# Introduction to Mathematics

## Week 2

## Linear and Quadratic Equations and Inequalities

### Unit 2.3

### Solving First-Degree Inequalities

# Inequalities

**Inequalities** are used to express the condition that two quantities are unequal.
Such statements are expressed using **inequality symbols < and >.** For example:

| Inequality | Interpretation |
|---|---|
| (*a*) $3 < 5$ | "3 is less than 5" |
| (*b*) $x > 100$ | "the value of $x$ is greater than 100" |
| (*c*) $0 < y < 10$ | "the value of $y$ is greater than zero and less than 10" |

These are strict inequalities because the items being compared can never be equal to each other.

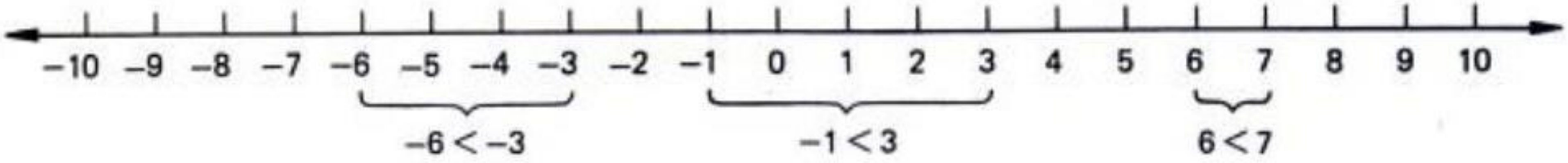
(a) Is **absolute inequality** because it is always true

(b) Is **conditional inequality** because it is true for a certain condition that is the numbers >100

(c) Is **double inequality**

# Inequalities and the Real Number Line

The real number line is very useful in understanding and solving inequalities




The following statements are equivalent to each other:

$a$ is less than $b$.
$b$ is greater than $a$.
$a < b$
$b > a$
$b - a > 0$
$a - b < 0$
$a$ lies to the left of $b$ on a real number line.
$b$ lies to the right of $a$ on a real number line.

$a < b$

# Inequalities

Another type of inequalities is non-strict inequality.

This includes equality sign along with less than or greater than signs:

$x \leq 2$

**≤** : This is called less than or equal to

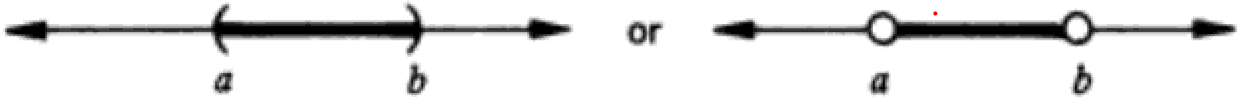
**≥** : This is called greater than or equal to

This means that the quantities being compared can also be equal to each other along with being less than or greater than.

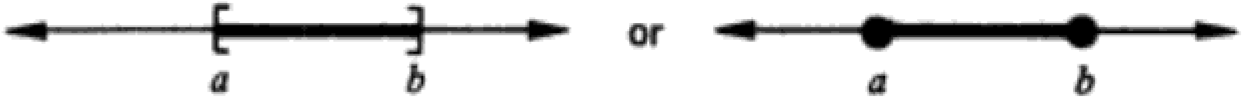
| Inequality | Interpretation |
|---|---|
| (*a*) $x + 3 \geq 15$ | "the quantity $(x + 3)$ is greater than *or* equal to 15" |
| (*b*) $y \leq x$ | "the value of $y$ is less than *or* equal to the value of $x$" |

# Inequalities and Interval Notation

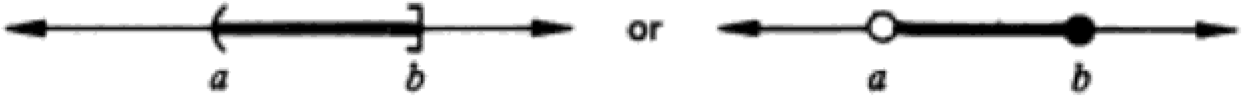
Open Interval: $(a, b) = \{x | a < x < b\}$

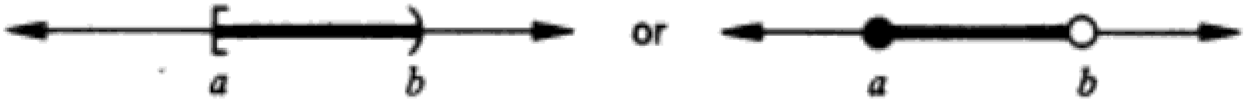



Closed Interval: $[a, b] = \{x | a \leq x \leq b\}$




Half-open Intervals: $(a, b] = \{x | a < x \leq b\}$ $[a, b) = \{x | a \leq x < b\}$




An **empty circle (○)** on the number line represents the **open interval**, means that endpoint is not included in the interval. Round bracket is used too.

A **solid circle (●)** means a **closed interval** and that endpoint is included in the interval. Square bracket is also used for this.

The above examples are finite intervals

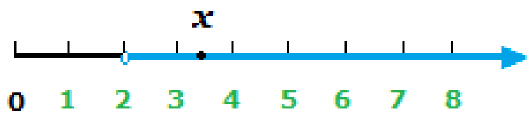
# Inequalities and Interval Notation

**Practice Questions:**

Sketch the following intervals on the number line:

**a) (-2, 1)**
**b) [1, 3]**
**c) [-3, 0)**

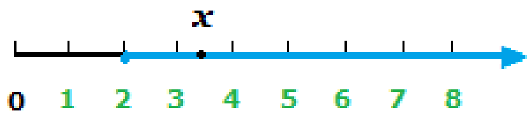
# Inequalities and Interval Notation




$x > 2$

$(2, \infty)$

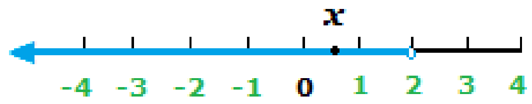
Infinite and Open




$x \geq 2$

$[2, \infty)$

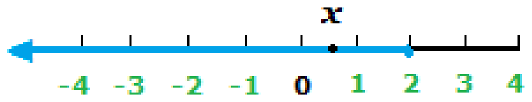
Infinite and Closed




$x < 2$

$(-\infty, 2)$

Infinite and Open




$x \leq 2$

$(-\infty, 2]$

Infinite and Closed

Infinity is always and open interval and expressed with round bracket

The above examples are infinite intervals

# Solving First-Degree Inequalities in One Variable

Inequalities are solved just like equations with just ONE DIFFERENCE:

When multiplying or dividing both sides on an inequality with a negative number, the sense of inequality MUST BE REVERSED.

Solve the inequality: $\mathbf{2x + 3 \geq -5}$

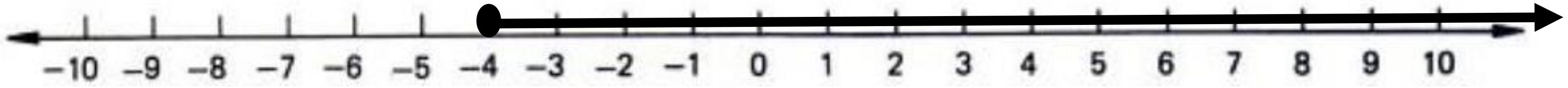
$\mathbf{2x \geq -5 - 3}$

$\mathbf{x \geq \frac{-8}{2}}$

$\mathbf{x \geq -4}$ **or** $[\mathbf{-4}, \infty)$

This solution means all values of x which are equal to or greater than -4 satisfy the given inequality.

Thus, the given inequality would always be true for all values of x equal to or greater than -4



# Solving First-Degree Inequalities in One Variable

Solve the inequality: $\mathbf{3x + 10 \le 5x - 4}$

*Adding 4 on both sides,*

$\mathbf{3x + 10 + 4 \le 5x - 4 + 4}$

$\mathbf{3x + 14 \le 5x}$

*Subtracting 5x from both sides*

$\mathbf{3x + 14 - 5x \le 5x - 5x}$

$\mathbf{-2x + 14 \le 0}$

*Subtracting 14 from both sides*

$\mathbf{-2x \le -14}$

*Dividing both sides by -2 (it would reverse the inequality)*

$\mathbf{x \ge 7}$ **or** $[7, \infty)$


This solution means all values of x which are equal to or greater than 7 satisfy the given inequality.

Thus, the given inequality would always be true for all values of x equal to or greater than 7

# Solving First-Degree Inequalities in One Variable

Solve the inequality: $\mathbf{6x - 10 \geq 6x + 4}$

*Adding 10 to both sides*

$\mathbf{6x \geq 6x + 14}$

*Subtracting 6x from both sides*

$\mathbf{0 \geq 14}$

***False Statement, No Solution***

Solve the inequality: $\mathbf{4x + 6 \geq 4x - 3}$

*Subtracting 6 from both sides*

$\mathbf{4x \geq 4x - 9}$

*Subtracting 4x from both sides*

$\mathbf{0 \geq -9}$

***The above statement is always true. This means all real values of x would make the inequality true.***

***So the solution set would be (- ∞,+ ∞)***

# Solving Double Inequalities

Solve the double inequality: $-2x + 1 \leq x \leq 6 - x$

**For double inequalities, we to solve each inequality separately first.**

**Left Inequality:**

$-2x + 1 \leq x$

Adding 2x to both sides

$1 \leq x + 2x$

$1 \leq 3x$

Dividing both sides by 3

$\frac{1}{3} \leq x \quad or \quad x \geq \frac{1}{3}$

**Right Inequality:**

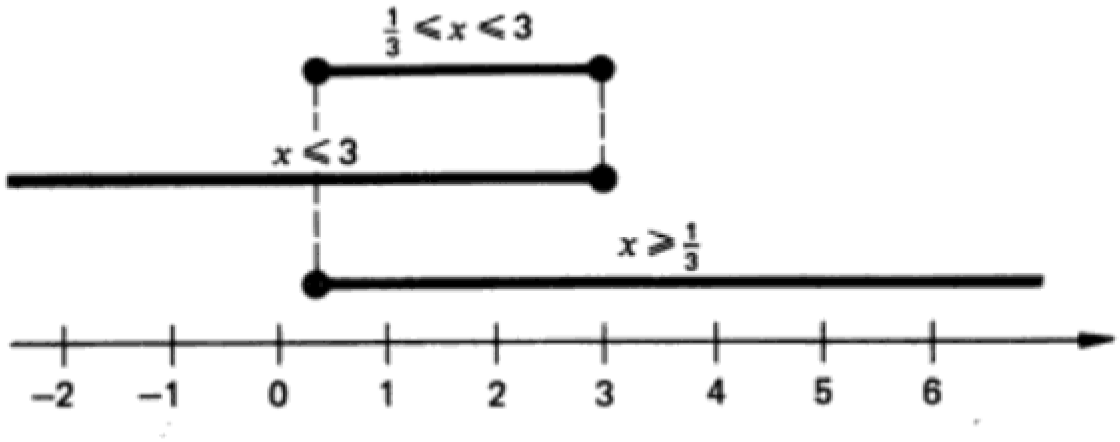
$x \leq 6 - x$

Adding x to both sides

$2x \leq 6$

Dividing both sides by 2

$x \leq 3$




Only those values of x would satisfy the given inequality which lie between 1/3 and 3, including both the endpoints, because only these values are common to the double inequality.

So the solution set would be:

$\frac{1}{3} \leq x \leq 3$ or $[\frac{1}{3}, 3]$

# Solving Double Inequalities

Solve the double inequality: $2x - 4 \leq x \leq 2x - 10$

**For double inequalities, we to solve each inequality separately first.**

**Left Inequality:**

$2x - 4 \leq x$

Subtracting x from both sides

$x - 4 \leq 0$

Adding 4 on both sides

$x \leq 4$

**Right Inequality:**

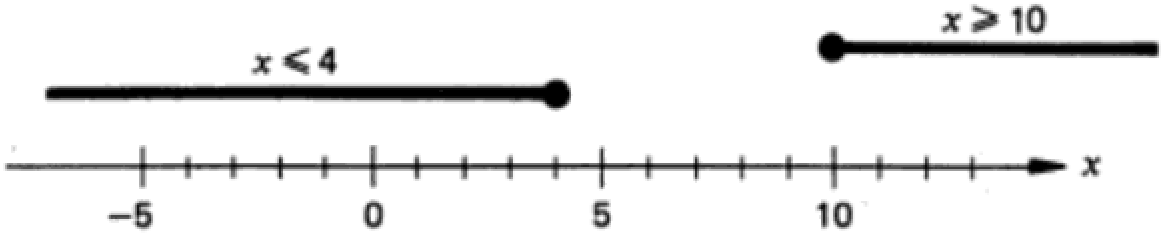
$x \leq 2x - 10$

Subtracting 2x from both sides

$-x \leq -10$

Multiplying both sides by -1 (would reverse inequality)

$x \geq 10$




Looking at the number line, we see that there are no common values in both the inequalities. So there is not solution to this inequality, means no real value would satisfy the given double inequality.

# Thank you