

Introduction to Mathematics

Applications of Linear Functions

Revenue, Cost and Profit

- **Revenue:** The income generated from sale of goods or services.
- **Cost:** The price paid or required for acquiring, producing, or maintaining something.
- **Profit:** A profit is an amount of money that you gain when you are paid more for something than it.

$$\text{Profit (P)} = \text{Revenue (R)} - \text{Cost (C)}$$

Cost can be broadly classified into variable cost and overhead cost. Variable cost varies with the volume of production while overhead cost is fixed, irrespective of the production volume.

Revenue, Cost and Profit Functions

- **Linear Revenue Functions:** $Total\ Revenue = (Price) * (Quantity\ Sold)$
- **Linear Profit Functions:** $= Profit = Total\ Revenue - Total\ Cost$

❖ If Total Revenue = $R(x)$ and Total Cost = $C(x)$

$x =$ The Quantity Produced and Sold, then

$$P(x) = R(x) - C(x)$$

- If $Total\ Revenue > Total\ Cost$, Profit is Positive (Net Gain or Net Profit)
- If $Total\ Revenue < Total\ Cost$, Profit is Negative (Net Loss or Deficit)

Revenue, Cost and Profit Functions

Example: A firm sells a single product for \$65 per unit. Variable costs per unit are \$20 for materials and \$27.50 for labor. Annual fixed costs are \$ 100,000. construct the profit function stated in terms of x , the number of units produced and sold. What profit is earned if annual sales are 20,000 units?

Solution:

$$R(x) = 65x$$

$$C(x) = 20x + 27.50x + 100,000$$

$$C(x) = 47.50x + 100,000$$

$$\diamond P(x) = R(x) - C(x)$$

$$P(x) = 65x - (47.50x + 100,000)$$

$$P(x) = 17.50x - 100,000$$

In order to find the value of profit at a sales of 20,000 . put $x = 20,000$ in profit function

$$\begin{aligned} P(20,000) &= 17.50(20,000) - 100,000 \\ &= 350,000 - 100,000 \\ &= 250,000 \end{aligned}$$

Revenue, Cost and Profit Functions

Example: A manufacturer of microcomputers produces three different models. The following table summarizes wholesale prices, material cost per unit, and labor cost per unit. Annual fixed costs are \$25 million.

	Microcomputer		
	Model 1	Model 2	Model 3
Wholesale price/unit	\$500	\$1,000	\$1,500
Material cost/unit	175	400	750
Labor cost/unit	100	150	225

- Determine a joint total revenue function for sales of the three different microcomputer models.
- Determine an annual total cost function for manufacturing the three models.
- Determine the profit function for sales of the three models.
- What is annual profit if the firm sells 20,000, 40,000 and 10,000 units, respectively, of the three models?

Revenue, Cost and Profit Functions

Solution:

Let x_1 , x_2 and x_3 represents the number of units of three different models respectively.

(a)

$$R(x_1, x_2, x_3) = 500x_1 + 1000x_2 + 1500x_3$$

(b)

$$C(x_1, x_2, x_3) = [(100+175)x_1 + (400+150)x_2 + (750+225)x_3] + 2,50,00,000$$

(c)

$$P(x_1, x_2, x_3) = R(x_1, x_2, x_3) - C(x_1, x_2, x_3)$$

$$P(x_1, x_2, x_3) = (500-275)x_1 + (1000-550)x_2 + (1500-975)x_3 - 25000000$$

$$P(x) = 225x_1 + 450x_2 + 525x_3 - 25000000$$

Example

- 7 Automobile Leasing** A car-leasing agency purchases new cars each year for use in the agency. The cars cost \$15,000 new. They are used for 3 years, after which they are sold for \$4,500. The owner of the agency estimates that the variable costs of operating the cars, exclusive of gasoline, are \$0.18 per mile. Cars are leased for a flat fee of \$0.33 per mile (gasoline not included).
- Formulate the total revenue function associated with renting one of the cars a total of x miles over a 3-year period.
 - Formulate the total cost function associated with renting a car for a total of x miles over 3 years.
 - Formulate the profit function.
 - What is profit if a car is leased for 60,000 miles over a 3-year period?
 - What mileage is required in order to earn zero profit for 3 years?

Thank you

Question Answers Session

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