Introduction to Mathematics

Week 3

Formation of Equation

- **Unit 3.4**

Formation of Equation of Line by Slope, Points & Intercepts

Topics In This Unit

- **Formation of Equation of Line using the Slope and Intercept**
- **Formation of Equation of Line using Slope and one Point**
- **Formation of Equation of Line using 2 points**

General Equation of a Line

The general equation (or the standard form) of the linear equation is

$$ax + by = c$$

It can be arranged in such a way so as to give us the general equation of a straight line

$$y = mx + c$$

Where x and y are the variables,

m is the gradient (or slope) of the line and

c is the y intercept value.

This is also called the slope intercept form of the linear equation.

For example, the equation 5x + 2y = 7 is a linear equation.

We can write it in the form of equation of straight line.

We can do it by making y the subject of the equation.

$$5x + 2y = 7$$

$$2y = 7 - 5x$$

$$y = \frac{7 - 5x}{2}$$

$$y = -\frac{5}{2}x + 7$$

The gradient "m" here is -5/2 and the y intercept is at y = 7

Word Problem Example

A police department estimates that the total cost C of owning and operating a patrol car can be estimated by the linear equation

$$C = 0.40x + 18,000$$

where C = *total cost, dollars*

and x = *number of miles driven*

This equation is in slope-intercept form with a slope of 0.40 and C intercept (which is equivalent to the y intercept) of (0, 18,000). The slope suggests that total cost increases at a rate of $0.40 for each additional mile driven. The C intercept indicates a cost of $18,000 if the car is driven zero miles.

Determining the Equation of a Straight Line

There are different ways to obtain an equation for the straight line. Depending on the information available, you can use these 3 methods to get the equation for a straight line in the form of y = mx + c

- Using Slope and Intercept
- Using Slope and Point
- Using Point and Point (2 Points)

Using Slope & Intercept

Determine the equation of the straight line which has a slope of -5 and a y intercept of $(0, 15)$.

SOLUTION

Substituting values of $m = -5$ and $k = 15$ into Eq. (2.12) gives

$$y = -5x + 15$$

Restated in the form of Eq. (2.1), an equivalent form of this equation is

$$5x + y = 15$$

Using Slope and Point

If given the slope and one point which lies on a line, we can substitute the known slope m and coordinates of the given point into Eq. (2.12) and solve for k.

Given that the slope of a straight line is -2 and one point lying on the line is $(2, 8)$ we can substitute these values into Eq. (2.12), yielding

$$8 = (-2)(2) + k$$

or

$$12 = k$$

Since $m = -2$ and $k = 12$, the slope-intercept equation is

$$y = -2x + 12$$

And, as before, we can rewrite this equation in the equivalent form

$$2x + y = 12$$

Using Two Points

A more likely situation is that some data points which lie on a line have been gathered and we wish to determine the equation of the line. Assume that we are given the coordinates of two points which lie on a straight line. We can determine the slope of the line by using the two-point formula [Eq. (2.10)]. As soon as we know the slope, the y intercept can be determined by using *either* of the two data points and proceeding as we did in the previous section.

To determine the equation of the straight line which passes through $(-4, 2)$ and the origin, we substitute the coordinates into the two-point formula, resulting in

$$m = \frac{0 - 2}{0 - (-4)}$$

$$= \frac{-2}{4} = -\frac{1}{2}$$

Substituting $m = -\frac{1}{2}$ and the coordinates $(-4, 2)$ into Eq. (2.13) yields

$$y - 2 = (-\tfrac{1}{2})[x - (-4)]$$

$$y - 2 = -\tfrac{1}{2}x - 2$$

$$y = -\tfrac{1}{2}x$$

Thus, the slope-intercept form of the equation is

$$y = -\tfrac{1}{2}x$$

Parallel and Perpendicular Lines

- Parallel Lines have the same slope
- For perpendicular lines, the product of the slopes is -1.

Review Questions

Attempt Section 2.1, 2.2, 2.3 and 2.4 from the book.

Helping Material

- https://www.khanacademy.org/math/algebra/x2f8bb11595b61c86:forms-of-linear-equations/x2f8bb11595b61c86:writing-slope-intercept-equations/v/equation-of-a-line-3
- https://www.youtube.com/watch?v=4il4haYASys
- https://www.youtube.com/watch?v=IIy4N2lAkDs
- https://www.youtube.com/watch?v=acsR7w0I__w

Thank you