Assignment 3

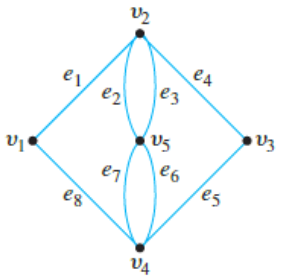
Friday, 13 September 2024 12:02 am

Answers to Test Yourself questions are located at the end of each section.

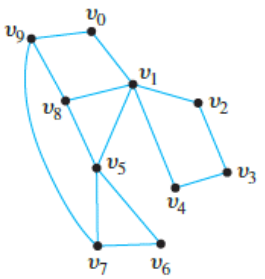
1. Let G be a graph and let v and w be vertices in G.
   (a) A walk from v to w is _____.
   (b) A trail from v to w is _____.
   (c) A path from v to w is _____.
   (d) A closed walk is _____.
   (e) A circuit is _____.
   (f) A simple circuit is _____.
   (g) A trivial walk is _____.
   (h) Vertices v and w are connected if, and only if, _____.
2. A graph is connected if, any only if, _____.
3. Removing an edge from a circuit in a graph does not _____.
4. An Euler circuit in a graph is _____.
5. A graph has a Euler circuit if, and only if, _____.
6. Given vertices v and w in a graph, there is an Euler trail from v to w if, and only if, _____.
7. A Hamiltonian circuit in a graph is _____.
8. If a graph G has a Hamiltonian circuit, then G has a subgraph H with the following properties: _____, _____, _____, and _____.
9. A traveling salesman problem involves finding a _____ that minimizes the total distance traveled for a graph in which each edge is marked with a distance.

Determine which of the graphs in 12–17 have Euler circuits. If the graph does not have an Euler circuit, explain why not. If it does have an Euler circuit, describe one.

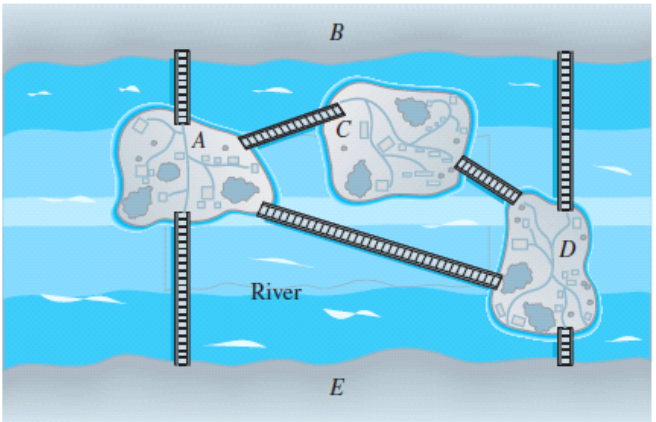
12.



13.

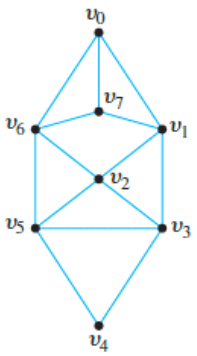


18. Is it possible to take a walk around the city whose map is shown below, starting and ending at the same point and crossing each bridge exactly once? If so, how can this be done?

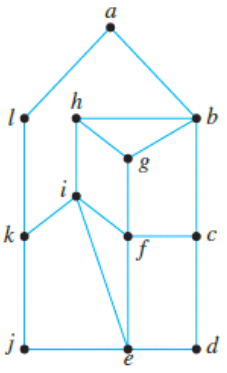


Find Hamiltonian circuits for each of the graphs in 29 and 3

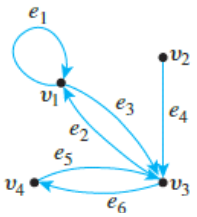
29.



30.



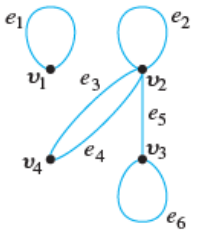
Find the adjacency matrices for the following directed graphs.



Find directed graphs that have the following adjacency matrices:

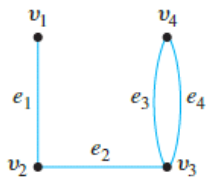
$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Find adjacency matrices for the following (undirected) graphs.

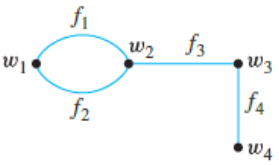


For each pair of graphs G and G' in 1–5, determine whether G and G' are isomorphic. If they are, give functions $g: V(G) \to V(G')$ and $h: E(G) \to E(G')$ that define the isomorphism. If they are not, give an invariant for graph isomorphism that they do not share.

1.

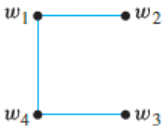


G

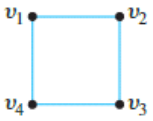


G'

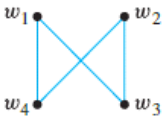
G



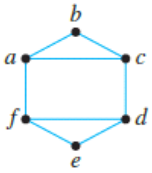
G'



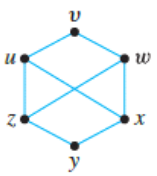
G



G'



G



G'

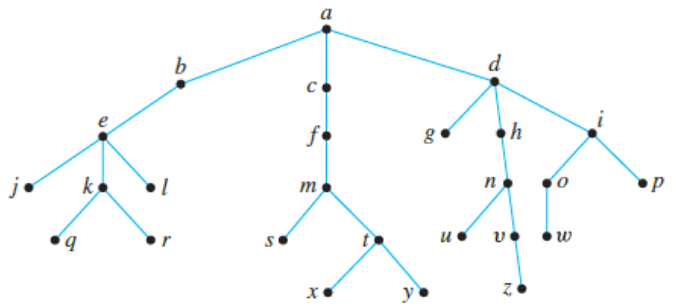
TEST YOURSELF

1. A circuit-free graph is a graph with _____.
2. A forest is a graph that is _____, and a tree is a graph that is _____.
3. A trivial tree is a graph that consists of _____.
4. Any tree with at least two vertices has at least one vertex of degree _____.
5. If a tree T has at least two vertices, then a terminal vertex (or leaf) in T is a vertex of degree _____ and an internal vertex (or branch vertex) in T is a vertex of degree _____.
6. For any positive integer n, any tree with n vertices has _____.
7. For any positive integer n, if G is a connected graph with n vertices and $n - 1$ edges then _____.

TEST YOURSELF

1. A rooted tree is a tree in which _____. The level of a vertex in a rooted tree is _____. The height of a rooted tree is _____.
2. A binary tree is a rooted tree in which _____.
3. A full binary tree is a rooted tree in which _____.
4. If k is a positive integer and T is a full binary tree with k internal vertices, then T has a total of _____ vertices and has _____ leaves.
5. If T is a binary tree that has t leaves and height h, then t and h are related by the inequality _____.

1. Consider the tree shown below with root a.
   a. What is the level of n?
   b. What is the level of a?
   c. What is the height of this rooted tree?
   d. What are the children of n?
   e. What is the parent of g?
   f. What are the siblings of j?
   g. What are the descendants of f ?
   h. How many leaves (terminal vertices) are on the tree?

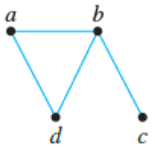


TEST YOURSELF

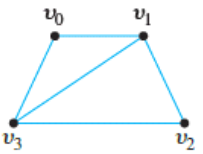
1. A spanning tree for a graph G is _____.
2. A weighted graph is a graph for which _____, and the total weight of the graph is _____.
3. A minimum spanning tree for a connected, weighted graph is _____.
4. In Kruskal's algorithm, the edges of a connected, weighted graph are examined one by one in order of _____ starting with _____.
5. In Prim's algorithm, a minimum spanning tree is built by expanding outward from an _____ in a sequence of _____.
6. In Dijkstra's algorithm, a vertex is in the fringe if it is _____ vertex in the tree that is being built up.
7. At each stage of Dijkstra's algorithm, the vertex that is added to the tree is a vertex in the fringe whose label is a _____.

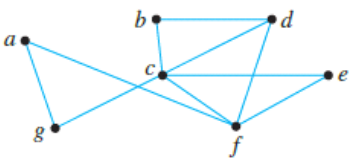
Find all possible spanning trees for each of the graphs in 1 and 2.

1.



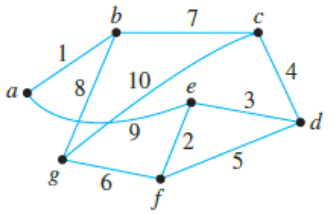
2.



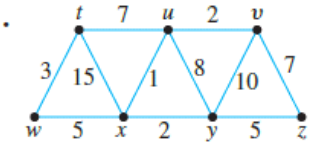


Use Kruskal's algorithm to find a minimum spanning tree for each of the graphs in 5 and 6. Indicate the order in which edges are added to form each tree.

5.



Use Prim's algorithm starting with vertex a or v_0 to find a minimum spanning tree for each of the graphs in 7 and 8. Indicate the order in which edges are added to form each tree.



Use Dijkstra's algorithm to find the shortest path from a to z for each of the graphs in 13–16. In each case make tables similar to Table 10.6.1 to show the action of the algorithm.