Lecture No.5   Argument

Before we discuss in detail about the argument, we first consider the following argument:

An interesting teacher keeps me awake. I stay awake in Discrete Mathematics class.
Therefore, my Discrete Mathematics teacher is interesting.

Is the above argument valid?

ARGUMENT:
An **argument** is a list of statements called **premises** (or **assumptions** or **hypotheses**) followed by a statement called the **conclusion.**
P1   Premise
P2   Premise
P3   Premise
. . . .
Pn   Premise
_____
∴C   Conclusion

NOTE: The symbol ∴ read "therefore" is normally placed just before the conclusion.

VALID AND INVALID ARGUMENT:
An argument is **valid** if the conclusion is true when all the premises are true.
Alternatively, an argument is valid if conjunction of its premises imply conclusion.
That is $(P_1 \land P_2 \land P_3 \land \ldots \land P_n) \to C$ is a tautology.
An argument is **invalid** if the conclusion is false when all the premises are true.
Alternatively, an argument is invalid if conjunction of its premises does not imply conclusion.
Critical Rows: The critical rows are those rows where the premises have truth value T.
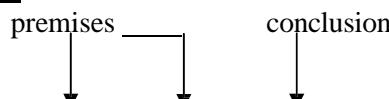
EXAMPLE: Show that the following argument form is valid:

$$p \to q$$
$$p$$
$$\therefore \ q$$

SOLUTION

premises _____ conclusion

p	q	p→q	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

critical row (← pointing to first data row)

Since the conclusion q is true when the premises p→q and p are True. Therefore, it is a valid argument.

EXAMPLE   Show that the following argument form is invalid:

p→q
q
∴ p

SOLUTION

p	q	p→q	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

In the second critical row, the conclusion is false when the premises p→q and q are true. Therefore, the argument is invalid.

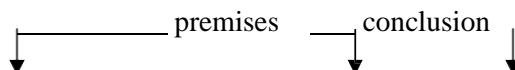**EXERCISE:**

Use truth table to determine the argument form

p ∨ q
p → ~q
p → r
 ∴ r

is valid or invalid.

p	q	r	p∨q	p→~q	p→r	r
T	T	T	T	F	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	T	F	F
F	T	T	T	T	T	T
F	T	F	T	T	T	F
F	F	T	F	T	T	T
F	F	F	F	T	T	F

In the third critical row, the conclusion is false when all the premises are true. Therefore, the argument is invalid.

The argument form is invalid
WORD PROBLEM

If Tariq is not on team A, then Hameed is on team B.
If Hameed is not on team B, then Tariq is on team A.
$\therefore$ Tariq is not on team A or Hameed is not on team B.

SOLUTION
Let

t = Tariq is on team A
h = Hameed is on team B

Then the argument is

$\sim t \rightarrow h$
$\sim h \rightarrow t$
$\therefore \sim t \vee \sim h$

t	h	$\sim t \rightarrow h$	$\sim h \rightarrow t$	$\sim t \vee \sim h$
T	T	T	T	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

Argument is invalid because there are three critical rows.

(Remember that the critical rows are those rows where the premises have truth value T) and in the first critical row conclusion has truth value F.

(Also remember that we say an argument is valid if in all critical rows conclusion has truth value T)

EXERCISE
If at least one of these two numbers is divisible by 6, then the product of these two numbers is divisible by 6.
Neither of these two numbers is divisible by 6.
$\therefore$ The product of these two numbers is not divisible by 6.
SOLUTION
Let   d =   at least one of these two numbers is divisible by 6.
        p =   product of these two numbers is divisible by 6.
Then the argument become in these symbols

$d \rightarrow p$
$\sim d$
$\therefore \quad \sim p$

We will made the truth table for premises and conclusion as given below

d	p	d → p	~d	~p
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

In the first critical row, the conclusion is false when the premises are true. Therefore, the argument is invalid.

EXERCISE

If I got an Eid bonus, I'll buy a stereo.
If I sell my motorcycle, I'll buy a stereo.
∴ If I get an Eid bonus or I sell my motorcycle, then I'll buy a stereo.

SOLUTION:

Let

e = I got an Eid bonus
s = I'll buy a stereo
m = I sell my motorcycle

The argument is

e → s
m → s

∴ e ∨ m → s

e	s	m	e →s	m →s	e∨m	e∨m →s
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	F	F	T	F
T	F	F	F	T	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	T
F	F	T	T	F	T	F
F	F	F	T	T	F	T

The argument is valid because in the five critical rows, the conclusion is true.

EXERCISE

An interesting teacher keeps me awake. I stay awake in Discrete Mathematics class.
Therefore, my Discrete Mathematics teacher is interesting.

Solution:

t = My teacher is interesting
a = I stay awake
m = I am in Discrete Mathematics class

The argument to be tested is

	t → a,
	a ∧ m
Therefore	m ∧ t

t	a	m	t → a	a ∧ m	m ∧ t
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	F	F	T
T	F	F	F	F	F
F	T	T	T	T	F
F	T	F	T	F	F
F	F	T	T	F	F
F	F	F	T	F	F

In the second critical row, the conclusion is false when the premises are true. Therefore, the argument is invalid