SOLUTION

Total revenue comes from the sale of crops planted at each of the three farms, or

$$\begin{aligned} R(x_1, x_2, x_3) &= r_1 x_1 + r_2 x_2 + r_3 x_3 \\ &= 1{,}300x_1 + 1{,}650x_2 + 1{,}200x_3 \end{aligned}$$

Total costs are the sum of those at the three farms plus the corporate fixed costs, or

$$\begin{aligned} C(x_1, x_2, x_3) &= c_1 x_1 + F_1 + c_2 x_2 + F_2 + c_3 x_3 + F_3 + 75{,}000 \\ &= 900x_1 + 150{,}000 + 1{,}100x_2 + 175{,}000 + 750x_3 + 125{,}000 + 75{,}000 \\ &= 900x_1 + 1{,}100x_2 + 750x_3 + 525{,}000 \end{aligned}$$

Total profit is a linear function computed as

$$\begin{aligned} P(x_1, x_2, x_3) &= R(x_1, x_2, x_3) - C(x_1, x_2, x_3) \\ &= 1{,}300x_1 + 1{,}650x_2 + 1{,}200x_3 - (900x_1 + 1{,}100x_2 + 750x_3 + 525{,}000) \\ &= 400x_1 + 550x_2 + 450x_3 - 525{,}000 \end{aligned}$$

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Section 5.1 Follow-up Exercises

1 Write the general form of a linear function involving five independent variables.

2 Assume that the salesperson in Example 1 (page 177) has a salary goal of \$800 per week. If product B is not available one week, how many units of product A must be sold to meet the salary goal? If product A is unavailable, how many units must be sold of product B?

3 Assume in Example 1 (page 177) that the salesperson receives a bonus when combined sales from the two products exceed 80 units. The bonus is \$2.50 per unit for each unit over 80. With this incentive program, the salary function must be described by two different linear functions. What are they, and when are they valid?

4 For Example 4 (page 181), how many units must be produced and sold in order to (*a*) earn a profit of \$1.5 million, and (*b*) earn zero profit (break even)?

5 A manufacturer of microcomputers produces three different models. The following table summarizes wholesale prices, material cost per unit, and labor cost per unit. Annual fixed costs are \$25 million.

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	Microcomputer		
	Model 1	**Model 2**	**Model 3**
Wholesale price/unit	\$500	\$1,000	\$1,500
Material cost/unit	175	400	750
Labor cost/unit	100	150	225

unction

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(*a*) Determine a joint total revenue function for sales of the three different microcomputer models.

(*b*) Determine an annual total cost function for manufacturing the three models.

(*c*) Determine the profit function for sales of the three models.

(*d*) What is annual profit if the firm sells 20,000, 40,000 and 10,000 units, respectively, of the three models?

$R = 0.33x$

6 For Example 5 (page 181), the board of directors has voted on the following planting program for the coming year: 1,000 acres will be planted at farm 1, 1,600 at farm 2, and 1,550 at farm 3.

(a) What are the expected profits for the program?

(b) A summer drought has resulted in the revenue yields per acre being reduced by 20, 30, and 10 percent, respectively, at the three farms. What is the profit expected from the previously mentioned planting program?

7 Automobile Leasing A car-leasing agency purchases new cars each year for use in the agency. The cars cost $15,000 new. They are used for 3 years, after which they are sold for $4,500. The owner of the agency estimates that the variable costs of operating the cars, exclusive of gasoline, are $0.18 per mile. Cars are leased for a flat fee of $0.33 per mile (gasoline not included).

(a) Formulate the total revenue function associated with renting one of the cars a total of x miles over a 3-year period.

(b) Formulate the total cost function associated with renting a car for a total of x miles over 3 years.

(c) Formulate the profit function.

(d) What is profit if a car is leased for 60,000 miles over a 3-year period?

(e) What mileage is required in order to earn zero profit for 3 years?

8 A company produces a product which it sells for $55 per unit. Each unit costs the firm $23 in variable expenses, and fixed costs on an annual basis are $400,000. If x equals the number of units produced and sold during the year:

(a) Formulate the linear total cost function.

(b) Formulate the linear total revenue function.

(c) Formulate the linear profit function.

(d) What does annual profit equal if 10,000 units are produced and sold during the year?

(e) What level of output is required in order to earn zero profit?

9 A gas station sells unleaded regular gasoline and unleaded premium. The price per gallon charged by the station is $1.299 for unleaded regular and $1.379 for unleaded premium. The cost per gallon from the supplier is $1.219 for unleaded regular and $1.289 for premium. If x_1 equals the number of gallons sold of regular and x_2 the number of gallons sold of premium:

(a) Formulate the revenue function from selling x_1 and x_2 gallons, respectively, of the two grades of gasoline.

(b) Formulate the total cost function from purchasing x_1 and x_2 gallons, respectively, of the two grades.

(c) Formulate the total profit function.

(d) What is total profit expected to equal if the station sells 100,000 gallons of unleaded regular and 40,000 of unleaded premium?

5.2 OTHER EXAMPLES OF LINEAR FUNCTIONS

In this section we will see, by example, other applications of linear functions.

EXAMPLE 6 **(Straight-Line Depreciation)** When organizations purchase equipment, vehicles, buildings, and other types of "capital assets," accountants usually allocate the cost of the item over the period the item is used. For a truck costing $20,000 and having a useful life of 5 years, accountants might allocate $4,000 a year as a cost of owning the truck. The cost allocated to any given period is called ***depreciation.***

Accountants also keep records of each major asset and its current, or "book," value. For

SOLUTION

If we define

$$S = \textit{Social Security taxes collected, billions of dollars}$$

$$t = \textit{time measured in years since 1980}$$

we want to determine the linear function having the form

$$S = f(t) = a_1 t + a_0$$

The two data points (t, S) are (0, 150) and (9, 352). By observation, the value of a_0 equals 150. Substituting the data point for 1989 into the slope-intercept form gives

$$352 = a_1(9) + 150$$

$$202 = 9a_1$$

$$22.44 = a_1$$

Thus, the linear approximating function is

$$S = f(t) = 22.44t + 150$$

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Section 5.2 Follow-up Exercises

1 A piece of machinery is purchased for \$80,000. Accountants have decided to use a straight-line depreciation method with the machine being fully depreciated after 6 years. Letting V equal the book value of the machine and t the age of the machine, determine the function $V = f(t)$. (Assume no salvage value.)

2 **Straight-Line Depreciation with Salvage Value** Many assets have a **resale,** or **salvage, value** even after they have served the purposes for which they were originally purchased. In such cases, the allocated cost over the life of the asset is the difference between the purchase cost and the salvage value. The cost allocated each time period is the allocated cost divided by the useful life. In Example 6, suppose that it is estimated that the truck (which cost \$20,000) can be resold for \$2,500 at the end of 5 years. The total cost to be allocated over the 5-year period is the purchase cost less the resale value, or \$20,000 − \$2,500 = \$17,500. Using straight-line depreciation, the annual depreciation will be

$$\frac{\text{Purchase cost} - \text{salvage value}}{\text{Useful life (in years)}} = \frac{20{,}000 - 2{,}500}{5}$$

$$= \frac{17{,}500}{5}$$

$$= 3{,}500$$

The function expressing the book value V as a function of time t is

$$V = f(t) = 20{,}000 - 3{,}500t \qquad 0 \le t \le 5$$

In Exercise 1 assume that the machine will have a salvage value of \$7,500 at the end of 6 years. Determine the function $V = f(t)$ for this situation.

3 A piece of machinery is purchased for \$300,000. Accountants have decided to use a straight-line depreciation method with the machine being fully depreciated after 8 years. Letting V equal the book value of the machine and t the age of the machine, determine the function $V = f(t)$. Assume there is no salvage value.

4 Assume in Exercise 3 that the machine can be resold after 8 years for \$28,000. Determine the function $V = f(t)$.

5 A company purchases cars for use by its executives. The purchase cost this year is \$25,000. The cars are kept 3 years, after which they are expected to have a resale value of \$5,600. If accountants use straight-line depreciation, determine the function which describes the book value V as a function of the age of the car t.

6 A police department believes that arrest rates R are a function of the number of plainclothes officers n assigned. The *arrest rate* is defined as the percentage of cases in which arrests have been made. It is believed that the relationship is linear and that each additional officer assigned to the plainclothes detail results in an increase in the arrest rate of 1.20 percent. If the current plainclothes force consists of 16 officers and the arrest rate is 36 percent:

(*a*) Define the function $R = f(n)$.
(*b*) Interpret the meaning of the R intercept.
(*c*) Determine the restricted domain and range for the function.
(*d*) Sketch the function.

7 Two points on a linear demand function are (\$20, 80,000) and (\$30, 62,500).

(*a*) Determine the demand function $q = f(p)$.
(*b*) Determine what price would result in demand of 50,000 units.
(*c*) Interpret the slope of the function.
(*d*) Define the restricted domain and range for the function.
(*e*) Sketch $f(p)$.

8 Two points (p, q) on a linear demand function are (\$24, 60,000) and (\$32, 44,400).

(*a*) Determine the demand function $q = f(p)$.
(*b*) What price would result in demand of 80,000 units?
(*c*) Interpret the slope of the function.
(*d*) Determine the restricted domain and range.
(*e*) Sketch $f(p)$.

9 Two points on a linear supply function are (\$4.00, 28,000) and (\$6.50, 55,000).

(*a*) Determine the supply function $q = f(p)$.
(*b*) What price would result in suppliers offering 45,000 units?
(*c*) Determine and interpret the p intercept.

10 Two points (p, q) on a linear supply function are (\$3.50, 116,000) and (\$5.00, 180,000).

(*a*) Determine the supply function $q = f(p)$.
(*b*) What price would result in suppliers offering 135,000 units for sale?
(*c*) Interpret the slope of the function.
(*d*) Determine and interpret the p intercept.
(*e*) Sketch $f(p)$.

11 **Alimony/Child Support** Recent surveys indicate that payment of alimony or child support tends to decline with time elapsed after the divorce decree. One survey uses the estimating function

$$p = f(t) = 90 - 12.5t$$

$m = \frac{1235 - 925}{88 - 80}$

where p equals the percentage of cases in which payments are made and t equals time measured in years after the divorce decree.

(a) Interpret the p intercept.
(b) Interpret the slope.
(c) In what percentage of cases is alimony/child support paid after 5 years?
(d) Sketch $f(t)$.

12 **Sports Injuries** A survey of high school and college football players suggests that the number of career-ending injuries in this sport is increasing. In 1980 the number of such injuries was 925; in 1988 the number was 1,235. If it is assumed that the injuries are increasing at a linear rate:

(a) Determine the function $n = f(t)$, where n equals the estimated number of injuries per year and t equals time measured in years since 1980.
(b) Interpret the meaning of the slope of this function.
(c) When is it expected that the number of such injuries will go over the 1,500 mark?

13 **Marriage Prospects** Data released by the Census Bureau in 1986 indicated the likelihood that never-married women would eventually marry. The data indicated that the older the woman, the less the likelihood of marriage. Specifically, two statistics indicated that women who were 45 and never-married had an 18 percent chance of marriage and women 25 years old had a 78 percent chance of marriage. Assume that a linear fit to these two data points provides a reasonable approximation for the function $p = f(a)$, where p *equals the probability of marriage* and a *equals the age of a never-married woman.*

(a) Determine the linear function $p = f(a)$.
(b) Interpret the slope and p intercept.
(c) Do the values in part b seem reasonable?
(d) If the restricted domain on this function is $20 \le a \le 50$, determine $f(20)$, $f(30)$, $f(40)$, and $f(50)$.

14 **Two-Income Families** Figure 5.8 illustrates the results of a survey regarding two-income families. The data reflect the percentage of married couples with wives who work for four different years. The percentage appears to be increasing approximately at a linear rate. Using the data points for 1960 and 1988:

(a) Determine the linear function $P = f(t)$, where P equals the estimated percentage of married couples with wives who work and t equals time measured in years *since 1950 (t = 0 corresponds to 1950).*

$P = mt + k$

$P = \frac{28}{38}t + 25$

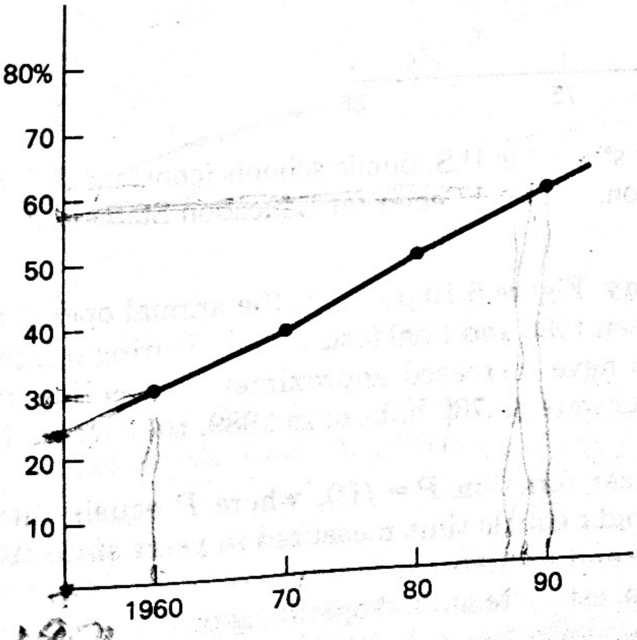

Figure 5.8 Percentage of married couples with wives who work.

$m = \frac{58 - 30}{88 - 60}$

(*b*) Interpret the meaning of the slope and the *P* intercept.
(*c*) When is it expected that the percentage will exceed 75 percent?

15 Education Expenditures Figure 5.9 illustrates the expenditures per student in U.S. public schools over a three-decade period. The expenditures are stated in "constant dollars" which filter out the effects of inflation. The increase in expenditures per student appears to have occurred approximately at a linear rate. In 1958, the expenditures per student were \$1,750; in 1984, the expenditures were \$3,812.50. Using these two data points:

(*a*) Determine the linear approximating function $E = f(t)$, where E equals the estimated expenditure per student in dollars and t equals time measured in years *since* 1955 ($t = 0$ corresponds to 1955).
(*b*) Interpret the slope and E intercept.
(*c*) According to this function, what are expenditures per student expected to equal in the year 2000?

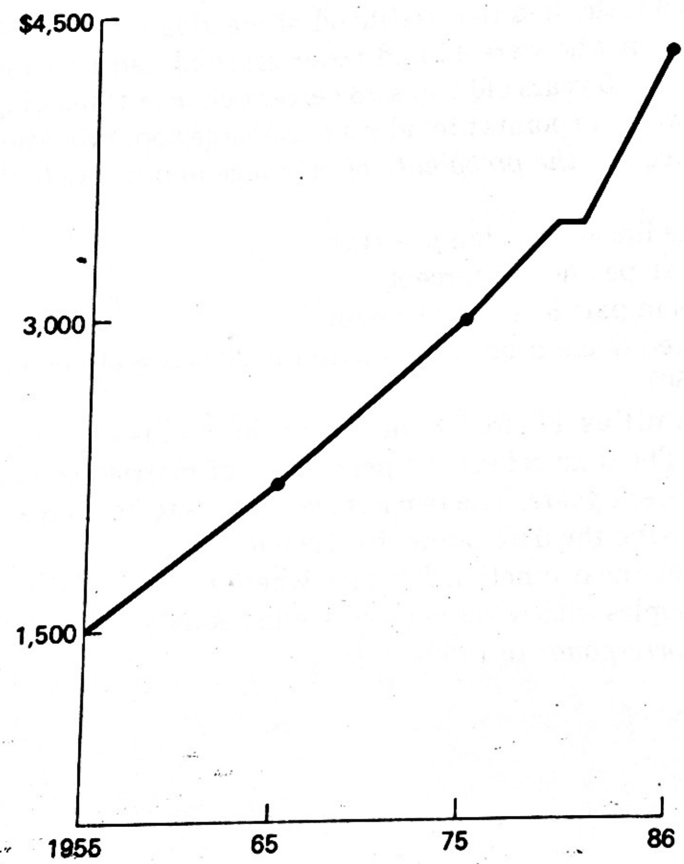

Figure 5.9 Expenditures per student in U.S. public schools (constant dollars). (Source: U.S. Dept. of Education, National Center for Education Statistics)

16 Walt Disney Company Figure 5.10 portrays the annual operating profits for Walt Disney Company between 1986 and 1990 (estimated). During this period, annual operating profits appear to have increased approximately in a linear manner. In 1987, annual operating profits were \$0.762 billion; in 1989, they were \$1.220 billion. Using these two data points:

(*a*) Determine the linear function $P = f(t)$, where P equals the estimated annual operating profits and t equals time measured in years *since* 1986.
(*b*) Interpret the slope and P intercept.
(*c*) Using this function, estimate annual operating profits for Walt Disney Company in the year 2000.

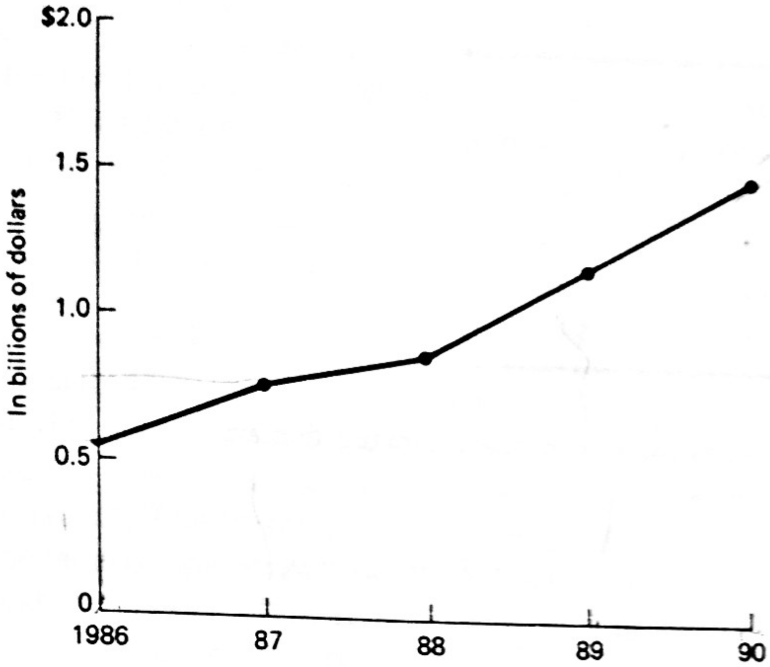

Figure 5.10 Annual operation profits, Walt Disney Company. (Data: Company Reports, Wertheim Schroder & Co.)

17 Economic Downturn A general trend of economic decline within New York City is reflected by Fig. 5.11. This figure indicates the vacancy rate for offices in Manhattan during the period 1985–1990. The increase in the vacancy rate appears to be approximately linear. The vacancy rate in 1986 was 9.4 percent; in 1989, the rate was 13.2 percent. Using these two data points:

(*a*) Determine the linear function $V = f(t)$, where V equals the estimated vacancy rate (in percent) and t equals time measured in years since 1985.

(*b*) Interpret the slope and V intercept.

(*c*) Using the function, estimate the vacancy rate in 1995.

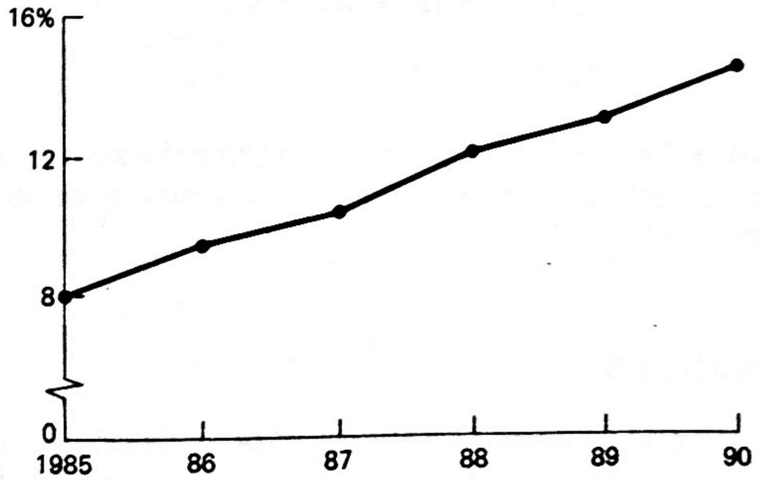

Figure 5.11 Percentage of Manhattan offices vacant. (Business Week, June 18, 1990)

18 Federal Income Taxes Table 5.3 contains the 1990 federal tax rates for a single person. Determine the function $T = f(x)$, where T equals the tax liability (in dollars) for a single person and x equals taxable income (in dollars).

TABLE 5.3

Taxable Income		
Over	**But not Over**	**Tax Rate**
$ 0	$19,450	15%
19,450	47,050	28
47,050	97,620	33
97,620		28

19 **Market Equilibrium** Given the following demand and supply functions for two competing products,

$$q_{d1} = 82 - 3p_1 + p_2$$
$$q_{s1} = 15p_1 - 5$$
$$q_{d2} = 92 + 2p_1 - 4p_2$$
$$q_{s2} = 32p_2 - 6$$

determine whether there are prices which bring the supply and demand levels into equilibrium for the two products. If so, what are the equilibrium quantities?

*20 **Market Equilibrium: Three Competing Products** The following are the demand and supply functions for three competing products.

$$q_{d1} = 46 - 10p_1 + 2p_2 + 2p_3$$
$$q_{s1} = 12p_1 - 16$$
$$q_{d2} = 30 + 2p_1 - 6p_2 + 4p_3$$
$$q_{s2} = 6p_2 - 22$$
$$q_{d3} = 38 + 2p_1 + 4p_2 - 8p_3$$
$$q_{s3} = 6p_3 - 10$$

Determine whether there are prices which would bring the supply and demand levels into equilibrium for each of the three products. If so, what are the equilibrium demand and supply quantities?

5.3 BREAK-EVEN MODELS

In this section we will discuss ***break-even models,*** a set of planning tools which can be, and has been, very useful in managing organizations. One significant indication of the performance of a company is reflected by the so-called bottom line of the income statement for the firm; that is, how much profit is earned! Break-even analysis focuses upon the profitability of a firm. Of specific concern in break-even analysis is identifying the level of operation or level of output that would result in a **zero profit.** This level of operations or output is called the ***break-even point.*** The break-even point is a useful reference point in the sense that it represents the

TABLE 5.4

	Product		
	A	B	C
Price/unit	$40	$30	$55
Variable cost/unit	30	21	43
Profit margin	$10	$ 9	$12

C. If a product mix unit can be defined, we can conduct break-even analysis using this as the measure of output.

Suppose that these three products have the price and cost attributes shown in Table 5.4. Combined fixed cost for the three products is $240,000. Since *1 unit of the product mix* consists of 3 units of *A*, 2 units of *B*, and 1 unit of *C*, the *profit contribution per unit of product mix equals*

$$3(\$10) + 2(\$9) + 1(\$12) = \$60$$

If we let *x equal the number of units of product mix*, the profit function for the three products is

44-33

$$P(x) = 60x - 240,000$$

The break-even point occurs when $P(x) = 0$, or

240000

$$60x - 240,000 = 0$$

$$60x = 240,000$$

$$x = 4,000$$

The firm will break even when it produces 4,000 product mix units, or 12,000 units of *A*, 8,000 units of *B*, and 4,000 units of *C*.

□

The analysis presented in Example 18 presumes that a product mix is known. If the product mix is not known exactly but can be approximated, this analysis can still be of value as a planning tool.

P.V ??

Section 5.3 Follow-up Exercises

1 A firm sells a product for $45 per unit. Variable costs per unit are $33 and fixed costs equal $450,000. How many units must be sold in order to break even?

2 An enterprising college student has decided to purchase a local car wash business. The purchase cost is $150,000. Car washes will be priced at $5.50, and variable cost per car (soap, water, labor, etc.) is expected to equal $1.50. How many cars must be washed in order to recover the $150,000 purchase price?

3 A charitable organization is planning a raffle to raise $10,000. Five hundred chances will be sold on a new car. The car will cost the organization $15,000. How much should each ticket cost if the organization wishes to net a profit of $10,000?

4 A publisher has a fixed cost of $250,000 associated with the production of a college mathematics book. The contribution to profit and fixed cost from the sale of each book is $6.25.
(a) Determine the number of books which must be sold in order to break even.
(b) What is the expected profit if 50,000 books are sold?

5 A local university football team has added a national power to next year's schedule. The other team has agreed to play the game for a guaranteed fee of $100,000 plus 25 percent of the gate receipts. Assume the ticket price is $12.
(a) Determine the number of tickets which must be sold to recover the $100,000 guarantee.
(b) If college officials hope to net a profit of $240,000 from the game, how many tickets must be sold?
(c) If a sellout of 50,000 fans is assured, what ticket price would allow the university to earn the desired profit of $240,000?
(d) Again assuming a sellout, what would total profit equal if the $12 price is charged?

6 **Make or Buy Decision** Assume that a manufacturer can purchase a needed component from a supplier at a cost of $9.50 per unit, or it can invest $60,000 in equipment and produce the item at a cost of $7.00 per unit.
(a) Determine the quantity for which total costs are equal for the *make* and *buy* alternatives.
(b) What is the minimum cost alternative if 15,000 units are required? What is the minimum cost?
(c) If the number of units required of the component is close to the break-even quantity, what factors might influence the final decision to make or buy?

7 A local civic arena is negotiating a contract with a touring ice-skating show, Icey Blades. Icey Blades charges a flat fee of $60,000 per night plus 40 percent of the gate receipts. The civic arena plans to charge one price for all seats, $12.50 per ticket.
(a) Determine the number of tickets which must be sold each night in order to break even.
(b) If the civic arena has a goal of clearing $15,000 each night, how many tickets must be sold?
(c) What would nightly profit equal if average attendance is 7,500 per night?

8 In the previous exercise, assume that past experience with this show indicates that average attendance should equal 7,500 persons.
(a) What ticket price would allow the civic arena to break even?
(b) What ticket price would allow them to earn a profit of $15,000?

9 **Equipment Selection** A firm has two equipment alternatives it can choose from in producing a new product. One automated piece of equipment costs $200,000 and produces items at a cost of $4 per unit. Another semiautomated piece of equipment costs $125,000 and produces items at a cost of $5.25 per unit.
(a) What volume of output makes the two pieces of equipment equally costly?
(b) If 80,000 units are to be produced, which piece of equipment is less costly? What is the minimum cost?

10 **Robotics** A manufacturer is interested in introducing the robotics technology into one of its production processes. The process is one which would provide a "hostile environment" for humans. To be more specific, the process involves exposure to extremely high temperatures as well as to potentially toxic fumes. Two robots which appear to have the capabilities for executing the functions of the production process have been identified. There appear to be no significant differences in the speeds at which the two models work. One robot costs $180,000 and has estimated maintenance

costs of \$100 per hour of operation. The second type of robot costs \$250,000 with maintenance costs estimated at \$80 per hour of operation.

(a) At what level of operation (total production hours) are the two robots equally costly? What is the associated cost?

(b) Define the levels of operation for which each robot would be the less costly.

11 **Computer Software Development** A firm has a computer which it uses for a variety of purposes. One of the major costs associated with the computer is software development (writing computer programs). The vice president for information systems wants to evaluate whether it is less costly to have his own programming staff or to have programs developed by a software development firm. The costs of both options are a function of the number of lines of code (program statements). The vice president estimates that in-house development costs \$1.50 per line of code. In addition, annual overhead costs for supporting the programmers equal \$30,000. Software developed outside the firm costs, on average, \$2.25 per line of code.

(a) How many lines of code per year make costs of the two options equal?

(b) If programming needs are estimated at 30,000 lines per year, what are the costs of the two options?

(c) In part *b* what would the in-house cost per line of code have to equal for the two options to be equally costly?

C2

1+12

12 **Sensitivity Analysis** Because the parameters (constants) used in mathematical models are frequently estimates, actual results may differ from those projected by the mathematical analysis. To account for some of the uncertainties which may exist in a problem, analysts often conduct *sensitivity analysis.* The objective is to assess how much a solution might change if there are changes in model parameters.

Assume in the previous exercise that software development costs by outside firms might actually fluctuate by ±20 percent from the \$2.25 per-line estimate.

(a) Recompute the break-even point if costs are 20 percent higher or lower and compare your result with the original answer.

(b) Along with the uncertainty in part *a*, in-house variable costs might increase by as much as 30 percent because of a new union contract. Determine the combined effects of these uncertainties.

13 **Video Games** A leading manufacturer of video games is about to introduce four new games. The accompanying table summarizes price and cost data. Combined fixed costs equal \$500,000. A marketing research study predicts that for each unit sold of Black Hole, 1.5 units of Haley's Comet, 3 units of Astervoids, and 4 units of PacPerson will be sold.

(a) How many product mix units must be sold to break even?

(b) How does this translate into sales of individual games?

	Game			
	PacPerson	**Astervoids**	**Haley's Comet**	**Black Hole**
Selling price	\$50	\$45	\$30	\$20
Variable cost/unit	20	15	10	10

14 A company produces three products which sell in a ratio of 4 units of product 2 and 2 units of product 3 for each unit sold of product 1. The following table summarizes price and cost data for the three products. If fixed costs are estimated at \$2.8 million, determine the number of units of each product needed to break even.

	Product		
	1	2	3
Selling price	\$40	\$32	\$55
Variable cost/unit	20	24	46

*15 A company is considering the purchase of a piece of equipment to be used to manufacture a new product. Four machines are being considered. The following table summarizes the purchase cost of each machine and the associated variable cost of production if the machine is used to produce the new product. Determine the ranges of output over which each machine would be the least costly alternative. Sketch the four cost functions.

	Purchase Cost	Variable Cost/Unit
Machine 1	\$ 80,000	\$10.00
Machine 2	120,000	9.00
Machine 3	200,000	7.50
Machine 4	300,000	5.50

❑ KEY TERMS AND CONCEPTS

break-even models 196
break-even point 196
contribution to fixed cost and profit 200
cost 178
demand function 184
depreciation (straight-line) 183
economies of scale 178
fixed cost 178
linear function 176
profit 180
profit margin (contribution) 200
revenue 179
salvage value 191
supply function 186
variable cost 178

❑ IMPORTANT FORMULAS

$$y = f(x) = a_1x + a_0 \tag{5.1}$$

$$y = f(x_1, x_2) = a_1x_1 + a_2x_2 + a_0 \tag{5.2}$$

$$y = f(x_1, x_2, \ldots, x_n) = a_1x_1 + a_2x_2 + \cdots + a_nx_n + a_0 \tag{5.3}$$

$$P(x) = R(x) - C(x) \tag{5.7}$$

$$\text{Profit margin} = p - v \qquad p > v \tag{5.11}$$

$$\left.\begin{aligned} R(x) &= C(x) & (5.10) \\ x_{BE} &= \frac{FC}{p-v} & (5.12) \end{aligned}\right\} \text{Break-even conditions}$$