Introduction to Mathematics

Week 3

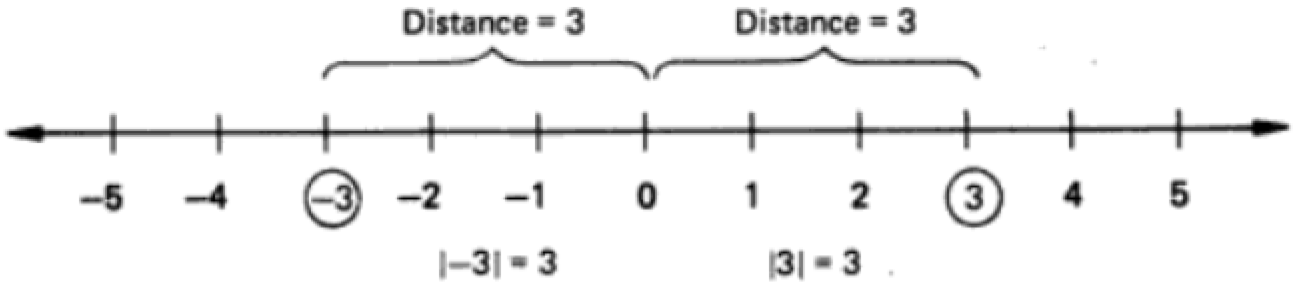
Formation of Equations

- <u>Unit 3.1</u>

<u>Absolute Value Representations</u>

Absolute Value

- The **absolute value** of a number is its distance from zero on a real number line. This distance can be either zero or greater than zero, but never less than zero.
- The absolute value of the number x is denoted by $|x|$
- For example by writing |3|, we are looking for the distance on the number line between the number 3 and the number 0. That distance is 3.
- So |3|= 3
- Similarly, |-3| is also 3, as the distance between the number -3 and 0, is 3.
- Hence |-3| = 3



Properties of Absolute Values

Property 1

$$|a| \geq 0$$

$$|-5| = 5 \geq 0$$

$$|10| = 10 \geq 0$$

$$|0| = 0 \geq 0$$

Property 2

$$|-a| = |a|$$

$$|-4| = |4| = 4$$

Property 3

$$|x - y| = |y - x|$$

$$|12 - 5| = |7| = 7$$

$$|5 - 12| = |-7| = 7$$

Property 4

$$|ab| = |a|\,|b|$$

$$|3(-5)| = |-15| = 15$$

Property 5

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

$$\left|\frac{-25}{10}\right| = \frac{|-25|}{|10|} = \frac{25}{10} = 2.5$$

Solving with Absolute Values

- Suppose we are asked to solve $|x| = 4$, how do we proceed?
- If it was $x = 4$, there is no need to solve, as the value of x is already highlighted.
- However, in solving $|x| = 4$, we cannot say the value of x is 4. It is either 4 or -4
- Using this logic, let's see how to solve with absolute values by looking at some examples.

Example

To solve the equation

$$|x - 5| = 3$$

we know that $x - 5 = \pm 3$. That is, either

$$x - 5 = 3 \qquad \text{or} \qquad x - 5 = -3$$

Solving both equations, we find

$$x = 8 \qquad \text{or} \qquad x = 2$$

To check this result, substitution of the two values into the original equation yields

$$\begin{aligned} |8 - 5| &= 3 \qquad \text{and} \qquad & |2 - 5| &= 3 \\ |3| &= 3 \qquad \text{and} \qquad & |-3| &= 3 \\ 3 &= 3 \qquad \text{and} \qquad & 3 &= 3 \end{aligned}$$

Example 2

$$|10 - 2x| = |x + 5|$$

Notice that the absolute sign is on both sides. Which means they are either of equal signs or opposite signs.

$$10 - 2x = \pm(x + 5)$$

Solving for x under the two conditions gives

$$\begin{aligned} 10 - 2x &= (x + 5) & \text{or} && 10 - 2x &= -(x + 5) \\ 5 &= 3x & \text{or} && 10 - 2x &= -x - 5 \\ \tfrac{5}{3} &= x & \text{or} && 15 &= x \end{aligned}$$

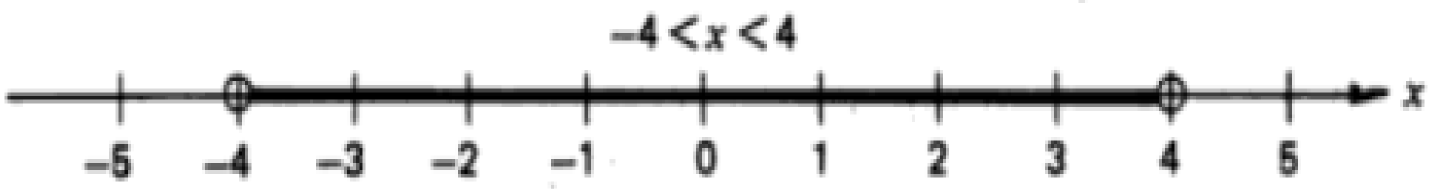
Therefore, the equation is satisfied when $x = \frac{5}{3}$ or 15.

Inequalities with Absolute Values(Examples)

Solve the inequality $|x| < 4$.

SOLUTION

Because $|x|$ represents the distance of x from 0 on a real number line, the solution to this inequality consists of all real numbers with distance from 0 on a real number line less than 4. Figure 1.10 indicates that the values satisfying the inequality are $-4 < x < 4$.



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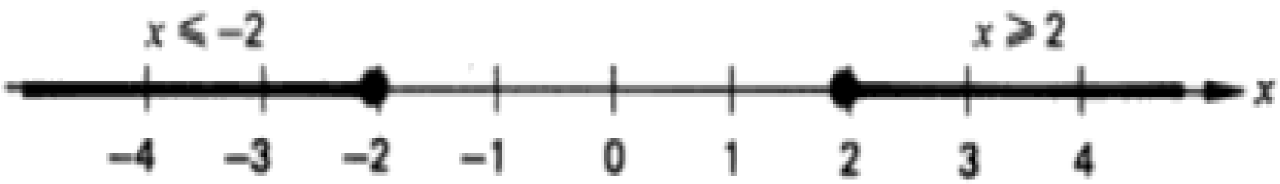
Example 2

Solve the inequality $|x| \geq 2$.

SOLUTION

The values of x which satisfy this inequality consist of all real numbers located 2 or more units from zero on a real number line. Figure 1.11 indicates that the values satisfying the inequality are $x \leq -2$ and $x \geq 2$.



Examples solved in video lectures

IMPORTANT!

- $|7x| \geq 21$
- $|5x + 3| < 7$

Practice Questions

- $|x - 6| = 5$
- $|5 - 3x| = |-2x + 7|$
- $|3x - 10| = |2x - 7|$
- $|2x - 3| > 5$
- $|y - 5| \geq 3$

Helping Material

- https://www.khanacademy.org/math/algebra-home/alg-absolute-value
- https://www.khanacademy.org/math/arithmetic/arith-review-negative-numbers/arith-review-abs-value/v/absolute-value-of-integers
- https://www.khanacademy.org/math/algebra-home/alg-absolute-value/alg-absolute-value-inequalities/v/absolute-value-inequalities

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Thank you