Changing between polar and Cartesian coordinates

Starter

1. By possibly considering complex number equivalents, convert these points from polar form, (r, θ) to Cartesian form (x, y):

(a) $\left(8, \frac{\pi}{6}\right)$ (b) $\left(6, -\frac{3\pi}{4}\right)$ (c) $\left(5, \frac{\pi}{2}\right)$

Working: (a) $\left(8, \frac{\pi}{6}\right) \equiv \left(8\cos\frac{\pi}{6}, 8\sin\frac{\pi}{6}\right) = (4\sqrt{3}, 4)$

(b) $\left(6, -\frac{3\pi}{4}\right) \equiv \left(6\cos\left(-\frac{3\pi}{4}\right), 6\sin\left(-\frac{3\pi}{4}\right)\right)$
$= (-3\sqrt{2}, -3\sqrt{2})$

(c) $\left(5, \frac{\pi}{2}\right) \equiv \left(5\cos\frac{\pi}{2}, 5\sin\frac{\pi}{2}\right) = (0, 5)$

2. By possibly considering complex number equivalents, convert these points from Cartesian form (x, y) to polar form, (r, θ):

(a) $(1, \sqrt{3})$ (b) $(-4, 4)$ (c) $(0, -6)$

Working: (a) $r = \sqrt{1^2 + (\sqrt{3})^2} = 2$

$\theta = \tan^{-1}\frac{\sqrt{3}}{1} = \frac{\pi}{3}$

$(1, \sqrt{3}) \equiv \left(2, \frac{\pi}{3}\right)$

(b) $r = \sqrt{(-4)^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

$(-4, 4)$ is in the 2nd quadrant so $\theta = \pi \ \tan^{-1}\frac{\sqrt{4}}{4} = \frac{3\pi}{4}$

$(-4, 4) \equiv \left(4\sqrt{2}, \frac{3\pi}{4}\right)$

(c) $r = 6$

The point is on the negative $y-$axis so $\theta = \frac{3\pi}{2}$

$(0, -6) \equiv \left(6, \frac{3\pi}{2}\right)$

E.g. 1 Convert $(x^2 + y^2)^2 = 4xy$ into polar form and state the family of curves from which it comes.

Working: $(x^2 + y^2)^2 = 4xy$:

$$
\begin{aligned}
(x^2 + y^2)^2 &= 4xy \\
(r^2)^2 &= 4r\cos\theta \times r\sin\theta \\
r^4 &= 2r^2\sin 2\theta \\
r^2 &= 2\sin 2\theta
\end{aligned}
$$

E.g. 2 Express these Cartesian equations in polar form:

(a) $y = x^2$ (b) $x \cos\alpha + y \sin\alpha = p$, where $p > 0$

Working: (a) $y = x^2$:

$$r \sin\theta = r^2 \cos^2\theta$$
$$r = \frac{\sin\theta}{\cos^2\theta}$$
$$r = \sec\theta \tan\theta$$

(b) $x \cos\alpha + y \sin\alpha = p$:

$$r\cos\theta\cos\alpha + r\sin\theta\sin\alpha = p$$
$$r(\cos\theta\cos\alpha + \sin\theta\sin\alpha) = p$$
$$r\cos(\theta - \alpha) = p$$

E.g. 3 Convert the polar equation $r = 2a\cos\theta$ to Cartesian form.

Working: $r = 2a\cos\theta$:

$$\sqrt{x^2 + y^2} = 2a \times \frac{x}{r}$$
$$x^2 + y^2 = 2ax$$

E.g. 4 Express these polar equations in Cartesian form:

(a) $r^2 = a^2 \sin 2\theta$ (b) $r = 2\sin\theta$ for $0 \le \theta < \pi$

Working: (a) $r^2 = a^2 \sin 2\theta$:

$$x^2 + y^2 = 2a^2 \cos\theta \sin\theta$$
$$x^2 + y^2 = 2a^2 \times \frac{x}{r} \times \frac{y}{r}$$
$$(x^2 + y^2)^2 = 2a^2xy$$

(b) $r = 2\sin\theta$:

$$\sqrt{x^2 + y^2} = 2 \times \frac{y}{r}$$
$$x^2 + y^2 = 2y$$

Video: Converting Cartesian to polar coordinates
Video: Converting polar to Cartesian coordinates
Video: Converting polar equations to Cartesian equations
Video: Converting Cartesian equations to polar equations

Solutions to Starter and E.g.s

Exercise

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