Introduction to Mathematics

Week 2

Linear and Quadratic Equations and Inequalities

Unit 2.4

Solving Second-Degree Inequalities

Solving Second-Degree Inequalities in One Variable

Quadratic Inequalities or Second-Order Inequalities are solved using factoring method or if factors are not possible, we use the quadratic formula to find the factors of the quadratic expression.

Please note that before applying factoring method or quadratic formula, there should be 0 on the right side of the inequality.

After finding the factors, we evaluate various conditions with both the factors involving the inequality.

Solving Second-Degree Inequalities in One Variable

Solve the quadratic inequality $\boldsymbol{x^2 - 5x + 6 \leq 0}$
By factoring method:

$$\boldsymbol{x^2 - 3x - 2x + 6 \leq 0}$$
$$\boldsymbol{x(x-3) - 2(x-3) \leq 0}$$
$$\boldsymbol{(x-3)(x-2) \leq 0}$$

Now the above inequality can be broken down in FOUR conditions.

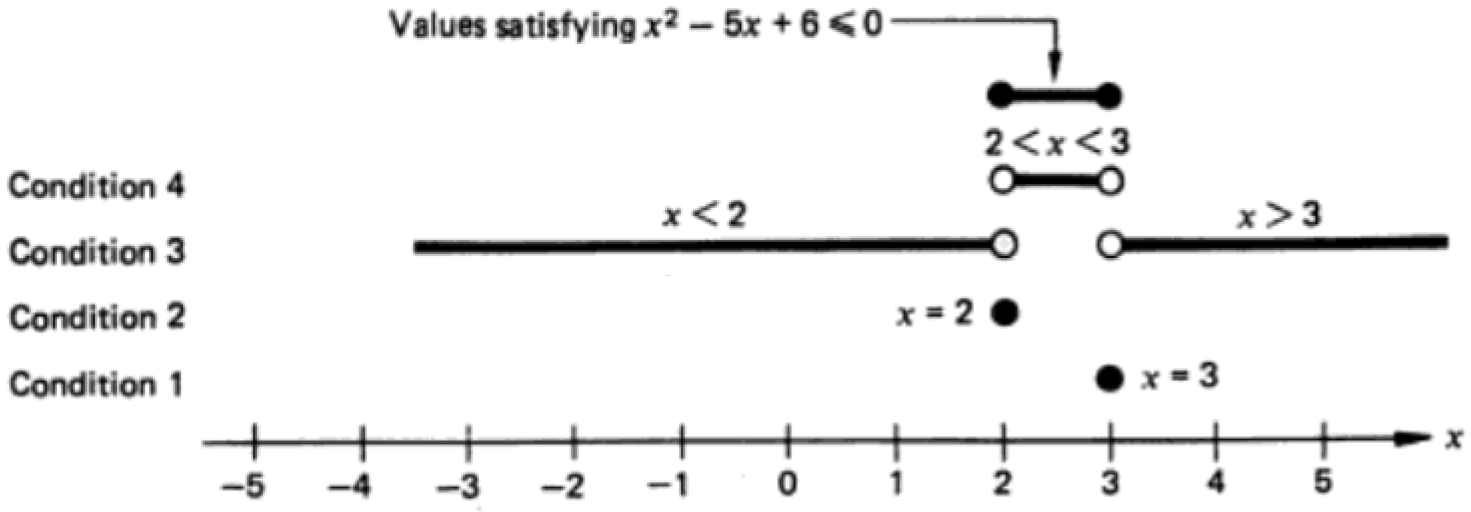
	x-3	x-2	Product
Condition 1	= 0	Any Value	0
Condition 2	Any Value	= 0	0
Condition 3	> 0	< 0	< 0
Condition 4	< 0	> 0	< 0

> 0 means positive and < 0 means negative

Number line and solution on next page

Solving Second-Degree Inequalities in One Variable

Condition 1: x – 3 = 0 so ***x* = *3***
Condition 2: x – 2 = 0 so ***x* = *2***
Condition 3: x – 3 > 0 and x – 2 < 0
x* > *3 and ***x* < *2***
Condition 4: x – 3 < 0 and x – 2 > 0
x* < *3 and ***x* > *2***



Plotting all four conditions and then deciding the common values to be the solution set, which is: 2 ≤ x ≤ 3, excluding the endpoints

Solving Second-Degree Inequalities in One Variable

Solve the quadratic inequality $\boldsymbol{x^2 - 2x - 15 > 0}$
By factoring method:

$\boldsymbol{x^2 - 5x + 3x - 15 > 0}$
$\boldsymbol{x(x-5) + 3(x-5) > 0}$
$\boldsymbol{(x-5)(x+3) > 0}$
Now the above inequality can be broken down in TWO conditions.

	x-5	x+3	Product
Condition 1	> 0	> 0	> 0
Condition 2	< 0	< 0	> 0

> 0 means positive and < 0 means negative

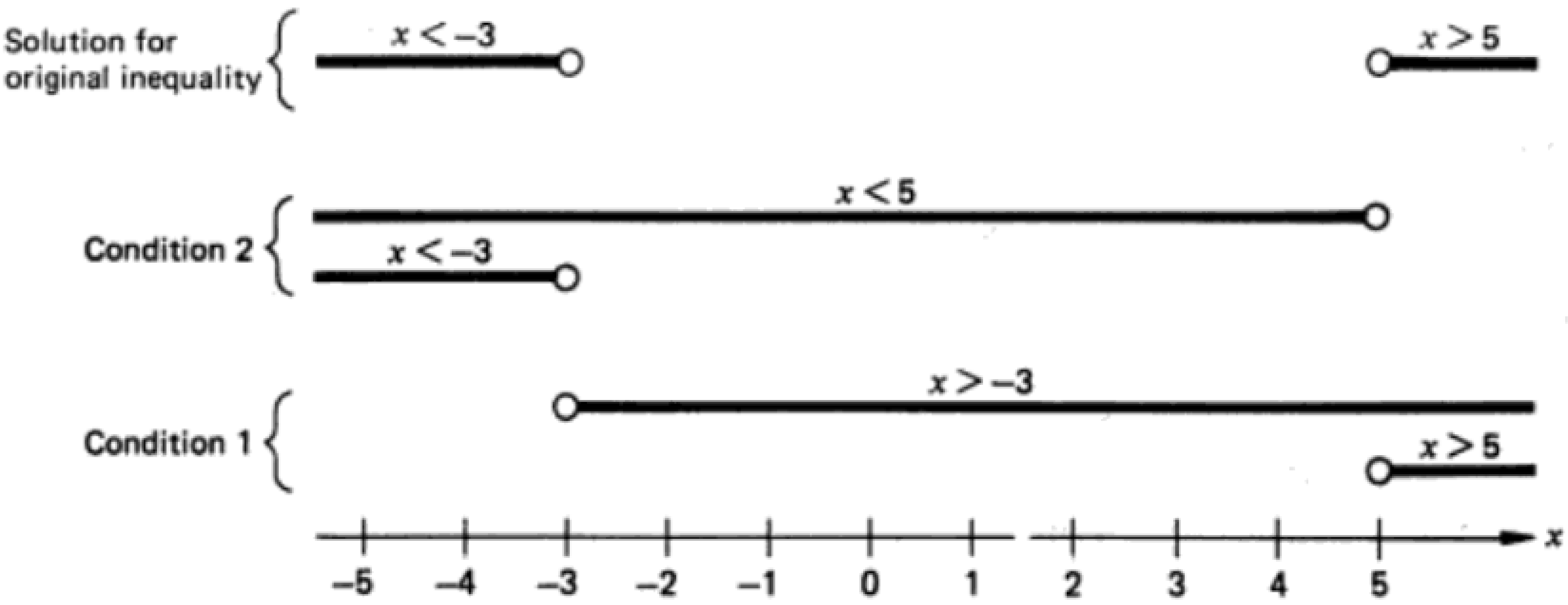
Solving Second-Degree Inequalities in One Variable

Condition 1: $x - 5 > 0$ and $x + 3 > 0$

x > 5 and ***x > -3***

Condition 2: $x - 5 < 0$ and $x + 3 < 0$

x < 5 and ***x < -3***



Solution Set:

Common points of both conditions are $x < -3$ and $x > 5$ with both endpoints excluded

Can also be written as:

$(-\infty, -3) \cup (5, \infty)$

Thank you