Assignment 1

Tuesday, September 27, 2022 11:24 AM

TEST YOURSELF

Answers to Test Yourself questions are located at the end of each section.

1. An *and* statement is true when, and only when, both components are ______.
2. An *or* statement is false when, and only when, both components are ______.
3. Two statement forms are logically equivalent when, and only when, they always have ______.
4. De Morgan's laws say (1) that the negation of an *and* statement is logically equivalent to the ______ statement in which each component is ______, and (2) that the negation of an *or* statement is logically equivalent to the ______ statement in which each component is ______.
5. A tautology is a statement that is always ______.
6. A contradiction is a statement that is always ______.

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*46. Let the symbol $\oplus$ denote *exclusive or*; so $p \oplus q \equiv (p \vee q) \wedge \sim(p \wedge q)$. Hence the truth table for $p \oplus q$ is as follows:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

a. Find simpler statement forms that are logically equivalent to $p \oplus p$ and $(p \oplus p) \oplus p$.

b. Is $(p \oplus q) \oplus r \equiv p \oplus (q \oplus r)$? Justify your answer.

c. Is $(p \oplus q) \wedge r \equiv (p \wedge r) \oplus (q \wedge r)$? Justify your answer.

In 48 and 49 below, a logical equivalence is derived from Theorem 2.1.1. Supply a reason for each step.

48. $(p \wedge \sim q) \vee (p \wedge q) \equiv p \wedge (\sim q \vee q)$ by (a)

$\equiv p \wedge (q \vee \sim q)$ by (b)

$\equiv p \wedge \mathbf{t}$ by (c)

$\equiv p$ by (d)

Therefore, $(p \wedge \sim q) \vee (p \wedge q) \equiv p$.

49. $(p \vee \sim q) \wedge (\sim p \vee \sim q)$

$\equiv (\sim q \vee p) \wedge (\sim q \vee \sim p)$ by (a)

$\equiv \sim q \vee (p \wedge \sim p)$ by (b)

$\equiv \sim q \vee \mathbf{c}$ by (c)

$\equiv \sim q$ by (d)

Therefore, $(p \vee \sim q) \wedge (\sim p \vee \sim q) \equiv \sim q$.

Use Theorem 2.1.1 to verify the logical equivalences in 50–54. Supply a reason for each step.

50. $(p \wedge \sim q) \vee p \equiv p$

51. $p \wedge (\sim q \vee p) \equiv p$

52. $\sim(p \vee \sim q) \vee (\sim p \wedge \sim q) \equiv \sim p$

53. $\sim((\sim p \wedge q) \vee (\sim p \wedge \sim q)) \vee (p \wedge q) \equiv p$

54. $(p \wedge (\sim(\sim p \vee q))) \vee (p \wedge q) \equiv p$

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Use De Morgan's laws to write negations for the statements in 25–30.

25. Hal is a math major and Hal's sister is a computer science major.

26. Sam is an orange belt and Kate is a red belt.

27. The connector is loose or the machine is unplugged.

28. The train is late or my watch is fast.

29. This computer program has a logical error in the first ten lines or it is being run with an incomplete data set.

30. The dollar is at an all-time high and the stock market is at a record low.

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