Introduction to Mathematics

Week 2

Linear and Quadratic Equations and Inequalities

Unit 2.1

Solution of First Degree Equations

Equation

An **equation** is a statement that tells about the equality of two algebraic expressions.

There may be one or more variables in those algebraic expressions.

Examples:

$3x - 10 = 22 - 5x$ **(One variable: x)**

$\frac{2r - 5s + 8t}{3} = 100$ **(Three variables: s, r and t)**

$w^2 - 5w = -16$ **(One variable: w)**

Solution of an Equation

Solution: The number(s) when substituted in place of the variable(s) in an equation and the equation becomes TRUE

The number (or numbers) or values of the variables that satisfy an equation are also called **Roots** of that equation.

Consider the following equation:

$\boldsymbol{3x - 10 = 22 - 5x}$

If we put x=0,

$\boldsymbol{-10 = 22}$ (Which is FALSE, so **x=0 is not a solution** of this equation)

Solution of an Equation

$$3x - 10 = 22 - 5x$$

If we put x=4,

$$3(4) - 10 = 22 - 5(4)$$

$$12 - 10 = 22 - 20$$

$2 = 2$ (Which is TRUE, thus x = 4 is a root of this equation)

A **solution** of an equation in one variable is a real number which, when substituted for the variable in the equation, makes the equation true.

Example: Is 3 a solution of $2x + 3 = 11$?

$$2x + 3 = 11$$

$$2(3) + 3 = 11$$

$$6 + 3 = 11$$

False equation

3 is not a solution of $2x + 3 = 11$.

Example: Is 4 a solution of $2x + 3 = 11$?

$$2x + 3 = 11$$

$$2(4) + 3 = 11$$

$$8 + 3 = 11$$

True equation

4 is a solution of $2x + 3 = 11$.

Types of Equations

Identity: An equation that is true for all values of the variables.
For example:

$$\mathbf{15 + 6x = \frac{1}{2}(32 + 12x) - 1}$$

In other words, an identity always yields 0=0

Conditional Equation: True for only one or more than one specific values of the variables.
For example:

$$\boldsymbol{x + 3 = 7}$$

The above equation is true only when we substitute x with 4.

Contradiction or False Statement: Which is never true. For example:

$$\boldsymbol{x = x + 4\ (This\ can\ never\ be\ true)}$$

Thus, we write it as:

$$\boldsymbol{x \neq x + 4}$$

Degree of a Polynomial

The degree of a polynomial or an equation is identified from the highest-degree term in that equation.

If we write the equation in the form:

Polynomial Expression = 0

For example:

2x – 4 = 0 (The highest degree is 1 here because here are two terms, one with x^1 and one with x^0)
Above equation is also called Linear Equation.

For example:

$4r^2 – r + 10 = 0$ (This is a second-order equation because the highest degree is 2 here)
The above equation is also called Quadratic Equation.

Solving an Equation

Solving an equation means the process of finding the roots of an equation, if any roots exist.

We have to manipulate equations using following rules to solve them:

SELECTED RULES FOR MANIPULATING EQUATIONS

I *Real-valued expressions which are equal can be added to or subtracted from both sides of an equation.*

II *Both sides of an equation may be multiplied or divided by any nonzero constant.*

III *Both sides of an equation may be multiplied by a quantity which involves variables.*

IV *Both sides of an equation may be squared.*

V *Both sides of an equation may be divided by an expression which involves variables provided the expression is not equal to 0.*

First Degree Equation in One Variable

$$3x = 2x - 5$$
$$5x - 4 = 12 + x$$

The above equations are First Degree Equations in One Variable.

We will solve such equations using the rules stated in the previous slide.

The objective would be to isolate the unknown variable on one side (left) and all other numbers on another side (right).

Solution of First Degree Equations in One Variable

Example: Solve $x + 1 = 3(x - 5)$.

$$x + 1 = 3(x - 5)$$
$$x + 1 = 3x - 15$$
$$x = 3x - 16$$
$$-2x = -16$$
$$x = 8$$

The solution or the root is 8.

Check the solution: $(8) + 1 = 3((8) - 5) \rightarrow 9 = 3(3)$ True

Solution of First Degree Equations in One Variable

Example: Solve $2x + 5 = 10 + 2x$

Subtracting 2x from both sides

$$2x + 5 - 2x = 10 + 2x - 2x$$
$$5 = 10$$

Which is a False Statement or Contradiction.

This means the given equation does not have any roots or solution.

Solution of First Degree Equations in One Variable

Example: Solve $x - 3 = \frac{2x - 6}{2}$.

Multiplying both sides of the equation by 2

$$2(x - 3) = 2x - 6$$
$$2x - 6 = 2x - 6$$

This is an Identity and thus all real values of x would satisfy the given equation.

Review Questions

(a) $4x - 10 = 8 - 2x$

(b) $x - 5 = -\dfrac{(-2x + 10)}{2}$

(c) $3x + 3 = 3x - 5$

Thank you