Introduction to Mathematics

Mathematics of Finance

Week 14

Credits

- This presentation has been made with the content from:
  - Applied Mathematics for Business, Economics & Social Sciences by Frank S. Budnick

Chapter # 08

Mathematics of Finance

Learning Objectives

The learning objectives the lecture are as follow:

- Calculation for
  a) Monthly mortgage payment
  b) Total payment
  c) Total interest
- Cash flow statement and calculation of NPV at the indicated interest rate.

Learning Outcomes

- Mortgage and its calculations
- Cost Benefit Analysis

Mortgage

Sooner or later, many of us succumb to the "American dream" of owning a home. However, interest rates, as well as high real estate prices in some areas, cause this to be an expensive dream. Aside from the numerous pleasures of home ownership, there is *at least* one time during each month when we cringe from the effects of owning a home. That time is when we sign a check for the monthly mortgage payment. And whether we realize it or not, we spend an incredible amount of money to realize our dream.

Given a mortgage loan, many homeowners do not realize how the amount of their mortgage payment is calculated. It is calculated in the same way as were the loan payments in the last section. That is, they are calculated by using Eq. (8.19). Interest is typically compounded monthly, and the interest rate per compounding period can equal unusual fractions or decimal answers. If the annual interest rate is 8.5 percent, the value of i is $0.085/12 = \frac{17}{24}$ of a percent, or 0.0070833. Obviously Table VI cannot be used for these interest rates.

Mortgage

Example: A person pays \$100,000 for a new house. A down payment of \$30,000 leaves a mortgage of \$70,000 with interest computed at 10.5 percent per year compounded monthly. Determine the monthly mortgage payment if the loan is to be repaid over (*a*) 20 years, (*b*) 25 years, and (*c*) 30 years. (*d*) Compute total interest under the three different loan periods.

SOLUTION

(*a*) From Table VII, the *monthly payment per dollar of mortgage* is 0.00998380 (corresponding to $n = 20 \times 12 = 240$ payments). Therefore,

$$\begin{aligned} R &= \$70{,}000(0.00998380) \\ &= \$698.87 \end{aligned}$$

(*b*) For 25 years (or 300 monthly payments),

$$\begin{aligned} R &= \$70{,}000(0.00944182) \\ &= \$659.27 \end{aligned}$$

(*c*) For 30 years (or 360 monthly payments)

$$\begin{aligned} R &= \$70{,}000(0.00914739) \\ &= \$640.32 \end{aligned}$$

Mortgage

Example Cont.:

(*d*) Total payments are

$$(240)(\$698.87) = \$167{,}728.80 \qquad \text{for 20 years}$$

$$(300)(\$659.27) = \$197{,}781.00 \qquad \text{for 25 years}$$

$$(360)(\$640.32) = \$230{,}515.20 \qquad \text{for 30 years}$$

Because these payments are all repaying a \$70,000 loan, interest on the loan is

$$\$167{,}728.80 - \$70{,}000 = \$97{,}728.80 \qquad \text{for 20 years}$$

$$\$197{,}781.00 - \$70{,}000 = \$127{,}781.00 \qquad \text{for 25 years}$$

$$\$230{,}515.20 - \$70{,}000 = \$160{,}515.20 \qquad \text{for 30 years}$$

Mortgage

Example # 2: In the previous problem, determine the effects of a decrease in the interest rate to 10 percent (*a*) on monthly payments for the 25-year mortgage and (*b*) on total interest for the 25-year mortgage.

SOLUTION

(*a*) For $i = 10.00$ and 25 years,

$$\begin{aligned} R &= (70{,}000)(0.00908700) \\ &= \$636.09 \end{aligned}$$

Therefore, monthly payments are less by an amount of

$$\$659.27 - \$636.09 = \$23.18$$

(*b*) Total payments over the 25 years will equal

$$300(636.09) = \$190{,}827.00$$

The total interest is $120,827, which is $6,954 less than with the 10.5 percent mortgage.

Mortgage

Example # 3: **(Maximum Affordable Loan)** A couple estimates that they can afford a monthly mortgage of $750. Current mortgage interest rates are 10.25 percent. If a 30-year mortgage is obtainable, what is the maximum mortgage loan this couple can afford?

SOLUTION

The formula for computing the monthly mortgage payment is

$$\text{Monthly payment} = \begin{pmatrix}\text{dollar amount of}\\ \text{mortgage loan}\end{pmatrix}\begin{pmatrix}\text{monthly payment per}\\ \text{dollar of mortgage loan,}\\ \text{Table VII}\end{pmatrix}$$

or

$$\boxed{R = A\begin{pmatrix}\text{Table VII}\\ \text{factor}\end{pmatrix}} \tag{8.20}$$

In this problem, A is the unknown. If Eq. (8.20) is rearranged,

$$A = \frac{R}{\text{Table VII factor}}$$

$$= \frac{750}{0.00896101} = \$83,695.92$$

Cost Benefit Analysis

When organizations evaluate the financial feasibility of investment decisions, the time value of money is an essential consideration. This is particularly true when a project involves cash flow patterns which extend over a number of years. This section will discuss one way in which such multiperiod investments can be evaluated.

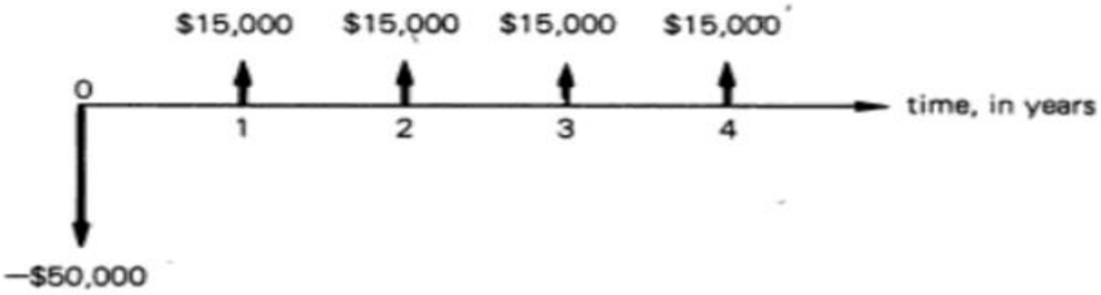
Discounted Cash Flow

Consider an investment decision characterized by the cash flow pattern shown in Fig. 8.12. An initial investment of $50,000 is expected to generate a net (after expenses) return of $15,000 at the end of 1 year and an equal return at the end of the following 3 years. Thus, a $50,000 investment is expected to return $60,000 over a 4-year period. Because the cash inflows occur over a 4-year period, the dollars during the different periods cannot be considered equivalent. To evaluate this project properly, the time value of the different cash flows must be accounted for.

One approach to evaluating a project like this is to translate all cash flows into equivalent dollars at a common base period. This is called a ***discounted cash flow method.*** For example, this project might be evaluated by restating all cash flows in terms of their equivalent values at $t = 0$, the time of the investment. The original $50,000 is stated in terms of dollars at $t = 0$. However, each of the $15,000 cash inflows must be restated in terms of its equivalent value at $t = 0$.

Cost Benefit Analysis

In order to discount all cash flows to a common base period, an interest rate must be assumed for the intervening period. Frequently this interest rate is an assumed minimum desired rate of return on investments. For example, management might state that a minimum desired rate of return on all investments is 10 percent per year. How this figure is obtained by management is another issue in



itself. Sometimes it is a reflection of the known rate of return which can be earned on alternative investments (e.g., bonds or money market funds).

Cost Benefit Analysis

Let's assume that the minimum desired rate of return for the project in Fig. 8.12 is 8 percent per year. Our discounted cash flow analysis will compute the ***net present value*** (NPV) of all cash flows associated with a project. The net present value is the algebraic sum of the present value of all cash flows associated with a project; cash *inflows* are treated as positive cash flows and cash *outflows* as negative cash flows. *If the net present value of all cash flows is* ***positive*** *at the assumed minimum rate of return, the actual rate of return from the project exceeds the minimum desired rate of return. If the net present value for all cash flows is* ***negative,*** *the actual rate of return from the project is less than the minimum desired rate of return.*

In our example, we discount the four \$15,000 figures at 8 percent. By computing the present value of these figures, we are, in effect, determining the amount of money we would have to invest today ($t = 0$) at 8 percent in order to generate those four cash flows. Given that the net cash return values are equal, we can treat this as the computation of the present value of an annuity. Using Table V and Eq. (8.17), with $n = 4$ and $i = 0.08$,

$$\begin{aligned} A &= 15{,}000(3.31213) \\ &= \$49{,}681.95 \end{aligned}$$

Cost Benefit Analysis

This value suggests that an investment of \$49,681.95 would generate an annual payment of 15,000 at the end of each of the following 4 years. In this example, an investment of \$50,000 is required.

The net present value for this project combines the present values of all cash flows at $t = 0$, or

$$\boxed{\text{NPV} = \begin{array}{c}\text{present value}\\ \text{of cash inflows}\end{array} - \begin{array}{c}\text{present value}\\ \text{of cash outflows}\end{array}} \tag{8.21}$$

Thus

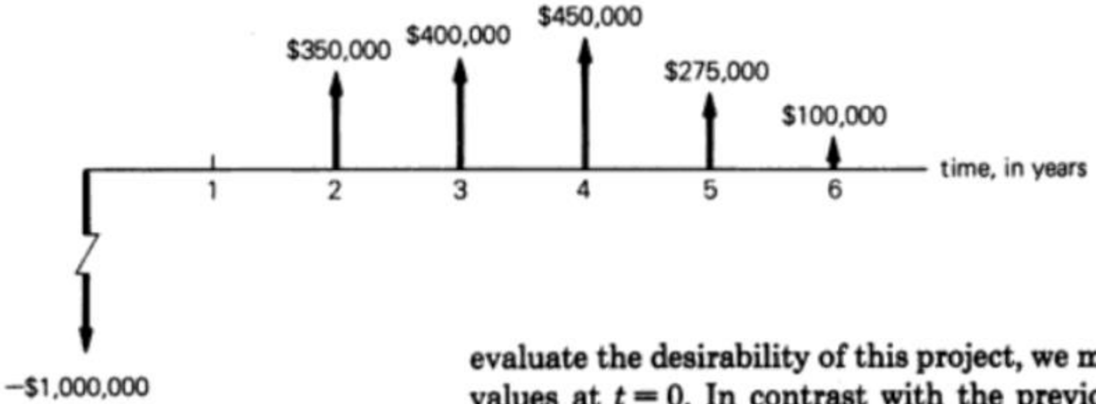
$$\begin{aligned}\text{NPV} &= \$49{,}681.80 - \$50{,}000\\ &= -\$318.20\end{aligned}$$

This negative value indicates that the project will result in a rate of return less than the minimum desired return of 8 percent per year, compounded annually.

Cost Benefit Analysis

Example # 4: **(Nonuniform Cash Flow Patterns)** The previous example resulted in net cash inflows which were equal over 4 years. The cash flow patterns for most investments tend to be irregular, both with regard to amount of money and timing of cash flows. Consider the cash flow pattern illustrated in Fig. 8.13. For this investment project, a $1 million investment results in no cash flow during the first year. However, at the end of each of the following 5 years the investment generates a stream of positive net returns. These returns are not equal to one another, increasing to a maximum of $450,000 at the end of the fourth year and decreasing finally to $100,000 at the end of the sixth year.

... ents is 12 percent. In order to



evaluate the desirability of this project, we must discount all cash flows to their equivalent values at $t = 0$. In contrast with the previous example, each net return figure must be discounted separately. The present value of each is computed as shown in Table 8.3.

Cost Benefit Analysis

n	Net Return	Present Value Factor $(1+0.12)^{-n}$	Present Value
2	$350,000	0.79719	$ 279,016.50
3	400,000	0.71178	284,712.00
4	450,000	0.63552	285,984.00
5	275,000	0.56743	156,043.25
6	100,000	0.50663	50,663.00
			$1,056,418.75

The net present value of all cash flows is

$$\text{NPV} = \$1{,}056{,}418.75 - \$1{,}000{,}000 = \$56{,}418.75$$

Because the net present value is positive, this project will result in a rate of return which exceeds the minimum desired rate of return of 12 percent per year, compounded annually. Another way of viewing this is that $1,056,418.75, invested at 12 percent per year, will generate the indicated net returns; this investment only requires $1,000,000.

Review Questions

1 Determine the present value of a series of 10 annual payments of $25,000 each which begins 1 year from today. Assume interest of 9 percent per year compounded annually.
2 Determine the present value of a series of 20 annual payments of $8,000 each which begins 1 year from today. Assume interest of 7 percent per year compounded annually.
3 Determine the present value of a series of 25 semiannual payments of $10,000 each which begins in 6 months. Assume interest of 10 percent per year compounded semiannually.
4 Determine the present value of a series of 15 payments of $5,000 each which begins 6 months from today. Assume interest of 9 percent per year compounded semiannually.
5 Determine the present value of a series of 30 quarterly payments of $500 which begins 3 months from today. Assume interest of 10 percent per year compounded quarterly.
6 Determine the present value of a series of 20 quarterly payments of $2,500 each which begins 3 months from today. Assume interest of 8 percent per year compounded quarterly.
7 Determine the present value of a series of 60 monthly payments of $2,500 each which begins 1 month from today. Assume interest of 12 percent per year compounded monthly.

Review Questions

8 Determine the present value of a series of 36 monthly payments of $5,000 each which begins 1 month from today. Assume interest of 18 percent per year compounded monthly.

9 A person wants to buy a life insurance policy which would yield a large enough sum of money to provide for 20 annual payments of $50,000 to surviving members of the family. The payments would begin 1 year from the time of death. It is assumed that interest could be earned on the sum received from the policy at a rate of 8 percent per year compounded annually.

(*a*) What amount of insurance should be taken out so as to ensure the desired annuity?

(*b*) How much interest will be earned on the policy benefits over the 20-year period?

10 Assume in Exercise 9 that semiannual payments of $25,000 are desired over the 20-year period, and interest is compounded semiannually.

(*a*) What amount of insurance should be taken out?

(*b*) How does this amount compare with that for Exercise 9?

(*c*) How much interest will be earned on the policy benefits?

(*d*) How does this compare with that for Exercise 9?

Review Questions

1 Mortgage loan of $80,000 at 10 percent per year for 20 years
2 Mortgage loan of $100,000 at 11.25 percent per year for 30 years
3 Mortgage loan of $90,000 at 9.5 percent per year for 25 years
4 Mortgage loan of $200,000 at 10.75 percent per year for 30 years
5 Mortgage loan of $90,000 at 10 percent per year for 25 years
6 Mortgage loan of $150,000 at 9.75 percent per year for 30 years
7 Mortgage loan of $120,000 at 10.5 percent per year for 20 years
8 Mortgage loan of $160,000 at 12 percent per year for 25 years

Learning Material

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