Mid Exam

Wednesday, 28 August 2024 9:58 am

Mid-Term Examination Summer-24

Subject:	**Discrete Structure**	**Date:**	**27-08-2024**
Instructor:	**Dr. Muhammad Sami Siddiqui**	**Day:**	**Tuesday**
Program:	**BS(CS)**	**Time Slot:**	**10:15 - 11:45**
No. of Students:	**55**	**Duration:**	**1.5 Hours**
Section Code:	**032407049**	**Max. Marks**	**25**

Instructions:

1. Attempt all questions in the answer sheet provided to you and return the question paper after the exam.
2. Please do not use pencils except for underlining or drawing diagrams.
3. Any attempt to use unfair means will disqualify you from the examination.
4. Students are not supposed to ask questions after the first fifteen minutes of the commencement of the exam.
5. The invigilator will not return the answer script to the candidate in any case once it is submitted.
6. All students must bring their stationery and calculators. Borrowing in the examination hall is strictly prohibited.
7. All students shall comply with any other instruction, written or oral, given by the examiner/invigilator in the examination hall.
8. Marks of each question are mentioned at the end of each question.
9. Please follow any other instructions the invigilator/examiner provided in the question paper.

Instructor's Signature

Question 1 **[6+3 =9 Marks]**

a) Consider the following logical statement:

$$(p \rightarrow q) \leftrightarrow r \quad \text{and} \quad p \rightarrow (q \leftrightarrow r)$$

i) **Rewrite** the above expression after replacing all conditional and bi-conditional operators.
ii) **Compute** the truth table and check whether they are logically equivalent.

b) **Infer** logic laws and show $\sim(p \rightarrow q) \rightarrow p$ is tautology.

[CLO 1]

Question 2 **[3+3+3+3+4=16 Marks]**

a) Several laptop users are surveyed to determine if they have a separate Mouse, Keyboard, or Webcam. **Draw** separate Venn diagrams and shade the areas that represent the following configurations.

i. Mouse and Keyboard but no Webcam
ii. Mouse but no Keyboard and no Webcam
iii. Keyboard or Mouse but no Webcam.
iv. no Mouse and no Keyboard

b) **Investigate** whether the function is well-defined from $f: Z^+ \rightarrow R$ or not and discuss the domain, co-domain, and range of well-defined functions.

i. $f(x) = \pm x$ ii. $f(x) = \frac{1}{x}$ iii. $f(x) = \sqrt{x}$ iv. $f(x) = \sqrt{x^2+1}$

c) Considering the above functions, **classify** which of the following functions are one-to-one or on-to. justify your answer.

d) **Express** the conditions for the inverse function and **compute** the inverse of $f(x) = \frac{x+1}{x}$?

e) Let F and G are both defined $\boldsymbol{R} \rightarrow \boldsymbol{R}$ by the formula $F(x) = 3x + 2$, $G(y) = \frac{y-2}{3}$, **solve** for $F°G$ and $G°F$ and conclude.

[CLO 2]

CLO No.	Mapped GA	BT (Domain-level)	Question No
1	2 (Knowledge for Solving Computing Problems)	C2	1
2	3 (Problem Analysis)	C3	2

BT=Bloom's Taxonomy; C= Cognitive Domain

Q1.

$((\sim P \wedge Q) \rightarrow R) \wedge (R \rightarrow (\sim P \wedge Q))$

$\sim(\sim(P \wedge Q)) \vee R) \wedge (\sim R \vee (\sim P \wedge Q))$

P	Q	R	((P → Q) ↔ R)
F	F	F	**F**
F	F	T	**T**
F	T	F	**F**
F	T	T	**T**
T	F	F	**T**
T	F	T	**F**
T	T	F	**F**
T	T	T	**T**

P	Q	R	(P → (Q ↔ R))
F	F	F	**T**
F	F	T	**T**
F	T	F	**T**
F	T	T	**T**
T	F	F	**T**
T	F	T	**F**
T	T	F	**F**
T	T	T	**T**

Not Logically Equivalent

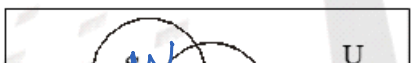
SOLUTION

STATEMENT	REASON
~(p→q) → p	Given statement form
≡ ~[~(p ∧ ~q)] → p	Implication law p→q ≡ ~(p ∧ ~q)
≡ (p ∧ ~q) → p	Double negation law
≡ ~(p ∧ ~q) ∨ p	Implication law p→q ≡ ~p ∨ q
≡ (~p ∨ q) ∨ p	De Morgan's law
≡ (q ∨ ~p) ∨ p	Commutative law of
∨≡ q ∨ (~p ∨ p)	Associative law of
∨≡ q ∨ t	Negation law
≡ t	Universal bound law

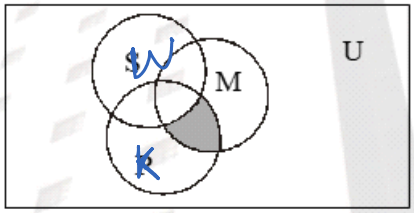
Q2 9)

(i)

Modem and printer but no Scanner is shaded.

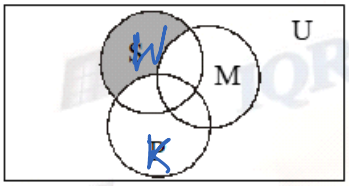


Modem and printer but no Scanner is shaded.



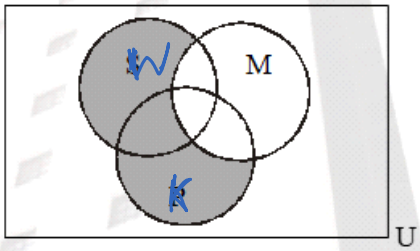
(ii)

Scanner but no printer and no modem is shaded.



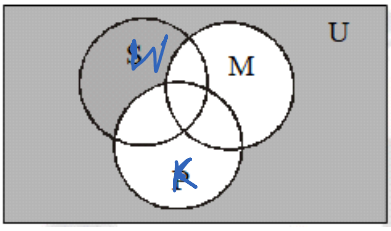
(iii)

scanner or printer but no modem is shaded.



(iv)

no modem and no printer is shaded.



b) i. two images $f(x) = x$, $f(x) = -x$

ii. $f(0) = \frac{1}{0} = \infty$

iii $f(x) = \sqrt{x}$ $x \in \mathbb{Z}^+ = \{0,1,2,3,\ldots\}$

iv. $f(x) = \sqrt{x^2+1}$ $x \in \mathbb{R}$

c) $\sqrt{x_1} = \sqrt{x_2} \Rightarrow x_1 = x_2$ one to one

$\sqrt{x_1^2+1} = \sqrt{x_2^2+1} \rightarrow x_1^2+1 = x_2^2+1$

$\sqrt{x_1^2+1} = \sqrt{x_2^2+1} \Rightarrow x_1^2+1 = x_2^2+1$

$\Rightarrow x_1^2 = x_2^2 \Rightarrow x_1 = x_2$ One to one

Not Onto Codom = $\mathbb{R}$

& Range = $[0,\infty)$ not Equal

Rang $[1,\infty)$

d) Inverse = One to one & Onto

$f(x) = \frac{x+1}{x} \Rightarrow y = \frac{x+1}{x} \Rightarrow x = \frac{1}{y-1}$

c) $F \circ G(x) = F(G(x)) = F\left(\frac{y-2}{3}\right)$

$3\left(\frac{y-2}{3}\right) + 2 \Rightarrow x$

$G \circ F(x) = G(F(x)) = x$

JA is inverse of eachother