Self-Assessment Quiz

Lecture 4: Mathematical Logic & Reasoning

Instructions: Answer all questions. For multiple-choice questions circle the correct option. For short answers, write concise responses.

Q1. (MCQ) Which of the following is a *tautology*?

A. $p \land \neg p$
B. $p \lor \neg p$
C. $(p \land q) \land \neg q$
D. $p \to (q \land \neg q)$

Q2. (MCQ) Which statement is a *contradiction*?

A. $p \lor q$
B. $p \land q$
C. $p \land \neg p$
D. $p \leftrightarrow q$

Q3. The formula $(p \lor q) \land \neg(p \land q)$ is equivalent to the exclusive-or $p \oplus q$. (True / False)

Q4. (Short answer) Rewrite the implication $p \to q$ using only $\neg$ and $\lor$ (i.e., without $\to$). Provide the equivalent formula.

Q5. (MCQ) Which of the following is the correct translation of the English sentence: "If it is raining and cold, then I take an umbrella" into propositional logic (let r = "it is raining", c = "it is cold", u = "I take an umbrella")?

A. $(r \land c) \to u$
B. $r \land (c \to u)$
C. $(r \to c) \land u$
D. $r \to (c \land u)$

Q6. (Short answer / Laws of logic) Using De Morgan's laws, transform the formula $\neg(p \land (q \lor \neg r))$ into an equivalent expression that uses only $\neg$, $\lor$, and $\land$ (show steps or final form).

Q7. (MCQ) The biconditional $p \leftrightarrow q$ is logically equivalent to:

A. $(p \to q) \land (q \to p)$
B. $(p \land q) \lor (\neg p \land \neg q)$
C. Both A and B
D. None of the above

Q8. (Short proof idea) Show concisely (no full truth table required) why the formula $\neg(p \to q) \to p$ is a tautology. (Hint: replace $p \to q$ by an equivalent expression and simplify.)

Q9. (Construct) Write a short truth table (list rows) for p, q and compute the truth value of $(p \wedge \neg q) \wedge (\neg p \vee q)$. Based on the table, is the formula a tautology, contradiction, or contingent?

Q10. (Application) Rewrite the statement $(p \wedge \neg q) \to r$ using only $\neg$ and $\wedge$ (i.e., eliminate $\to$ and $\vee$). Provide the equivalent formula.

Answer Key (Do not show to students until grading)

Q1. **B.** $p \vee \neg p$ is a tautology (law of excluded middle).

Q2. **C.** $p \wedge \neg p$ is a contradiction (negation law).

Q3. **True.** $(p \vee q) \wedge \neg(p \wedge q)$ is true exactly when exactly one of p, q is true, i.e. exclusive-or.

Q4. **Answer:** $p \to q \equiv \neg p \vee q$.

Q5. **A.** $(r \wedge c) \to u$ is the direct translation.

Q6. **Solution (De Morgan):**

$$\neg\big(p \wedge (q \vee \neg r)\big) \equiv \neg p \vee \neg(q \vee \neg r) \equiv \neg p \vee (\neg q \wedge r).$$

(One may also distribute if desired.)

Q7. **C.** Both representations are equivalent:

$$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p) \equiv (p \wedge q) \vee (\neg p \wedge \neg q).$$

Q8. **Sketch:** Start with $\neg(p \to q) \to p$.

$$p \to q \equiv \neg p \vee q \quad \Rightarrow \quad \neg(p \to q) \equiv \neg(\neg p \vee q) \equiv p \wedge \neg q.$$

Thus $\neg(p \to q) \to p \equiv (p \wedge \neg q) \to p$. But $(p \wedge \neg q) \to p$ is always true because whenever $(p \wedge \neg q)$ holds then p holds; otherwise the implication is true vacuously. Hence the formula is a tautology.

Q9. **Truth table rows and result:**

p	q	$p \wedge \neg q$	$\neg p \vee q$	$(p \wedge \neg q) \wedge (\neg p \vee q)$
T	T	F	T	F
T	F	T	F	F
F	T	F	T	F
F	F	F	T	F

All rows evaluate to **F** $\Rightarrow$ the formula is a **contradiction**.

Q10. **Rewrite:** $(p \wedge \neg q) \to r \equiv \neg(p \wedge \neg q) \vee r$. To use only $\neg$ and $\wedge$ (eliminate $\vee$) apply $a \vee b \equiv \neg(\neg a \wedge \neg b)$:

$$\neg(p \wedge \neg q) \vee r \equiv \neg\big(\neg\neg(p \wedge \neg q) \wedge \neg r\big) \equiv \neg\big((p \wedge \neg q) \wedge \neg r\big).$$

So an equivalent using only $\neg$ and $\wedge$ is $\boxed{\neg\big((p \wedge \neg q) \wedge \neg r\big)}$.

Notes for instructor: Questions cover tautology/contradiction, truth tables, logical equivalence, De Morgan, implication elimination, biconditional, and short reasoning — matching Lecture 4 learning outcomes.