Self-Assessment Quiz

Inclusion–Exclusion, Onto Functions, and Derangements

Ungraded Practice — Based on Lecture Notes

Instructions: Choose the best answer for each question. This quiz is for self-study only.

Q1. The Inclusion–Exclusion Principle is used to:

(a) Find the probability of independent events.
(b) Count the number of elements in a union of sets when overlaps exist.
(c) Determine permutations of distinct elements.
(d) Compute combinations without repetition.

Q2. For two finite sets A and B, the number of elements in their union is given by:

(a) $|A| + |B|$
(b) $|A| + |B| + |A \cap B|$
(c) $|A| + |B| - |A \cap B|$
(d) $|A| - |B| + |A \cap B|$

Q3. If $|A| = 25$, $|B| = 13$, and $|A \cap B| = 8$, then $|A \cup B| =$:

(a) 30
(b) 35
(c) 38
(d) 42

Q4. The general Inclusion–Exclusion formula for three sets A, B, C is:

(a) $|A| + |B| + |C|$
(b) $|A| + |B| + |C| - |A \cap B| - |B \cap C| - |A \cap C| + |A \cap B \cap C|$
(c) $|A \cup B| + |B \cup C| + |C \cup A|$
(d) $|A \cap B \cap C|$

Q5. Suppose $|A| = 1232$, $|B| = 879$, $|C| = 114$, $|A \cap B| = 103$, $|A \cap C| = 23$, $|B \cap C| = 14$, and $|A \cup B \cup C| = 2092$. The number of students who took all three is:

(a) 9

(b) 11

(c) 13

(d) 15

Q6. In the example with 67 students enjoying Math, Science, and English, the Inclusion–Exclusion principle helps to:

(a) Find the number who like only one subject.

(b) Find the number who like all three.

(c) Find those who like at least one subject.

(d) All of the above.

Q7. The general Inclusion–Exclusion formula for n finite sets $A_1, A_2, \ldots, A_n$ is:

(a) $|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum_i |A_i|$

(b) $|A_1 \cup A_2 \cup \cdots \cup A_n| = \sum |A_i| - \sum |A_i \cap A_j| + \sum |A_i \cap A_j \cap A_k| - \cdots + (-1)^{n+1} |A_1 \cap \cdots \cap A_n|$

(c) $|A_1 \cup \cdots \cup A_n| = |A_1| + |A_2| + \cdots + |A_n|$

(d) None of the above.

Q8. Using Inclusion–Exclusion, the number of integers ≤ 1000 that are multiples of at least one of 4, 6, or 15 is found by:

(a) Adding the counts of multiples of each number only.

(b) Subtracting the overlap of multiples counted more than once.

(c) Adding all pairwise intersections and ignoring triple intersections.

(d) Counting only even numbers.

Q9. For sets A, B, C, $|A \cup B \cup C|$ will always be:

(a) Greater than or equal to each individual $|A|, |B|, |C|$.

(b) Less than each individual $|A|, |B|, |C|$.

(c) Equal to the intersection $|A \cap B \cap C|$.

(d) Zero.

Q10. The number of **onto functions** from a set with m elements to a set with n elements is given by:

(a) n^m

(b) $n^m - C(n,1)(n-1)^m + C(n,2)(n-2)^m - \cdots + (-1)^{n-1}C(n,n-1)\,1^m$
(c) $(n-1)^m$
(d) m^n

Q11. The number of onto functions from a 6-element set to a 3-element set equals:

(a) 3^6
(b) $3^6 - 3(2^6) + 3(1^6)$
(c) $3^6 - 6(2^3) + 3(1^3)$
(d) 6^3

Q12. A **derangement** is:

(a) A permutation where all elements remain fixed.
(b) A permutation with at least one element in its original position.
(c) A permutation with no element in its original position.
(d) A random arrangement of identical objects.

Q13. The number of derangements of n distinct objects is given by:

(a) $D_n = n!$
(b) $D_n = n!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^n \frac{1}{n!}\right)$
(c) $D_n = n^n$
(d) $D_n = n!\left(1 + \frac{1}{n}\right)$

Q14. The probability that no card is placed in its own box when 4 cards are randomly distributed among 4 boxes is:

(a) $\dfrac{D_4}{4!} = \dfrac{9}{24}$
(b) $\dfrac{D_4}{4!} = \dfrac{9}{24} = \dfrac{3}{8}$
(c) $\dfrac{D_4}{4!} = \dfrac{9}{24} = \dfrac{3}{8} \approx 0.375$
(d) $\dfrac{D_4}{4!} = \dfrac{1}{4}$

Q15. The principle connecting Inclusion–Exclusion, onto functions, and derangements is:

(a) All three apply addition rules only.
(b) All three are examples of alternating sum formulas.
(c) They all depend on geometric sequences.
(d) They are unrelated topics.

Answer Key

Q1	(b)
Q2	(c)
Q3	(b)
Q4	(b)
Q5	(c)
Q6	(d)
Q7	(b)
Q8	(b)
Q9	(a)
Q10	(b)
Q11	(b)
Q12	(c)
Q13	(b)
Q14	(c)
Q15	(b)