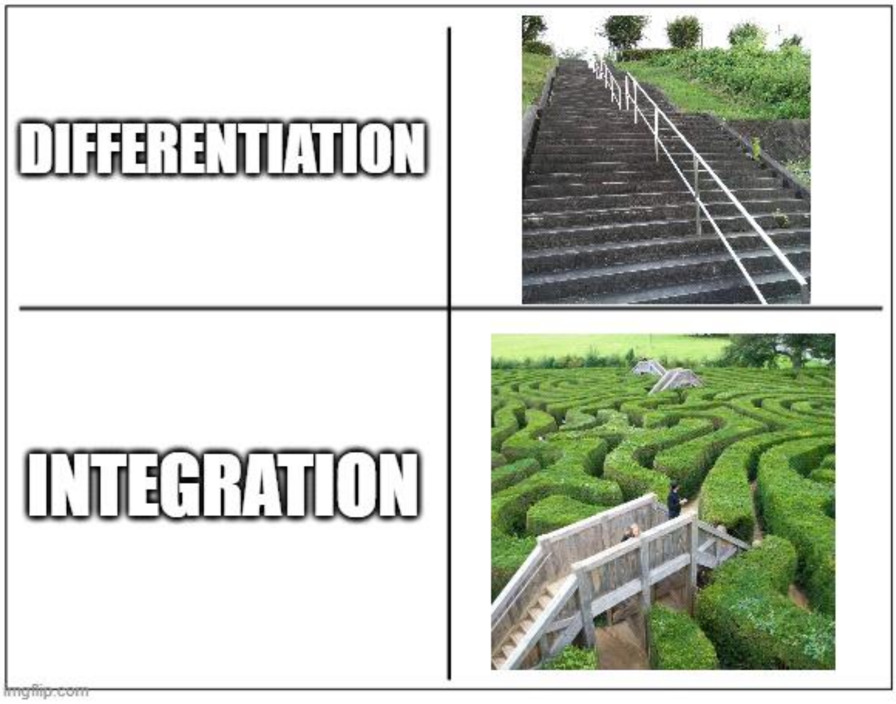
Integration By Parts

Sunday, 22 December 2024 12:46 am



7.1 Integration by Parts

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

$$\int [f(x)g'(x) + g(x)f'(x)]\,dx = f(x)g(x)$$

$$\int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx = f(x)g(x)$$

$$\boxed{\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx}$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

EXAMPLE 1 Find $\int x \sin x\,dx$.

$$u = x \qquad dv = \sin x\,dx$$

$$du = dx \qquad v = -\cos x$$

$$\int x \sin x\,dx = \int \overbrace{x}^{u}\ \overbrace{\sin x\,dx}^{dv} = \overbrace{x}^{u}\ \overbrace{(-\cos x)}^{v} - \int \overbrace{(-\cos x)}^{v}\ \overbrace{dx}^{du}$$

$$= -x \cos x + \int \cos x\,dx$$

$$= -x \cos x + \sin x + C$$

V EXAMPLE 2 Evaluate $\int \ln x\,dx$.

$$u = \ln x \qquad dv = dx$$

$$du = \frac{1}{x}\,dx \qquad v = x$$

$$\int \ln x\,dx = x \ln x - \int x \frac{dx}{x}$$

$$= x \ln x - \int dx$$

$$= x \ln x - x + C$$

V EXAMPLE 3 Find $\int t^2 e^t\,dt$.

$$u = t^2 \qquad dv = e^t\,dt$$

$$du = 2t\,dt \qquad v = e^t$$

$$\int t^2 e^t\,dt = t^2 e^t - 2\int te^t\,dt$$

$$\int te^t\,dt = te^t - \int e^t\,dt$$

$$= te^t - e^t + C$$

$$\int t^2 e^t\,dt = t^2 e^t - 2\int te^t\,dt$$

$$= t^2 e^t - 2(te^t - e^t + C)$$

$$= t^2 e^t - 2te^t + 2e^t + C_1 \qquad \text{where } C_1 = -2C$$

V EXAMPLE 4 Evaluate $\int e^x \sin x\,dx$.

SOLUTION Neither e^x nor $\sin x$ becomes simpler when differentiated, but we try choosing $u = e^x$ and $dv = \sin x\,dx$ anyway. Then $du = e^x\,dx$ and $v = -\cos x$, so integration by parts gives

$$\boxed{4} \qquad \int e^x \sin x\,dx = -e^x \cos x + \int e^x \cos x\,dx$$

$$\int e^x \cos x\,dx = e^x \sin x - \int e^x \sin x\,dx$$

$$\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x - \int e^x \sin x\,dx$$

$$2\int e^x \sin x\,dx = -e^x \cos x + e^x \sin x$$

$$\int e^x \sin x\,dx = \tfrac{1}{2}e^x(\sin x - \cos x) + C$$

Performing Integration By Parts

Integration By Parts

$$\int f(x)\,g'(x)\,dx = f(x)\,g(x) - \int g(x)\,f'(x)\,dx$$

$$f(x) = u \qquad g(x) = v$$

$$f'(x)\,dx = du \qquad g'(x)\,dx = dv$$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

EXAMPLE 6 Prove the reduction formula

$$\boxed{7} \qquad \int \sin^n x\,dx = -\frac{1}{n}\cos x \sin^{n-1} x + \frac{n-1}{n}\int \sin^{n-2} x\,dx$$

. (a) Use the reduction formula in Example 6 to show that

$$\int \sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

(b) Use part (a) and the reduction formula to evaluate $\int \sin^4 x \, dx$.

$$\int \frac{\ln x}{x^2} \, dx$$

$$\int (\ln x)^2 \, dx$$