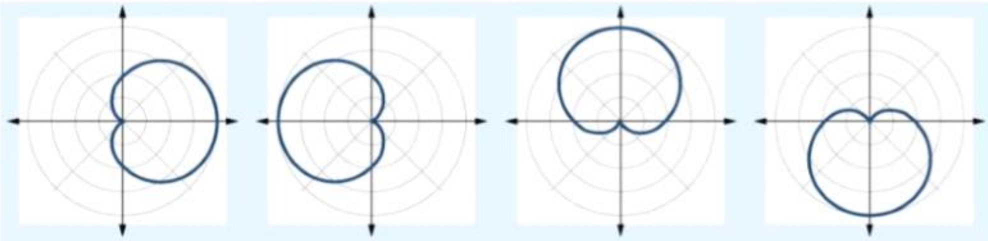
Parametric Curve & Trace

Monday, 12 January 2026 11:25 am

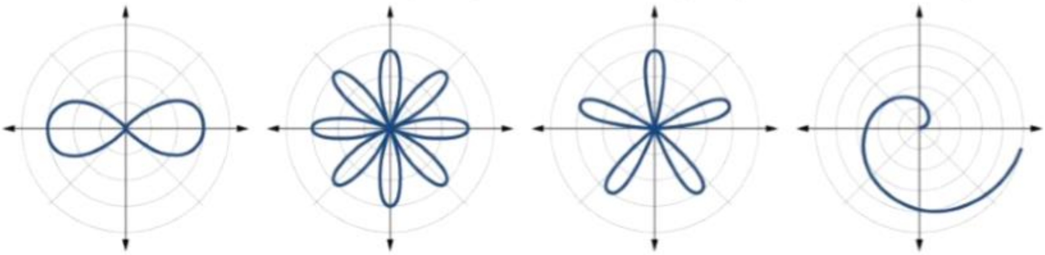
Famous Polar Graphs

Friday, 26 January 2024 11:06 am



$r = a + b\cos\theta$ (a) | $r = a - b\cos\theta$ (b) | $r = a + b\sin\theta$ (c) | $r = a - b\sin\theta$ (d)

Lemniscate | Rose Curve (*n even*) | Rose Curve (*n odd*) | Archimedes' Spiral

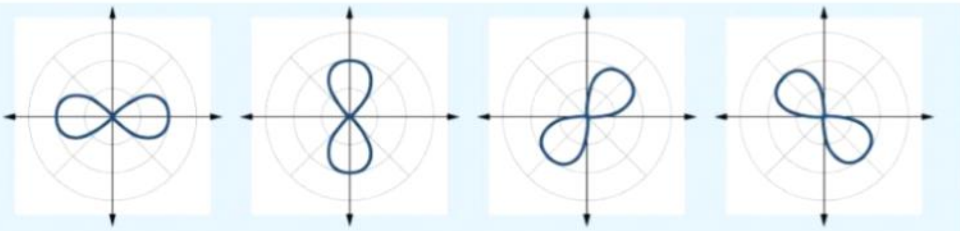


(a) $r^2 = a^2\cos 2\theta$, $r^2 = a^2\sin 2\theta$, $a \neq 0$

(b) $r = a\cos n\theta$, $r = a\sin n\theta$, n even, $2n$ petals

(c) $r = a\cos n\theta$, $r = a\sin n\theta$, n odd, n petals

(d) $r = \theta$, $\theta \geq 0$



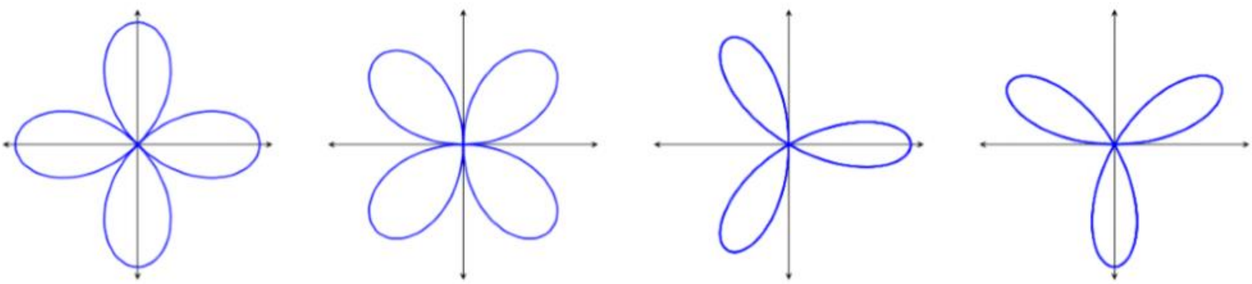
$r^2 = a^2 cos(2\theta)$ (a) | $r^2 = -a^2 cos(2\theta)$ (b) | $r^2 = a^2 sin(2\theta)$ (c) | $r^2 = -a^2 sin(2\theta)$ (d)

Rose Curves

Symmetric about x-axis: $r = a\cos(n\theta)$; Symmetric about y-axis: $r = a\sin(n\theta)$

Curve contains $2n$ petals when n is even and n petals when n is odd.

$r = a\cos(2\theta)$ | $r = a\sin(2\theta)$ | $r = a\cos(3\theta)$ | $r = a\sin(3\theta)$



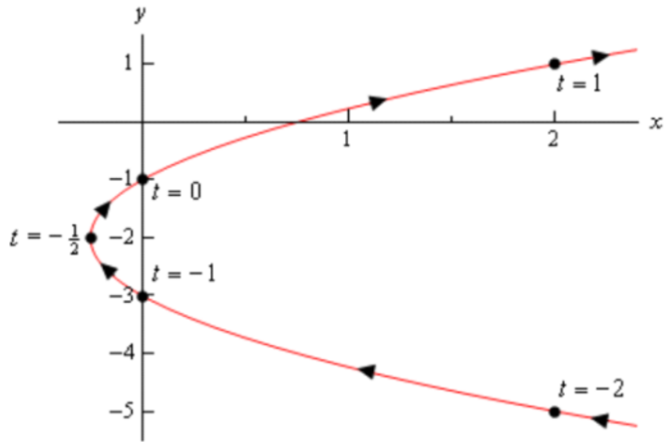
Key Reasons for Using Parametric Equations:

- **Modeling Motion & Time:** They excel at describing how an object's position (x, y) changes over time (t), capturing direction and speed, crucial in physics and kinematics.
- **Describing Non-Function Curves:** They can represent curves that fail the vertical line test (like circles, ellipses, or self-intersecting paths) that traditional $y = f(x)$ equations can't.
- **Simplifying Complex Systems:** They break down complex movements into simpler, independent components, making calculations and analysis easier (e.g., projectile motion with independent horizontal and vertical components).
- **Adding Direction & Control:** The parameter (like 't') acts as a clock, allowing you to trace the curve in a specific order and control the drawing speed, essential for computer graphics and animations.
- **Computer Graphics & Imaging:** They provide a natural way to define curves for rendering and are used in medical imaging (MRI/CT) to reconstruct 3D shapes.
- **Versatility:** Multiple different parametric equations can describe the same curve, offering flexibility in how you represent a path.

Example 1 Sketch the parametric curve for the following set of parametric equations.

$$x = t^2 + t \qquad y = 2t - 1$$

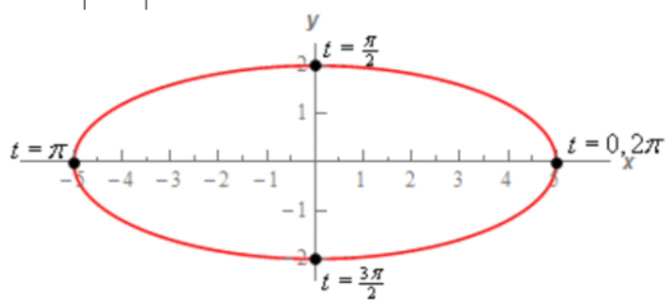
t	x	y
-2	2	-5
-1	0	-3
$-\frac{1}{2}$	$-\frac{1}{4}$	-2
0	0	-1
1	2	1



Example 3 Eliminate the parameter from the following set of parametric equations.

$$x = t^2 + t \qquad y = 2t - 1$$

t	x	y
0	5	0
$\frac{\pi}{2}$	0	2
π	-5	0
$\frac{3\pi}{2}$	0	-2
2π	5	0



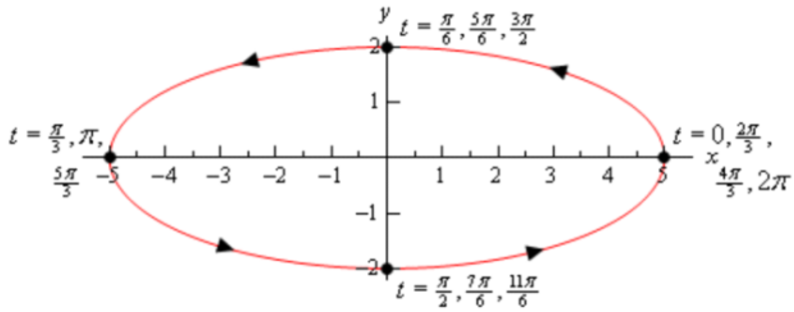
$$\cos t = \frac{x}{5} \qquad \sin t = \frac{y}{2}$$

and these equations we get,

$$1 = \cos^2 t + \sin^2 t = \left(\frac{x}{5}\right)^2 + \left(\frac{y}{2}\right)^2 = \frac{x^2}{25} + \frac{y^2}{4}$$

Example 5 Sketch the parametric curve for the following set of parametric equations. Clearly indicate direction of motion.

$$x = 5\cos(3t) \qquad y = 2\sin(3t) \qquad 0 \le t \le 2\pi$$



$x = \sin^2 t \qquad y = 2\cos t$

$x = 3\cos(2t) \qquad y = 1 + \cos^2(2t)$