Logic 1

Saturday, December 3, 2022 4:55 PM

Logical Form and Logical Equivalence

Logic is a science of the necessary laws of thought, without which no employment of the understanding and the reason takes place. —Immanuel Kant, 1785

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Definition

A **statement** (or **proposition**) is a sentence that is true or false but not both.

For example, "Two plus two equals four" and "Two plus two equals five" are both statements, the first because it is true and the second because it is false. On the other hand, the truth or falsity of

$$x^2 + 2 = 11$$

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depends on the value of x. For some values of x, it is true ($x = 3$ and $x = -3$), whereas for other values it is false. Similarly, the truth or falsity of

$$x + y > 0$$

depends on the values of x and y. For instance, when $x = -1$ and $y = 2$ it is true, whereas when $x = -1$ and $y = 1$ it is false. In Section 3.1 we will discuss ways to transform sentences of these forms into statements.

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Compound Statements

LOGICALOPERATIONS OR LOGICAL CONNECTIVES :

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Sr. No.	Connective	Symbol	Compound statement
1	AND	$\wedge$	Conjunction
2	OR	$\vee$	Disjunction
3	NOT	$\neg$	Negation
4	If....then	$\rightarrow$	Conditional or implication
5	If and only if (iff)	$\leftrightarrow$	Biconditional

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p but q	means	p and q
neither p nor q	means	$\sim p$ and $\sim q$.

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And, Or, and Inequalities

Suppose x is a particular real number. Let p, q, and r symbolize "$0 < x$," "$x < 3$," and "$x = 3$," respectively. Write the following inequalities symbolically:

a. $x \leq 3$ b. $0 < x < 3$ c. $0 < x \leq 3$

Solution

a. $q \vee r$ b. $p \wedge q$ c. $p \wedge (q \vee r)$

Truth Values

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Definition

If p is a statement variable, the **negation** of p is "not p" or "It is not the case that p" and is denoted $\sim p$. It has opposite truth value from p: if p is true, $\sim p$ is false; if p is false, $\sim p$ is true.

The truth values for negation are summarized in a *truth table*.

Truth Table for $\sim p$

p	$\sim p$
T	F
F	T

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Definition

If p and q are statement variables, the **conjunction** of p and q is "p and q," denoted $p \wedge q$. It is true when, and only when, both p and q are true. If either p or q is false, or if both are false, $p \wedge q$ is false.

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Truth Table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

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Definition

If p and q are statement variables, the **disjunction** of p and q is "p or q," denoted $p \vee q$. It is true when either p is true, or q is true, or both p and q are true; it is false only when both p and q are false.

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Here is the truth table for disjunction:

Truth Table for $p \vee q$

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Definition

A **statement form** (or **propositional form**) is an expression made up of statement variables (such as p, q, and r) and logical connectives (such as $\sim$, $\wedge$, and $\vee$) that becomes a statement when actual statements are substituted for the component statement variables. The **truth table** for a given statement form displays the truth values that correspond to all possible combinations of truth values for its component statement variables.

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Truth Table for *Exclusive Or:* $(p \vee q) \wedge \sim(p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$\sim(p \wedge q)$	$(p \vee q) \wedge \sim(p \wedge q)$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

Conditional or Implication: (If…then)

If two statements are combined by using the logical connective 'if…then' then the resulting statement is called a conditional statement.

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If P and Q are two statements forming the implication "if P then Q" then we denotes this implication $P \to Q$.
In the implication $P \to Q$,
P is called antecedent or hypothesis
Q is called consequent or conclusion.

The statement $P \to Q$ is true in all cases except when P is true and Q is false.

The truth table for implication is as follows.

P	Q	$P \to Q$
T	T	T
T	F	F
F	T	T
F	F	T

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1. **Converse:** If $P \to Q$ is an implication then $Q \to P$ is called the converse of $P \to Q$.
2. **Contra positive :** If $P \to Q$ is an implication then the implication $\neg Q \to \neg P$ is called it's contra positive.
3. **Inverse:** If $P \to Q$ is an implication then $\neg P \to \neg Q$ is called its inverse.

Example 6:

Let P: You are good in Mathematics.
Q: You are good in Logic.

Then, $P \to Q$: If you are good in Mathematics then you are good in Logic.

1) Converse: $(Q \to P)$
If you are good in Logic then you are good in Mathematics.
2) Contra positive: $\neg Q \to \neg P$
If you are not good in Logic then you are not good in Mathematics.
3) Inverse: $(\neg P \to \neg Q)$
If you are not good in Mathematics then you are not good in Logic.

Biconditional Statement: Let P and Q be propositions. The biconditional statement $P \leftrightarrow Q$ is the proposition "P if and only if Q". The biconditional statement is true when P and Q have same truth values and is false otherwise.

Biconditional statements are also called bi-implications. It is also read as p is necessary and sufficient condition for Q.

The truth table for biconditional statement is as follows.

P	Q	$P \leftrightarrow Q$
T	T	T
T	F	F
F	T	F
F	F	T

Logical Equivalence

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p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$p \wedge q$ and $q \wedge p$ always have the same truth values, so they are logically equivalent

Double Negative Property: $\sim(\sim p) \equiv p$

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Example 8: If P: "This book is good."
Q: "This book is costly."
Write the following statements in symbolic form.

a) This book is good & costly.
b) This book is not good but costly.
c) This book is cheap but good.
d) This book is neither good nor costly.
e) If this book is good then it is costly.

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