Welcome!!

Discussion Topics

01   Annuities

02   Future Value of Annuity

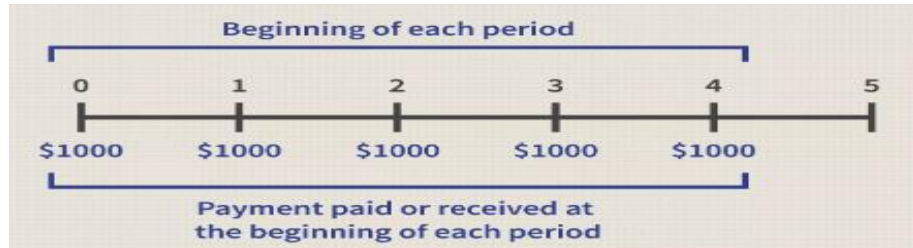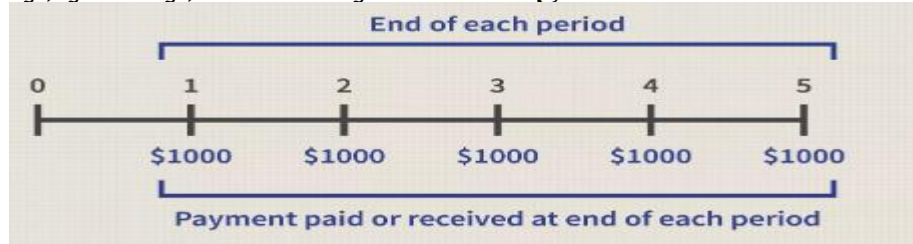**03**   Present Value of Annuity

04   Practice Problems

Annuities

❑ An annuity is a series of same size payments made at equal intervals. The payments (deposits) may be made weekly, monthly, quarterly, yearly, or at any other regular interval of time.
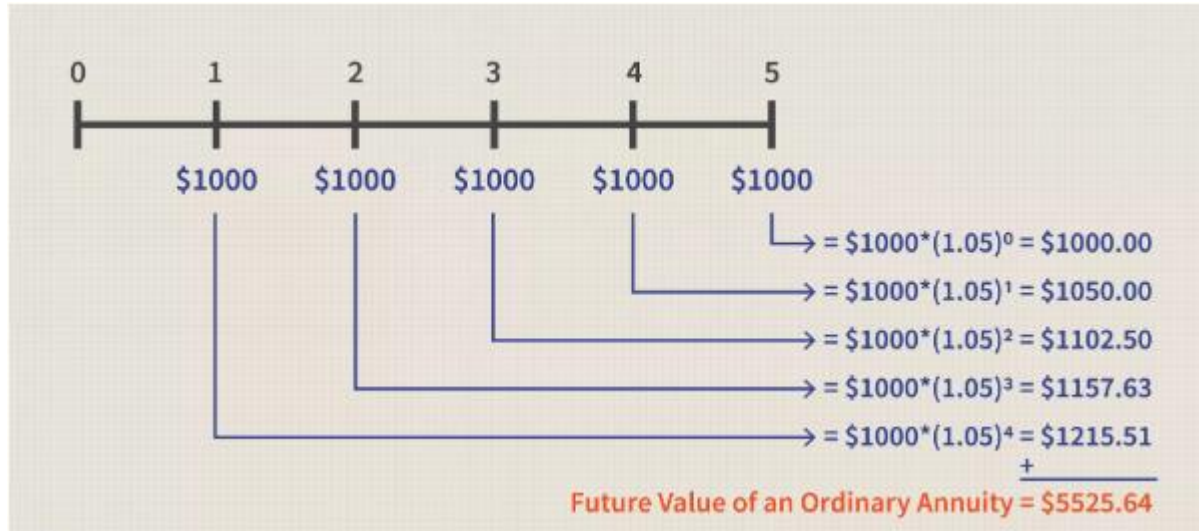
❑ Examples:

▪ Regular deposits to a savings account.

▪ Monthly home mortgage payments.

▪ Monthly insurance payments.

Future Value of an Annuity

❑ Future value (FV) of an annuity is a measure of how much a series of regular payments will be worth at some point in the future, given a specified interest rate.

$$FVA = PMT \left[\frac{(1+i)^n - 1}{i} \right]$$

FVA   =   future value of the annuity

PMT   =   payment per period

i   =   interest rate

n   =   number of periods

The time line shows payments of $1000 at periods 1, 2, 3, 4, 5:

= $1000*(1.05)^0 = $1000.00
= $1000*(1.05)^1 = $1050.00
= $1000*(1.05)^2 = $1102.50
= $1000*(1.05)^3 = $1157.63
= $1000*(1.05)^4 = $1215.51
+
Future Value of an Ordinary Annuity = $5525.64

Future Value of an Annuity (Example)

❑ A person wishes to deposit $100 per in a saving account which earns interest of 5% per year compounded annually. Assume the first deposit is made at the end of this current year and additional deposits at the end of each year for the total 20 years period. To what sum will the investment grow at the time of 20th deposit?

$$FVA = \$100 \left[\frac{(1+0.05)^{20} - 1}{0.05} \right]$$

$$FVA = \$100 \left[\frac{2.65329 - 1}{0.05} \right]$$

$$FVA = \$100 \left[\frac{1.65329 - 1}{0.05} \right]$$

$$FVA = \$100 \times 33.0659$$

Future Value of an Annuity (Example)

❑ A mother wishes to set up a saving account for her son's education. She plans investing $750 when her son is 6 months old and every 6 months thereafter. The account earns interest at a rate of 8 percent per year, compounded semiannually.

a. To what amount will the account grow by the time of her son's 18th birthday?
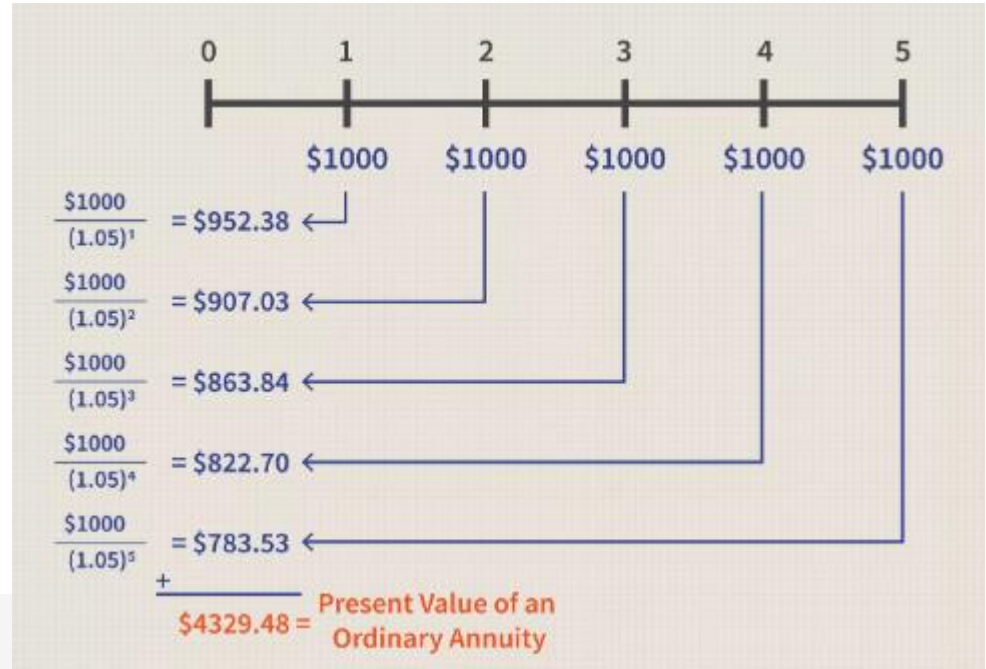b. How much interest will be earned during this period?

$$R = \$750$$
$$i = 8\%$$

Sol:

$$i_{actual} = \frac{8\%}{2} = 4\%$$

$$\boxed{i_{actual} = 0.04}$$

a.
$$FVA = \$750\left[\frac{(1+0.04)^{36} - 1}{0.04}\right]$$

$$\boxed{FVA = 58198.73\ \$}$$

b.
$$58198.73 - (750 \times 36)$$

$$\boxed{Interest = 31198.73\ \$}$$

Present Value of an Annuity

❑ In contrast to the future value calculation, a present value (PV) calculation tells you how much money would be required now to produce a series of payments in the future, again assuming a set interest rate.

$$PVA \quad = \quad PMT \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

where:

PVA = present value of the annuity

PMT = payment per period

i = interest rate

n = number of periods

$\dfrac{\$1000}{(1.05)^1} = \$952.38$

$\dfrac{\$1000}{(1.05)^2} = \$907.03$

$\dfrac{\$1000}{(1.05)^3} = \$863.84$

$\dfrac{\$1000}{(1.05)^4} = \$822.70$

$\dfrac{\$1000}{(1.05)^5} = \$783.53$

$+$

$\$4329.48 =$ Present Value of an Ordinary Annuity

Example: Present Value of an Annuity

❑ A person recently won a state lottery. The terms of the lottery are that winner will receive annual payments of $1,000 at the end of this year and each of the following 4 years. If the winner could invest money today at the rate of 5% per year compounded annually, what is the present value of the five payments?

$$PVA \quad = \quad PMT \left[\frac{(1+0.05)^5 - 1}{0.05\,(1+0.05)^5} \right]$$

$$PVA \quad = \quad \$1,000 \times 4.329477$$

$$PVA \quad = \quad \$4,329.48$$

Mortgage

❑ When a person purchases some property by paying partial amount and leaves the remaining amount (mortgage) to be paid over years, the borrower repays the remaining amount (loan) with interest, until he or she owns the property.

Example: Mortgage

❑ A person pays $100,000 for a new house. A down payment of $30,000 leaves a mortgage of $70,000 with interest computed at 10.5% per year compounded monthly. Determine the monthly mortgage payment if the loan is to be repaid over in 20 years? Also compute the total interest for this loan period.

❑ **Solution:**

n = 20*12 = 240 Payments
using the formula,
where PVA = $70,000

Total Interest = $698.87 * 240 - $70,000
Total Interest = $97,728.80

$$PVA = PMT \left[\frac{(1+i)^n - 1}{i(1+i)^n} \right]$$

PMT = $698.87

Thank you

Question Answers Session