BS(SE)V1.

IQRA UNIVERSITY IU

FACULTY OF ENGINEERING SCIENCES AND TECHNOLOGY

Department: **Computer Science** Program: **BS**

DISCRETE STRUCTURES

Announced date: 31/8/2024 Due Date: 08/8/2024 Max Marks:05

ASSIGNMENT# 2			
Mapped CLO	**Mapped GA**	**Mapped Learning Level**	**SDG**
CLO 2	GA 3 (Problem analysis)	C3 (Knowledge for solving computing problems)	4 & 9

ASSIGNMENT 2

Question 1

1. Given a function f from a set X to a set Y, $f(x)$ is ______.
2. Given a function f from a set X to a set Y, if $f(x) = y$ then y is called ______ or ______ or ______.
3. Given a function f from a set X to a set Y, the range of f (or the image of X under f) is ______.
4. Given a function f from a set X to a set Y, if $f(x) = y$ then x is called ______ or ______.
5. Given a function f from a set X to a set Y, if $y \in Y$ then $f^{-1}(y) =$ ______ and is called ______.

FACULTY OF ENGINEERING SCIENCES AND TECHNOLOGY

6. Given functions f and g from a set X to a set Y, $f = g$ if, and only if, ______.
7. Given positive real numbers x and b with $b \neq 1$, $\log_b(x) =$ ______.
8. Given a function f from a set X to a set Y and a subset A of X, $f(A) =$ ______.
9. Given a function f from a set X to a set Y and a subset C of Y, $f^{-1}(C) =$ ______.

Question 2

Indicate whether the statements in parts (a)–(d) are true or false for all functions. Justify your answers.

a. If two elements in the domain of a function are equal, then their images in the co-domain are equal.

b. If two elements in the co-domain of a function are equal, then their preimages in the domain are also equal.

c. A function can have the same output for more than one input.

d. A function can have the same input for more than one output.

Question 3

1. If F is a function from a set X to a set Y, then F is one-to-one if, and only if, ______.
2. If F is a function from a set X to a set Y, then F is not one-to-one if, and only if, ______.
3. If F is a function from a set X to a set Y, then F is onto if, and only if, ______.
4. If F is a function from a set X to a set Y, then F is not onto if, and only if, ______.
5. The following two statements are ______:

$$\forall u, v \in U, \text{ if } H(u) = H(v) \text{ then } u = v.$$

$$\forall u, v \in U, \text{ if } u \neq v \text{ then } H(u) \neq H(v).$$

6. Given a function $F: X \to Y$ where X is an infinite set, to prove that F is one-to-one, you suppose that ____ and then you show that ____.
7. Given a function $F: X \to Y$ where X is an infinite set, to prove that F is onto, you suppose that ____ and then you show that ____.
8. Given a function $F: X \to Y$, to prove that F is not one-to-one, you ____.
9. Given a function $F: X \to Y$, to prove that F is not onto, you ____.
10. A one-to-one correspondence from a set X to a set Y is a ____ that is ____.
11. If F is a one-to-one correspondence from a set X to a set Y and y is in Y, then $F^{-1}(y)$ is ____.

Question 4

5. All but two of the following statements are correct ways to express the fact that a function f is onto. Find the two that are incorrect.
   a. f is onto $\Leftrightarrow$ every element in its co-domain is the image of some element in its domain.
   b. f is onto $\Leftrightarrow$ every element in its domain has a corresponding image in its co-domain.
   c. f is onto $\Leftrightarrow$ $\forall y \in Y, \exists x \in X$ such that $f(x) = y$.

d. f is onto $\Leftrightarrow$ $\forall x \in X, \exists y \in Y$ such that $f(x) = y$.
e. f is onto $\Leftrightarrow$ the range of f is the same as the co-domain of f.

Question 5

a. Define $f: \mathbf{Z} \to \mathbf{Z}$ by the rule $f(n) = 2n$, for every integer n.
   (i) Is f one-to-one? Prove or give a counterexample.
   (ii) Is f onto? Prove or give a counterexample.

b. Let $2\mathbf{Z}$ denote the set of all even integers. That is, $2\mathbf{Z} = \{n \in \mathbf{Z} \mid n = 2k, \text{ for some integer } k\}$. Define $h: \mathbf{Z} \to 2\mathbf{Z}$ by the rule $h(n) = 2n$, for each integer n. Is h onto? Prove or give a counterexample.

Question 6

a. Define $g : \mathbf{Z} \to \mathbf{Z}$ by the rule $g(n) = 4n - 5$, for each integer n.

(i) Is g one-to-one? Prove or give a counterexample.

(ii) Is g onto? Prove or give a counterexample.

b. Define $G : \mathbf{R} \to \mathbf{R}$ by the rule $G(x) = 4x - 5$ for every real number x. Is G onto? Prove or give a counterexample.

Question 7

1. If f is a function from X to Y', g is a function from Y to Z, and $Y' \subseteq Y$, then $g \circ f$ is a function from _____ to_____, and $(g \circ f)(x) =$ _____ for every x in X.

2. If f is a function from X to Y and I_x and I_y are the identity functions from X to X and Y to Y, respectively, then $f \circ I_x =$ _____ and $I_y \circ f =$ _____.

3. If f is a one-to-one correspondence from X to Y, then $f^{-1} \circ f =$ _____ and $f \circ f^{-1} =$ _____.

4. If f is a one-to-one function from X to Y and g is a one-to-one function from Y to Z, you prove that $g \circ f$ is one-to-one by supposing that _____ and then showing that _____.

5. If f is an onto function from X to Y and g is an onto function from Y to Z, you prove that $g \circ f$ is onto by supposing that _____ and then showing that _____.

Question 8

In 3 and 4, functions F and G are defined by formulas. Find $G \circ F$ and $F \circ G$ and determine whether $G \circ F$ equals $F \circ G$.

3. $F(x) = x^3$ and $G(x) = x - 1$, for each real number x.

4. $F(x) = x^5$ and $G(x) = x^{1/5}$ for each real number x.

Question 9

1. If R is a relation from A to B, $x \in A$, and $y \in B$, the notation $x\,R\,y$ means that ______.
2. If R is a relation from A to B, $x \in A$, and $y \in B$, the notation $x \not{R}\, y$ means that ______.
3. If R is a relation from A to B, $x \in A$, and $y \in B$, then $(y, x) \in R^{-1}$ if, and only if, ______.
4. A relation on a set A is a relation from ______ to ______.
5. If R is a relation on a set A, the directed graph of R has an arrow from x to y if, and only if, ______.

Question 10

Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$ and let R be the "less than" relation. That is, for every ordered pair $(x, y) \in A \times B$,

$$x\,R\,y \quad \Leftrightarrow \quad x < y.$$

State explicitly which ordered pairs are in R and R^{-1}.

Question 11

Let $A = \{2, 3, 4, 5, 6, 7, 8\}$ and define a relation R on A as follows: For every $x, y \in A$,

$$x\,R\,y \quad \Leftrightarrow \quad x \mid y.$$

Question 12

16. Let $A = \{5, 6, 7, 8, 9, 10\}$ and define a relation S on A as follows: For every $x, y \in A$,

$$x\,S\,y \quad \Leftrightarrow \quad 2 \mid (x - y).$$

Question 13

In 9–33, determine whether the given relation is reflexive, symmetric, transitive, or none of these. Justify your answers.

9. R is the "greater than or equal to" relation on the set of real numbers: For every $x, y \in \mathbf{R}$, $x\,R\,y \Leftrightarrow x \geq y$.

11. D is the relation defined on $\mathbf{R}$ as follows: For every $x, y \in \mathbf{R}$, $x\,D\,y \Leftrightarrow xy \geq 0$.

FACULTY OF ENGINEERING SCIENCES AND TECHNOLOGY

Question 14

In 1–8, a number of relations are defined on the set $A = \{0, 1, 2, 3\}$. For each relation:

a. Draw the directed graph.
b. Determine whether the relation is reflexive.
c. Determine whether the relation is symmetric.
d. Determine whether the relation is transitive.

Give a counterexample in each case in which the relation does not satisfy one of the properties.

1. $R_1 = \{(0, 0), (0, 1), (0, 3), (1, 1), (1, 0), (2, 3), (3, 3)\}$
2. $R_2 = \{(0, 0), (0, 1), (1, 1), (1, 2), (2, 2), (2, 3)\}$
3. $R_3 = \{(2, 3), (3, 2)\}$
4. $R_4 = \{(1, 2), (2, 1), (1, 3), (3, 1)\}$
5. $R_5 = \{(0, 0), (0, 1), (0, 2), (1, 2)\}$
6. $R_6 = \{(0, 1), (0, 2)\}$
7. $R_7 = \{(0, 3), (2, 3)\}$
8. $R_8 = \{(0, 0), (1, 1)\}$