Introduction to Mathematics

Annuities

Learning Objectives

Understanding of Annuities & their Future Values
Understanding of Annuities & their Present Values

Learning Outcomes

Students will be able to find Annuities & their Future Values
Students will be able to find Annuities & their Present Values

Annuities and their Future value

An annuity is a series of periodic payments, e.g. monthly car payment, regular deposits to savings accounts, insurance payments.

We assume that an annuity involves a series of equal payments.

All payments are made at the end of a compounding period, e.g. a series of payments R, each of which equals $\$1000$ at the end of each period, earn full interest in the next period and does not qualify for the interest in the previous period.

Example

A person plans to deposit \$ 1000 in a tax-exempt savings plan at the end of this year and an equal sum at the end of each year following year. If interest is expected to be earned 6 % per year compounded annually, to what sum will the investment grow at the time of the 4^{th} deposit?

Solution: We can determine the value of S_4 by applying the compound amount formula to each deposit, determining its value at the time of the 4-th deposit. These compound amounts may be summed for the four deposits to determine S_4.

First deposit earns interest for 3 years, 4th deposit earns no interest. The interest earned on first 3 deposits is $ 374.62. Thus $S_4 = 4374.62$.

Formula

The procedure used in the above example is not practical when dealing with large number of payments. In general to compute the sum S_n, we use the following formula.

$$S_n = \frac{R[(1+i)^n - 1]}{i} = RS_{\overline{n}i}$$

The special symbol $S_{\overline{\mathrm{n}}\mathrm{i}}$, which is pronounced as "S sub n angle i", is frequently used to abbreviate the series compound amount factor $\frac{[(1+i)^n - 1]}{i}$.

Now we solve the last example by this formula. We have

$$S_4 = 1000\, S_{\overline{4}\,(.06)} = 4374.62,$$

by using the table III at the page T-10 in book.

Example

A boy plans to deposit \$ 50 in a savings account for the next 6 years. Interest is earned at the rate of 8% per year compounded quarterly. What should her account balance be 6 years from now? How much interest will he earn?

Solution: Here $R = 50, i = \frac{.08}{4} = 0.02, n = 6 \times 4 = 24$. We need to find

$$S_4 = 50 \,.\, S_{\overline{24}(0.02)} = 50(30.421) = \$1521.09.$$

Annuities and their Present value

There are applications which relate an annuity to its present value equivalent. e.g. we may be interested in knowing the size of a deposit which will generate a series of payments (an annuity) for college, retirement years, or given that a loan has been made, we may be interested in knowing the series of payments (annuity) necessary to repay the loan with interest.

The present value of an annuity is an amount of money today which is equivalent to a series of equal payments in the future. An assumption is that: the final withdrawal would deplete the investment completely.

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Example

A person recently won a state lottery. The terms of the lottery are that the winner will receive annual payments of $ 20,000 at the end of this year and each of the following 3 years. If the winner could invest money today at the rate of 8 % per year compounded annually, what is the present value of the four payments?

Solution: If *A* defines the present value of the annuity, we might determine the value of *A* by computing the present value of each 20000 payment. Here $S = 20000, i = 0.08$ then using $P = \frac{S}{(1+i)^n}$

For $n = 1 \; => \; P = \$18518.6$

For $n = 2 \; => \; P = \$17146.8$

For $n = 3 \; => \; P = \$15876.6$

For $n = 4 \; => \; P = \$14700.6$

Calculating the sum we get $A = \$66242.6$.

As with the future value of an annuity, we can find the general formula for the present value of an annuity. In case of large number of payments the method of example is not practical.

Formula

If

$R =$ Amount of an annuity

$i =$ Interest rate per compounding period

$n =$ Number of annuity payments

$A =$ Present value of an annuity

then

$$A = R\left[\frac{(1+i)^n - 1}{i(i+1)^n}\right] = Ra_{\bar{n}i}$$

The above equation is used to compute the present value A of an annuity consisting of n equal payments, each made at the end of n periods

Review Question

Referring to the below example, Find the value of grown investment at the time of 6th deposit ?

Example

A person plans to deposit $ 1000 in a tax-exempt savings plan at the end of this year and an equal sum at the end of each year following year. If interest is expected to be earned 6 % per year compounded annually, to what sum will the investment grow at the time of the 4^{th} deposit?

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Thank you