If 3,000 masks of each type were supplied during a given week,

$$\begin{aligned} f(3{,}000,\ 3{,}000) &= 200(3{,}000) + 250(3{,}000) - 75{,}000 \\ &= 600{,}000 + 750{,}000 - 75{,}000 \\ &= \$1{,}275{,}000 \end{aligned}$$

□

POINT FOR THOUGHT & DISCUSSION

Examine the sales function for the condition $x_1 + x_2 > 5{,}000$. Although the \$25 bonus applies only to units in excess of 5,000, it appears that all units receive the \$25 bonus. Also, where is the \$50,000 bonus in the function? What does the $-75{,}000$ represent? The rearrangement and simplification of this function appear to distort the logic of the relationships. Clarify the logic for us!

For the remainder of this chapter, the functions discussed will contain one independent variable. Later in the book we will return to functions involving more than one independent variable.

Section 4.1 Follow-up Exercises

In Exercises 1–16, determine $f(0)$, $f(-2)$, and $f(a+b)$.

1 $f(x) = 5x - 10$
3 $f(x) = -x + 4$
5 $f(x) = mx + b$
7 $f(x) = x^2 - 9$
9 $f(t) = t^2 + t - 5$
11 $f(u) = u^3 - 10$
13 $f(n) = n^4$
15 $f(x) = x^3 - 2x + 4$
2 $f(x) = 3x + 5$
4 $f(x) = -x/2$
6 $f(x) = mx$
8 $f(x) = -x^2 + 2x$
10 $f(r) = tr^2 - ur + v$
12 $f(u) = -2u^3 + 5u$
14 $f(t) = 100$
16 $f(x) = 25 - x^2/2$

In Exercises 17–40, determine the domain of the function.

17 $f(x) = -10$
19 $f(x) = 5x - 10$
21 $f(x) = mx + b$
23 $f(x) = 25 - x^2$
25 $f(x) = \sqrt{x+4}$
27 $f(t) = \sqrt{-t-8}$
29 $f(r) = \sqrt{r^2+9}$
31 $f(x) = 10/(4 - x)$
33 $f(u) = (3u - 5)/(-u^2 + 2u + 5)$
35 $f(x) = \sqrt{2.5x - 20}/(x^3 + 2x^2 - 15x)$
37 $g(h) = \sqrt{h^2 - 4}/(h^3 + h^2 - 6h)$
39 $f(x) = \sqrt{x^2 + 8x + 15}$
18 $f(x) = 25$
20 $f(x) = -x + 3$
22 $f(x) = -ax$
24 $f(x) = x^2 - 4$
26 $f(x) = \sqrt{-2x + 25}$
28 $f(t) = \sqrt{9 - t^2}$
30 $f(r) = \sqrt{25 - r^2}$
32 $f(x) = (x - 4)/(x^2 - 6x - 16)$
34 $f(t) = \sqrt{-t - 10}/(-3t^3 + 5t^2 + 10t)$
36 $h(v) = \sqrt{10 - v/3}/(v^5 - 81v)$
38 $f(x) = \sqrt{x^2 - x - 6}$
40 $h(r) = \sqrt{r^2 - 16}$

0 5 5

41 The function $C(x) = 15x + 80{,}000$ expresses the total cost $C(x)$ (in dollars) of manufacturing x units of a product. If the maximum number of units which can be produced equals 50,000, state the restricted domain and range for this cost function.

42 **Demand Function** The function $q = f(p) = 280{,}000 - 35p$ is a *demand function* which expresses the quantity demanded of a product q as a function of the price charged for the product p, stated in dollars. Determine the restricted domain and range for this function.

43 **Demand Function** The function $q = f(p) = 180{,}000 - 30p$ is a demand function which expresses the quantity demanded of a product q as a function of the price charged for the product p, stated in dollars. Determine the restricted domain and range for this function.

44 **Insurance Premiums** An insurance company has a simplified method for determining the annual premium for a term life insurance policy. A flat annual fee of \$150 is charged for all policies plus \$2.50 for each thousand dollars of the amount of the policy. For example, a \$20,000 policy would cost \$150 for the fixed fee plus \$50, which corresponds to the face value of the policy. If p equals the annual premium in dollars and x equals the face value of the policy (stated in thousands of dollars), determine the function which can be used to compute annual premiums.

45 In Exercise 44, assume that the smallest policy which will be issued is a \$10,000 policy and the largest is a \$500,000 policy. Determine the restricted domain and range for the function found in Exercise 44.

46 The local electric company uses the following method for computing monthly electric bills for one class of customers. A monthly service charge of \$5 is assessed for each customer. In addition, the company charges \$0.095 per kilowatt hour. If c equals the monthly charge stated in dollars and k equals the number of kilowatt hours used during a month:

(*a*) Determine the function which expresses a customer's monthly charge as a function of the number of kilowatt hours.

(*b*) Use this function to compute the monthly charge for a customer who uses 850 kilowatt hours.

47 Referring to Exercise 46, assume that the method for computing customer bills applies for customers who use between 200 and 1,500 kilowatt hours per month. Determine the restricted domain and range for the function in that exercise.

48 **Auto Leasing** A car rental agency leases automobiles at a rate of \$15 per day plus \$0.08 per mile driven. If y equals the cost in dollars of renting a car for one day and x equals the number of miles driven in one day:

(*a*) Determine the function $y = f(x)$ which expresses the daily cost of renting a car.

(*b*) What is $f(300)$? What does $f(300)$ represent?

(*c*) Comment on the restricted domain of this function.

49 In manufacturing a product, a firm incurs costs of two types. Fixed annual costs of \$250,000 are incurred regardless of the number of units produced. In addition, each unit produced costs the firm \$6. If C equals total annual cost in dollars and x equals the number of units produced during a year:

(*a*) Determine the function $C = f(x)$ which expresses annual cost.

(*b*) What is $f(200{,}000)$? What does $f(200{,}000)$ represent?

(*c*) State the restricted domain and restricted range of the function if maximum production capacity is 300,000 units per year.

50 **Wage Incentive Plan** A producer of a perishable product offers a wage incentive to drivers of its trucks. A standard delivery takes an average of 20 hours. Drivers are paid at the rate of \$10 per hour up to a maximum of 20 hours. There is an incentive for drivers to make the trip in less (but not too much less!) than 20 hours. For each hour under 20, the

$y = \begin{cases} 200 \\ 200 - 2(x - 60) \end{cases}$

hourly wage increases by \$2.50. (The \$2.50-per-hour increase in wages applies for fractions of hours. That is, if a trip takes 19.5 hours, the hourly wage increases by 0.5 × \$2.50, or \$1.25.) Determine the function $w = f(n)$, where w equals the hourly wage (in dollars) and n equals the number of hours to complete the trip.

51 Membership Drive A small health club is trying to stimulate new memberships. For a limited time, the normal annual fee of \$300 per year will be reduced to \$200. As an additional incentive, for each new member in excess of 60, the annual charge for each new member will be further reduced by \$2. Determine the function $p = f(n)$, where p equals the membership fee for new members and n equals the number of new members.

52 Given $f(x, y) = x^2 - 6xy + 2y^2$, determine (*a*) $f(0, 0)$, (*b*) $f(-1, 2)$, and (*c*) $f(5, 10)$.

53 Given $g(u, v) = 2u^2 + 5uv + v^3$, determine (*a*) $g(0, 0)$, (*b*) $g(-5, 2)$, (*c*) $g(5, 10)$, and (*d*) $g(x, y)$.

54 Given $v(h, g) = h^2/2 - 5hg + g^2 + 10$, determine (*a*) $v(0, 0)$, (*b*) $v(4, 2)$, and (*c*) $v(-2, -5)$.

55 Given $f(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3)^2$, determine (*a*) $f(1, 1, 1)$, (*b*) $f(2, 3, -1)$, and (*c*) $f(2, 0, -4)$.

56 Given $f(x_1, x_2, x_3) = x_1^3 + 2x_1^2x_2 - 3x_2x_3 - 10$, determine (*a*) $f(0, 2, -3)$, (*b*) $f(-2, 1, 5)$, and (*c*) $f(3, 0, -5)$.

57 Given $f(x_1, x_2, x_3, x_4) = 2x_1x_2 - 5x_2x_4 + x_1x_3x_4$, determine (*a*) $f(0, 1, 0, 1)$ and (*b*) $f(2, 1, 2, -3)$.

58 Given $f(a, b, c, d) = 4ab - a^2bd + 2c^2d$, determine (*a*) $f(1, 2, 3, 4)$ and (*b*) $f(2, 0, 1, 5)$.

59 Given $f(x_1, x_2, x_3, x_4) = x_1x_2 - 5x_3x_4$, determine (*a*) $f(1, 10, 4, -5)$, (*b*) $f(2, 2, 2, 2)$, and (*c*) $f(a, b, c, d)$.

60 A company estimates that the number of units it sells each year is a function of the expenditures on radio and TV advertising. The specific function is

$$z = f(x, y) = 20{,}000x + 40{,}000y - 20x^2 - 30y^2 - 10xy$$

where z equals the number of units sold annually, x equals the amount spent for TV advertising, and y equals the amount spent for radio advertising (both in thousands of dollars).

(*a*) Determine the expected annual sales if \$50,000 is spent on TV advertising and \$20,000 is spent on radio advertising.

(*b*) What are expected sales if \$80,000 and \$100,000 are spent, respectively?

61 Pricing Model A manufacturer sells two related products, the demand for which is characterized by the following two demand functions:

$$q_1 = f_1(p_1, p_2) = 250 - 4p_1 - p_2$$

$$q_2 = f_2(p_1, p_2) = 200 - p_1 - 3p_2$$

where p_j equals the price (in dollars) of product j and q_j equals the demand (in thousands of units) for product j.

(*a*) How many units are expected to be demanded of each product if \$20/unit is charged for product 1 and \$40/unit is charged for product 2?

(*b*) How many units are expected if the unit prices are \$40 and \$30, respectively?

62 Family Shelter A women's resource center which provides housing for women and children who come from abusive homes is undertaking a grass roots fund-raising effort within the community. One component of the campaign is the sale of two types of candy bars. The profit from the candy is \$0.50 and \$0.75 per bar, respectively, for the two types. The supplier of the candy has offered an incentive if the total number of candy bars sold exceeds 2,000. For each bar over 2,000, an additional \$0.25 will be earned by the center. Determine the function $P = f(x_1, x_2)$, where P equals the total profit in dollars and x_j equals the number of bars sold of type j. If 750 and 900 bars, respectively, are sold, what is the profit expected to equal? If 1,500 and 2,250, respectively, are sold?

63 A salesperson is paid a base weekly salary and earns a commission on each unit sold of three different products. The base salary is \$60 and the commissions per unit sold are \$2.50, \$4.00, and \$3.00, respectively. If S equals the salesperson's weekly salary and x_j equals the number of units sold of product j during a given week, determine the salary function $S = f(x_1, x_2, x_3)$. What weekly salary would be earned if the salesperson sells 20, 35, and 15 units, respectively, of the three products?

64 In the previous exercise, assume that the salesperson can earn a bonus if combined sales for the three products exceeds 50 units for the week. The bonus equals \$25 plus \$1.25 additional commission for all units sold in excess of 50. Determine the weekly salary function $S = f(x_1, x_2, x_3)$. What salary would be earned for the 20, 35, and 15 units sold in the previous exercise?

4.2 TYPES OF FUNCTIONS

Functions can be classified according to their structural characteristics. A discussion of some of the more common functions follows. A more thorough treatment of these functions is provided in Chaps. 5 and 6. *Exponential* and *logarithmic* functions will be discussed in Chap. 7.

Constant Functions

A ***constant function*** has the general form

$$\boxed{y = f(x) = a_0} \tag{4.3}$$

where a_0 is real. For example, the function

$$y = f(x) = 20$$

is a constant function. Regardless of the value of x, the range consists of the single value 20. That is,

$$f(-10) = 20$$

$$f(1{,}000) = 20$$

$$f(a + b) = 20$$

Done!

As shown in Fig. 4.8, every value in the domain maps into the same value in the range for constant functions.

EXAMPLE 11 **(Marginal Revenue)** An important concept in economics is that of ***marginal revenue.*** Marginal revenue is the *additional revenue derived from selling one more unit of a product or service.* If each unit of a product sells at the same price, the marginal revenue is always equal to the price. For example, if a product is sold for \$7.50 per unit, the marginal revenue function can be stated as the *constant function*

SOLUTION

(a) $y = g(h(x)) = 2(x^2 - 2x + 5)^3$
(b) $g(h(2)) = 2[(2)^2 - 2(2) + 5]^3 = 2(5)^3 = 2(125) = 250$
(c) $g(h(-3)) = 2[(-3)^2 - 2(-3) + 5]^3 = 2(20)^3 = 2(8,000) = 16,000$

4.2 done

variable

Section 4.2 Follow-up Exercises

In the following exercises, classify (if possible) each function by type (constant, linear, quadratic, cubic, polynomial, rational).

done

1 $f(x) = 2^x$ → Exponential Function
2 $f(x) = -24$
3 $f(x) = (x - 5)/2$
4 $f(x) = x^2 - 25$
5 $f(x) = 2x^0$
6 $f(x) = x^5 + 2x^3 - 100$
7 $f(x) = 10 - x/4$
8 $f(x) = 10/x$
9 $f(x) = \log_{10} x$ — logarithmic Function
10 $f(x) = (x^4 - 5x^2)/(x^6 + 5)$
11 $g(h) = -25/h^5$
12 $h(s) = 3 - 4s + s^2 - s^3/4$
13 $v(t) = x^2/\sqrt{x^3}$
14 $f(u) = (5u - 3)^0/4$
15 $f(n) = 50/(4)^3$
16 $g(h) = \sqrt{100/(5)^2}$
17 $f(x) = 10^x$
18 $f(x) = x^{16}/\sqrt{x}$
19 $f(t) = t^5/(36 - t^8)$
20 $f(x) = 3^{2x}$
21 $f(x) = \log_{10}(x + 5)$
22 $v(h) = \log_e h$
23 $f(x) = [(x - 9)^0]^3$
24 $f(x) = [(x + 4)^5]^0$

25 Given the general form of a constant function stated by Eq. (4.3), determine the domain for these functions.

26 Given the general form of a polynomial function stated by Eq. (4.7), determine the domain for such functions.

27 Given the general form of a rational function stated by Eq. (4.8), determine the domain for these functions.

Profit Discuss
Revenue - cost

28 Total profit from planting x_j acres at farm j is expressed by the function

$$P(x_1, x_2, x_3) = 500x_1 + 650x_2 + 450x_3 - 300,000$$

(a) What is total profit if 200 acres are planted at farm 1, 250 acres at farm 2, and 150 acres at farm 3?
(b) What is total profit if 500, 300, and 700 acres are planted, respectively, at the three farms?
(c) Identify one combination of plantings which would result in profit equaling zero.

done

29 The value of a truck is estimated by the function

$$V = f(t) = 20,000 - 3,000t$$

where V equals the value stated in dollars and t equals the age of the truck expressed in years.
(a) What class of function is this?
(b) What is the value after 3 years?
(c) When will the value equal 0?

30 A police department has determined that the number of serious crimes which occur per

(*b*) What is the salvage value expected to equal after 10,000 hours of flight time?
(*c*) How many hours would the plane have to be flown for the salvage value to equal zero?
(*d*) What interpretation would you give to the *y* intercept? Why do you think this does not equal 75?

35 The demand function for a product is

done

$$q_d = p^2 - 90p + 2{,}025 \qquad 0 \leq p \leq 45$$

45

where q_d equals the number of units demanded and p equals the price per unit, stated in dollars.
(*a*) What type of function is this? Quadratic
(*b*) How many units will be demanded at a price of \$30?
(*c*) What price(s) would result in zero demand for the product?

36 An epidemic is spreading through a herd of beef cattle. The number of cattle expected to be afflicted by the disease is estimated by the function

$$n = f(t) = 0.08t^3 + 5$$

where n equals the number of cattle afflicted and t equals the number of days since the disease was first detected. How many cattle are expected to be afflicted after 10 days? After 20 days?

37 Given $f(x) = x^2 - 3$ and $g(x) = 10 - 2x$, determine (*a*) $f(x) + g(x)$, (*b*) $f(x) \cdot g(x)$, and (*c*) $f(x)/g(x)$.

38 Given $f(x) = \sqrt{x}$ and $g(x) = 3/(x - 1)$, determine (*a*) $f(x) - g(x)$, (*b*) $f(x) \cdot g(x)$, and (*c*) $f(x)/g(x)$.

39 If $y = g(u) = u^2 - 4u + 10$ and $u = h(x) = x - 4$, determine (*a*) $g(h(x))$, (*b*) $g(h(-2))$, and (*c*) $g(h(1))$.

40 Given $y = g(u) = 3u^2 + 4u$ and $u = h(x) = x + 8$, determine (*a*) $g(h(x))$, (*b*) $g(h(-2))$, and (*c*) $g(h(1))$.

41 If $y = g(u) = u^2 + 2u$ and $u = h(x) = x^3$, determine (*a*) $g(h(x))$, (*b*) $g(h(0))$, and (*c*) $g(h(2))$.

42 Given $c = h(s) = s^2 - 8s + 5$ and $s = f(t) = 10$, determine (*a*) $h(f(t))$, (*b*) $h(f(3))$, and (*c*) $h(f(-2))$.

43 Given $y = g(u) = (2)^u$ and $u = h(x) = x + 2$, determine (*a*) $g(h(x))$, (*b*) $g(h(3))$, and (*c*) $g(h(-2))$.

44 Given $y = g(u) = (u - 5)^2$ and $u = h(x) = x^2 + 1$, determine (*a*) $g(h(x))$, (*b*) $g(h(5))$, and (*c*) $g(h(-3))$.

4.3 GRAPHICAL REPRESENTATION OF FUNCTIONS

Throughout this book the visual model is used as often as possible to reinforce your understanding of different mathematical concepts. The visual model will most frequently take the form of a graphical representation. In this section we discuss the graphical representation of functions involving two variables.

Graphing Functions in Two Dimensions

Functions of one or two independent variables can be represented graphically. This graphical portrayal brings an added dimension to the understanding of mathemat-

Throughout the text we will continue to gain knowledge about mathematical functions. You will soon come to recognize the structural differences between linear functions and the various nonlinear functions, and with this knowledge will come greater facility and ease in determining a visual or graphical counterpart. For example, our discussions of linear equations in Chap. 2 enable us to recognize that the functions in Examples 18 and 21 are linear; hence, we know that they will sketch as straight lines which can be defined by two points.

Section 4.3 Follow-up Exercises

Sketch each of the following functions.

1 $f(x) = 8 - 3x$

2 $f(x) = 4 - x/2$

3 $f(x) = x^2 - 2x + 1$

4 $f(x) = x^2 - 9$

5 $f(x) = x^2 + 5x$

6 $f(x) = -x^2 + 4$

7 $f(x) = x^3 + 2$

8 $f(x) = -x^3 - 1$

9 $f(x) = x^4$

10 $f(x) = -x^4 + 2$

11 $f(x) = \begin{cases} x + 2 & x \geq 0 \\ -x + 2 & x < 0 \end{cases}$

12 $f(x) = \begin{cases} x^2 & x \geq 0 \\ -x - 2 & x < 0 \end{cases}$

13 $f(x) = \begin{cases} 4 & x \leq -2 \\ |x| & -2 < x < 2 \\ -4 & x \geq 2 \end{cases}$

14 $f(x) = \begin{cases} -x^2 + 4 & -2 < x < 2 \\ -x - 3 & x \leq -2 \\ x - 3 & x \geq 2 \end{cases}$

15 **Demand Function** In Example 13 we examined the quadratic demand function

$$q_d = f(p) = p^2 - 70p + 1{,}225$$

Sketch this function if the restricted domain is $0 \leq p \leq 20$.

16 **Epidemic Control** In Example 14 we examined the cubic function

$$n = f(t) = 0.05t^3 + 1.4$$

where n was the number of persons (in hundreds) expected to have caught a disease t days after it was detected by the health department. Sketch this function, assuming a restricted domain $0 \leq t \leq 30$.

17 In Fig. 4.16, identify those graphs which represent functions.

18 In Fig. 4.17, identify those graphs which represent functions.

19 Given the graph of some function $f(x)$ in Fig. 4.18, explain how the graph would change if we wanted to sketch $f(x) + c$, where c is a positive real number. What would the graph of $f(x) - c$ look like? [*Hint:* Graph $f(x) = x^2$ and compare with the graphs of $g(x) = x^2 + 1$ and $h(x) = x^2 - 1$.]

20 Given the graph of some function $f(x)$ in Fig. 4.19, explain how the graph would change if we wanted to sketch $-f(x)$. [*Hint:* Graph $f(x) = x^2$ and compare with the graph of $g(x) = -x^2$.]

Figure 4.16

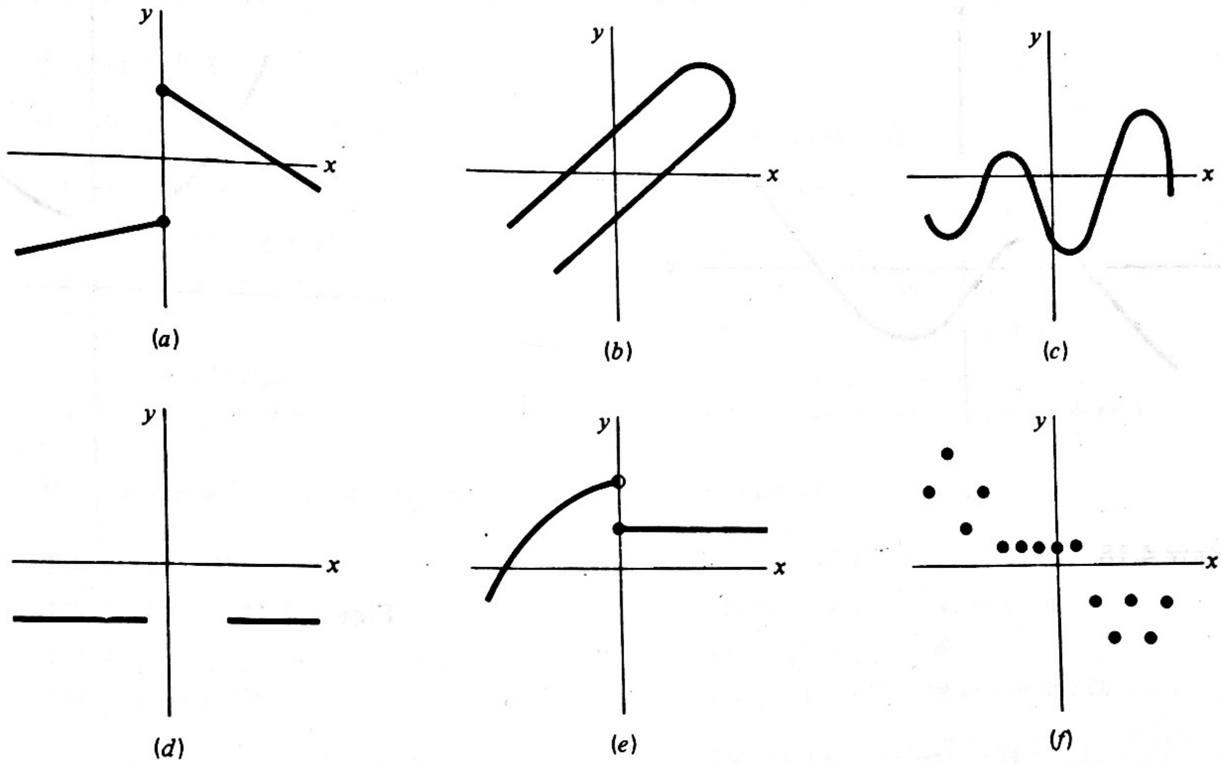
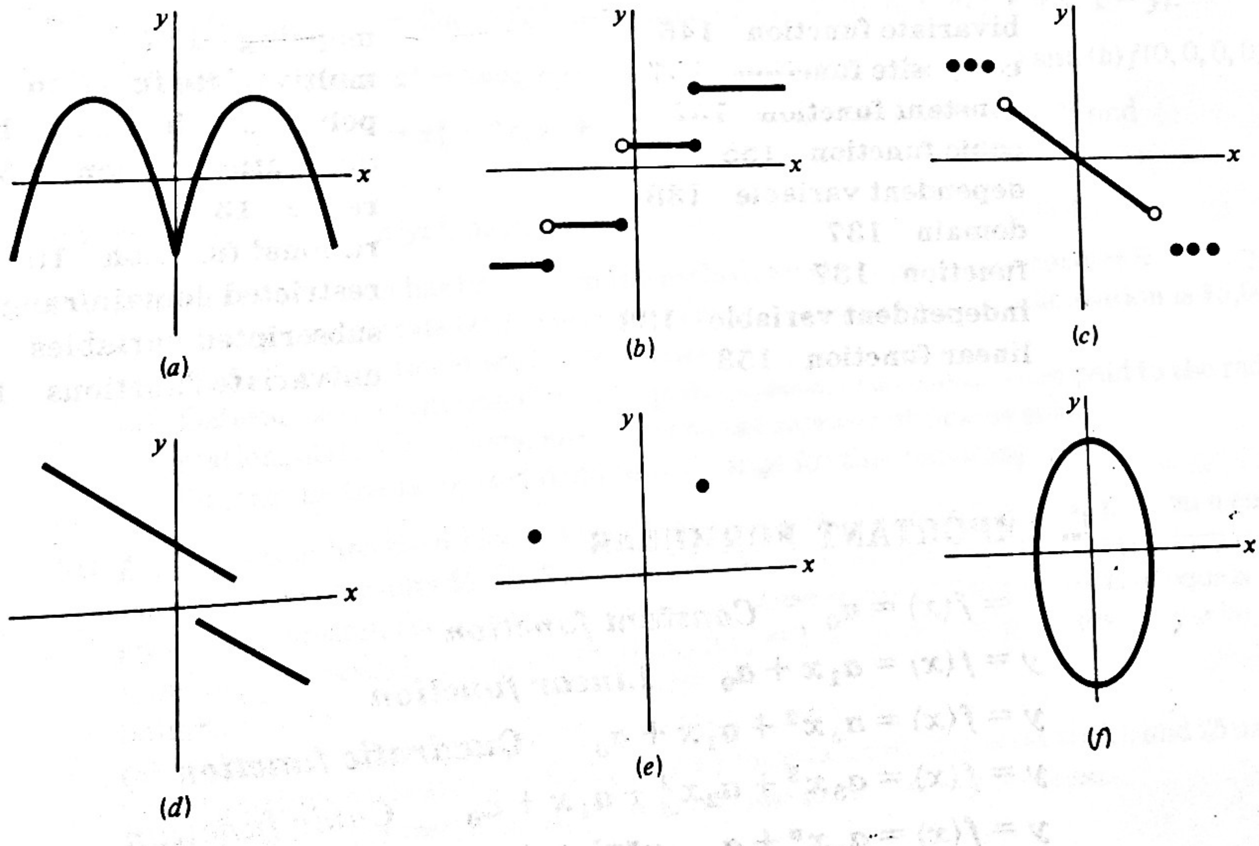


Figure 4.17



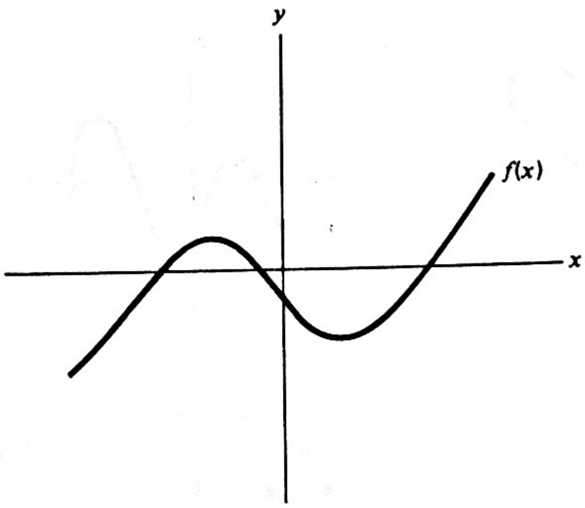


Figure 4.18

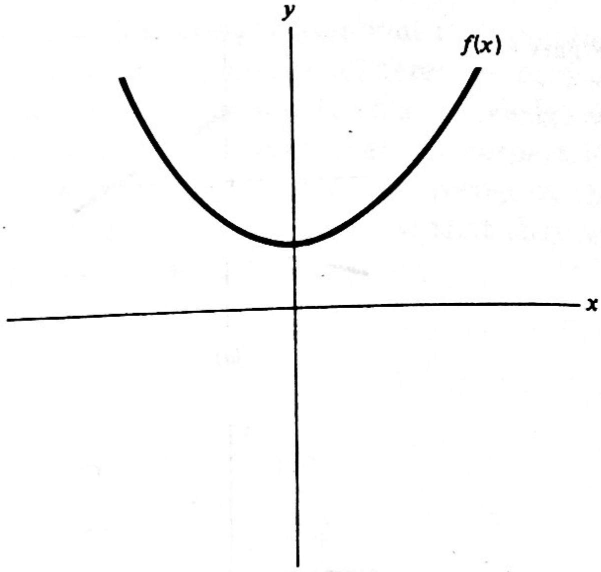


Figure 4.19

❑ KEY TERMS AND CONCEPTS

bivariate function 145
composite function 157
constant function 152
cubic function 155
dependent variable 138
domain 137
function 137
independent variable 138
linear function 153
mapping 137
multivariate function 145
polynomial function 155
quadratic function 154
range 137
rational function 156
restricted domain/range 144
subscripted variables 147
univariate functions 145

❑ IMPORTANT FORMULAS

$y = f(x) = a_0$ *Constant function* (4.3)

$y = f(x) = a_1x + a_0$ *Linear function* (4.4)

$y = f(x) = a_2x^2 + a_1x + a_0$ *Quadratic function* (4.5)

$y = f(x) = a_3x^3 + a_2x^2 + a_1x + a_0$ *Cubic function* (4.6)

$y = f(x) = a_nx^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ *Polynomial function* (4.7)

$y = f(x) = \dfrac{g(x)}{h(x)}$ *For g, h polynomials* *Rational function* (4.8)