# Introduction to Mathematics

## System for Linear Equations

**Week 8**

# Introduction

## System of Linear Equations

- Two Variables(2x2)
- Three Variables(3x3)

# Learning Objectives

- Determine the nature of solution using slope intercept form.
- Determine the nature of solution using graphical method.
- Solve the system of equation in two variables.
- Solve the system of equation in three variables.

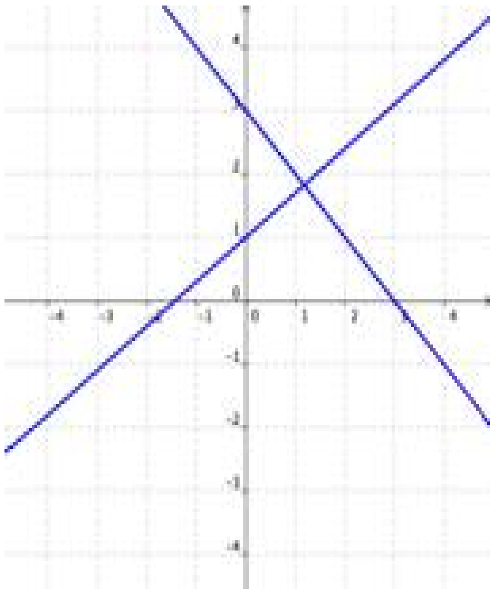
# Linear Functions

- A system of equations is a collection of two or more equations with a same set of unknowns. In solving a system of equations, we try to find values for each of the unknowns that will satisfy every equation in the system.

- The equations in the system can be linear or non-linear. This lecture reviews systems of linear equations.

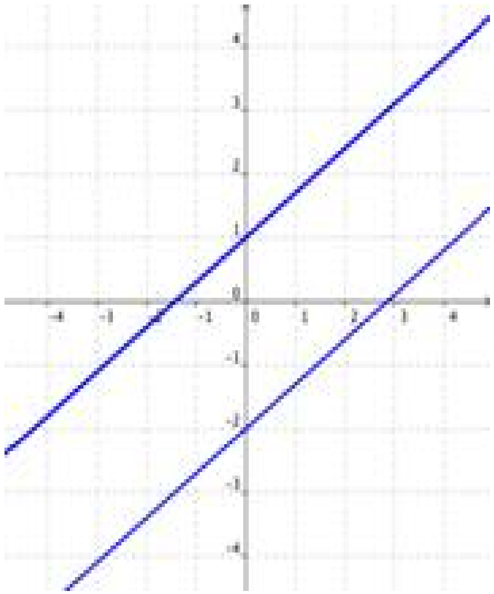
# Graphical Interpretation of solutions

### One Solution



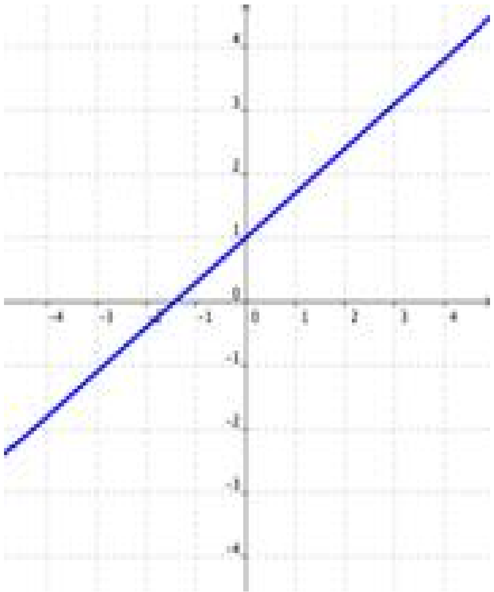
If the graphs of the equations intersect, then there is one solution that is true for both equations.

### No Solutions



If the graphs of the equations do not intersect (for example, if they are parallel), then there are no solutions that are true for both equations.

### Infinite Solutions



If the graphs of the equations are the same, then there are an infinite number of solutions that are true for both equations.

# Interpretation of solution using slope-intercept method

**From Figure 1**

**Unique Solution :**

If $m1 \neq m2$

**From Figure 2**

**No Solution :**

If $m1 = m2$, but $k1 \neq k2$

**From Figure 3**

**Infinite Solution :** ( Same Equations)

If $m1 = m2$, and $k1 = k2$

# Exercise 3.1 (Page 96)

**Q.3** $4x - 2y = 8$--------(1)

$x + 2y = 12$---------(2)

**Slope intercept form of equation (1) y=mx + k**

- Solution:

$$-2y = -4x + 8$$

$$y = -\frac{4}{-2}x + \frac{8}{-2}$$

$$y = 2x - 4$$

Slope = 2, K= - 4

Unique Solution m1 $\neq$ m2

**Slope intercept form of equation (2) y=mx + k**

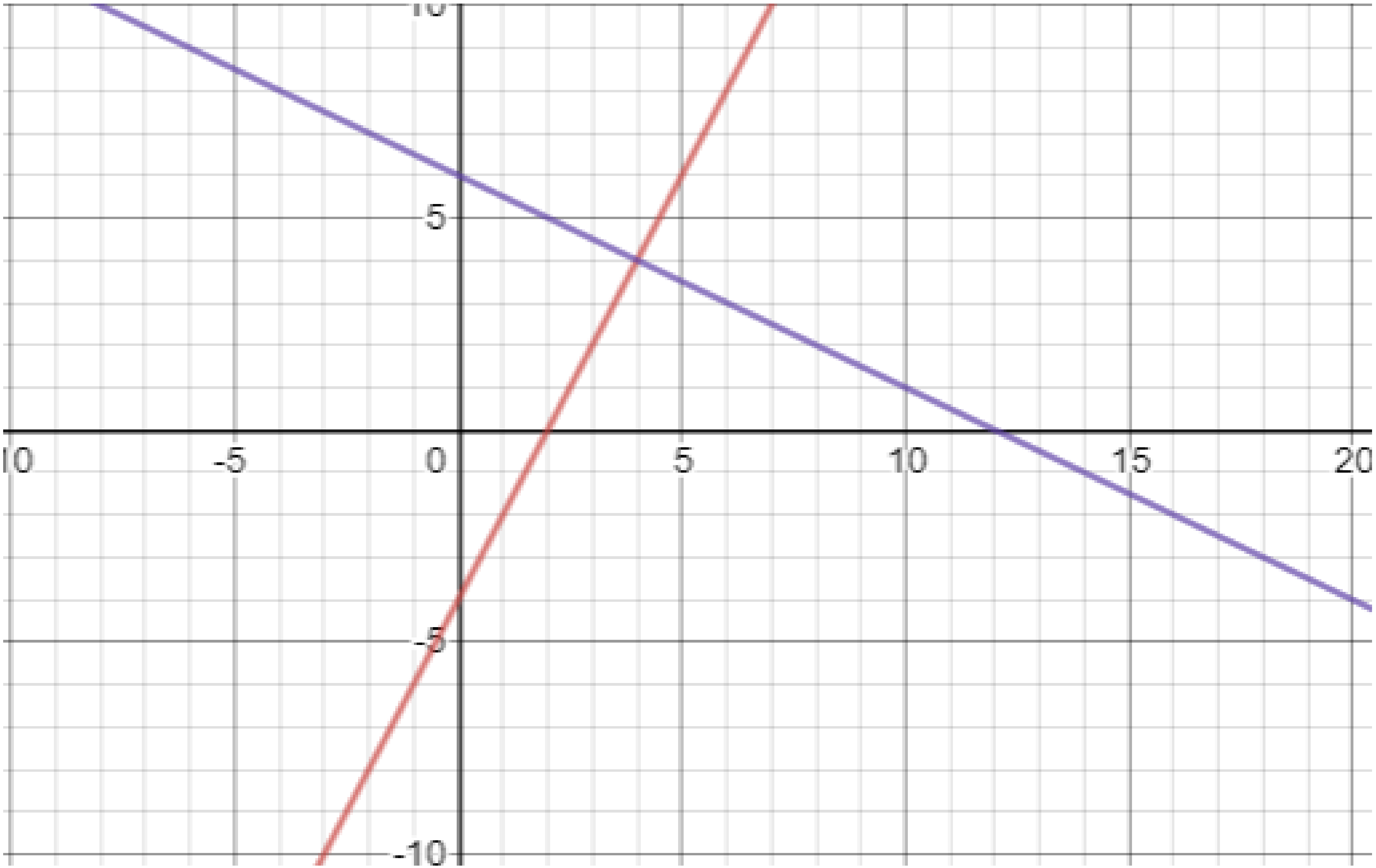
- Solution:

$$2y = -x + 12$$

$$y = -\frac{1}{2}x + \frac{12}{2}$$

$$y = -\frac{1}{2}x + 6$$

Slope -1/2, K= 6




**Unique solution**

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# Q.14 $x - 2y = 0$ --------(1)

$$-3x + 6y = 5\text{---------}(2)$$

**Slope intercept form of equation (1) y=mx + k**

- Solution:

$$-2y = -x$$

$$Y = -\frac{1}{-2}x$$

$$Y = \frac{1}{2}x$$

Slope =1/2, K= 0

**Slope intercept form of equation (2) y=mx + k**

- Solution:

$$6y = 3x + 5$$

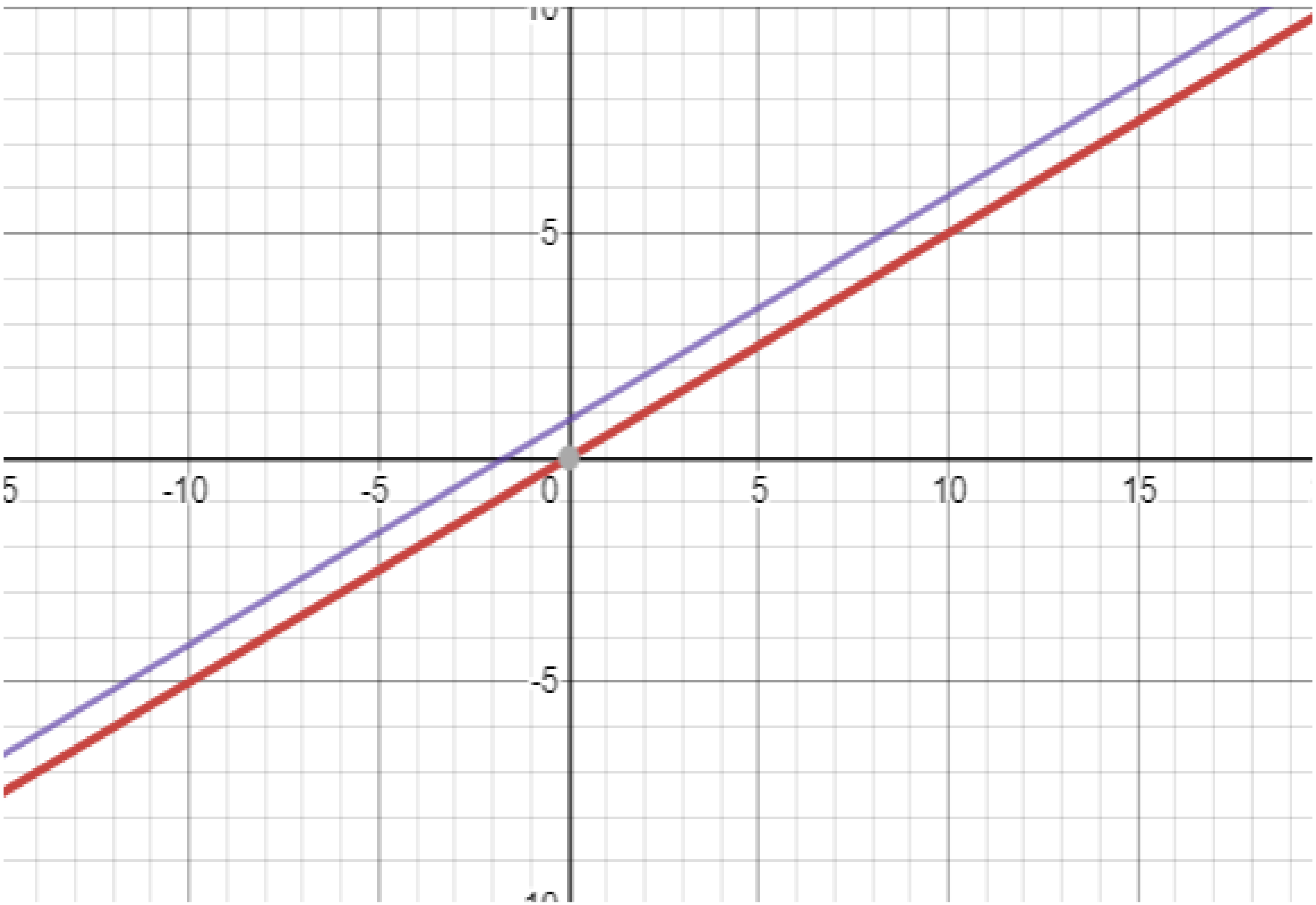
$$Y = \frac{3}{6}x + \frac{5}{6}$$

$$Y = \frac{1}{2}x + \frac{5}{6}$$

Slope 1/2, K= 5/ 6

**<u>No Solution :</u>**

Since m1= m2, but k1 =/= k2




**No solution**

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# Q.17 $4x - 2y = 10$ -------(1)

$$-2x + y = -5 ---------(2)$$

**Slope intercept form of equation (1) y=mx + k**

- Solution:

$$-2y = -4x + 10$$

$$Y = -\frac{4}{-2}x + \frac{10}{-2}$$

$$Y = 2x - 5$$

Slope =2, K= -5

<u>Infinitely many solutions</u>

Since m1= m2,and k1 = k2

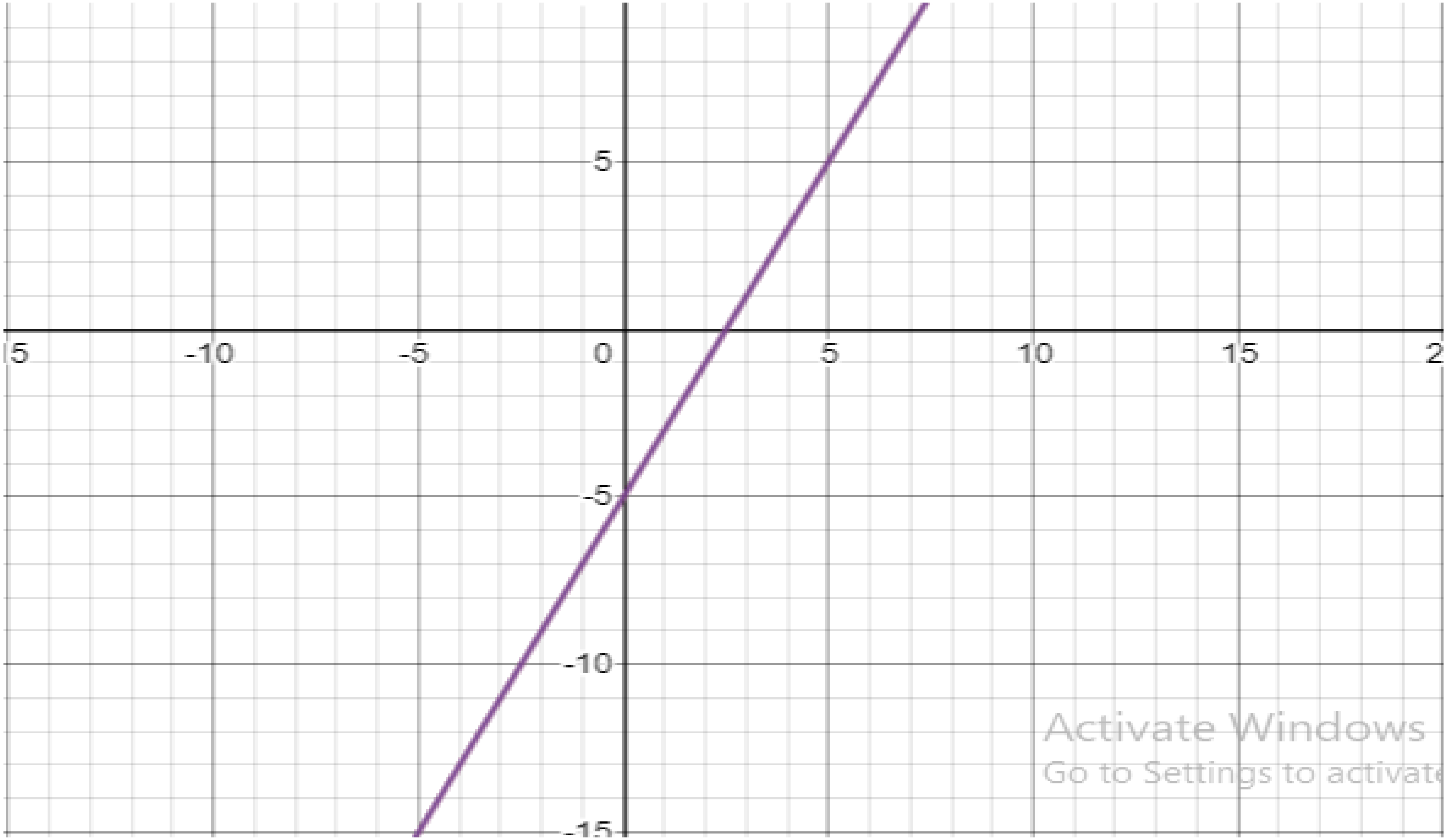
**Slope intercept form of equation (2) y=mx + k**

- Solution:

$$y = 2x - 5$$

$$y = 2x - 5$$

Slope =2, K= -5




**Infinitely Many Solutions**

**Q. 11.**

$$2x - 3y = -13$$
$$4x + 2y = -2$$

We would solve it through elimination,

Multiply eq 1 by 2 and subtract eq.2 from it

$$2(2x - 3y) = -(13)2$$
$$4x - 6y = -26$$
$$\underline{-\ 4x + 2y = -2}$$
$$-8y = -24$$

y = 3

- Put the value of y in Eq. 1

$$2x - 3y = -13$$
$$2x - 3(3) = -13$$
$$2x = -13 + 9$$
$$x = -\frac{4}{2}$$

x = -2

Solution set = ( -2 , 3) implies unique solution

## Q. 16.

$x - 2y = 4$---(1)

$-4x + 8y = -10$—(2)

We would solve it through elimination, Multiply equation 1 by 4 and subtract equation 2 from it

$$4(x - 2y) = 4(4)$$

$$4x - 8y = 16$$

$$\underline{-4x + 8y = -10}$$

$$0x + 0y = 6$$

$$0 = 6$$

**No Solution**

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- **Q. 13.**

$-x + 2y = -2$

$3x = 6y + 6$

First write the equation in standard form

$3x - 6y = 6$--------(2)

We would solve it through elimination, Multiply equation 1 by 3 and add equation 2 to it

$3(-x + 2y) = 3(-2)$

$-3x + 6y = -6$

$\underline{\quad 3x - 6y = 6-}$

$0x + 0y = 0$

$0 = 0$

**Infinitely Many Solutions**

# Practice Questions

1\. $\begin{cases} x + 3y = -9 \\ 4y = x + 16 \end{cases}$ **(12,1)**

2\. $\begin{cases} 2x + 3y = 3 \\ -6x - 12y = -11 \end{cases}$ $\left(\frac{1}{2}, \frac{2}{3}\right)$

3\. $\begin{cases} 3m + 6n = 12 \\ 4m + 5n = 28 \end{cases}$ (12, – 4)

4\. $\begin{cases} -3x + 6y = 18 \\ y = \frac{1}{2}x + 3 \end{cases}$ Many solutions Same Line!

5\. $\begin{cases} 5x - \frac{28}{3}y = 2 \\ -3x = 4y \end{cases}$ $\left(\frac{1}{6}, -\frac{1}{8}\right)$

6\. $\begin{cases} \frac{1}{2}x - \frac{1}{3}y = \frac{1}{6} \\ 6x - 4y = 1 \end{cases}$ NO solution Parallel Lines!

# Exercise 3.4 (Page 118)

**Q.1** A company produces three products which must be processed through three departments. The following table summarizes the labor hours required per unit each department. The monthly labor hour capacities for the three departments are1800, 1450, and 1900 hours. Determine whether there is a combination of the three products which could be produced monthly so as to consume full monthly labor availabilities of all departments.

| Department | Product 1 | Product 2 | Product 3 |
|---|---|---|---|
| A | 3 | 2 | 5 |
| B | 4 | 1 | 3 |
| C | 2 | 4 | 1 |

# Exercise 3.4

Solution.1

a= no of units of product 1

b= no of units of product 2

c= no of units of product 3

$3a + 2b + 5c = 1800$

$4a + b + 3c = 1450$

$2a + 4b + c = 1900$

**Solution**

a = 200 units

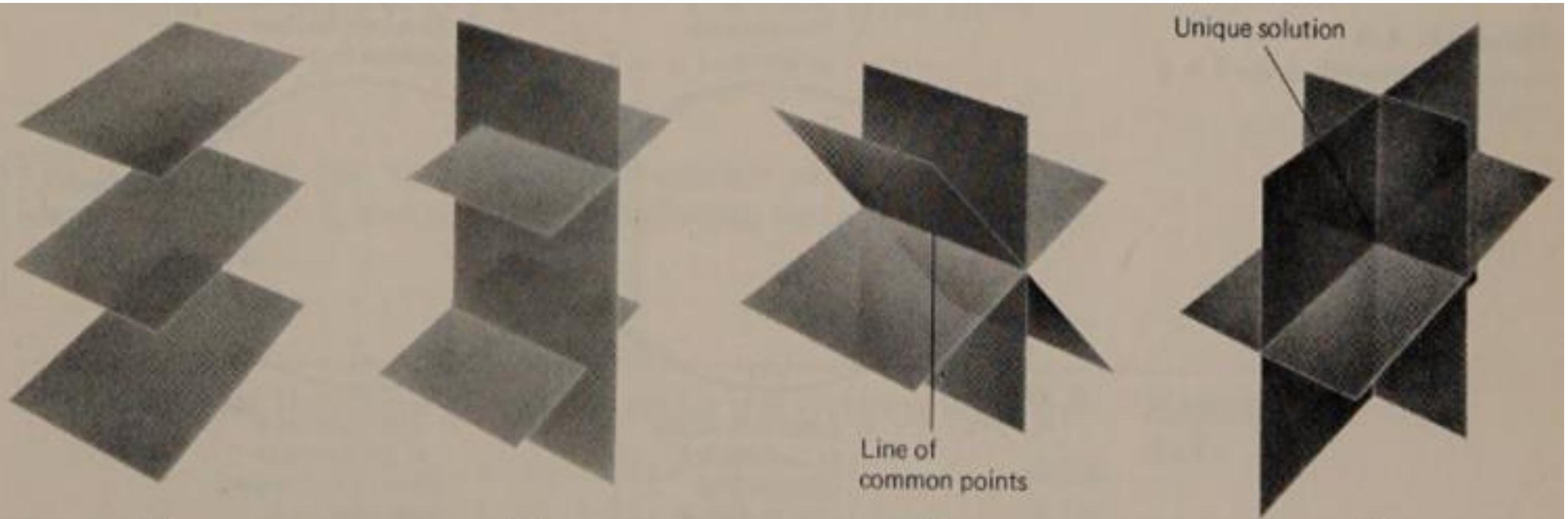
b = 350 units

c= 100 units

# Solution of 3x3 System of Equations

# 3x3 System of Equations

- A 3x3 system of equations will consist of 3 equations with 3 variables, for example, x,y,z.
- With three variables, each equation graphs as a plane in three dimensions (as opposed to a line for an equation with two variables).
- It is possible to have all 3 type of solution sets
- Unique Solution Set
- No Solution
- Infinite Number of Solutions.




(a)
Parallel planes
(no solution)

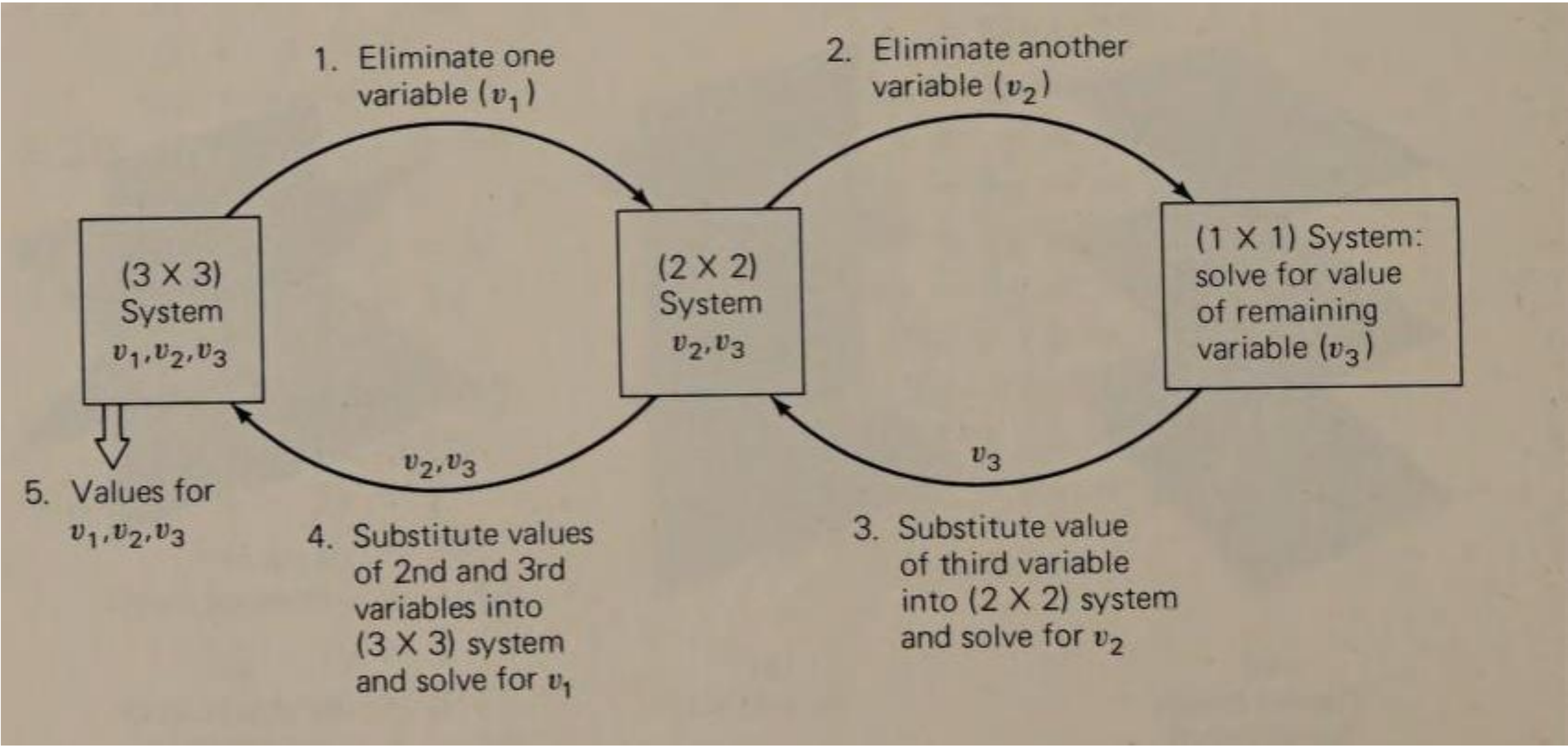
(b)
No solution

(c)
Infinite number
of solutions

(d)
Unique
solution

# Solution By Elimination Method

- The Elimination Method for 3 x 3 system is similar to that of a 2 x 2 system.
- We'll start with a 3 x 3 system and reduce it to an equivalent system with 2 variables and 2 equations (a 2 x 2 system).
- Then we'll eliminate a second variable to get a 1x1 system and get the value of first variable.
- This value is substituted into the 2 x 2 system and finally the 3 x 3 system to get the value of the other two variables.
- If, during any phase of the elimination an identity equation comes up, then there will be infinite solutions.


1. Eliminate one variable ($v_1$)
2. Eliminate another variable ($v_2$)
(3 X 3) System $v_1, v_2, v_3$
(2 X 2) System $v_2, v_3$
(1 X 1) System: solve for value of remaining variable ($v_3$)
$v_2, v_3$
$v_3$
5. Values for $v_1, v_2, v_3$
4. Substitute values of 2nd and 3rd variables into (3 X 3) system and solve for $v_1$
3. Substitute value of third variable into (2 X 2) system and solve for $v_2$

# Example

- Determine the Solution set for the following system of equations

$$x_1 + x_2 + x_3 = 6 \qquad \text{(Eq. 1)}$$
$$2x_1 - x_2 + 3x_3 = 4 \qquad \text{(Eq. 2)}$$
$$4x_1 + 5x_2 - 10x_3 = 13 \qquad \text{(Eq. 3)}$$

Let's try to eliminate variable $x_2$ first. We can see that if Eq.1 and Eq. 2 are added, the positive and negative $x_2$ will add to give zero, hence eliminating the variable. So, adding Eq. 1 and Eq. 2:

$$x_1 + x_2 + x_3 = 6$$
$$\underline{2x_1 - x_2 + 3x_3 = 4}$$
$$3x_1 \qquad + 4x_3 = 10 \qquad \text{Eq. 4}$$

We need another equation. Multiplying Eq. 2 with +5 and adding to Eq. 3 yields a new equation. This is shown in the next slide

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- $$10x_1 - 5x_2 + 15x_3 = 20 \quad \text{(Eq. 2 multiplied by 5)}$$
  $$\underline{4x_1 + 5x_2 - 10x_3 = 13} \quad \text{(Eq. 3)}$$
  $$14x_1 \qquad + 5x_3 = 33 \quad \text{(Eq. 5)}$$

Since $x_2$ has been eliminated, Eq. 4 and Eq. 5 constitute an equivalent 2 x 2 system, which is shown below.

$$3x_1 \quad + 4x_3 = 10 \qquad \text{(Eq. 4)}$$
$$14x_1 + 5x_3 = 33 \qquad \text{(Eq. 5)}$$

Using elimination methods discussed for the 2 x 2 system in the previous lessons, we can eliminate $x_3$ by multiplying Eq. 4 with +5 and Eq. 5 with -4 respectively, and adding the two results.

$$15x_1 \quad + 20x_3 = 50 \qquad \text{(Eq 4. multiplied by 5)}$$
$$\underline{-56x_1 - 20x_3 = -132} \quad \text{(Eq. 5 multiplied by -4)}$$
$$-41x_1 \qquad\qquad = -82$$
$$\boldsymbol{x_1 = 2}$$

- This value of $x_1$ is substituted into Eq. 4 to get the value of $x_3$

$$3x_1 + 4x_3 = 10$$

$$3(2) + 4x_3 = 10$$

$$\boldsymbol{x_3 = 1}$$

Now, substituting value of $x_1$ and $x_3$ into Eq. 1

$$2 + x_2 + 1 = 6$$

$$\boldsymbol{x_2 = 3}$$

You can further verify your answer by substituting the values of $x_1, x_2, x_3$ into Eq.2 and 3.

Further examples :

https://mathbitsnotebook.com/Algebra2/Polynomials/POSystems.html

# Practice Question

$$2x + 5y + 2z = -38$$
$$3x - 2y + 4z = 17$$
$$-6x + y - 7z = -12$$

For help in solution:
http://tutorial.math.lamar.edu/Solutions/Alg/SystemsThreeVrble/Prob1.aspx

# Learning Material

- https://www.khanacademy.org/math/algebra-home/alg-system-of-equations/alg-systems-with-three-variables/v/systems-of-three-variables?modal=1
- http://sites.science.oregonstate.edu/math/home/programs/undergrad/CalculusQuestStudyGuides/vcalc/gauss/gauss.html
- https://www.purplemath.com/modules/systlin6.htm

# Thank you