Introduction to Mathematics

Week 1
Review of Basic Arithmetic Operations & Algebra

- **Unit 1.2**

Ratio & Proportions

Topics in this Unit

- Ratios

- Proportions

Ratios

Ratios are comparisons made between two sets of numbers.

- For example: There are eight girls and seven boys in a class.
- The ratio of girls to boys is 8 to 7.

Ratios are used everyday. They are used for:

- Miles per hour
- The cost of items per pound, gallon, etc.
- Hourly rate of pay

THERE ARE 3 WAYS TO WRITE RATIOS.

1. Write the ratio using the word "to" between the two numbers being compared.

For example: There are 8 girls and 5 boys in my class. What is the ratio of girls to boys?

The ratio is: 8 girls to 5 boys

8 to 5

2. Write a ratio using a colon between the two numbers being compared.

For example: There are 3 apples and 4 oranges in the basket. What is the ratio of apples to oranges?

The ratio is: 3 apples to 4 oranges.

3:4

3. Write a ratio as a fraction.

For example: Hunter and Brandon were playing football. Brandon scored 5 goals and Hunter scored 6 goals.

What was the ratio of goals Hunter scored to the goals Brandon scored?

The ratio of goals scored was: 6 goals to 5 goals

$$\frac{6}{5}$$

Reducing Ratios

Ratios can be reduced without changing their relationship.

2 boys to 4 girls =

1 boy to 2 girls =

All Ratios Are Written In Lowest Terms.

Steps:
  1. Read the word problem.
  2. Set up the ratio.

For example:

You scored 40 answers correct out of 45 problems on a test. Write the ratio of correct answers to total questions in lowest form.

Step 1: Read the problem. What does it want to know?

40 to 45   40 : 45 $\frac{40}{45}$

3. Reduce the ratio if necessary.

Reduce means to break down a fraction or ratio into the lowest form possible.

Reduce = smaller number; operation will always be division.

HINT: When having to reduce ratios, it is better to set up the ratio in the vertical form. (Fraction Form)

40 to 45   = $\dfrac{40}{45}$

Look at the numbers in the ratio. What ONE number can you divide BOTH numbers by?

Factors of 40: 1, 2, 4, 5, 8, 10, 20, 40
Factors of 45:   1, 3, 5, 9, 15, 45

$$\dfrac{40}{45} \begin{array}{c} \div \\ \div \end{array} \begin{array}{c} 5 \\ 5 \end{array} \begin{array}{c} = \dfrac{8}{9} \\ = \end{array}$$

Example

1. There are 26 black cards in a deck of playing cards. If there are 52 cards in a deck, what is the ratio of black cards to the deck of cards?

Step 1: Read the problem. (What does it want to know?)

Step 2: Set up the ratio.

26 black cards to 52 cards

Step 3: Can the ratio be reduced? If so, set it up like a fraction.

$$\frac{26}{52} \div \frac{26}{26} = \frac{1}{2}$$

What is the largest number that will go into both the top number and the bottom number evenly? (It can not be the number one!)

Proportions

- A proportion is an equation that says one ratio is equal to another.

$$\text{ratio} \quad \frac{3}{4} \; = \; \frac{39}{52} \; \text{ratio}$$

Determining True Proportions

If the cross products are equal, then it is a true proportion.

$$\frac{4}{5} \quad \diagdown\!\!\!\!\diagup \quad \frac{20}{25}$$

$$20 \times 5 \;=\; 4 \times 25$$

$$100 \quad = \quad 100 \;\checkmark$$

The cross products were equal, therefore $\underline{4}$ And $\underline{20}$ makes a true proportion.
 5 25

Example

Directions: Solve to see if each problem is a true proportion.

1. $\dfrac{3}{5} = \dfrac{15}{25}$

2. $\dfrac{6}{8} = \dfrac{57}{76}$

3. $\dfrac{7}{12} = \dfrac{37}{60}$

Directions: Solve to see if each problem is a true proportion.

1. $\frac{3}{5} = \frac{15}{25}$

2. $\frac{6}{8} = \frac{57}{76}$

3. $\frac{7}{12} = \frac{37}{60}$

$15 \times 5 = 3 \times 25$

$75 = 75$

true

$57 \times 8 = 6 \times 76$

$456 = 456$

true

$7 \times 60 = 37 \times 12$

$420 \neq 444$

false

Solving Proportion with Variables

What is a variable?   A variable is any letter that takes place of a missing number or information.

Eric rode his bicycle a total of 52 miles in 4 hours. Riding at this same rate, how far can he travel in 7 hours?

Look for the two sets of ratios to make up a proportion.

You have 52 miles in 4 hours. This is the first ratio.

Next, the problem states "how far can he travel in 7 hours. The problem is missing the miles. Therefore, the miles becomes the variable.

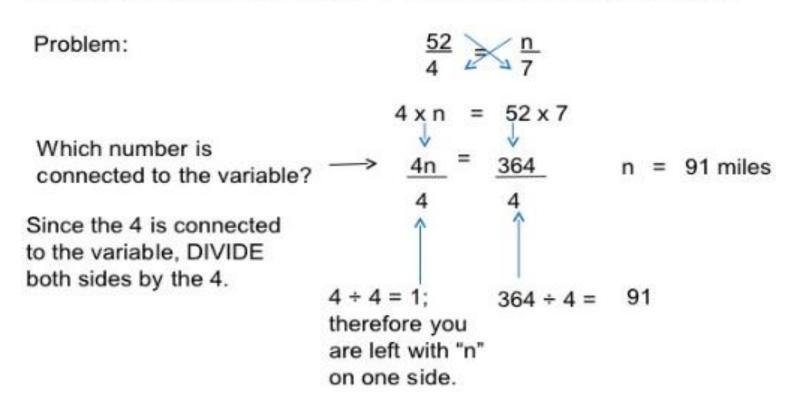Set 1 ⟶ $\dfrac{52 \text{ miles}}{4 \text{ hours}}$ Set 2 ⟶ $\dfrac{n \text{ miles}}{7 \text{ hours}}$

The proportion should be set equal to each other. ⟶ $\dfrac{52}{4} = \dfrac{n}{7}$

HINT: The order of the ratio does matter!

When solving proportions, follow these rules:

1. **Cross multiply.**
2. **Divide BOTH sides by the number connected to the variable.**
3. **Check the answer to see if it makes a true proportion.**

Problem:

$$\frac{52}{4} = \frac{n}{7}$$

$$4 \times n = 52 \times 7$$

Which number is connected to the variable? $\longrightarrow$

$$\frac{4n}{4} = \frac{364}{4} \qquad n = 91 \text{ miles}$$

Since the 4 is connected to the variable, DIVIDE both sides by the 4.

$4 \div 4 = 1$; therefore you are left with "n" on one side.

$364 \div 4 = 91$

If it comes out even, then the answer is correct.

Check your answer!

$$\frac{52}{4} \qquad \frac{91}{7}$$

$$52 \times 7 = 91 \times 4$$
$$364 = 364 \quad \checkmark$$

Example

1. Justin's car uses 40 gallons of gas to drive 250 miles. At this rate, approximately, how many gallons of gas will he need for a trip of 600 miles.

$$\frac{40 \text{ gal}}{250 \text{ mi}} = \frac{x \text{ gal}}{600 \text{ mi}} \qquad \frac{40}{250} = \frac{x}{600}$$

$$250x = 24000$$

$$\frac{250x}{250} = \frac{24000}{250}$$

$$x = 96$$

Check:

$$\frac{40}{250} = \frac{96}{600}$$

$$24000 = 24000$$

Thank you