Introduction to Mathematics

Week 2

Linear and Quadratic Equations and Inequalities

Unit 2.2

Solution of Second Degree Equations

Second-Degree Equations in One Variable

A second-degree equation involving one variable x would have the generalized form:

$$ax^2 + bx + c = 0$$

Where a, b and c are constant and are called coefficients, given that a ≠ 0.

Second-degree equations are also called **Quadratic Equations**.

Examples:

$$6x^2 - 2x + 1 = 0$$

$$3x^2 = 12$$

$$2x^2 - 1 = 5x + 9$$

Solving Quadratic Equations

A quadratic equation can have:

- **No real roots**
- **One real root**
- **Two real roots**

We will study two methods of solving quadratic equations:

- **Factoring method**
- **Quadratic formula**

Factoring Method

Solve the following equation using factoring method:

$$\boldsymbol{x^2 - 4x \ = 0}$$

Taking x common from the left side

$$\boldsymbol{x(x - 4) = 0}$$

The above statement means the two factors on the left are multiplied to yield the result 0.

So either **x = 0** or **x – 4 = 0**

This equation has two real roots:

x = 0 and ***x = 4***

Factoring Method

Solve the following equation using factoring method:

$$\boldsymbol{x^2 - 25 = 0}$$

One of the methods to solve this equation can be to break the expression on left using the formula:

a^2-b^2=(a+b)(a-b)

$$\boldsymbol{(x+5)(x-5) = 0}$$

So, here are two factors: x + 5 and x – 5, whose product is equating with 0,

Either **x + 5 = 0** or **x – 5 = 0**

Thus, the two real roots of this equation are:

x* = - *5 and ***x* = *5***

Both the above roots, when substituted in the given equation yield 0=0 which is true.

Factoring Method

Solve the following equation using factoring method:

$$\boldsymbol{x^2 + 25 = 0}$$

We can move +25 to the right side to find if the roots can be found using taking square root on both sides:

$$\boldsymbol{x^2 = -25}$$

Taking square root on both sides of the above equation:

$$\boldsymbol{\sqrt{x^2} = \sqrt{-25}}$$

$$\boldsymbol{x = \sqrt{-25}}$$

The right side contains negative sign in the square root, which means a root is not real but imaginary.
We state that the real roots do not exist.

Factoring Method

Solve the following equation using factoring method:

$$\boldsymbol{x^2 + 6x + 9 = 0}$$

Compare the above equation with the generalized form: $\boldsymbol{ax^2 + bx + c = 0}$

We find, a = 1, b = 6 and c = 9

To find the factors on the left, if a = 1, we directly find the factors of c (9) which can add or subtract up to b (+6)

$$\boldsymbol{x^2 + 3x + 3x + 9 = 0}$$

$$\boldsymbol{x(x + 3) + 3(x + 3) = 0}$$

This means $\boldsymbol{(x + 3)(x + 3) = 0}$

This equation has only one root which is x = -3

Alternate method: Convert the equation in $(a+b)^2$ formula.

Factoring Method

Solve the following equation using factoring method:

$$\boldsymbol{x^2 + 3x - 10 = 0}$$

We find, a = 1, b = 3 and c = -10

To find the factors on the left, if a = 1, we directly find the factors of c (10) which can add or subtract up to b (+3)

The suitable factors of 10 should be: +5 and -2,

$$\boldsymbol{x^2 + 5x - 2 - 10 = 0}$$

$$\boldsymbol{x(x + 5) - 2(x + 5) = 0}$$

This means $\boldsymbol{(x + 5)(x - 2) = 0}$

Either $\boldsymbol{(x + 5) = 0}$ ***; or*** $\boldsymbol{(x - 2) = 0}$

The two real roots are:

x = -5 *and* ***x = 2***

Factoring Method

Solve the following equation using factoring method:

$$4y^2 + 18y - 10 = 0$$

We find, a = 4, b = 18 and c = -10

Since, a ≠ 1, we have to multiply a and c (4 and 10) and then find the factors of this product (40) that would add or subtract up to b (18).

The suitable factors of 40 should be: +20 and -2,

$$4y^2 + 20y - 2y - 10 = 0$$

$$4y(y + 5) - 2(y + 5) = 0$$

$$(4y - 2)(y + 5) = 0$$

Either $(4y - 2) = 0$ ***; or*** $(y + 5) = 0$

One root is y = 1/2

Second root is y = -5

We can write the solution set as: {1/2, -5}

Quadratic Formula Method

Solve the following equation using quadratic formula:
$\boldsymbol{x^2 - 2x - 48 = 0}$

We find, a = 1, b = -2 and c = -48

The quadratic formula is:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(-48)}}{2(1)}$$

$$x = \frac{2 \pm \sqrt{4 + 192}}{2}$$

$$x = \frac{2 \pm \sqrt{196}}{2}$$

$$x = \frac{2 \pm \sqrt{196}}{2}$$

$$x = \frac{2 \pm 14}{2}$$

$$x = \frac{2 + 14}{2} \; or \;\; x = \frac{2 - 14}{2}$$

$$x = \frac{2 + 14}{2} \; or \;\; x = \frac{2 - 14}{2}$$

$$Either \;\; x = 8 \; or \;\; x = -6$$

Review Questions

Solve the following equation using factoring and quadratic formula:

(a) $x^2 + 3x + 2 = 0$
(b) $3x^2 - 2x + 5 = 0$
(c) $x^2 + 10x + 25 = 0$

Thank you