Monday, 10 June 2024 4:29 pm

Sketching Region of Integration

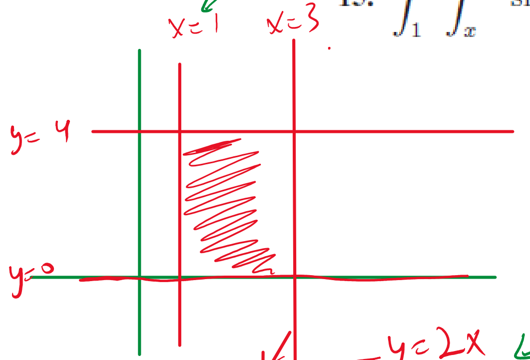
For Exercises 13–16, sketch the region of integration and evaluate the integral.

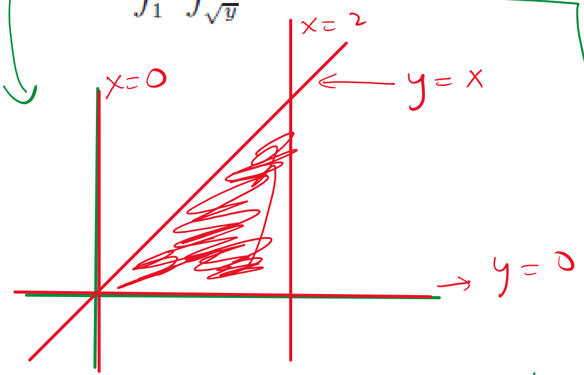
13. $\displaystyle\int_1^3\int_0^4 e^{x+y}\,dy\,dx$

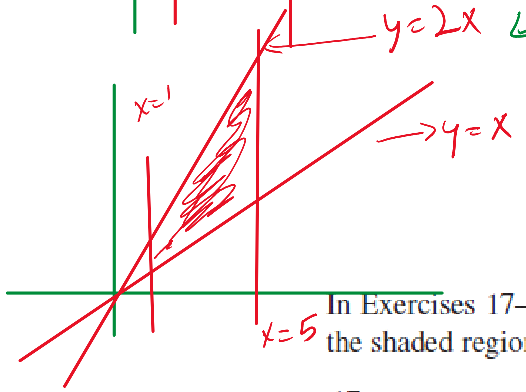
14. $\displaystyle\int_0^2\int_0^x e^{x^2}\,dy\,dx$

15. $\displaystyle\int_1^5\int_x^{2x} \sin x\,dy\,dx$

16. $\displaystyle\int_1^4\int_{\sqrt{y}}^{y} x^2y^3\,dx\,dy$





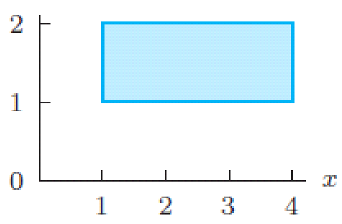


y=4
y=1
x=√y
x=y

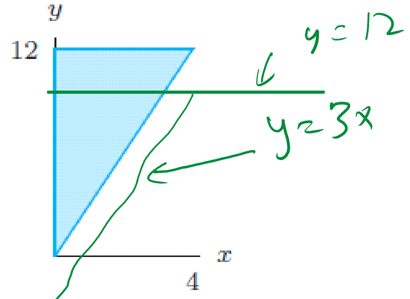
Use Desmos for Graphing

In Exercises 17–22, write $\int_R f\,dA$ as an iterated integral for the shaded region R.

17.



18.



$$\int_1^2\int_1^4 f(x,y)\,dx\,dy$$

$$\int_0^4\int_{3x}^{12} f(x,y)\,dx\,dy$$

Double Integrals Over General Region

$$\iint_D f(x,y)\,dA$$

where D is any region.

There are two types of regions that we need to look at. Here is a sketch of both of them.

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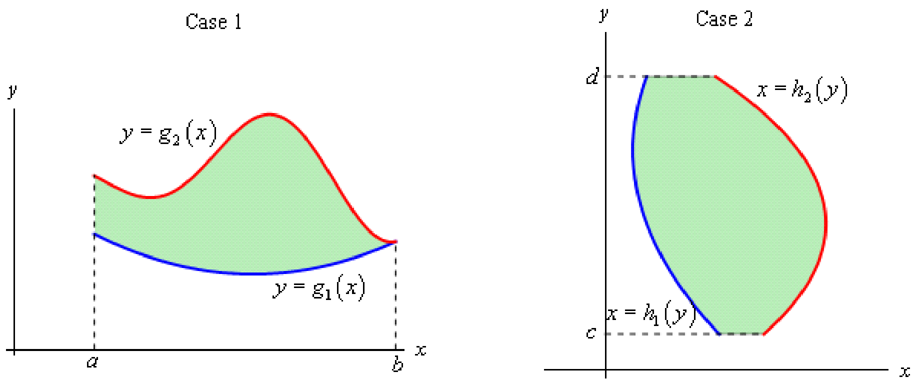


We will often use *set builder notation* to describe these regions. Here is the definition for the region in Case 1

$$D = \{(x,y)\,|a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$$

and here is the definition for the region in Case 2.

$$D = \{(x,y)\,|h_1(y) \le x \le h_2(y),\ c \le y \le d\}$$

In Case 1 where $D = \{(x,y)\,|a \le x \le b,\ g_1(x) \le y \le g_2(x)\}$ the integral is defined to be,

$$\iint_D f(x,y)\, dA = \int_a^b \int_{g_1(x)}^{g_2(x)} f(x,y)\, dy\, dx$$

In Case 2 where $D = \{(x,y)\,|h_1(y) \le x \le h_2(y),\ c \le y \le d\}$ the integral is defined to be,

$$\iint_D f(x,y)\, dA = \int_c^d \int_{h_1(y)}^{h_2(y)} f(x,y)\, dx\, dy$$

LIMITS ON ITERATED INTEGRALS:

- The limits on the outer integral must be constants.
- The limits on the inner integral can involve only the variable in the outer integral. For example, if the inner integral is with respect to x, its limits can be functions of y.

Find the mass M of a metal plate R bounded by $y = x$ and $y = x^2$, with density given by $\delta(x,y) = 1 + xy$ kg/meter^2. (See Figure 16.17.)

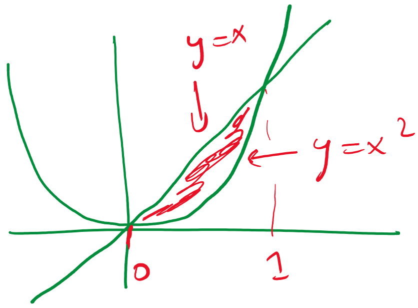
Density in 2D $\quad D = \dfrac{M\ (\text{Mass})}{A\ (\text{Area})} \Rightarrow M = D \times A \rightarrow \iint dA$

$y = x$

$$M = \int_0^1 \int_{x^2}^{x} \delta(x,y)\,dy\,dx$$

$$\int_0^1 \int_{x^2}^{x} (1+xy)\,dy\,dx$$

$$\int_0^1 \left(y + \frac{xy^2}{2}\right)\Big|_{x^2}^{x} dx$$



$y=x$ & $y=x^2$

$\Rightarrow x=x^2 \Rightarrow x-x^2=0$

$\Rightarrow x(1-x)=0$

$\Rightarrow x=0,\ x=1$

$$\Rightarrow \int_0^1 \left(\left(x + \frac{x(x)^2}{2}\right) - \left(x^2 + \frac{x(x^2)^2}{2}\right)\right) dy$$

$$\Rightarrow \int_0^1 \left(x + \frac{x^3}{2} - x^2 - \frac{x^5}{2}\right) dy$$

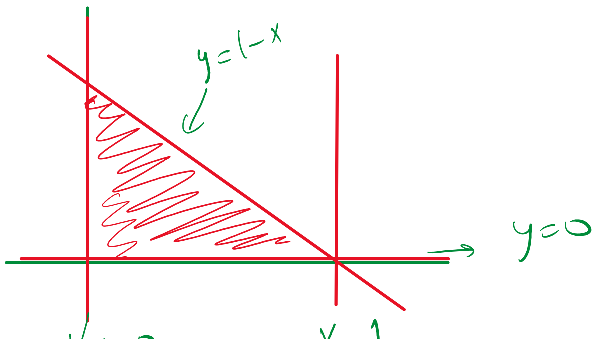
$$\Rightarrow \frac{x^2}{2} + \frac{x^4}{8} - \frac{x^3}{3} - \frac{x^6}{12}\Big|_0^1 \Rightarrow \frac{1}{2} + \frac{1}{8} - \frac{1}{3} - \frac{1}{12}$$

$\Rightarrow$

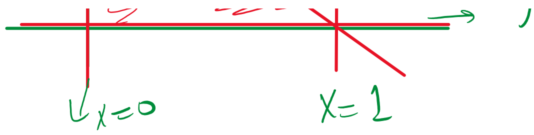
$x=0,\ x=1$

Sketch the region of integration for $\int_0^1 \int_0^{1-x} (1-x-y)\,dy\,dx$:

$y=0,\ y=1-x$



$$\int^1 \int^{1-x} (1-x-y)\,dy\,dx$$



$$\int_0^1\int_0^{1-x}(1-x-y)\,dy\,dx$$

Do Integration By yourself

Reversing the Order of Integration

It is sometimes helpful to reverse the order of integration in an iterated integral. An integral which is difficult or impossible with the integration in one order can be quite straightforward in the other. The next example is such a case.

Evaluate $\displaystyle\int_0^6\int_{x/3}^2 x\sqrt{y^3+1}\,dy\,dx$ using the region sketched in Figure

Sketch Region & compute

Properties

1. $\displaystyle\iint\limits_D f(x,y)+g(x,y)\,dA = \iint\limits_D f(x,y)\,dA + \iint\limits_D g(x,y)\,dA$

2. $\displaystyle\iint\limits_D cf(x,y)\,dA = c\iint\limits_D f(x,y)\,dA$, where c is any constant.

3. If the region D can be split into two separate regions D_1 and D_2 then the integral can be written as

$$\iint\limits_D f(x,y)\,dA = \iint\limits_{D_1} f(x,y)\,dA + \iint\limits_{D_2} f(x,y)\,dA$$