Introduction to Mathematics

Mathematics of Finance

Week 12

Learning Objectives

- Interest and its computation
- Single payment computations
- Annuities and their future values

Learning Outcomes

After going through this lectures, we should be able to:

- Know the application of mathematical equation involving single and multiple variables in linear and quadratic form.
- Convert the business, economic and financial problem into a mathematical equation and how to solve them.
- Apply the concept of mathematics in various fields of economics, finance, marketing, human resource management, etc.
- Develop hypothetical cases and their mathematical functions close to real business and economic activities.

Interest And Its Computation

Simple Interest

Interest, in finance and economics, is payment from a borrower or deposit-taking **financial** institution to a lender or depositor of an amount above repayment of the principal sum (that is, the amount borrowed), at a particular rate. It is distinct from a fee which the borrower may pay the lender or some third party.

- **Definition:** The Interest is a fee which is paid for having the use of money.
- We pay interest on loans for having the use of bank's money. Similarly, the bank pays us interest on money invested in savings accounts because the bank has temporary access to our money.
- 1. Interest is usually paid in proportion to the principal amount and the period of time over which the money is used.
- 2. The interest rate specifies the rate at which the interest accumulates.
- 3. It is typically stated as a percentage of the principal amount per period of time, e.g. 18 % per year, 12 % quarterly or 13.5 % per month.

Formula

Simple Interest

$$I = P * i * n$$

Where,

- $I = Simple\ Interest\ (in\ Rs)$
- $P = Principal\ (in\ Rs)$
- $i = Interest\ Rate$
- $n = Number\ of\ time\ period$

Definition: The amount of money that is lent or invested is called the principal.

Exercise 8.1 (Page 306-7)

Q1: A company has issued a 5-year loan of $90, 000.to a new vice president to finance a home improvement project. The terms of the loan are that it is to be paid back in full at the end of 5 years with simple interest at the rate of 8% per year. Determine the interest which must be paid on the loan for the 5 year period.

Solution:

- $P = 90{,}000$
- $i = 8\%\ or\ 0.08$
- $n = 5$
- $I = P * i * n$
- $I = 90{,}000 * (.08) * (5)$
- $I = \$36000$

Compound Interest

- Compound Interest is the addition of **interest** to the principal sum of a loan or deposit, or in other words, **interest** on **interest**.

- When calculating compound interest, the number of compounding periods makes a significant difference. Generally, the higher the number of compounding periods, the greater the amount of compound interest.

$$S = P + I$$

- Example:
- Assume that we have deposited $ 8000 in a credit union which pays interest of 8% per year compounded quarterly. The amount of interest at the end of 1 quarter would be:
- $I = 8000(0.08)(0.25) = 160.$
- Here n=0.25 year, with interest left in the account, the principal on which interest is earned in the second quarter is the original principal plus $160 earned during the first quarter
- $P = P + I = 8000 + 160 = 8160.$
- The interest earned during second quarter is
- $I = 8160(.25)(.08) = 163.2.$

- Continuing this way, after the year the total interest earned would be $ 659.46. At the end of one year, the compound amount would be $ 8659.49.
- The simple interest after 1 year would be:
- $I = 800(.08)(1) = 640$
- The difference between simple and compound interest is $659.49 - 640 = 19.46$.

- **Ex. 8.1; Q5:** A $10, 000 certificate of deposit interest of 8% per year, compounded semiannually. Complete the following table with regard to semi annual compounding. What is the total interest over the 2-year period?

Semi Annual Period	Principal	Interest	Compound Amount
1	10,000	400	10,400
2			
3			
4			

Solution

P = 10,000 i = 8% / year n = 2 years

Semi Annual Period (t)	Principal	Interest	Compound Amount
1	10,000	400	10,400
2	10,400	416	10,816
3	10,816	433	11,249
4	11,249	450	11,699

Continuous compounding

- The continuous compounding can be thought of as occurring infinite number of times. It can be computed by the following formula.

$$S = Pe^{it}$$, where

- i= Rate of interest, t= Time period, P = Principal.

Example

- Compute the growth in a $ 10,000 investment which earns interest at 10 per cent per year over the period of 10 years.
- Solution:
- Here i= 0.1, t= 10, and P = 10,000.
- Hence $S = (10{,}000)e^{0.1(10)}$ = $ 27,183

Note: The value of an investment increases with increased frequency of compounding. If we compute the interest by using different frequency, we get the following.

1. Simple interest in 10 years will raise the amount to $ 20,000.
2. Annual compounding in 10 years will raise the amount to $ 25,937.
3. Semi-annual compounding in 10 years will raise the amount to $ 26,533.
4. Quarterly compounding in 10 years will raise the amount to $ 26,830.

Single –Payment Computations

Compound Amount

If an amount of money P earns interest compounded at a rate of i percent, it will grow after n periods to the compound amount S, where

$$S = P(1 + i)^n$$

Exercise 8.2 (Page 317)

Q1: A sum of $8, 000 is invested in a saving account which pays interest at a rate of 9% per year compounded annually. If the amount is kept on deposit for 6 years, what will the compound amount equal? How much interest will be earned 6 years?

IQRA UNIVERSITY IU

Solution

Compound Amount

$$S = P(1 + i)^n$$

- $P = 8{,}000$
- $i = 9\%\ yearly\ or\ .09$
- $n = 6\ years$
- $S = ?$

$S = P(1 + i)^n$

S = 8000 (1+ .09)6

S = $13416

Interest = $S - P$

Interest earned = 13416-8000 =$ 5416

Present Value

- $S = P(1 + i)^n$
- $P = \frac{S}{(1+i)^n}$

Q.15: If a saving account awards interest of 6% per year compounded quarterly, what amount must be deposited today in order to accumulate $20, 000 after 5 years? How much interest will be earned during these five years?

IQRA UNIVERSITY IU

Solution:

- $P = \frac{S}{(1+i)^n}$
- $S = 20000$
- $i = 6\%\ yearly\ or\ .06 \quad 0r \quad 3\%\ semiannually$
- $n = 5\ years = 10\ periods$
- $P = ?$
- $P = \frac{S}{(1+i)^n}$
- $P = \frac{2000}{(1+0.03)^{10}}$
- $P = \$14881$
- $Interest = Compound\ Amount - Principal$

$= 20000 - 14881 = \$\,5119$

Q.21: A sum of $80, 000 earns interest at a rate of 8% per year compounded semiannually. How long will it take the investment to grow to $ 150,000?

IQRA UNIVERSITY IU

Solution:

- $P = 80,000$
- $i = 8\%\ yearly\ or\ .08 \quad 0r \quad 4\%\ semiannually$
- $S = 150,000$
- $n = ?$
- $S = P(1+i)^n$
- $150,000 = 80{,}000(1+0.04)^n$
- $1.875 = 1.04^n$
- $Taking\ \log\ on\ both\ sides$
- $\log(1.875) = \log(1.04)^n$
- $\log(1.875) = nlog(1.040) \quad using\ power\ property$
- $n = 16.027\ periods\ or\ 8\ years$

Review Question

- A principal of $20,000 is available to be invested. One alternative is to invest in Bank A that offers simple interest of 10 percent. Another option is to invest in Bank B that compounds semi annually with an interest rate of 8 percent. Compute the interests generated in both banks if the principal is to be invested for 2 years. Which seems to be the better option?

Video Links:

- Video 1:

https://www.youtube.com/watch?v=5wpsLW5JEms

- Video 2:

https://www.youtube.com/watch?v=ST_iw0x1Kow

Web Resources:

https://www.investopedia.com/ask/answers/042315/what-difference-between-compounding-interest-and-simple-interest.asp

Learning Material

- https://www.pearson.com/content/dam/one-dot-com/one-dot-com/us/en/higher-ed/en/products-services/course-products/lial-applied-mathematics-info/pdf/LGR-Finite-Ch5.pdf
- https://www.pearsonhighered.com/assets/samplechapter/0/1/3/4/0134437764.pdf
- http://www.railassociation.ir/Download/Article/Books/Basic%20Mathematics%20for%20Economists.pdf
- http://mongmara.yolasite.com/resources/Math4BusinessandEconomics/Applied%20Mathematics%20for%20Business%20and%20Economics.pdf

Thank you