's rule. **(4)**

i. $\lim\limits_{x\to 0^+} x\, lnx$ ii. $\lim\limits_{x\to 0} \frac{e^x - 1 - x}{x^2}$

[CLO 2]

$$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}} \underset{\text{L'Hop}}{\overset{\text{Apply}}{=}} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = -x \xrightarrow{\text{Limit}} 0$$

$$\lim_{x\to 0} \frac{e^x - 1 - x}{x^2} \overset{\text{Apply}}{=} \frac{e^x - 1}{2x} \overset{\text{Apply}}{=} \frac{e^x}{2} \overset{\text{Limit}}{=} \frac{1}{2}$$