Introduction to Mathematics

Lecture 4

Some of the material in these slides is from:
Calculus 10/E by Howard Anton, Irl Bivens, and Stephen Davis
Applied Mathematics for Business, Economics and Social Sciences by Frank. S Budnick

Lecture Plan:

- Functions
- Domain and Range of a Function

Learning Outcomes:

- Determine whether a relation is a function
- Determine the domain of a function and the range of a function
- Determine whether a graph is that of a function by using a vertical line test
- To apply the concept of Restricted Domain and Restricted Range

Functions

1.1.1 DEFINITION. If a variable y depends on a variable x in such a way that each value of x determines exactly one value of y, then we say that ***y* is a function of *x***.

1.1.2 DEFINITION. A ***function*** f is a rule that associates a unique output with each input. If the input is denoted by x, then the output is denoted by $f(x)$ (read "f of x").

Functions

- It is like a machine that has an input and an output and the output is related somehow to the input.
- We will see many ways to think about functions, but there are always three main parts:
  - The input
  - The relationship
  - The output

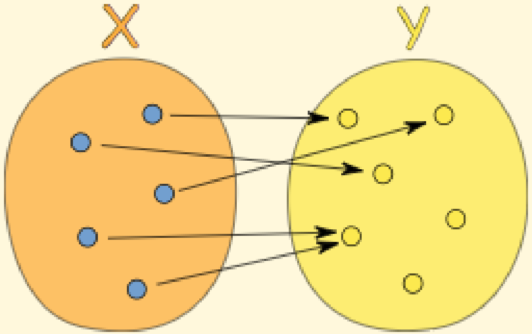
Example: "Multiply by 2" is a very simple function.

Here are the three parts:

Input	***Relationship***	**Output**
0	× 2	0
1	× 2	2
7	× 2	14
10	× 2	20
...	...	...

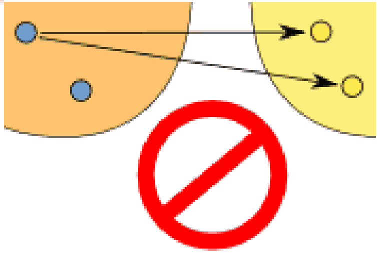
For an input of 50, what is the output?

Functions

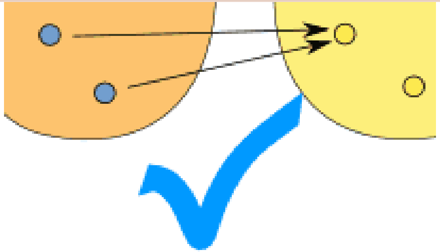


Formal Definition of a Function

A function relates **each element** of a set with **exactly one** element of another set (possibly the same set).



(one-to-many)

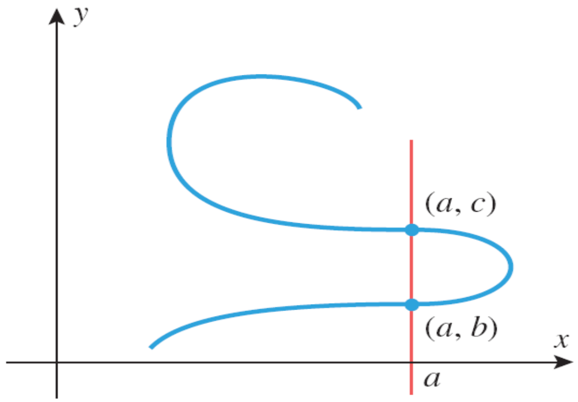


(many-to-one)

Functions

THE VERTICAL LINE TEST

1.1.3 THE VERTICAL LINE TEST. *A curve in the xy-plane is the graph of some function f if and only if no vertical line intersects the curve more than once.*



This curve cannot be the graph of a function

Practice Questions - Exercise 4.1 – Evaluating the Value of the Function

In Exercises 1–16, determine $f(0)$, $f(-2)$, and $f(a+b)$.

1 $f(x) = 5x - 10$

2 $f(x) = 3x + 5$

3 $f(x) = -x + 4$

4 $f(x) = -x/2$

5 $f(x) = mx + b$

6 $f(x) = mx$

7 $f(x) = x^2 - 9$

8 $f(x) = -x^2 + 2x$

9 $f(t) = t^2 + t - 5$

10 $f(r) = tr^2 - ur + v$

11 $f(u) = u^3 - 10$

12 $f(u) = -2u^3 + 5u$

13 $f(n) = n^4$

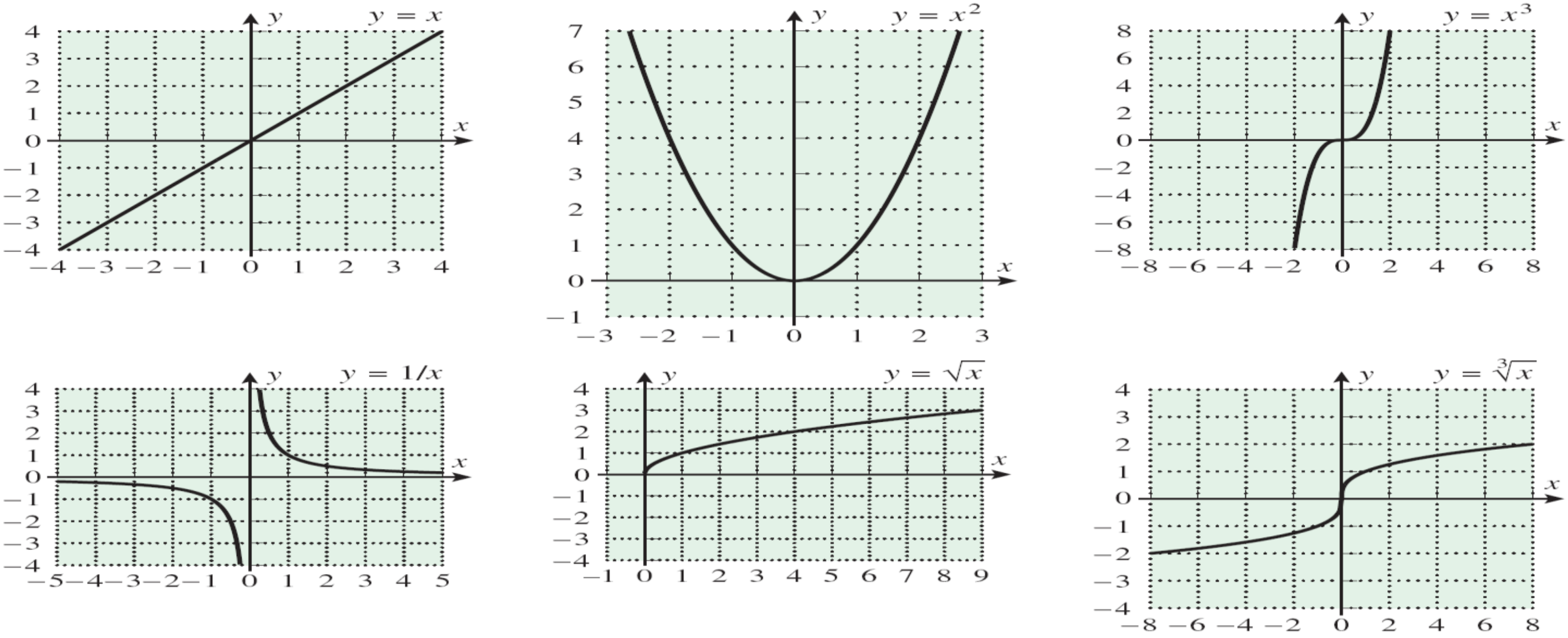
14 $f(t) = 100$

15 $f(x) = x^3 - 2x + 4$

16 $f(x) = 25 - x^2/2$

Functions

GRAPHS OF FUNCTIONS



EXAMPLE 1.2.4

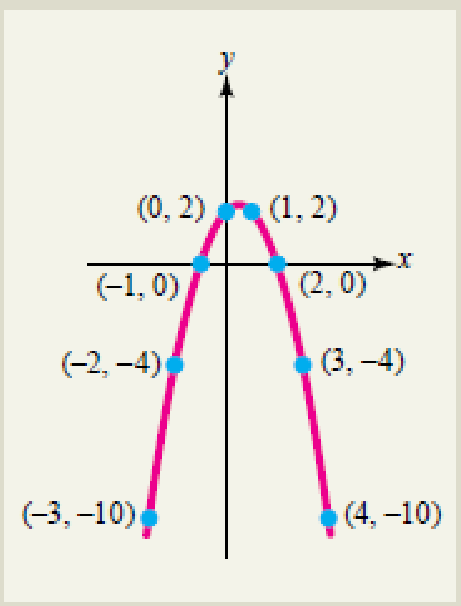
Graph the function $f(x) = -x^2 + x + 2$. Include all x and y intercepts.

Solution

The y intercept is $f(0) = 2$. To find the x intercepts, solve the equation $f(x) = 0$. Factoring, we find that

$$\begin{aligned} -x^2 + x + 2 &= 0 && \text{factor} \\ -(x+1)(x-2) &= 0 && uv = 0 \text{ if and only if } u = 0 \text{ or } v = 0 \\ x = -1,\ x &= 2 \end{aligned}$$

Thus, the x intercepts are $(-1, 0)$ and $(2, 0)$.



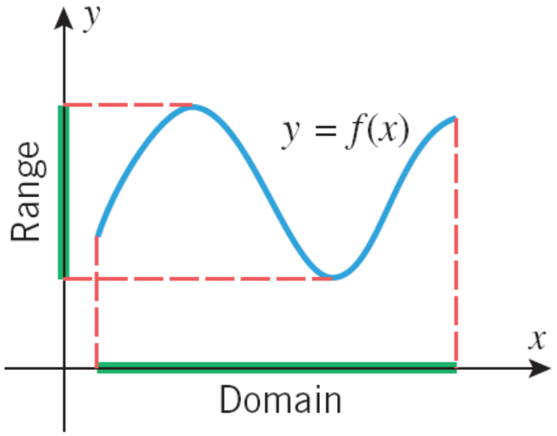
Next, make a table of values and plot the corresponding points $(x, f(x))$.

x	-3	-2	-1	0	1	2	3	4
$f(x)$	-10	-4	0	2	2	0	-4	-10

Domain and Range

If **x** and **y** are related by the equation **y = f(x),** then the set of all allowable inputs **(x-values)** is called the **domain** of **f**, and the set of outputs **(y-values)** that result when **x** varies over the domain is called the **range** of **f**.

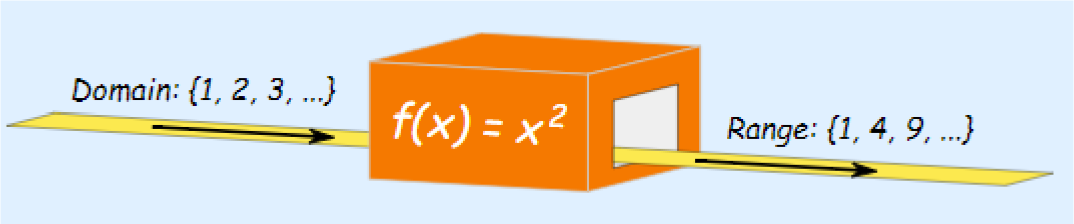
The projection of y = f(x) on the x-axis is the set of allowable x-values for f, and the projection on the y-axis is the set of co rresponding y-values.

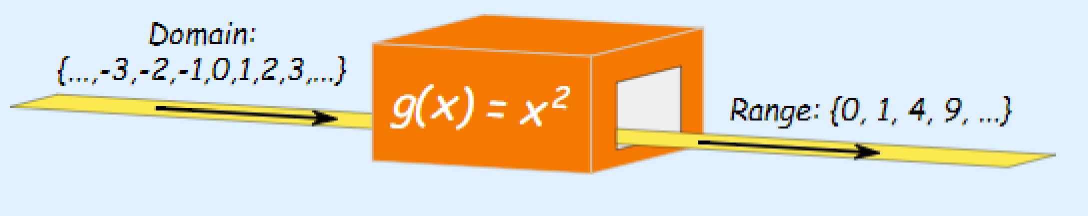


Domain and Range

- The domain is all the values that go into a function (x-values), and the range is all the values that come out (y-values).
- Now, what comes **out** *(the Range)* depends on what we put **in** *(the Domain)* ...

... but **WE** can define the Domain!





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Practice Questions – Exercise 4.1 – Domain of the Function

In Exercises 17–40, determine the domain of the function.

17 $f(x) = -10$

18 $f(x) = 25$

19 $f(x) = 5x - 10$

20 $f(x) = -x + 3$

21 $f(x) = mx + b$

22 $f(x) = -ax$

23 $f(x) = 25 - x^2$

24 $f(x) = x^2 - 4$

25 $f(x) = \sqrt{x + 4}$

26 $f(x) = \sqrt{-2x + 25}$

27 $f(t) = \sqrt{-t - 8}$

28 $f(t) = \sqrt{9 - t^2}$

29 $f(r) = \sqrt{r^2 + 9}$

30 $f(r) = \sqrt{25 - r^2}$

31 $f(x) = 10/(4 - x)$

32 $f(x) = (x - 4)/(x^2 - 6x - 16)$

33 $f(u) = (3u - 5)/(-u^2 + 2u + 5)$

34 $f(t) = \sqrt{-t - 10}/(-3t^3 + 5t^2 + 10t)$

35 $f(x) = \sqrt{2.5x - 20}/(x^3 + 2x^2 - 15x)$

36 $h(v) = \sqrt{10 - v/3}/(v^5 - 81v)$

37 $g(h) = \sqrt{h^2 - 4}/(h^3 + h^2 - 6h)$

38 $f(x) = \sqrt{x^2 - x - 6}$

39 $f(x) = \sqrt{x^2 + 8x + 15}$

40 $h(r) = \sqrt{r^2 - 16}$

Thank you