Assignment 1

Friday, 2 August 2024 10:08 am

Q1.

The temperature T (in °C) at any point in the region $-10 \leq x \leq 10$, $-10 \leq y \leq 10$ is given by the function

$$T(x, y) = 100 - x^2 - y^2.$$

(a) Sketch isothermal curves (curves of constant temperature) for $T = 100°\text{C}$, $T = 75°\text{C}$, $T = 50°\text{C}$, $T = 25°\text{C}$, and $T = 0°\text{C}$.

(b) A heat-seeking bug is put down at a point on the xy-plane. In which direction should it move to increase its temperature fastest? How is that direction related to the level curve through that point?

Q2.

Match the functions (a)–(d) with the shapes of their level curves (I)–(IV). Sketch each contour diagram.

(a) $f(x, y) = x^2$ **(b)** $f(x, y) = x^2 + 2y^2$

(c) $f(x, y) = y - x^2$ **(d)** $f(x, y) = x^2 - y^2$

I. Lines II. Parabolas

III. Hyperbolas IV. Ellipses

(Hint: Use https://www.desmos.com/calculator)

Q3.

A manufacturer sells two goods, one at a price of \$3000 a unit and the other at a price of \$12,000 a unit. A quantity q_1 of the first good and q_2 of the second good are sold at a total cost of \$4000 to the manufacturer.

(a) Express the manufacturer's profit, π, as a function of q_1 and q_2.

(b) Sketch curves of constant profit in the $q_1 q_2$-plane for $\pi = 10{,}000$, $\pi = 20{,}000$, and $\pi = 30{,}000$ and the break-even curve $\pi = 0$.

Q4.

Show that the function f does not have a limit at $(0, 0)$ by examining the limits of f as $(x, y) \to (0, 0)$ along the line $y = x$ and along the parabola $y = x^2$:

$$f(x, y) = \frac{x^2 y}{x^4 + y^2}, \qquad (x, y) \neq (0, 0).$$

Q5.

By approaching the origin along the positive x-axis and the positive y-axis, show that the following limit does not exist:

$$\lim_{(x,y)\to(0,0)} \frac{x+y^2}{2x+y}.$$

Q6.

Explain why the following function is not continuous along the line $y = 0$:

$$f(x,y) = \begin{cases} 1-x, & y \geq 0, \\ -2, & y < 0. \end{cases}$$

Q7.

What value of c makes the following function continuous at $(0,0)$?

$$f(x,y) = \begin{cases} x^2 + y^2 + 1 & \text{if } (x,y) \neq (0,0) \\ c & \text{if } (x,y) = (0,0) \end{cases}$$

Q8.

. Let $f(x,y) = \begin{cases} \dfrac{|x|}{x} y & \text{for } x \neq 0 \\ 0 & \text{for } x = 0. \end{cases}$

Is $f(x,y)$ continuous

(a) On the x-axis? **(b)** On the y-axis?

(c) At $(0,0)$?

Q9.

find the domain of the given function.

$$f(x,y) = \frac{1}{x} + \sqrt{y+4} - \sqrt{x+1}$$

$$f(x,y) = \ln(2x - 3y + 1)$$

$$f(x,y,z) = \frac{1}{x^2 + y^2 + 4z}$$

Example 1 Let $f(x, y) = \dfrac{x^2}{y+1}$. Find $f_x(3, 2)$ algebraically.

Example 2 Compute the partial derivatives with respect to x and with respect to y for the following functions.
(a) $f(x, y) = y^2 e^{3x}$ (b) $z = (3xy + 2x)^5$ (c) $g(x, y) = e^{x+3y} \sin(xy)$

Example 3 Find all the partial derivatives of $f(x, y, z) = \dfrac{x^2 y^3}{z}$.

Money in a bank account earns interest at a continuous rate, r. The amount of money, $\$B$, in the account depends on the amount deposited, $\$P$, and the time, t, it has been in the bank according to the formula

$$B = Pe^{rt}.$$

Find $\partial B/\partial t$ and $\partial B/\partial P$ and interpret each in financial terms.

The acceleration g due to gravity, at a distance r from the center of a planet of mass m, is given by

$$g = \frac{Gm}{r^2},$$

where G is the universal gravitational constant.

(a) Find $\partial g/\partial m$ and $\partial g/\partial r$.
(b) Interpret each of the partial derivatives you found in part (a) as the slope of a graph in the plane and sketch the graph.

The Dubois formula relates a person's surface area, s, in m^2, to weight, w, in kg, and height, h, in cm, by

$$s = f(w, h) = 0.01w^{0.25}h^{0.75}.$$

Find $f(65, 160)$, $f_w(65, 160)$, and $f_h(65, 160)$. Interpret your answers in terms of surface area, height, and weight.

The energy, E, of a body of mass m moving with speed v is given by the formula

$$E = mc^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right).$$

The speed, v, is nonnegative and less than the speed of light, c, which is a constant.

(a) Find $\partial E/\partial m$. What would you expect the sign of $\partial E/\partial m$ to be? Explain.
(b) Find $\partial E/\partial v$. Explain what you would expect the sign of $\partial E/\partial v$ to be and why.

Example 1 Determine if $f(x, y) = \dfrac{x^2}{y^3}$ is increasing or decreasing at $(2, 5)$,

(a) if we allow x to vary and hold y fixed.

(b) if we allow y to vary and hold x fixed.

Determine if $f(x, y) = x\ln(4y) + \sqrt{x + y}$ is increasing or decreasing at $(-3, 6)$ if

(a) we allow x to vary and hold y fixed.

(b) we allow y to vary and hold x fixed.

Find all the second order derivatives for $f(x, y) = \cos(2x) - x^2\mathbf{e}^{5y} + 3y^2$.

Example 1 Find y' for $xy = 1$.

Example 4 Find the equation of the tangent line to

$$x^2 + y^2 = 9$$

at the point $\left(2,\ \sqrt{5}\right)$.

Example 5 Find y' for each of the following.

(a) $x^3y^5 + 3x = 8y^3 + 1$

Example 2 Suppose that $z = f(x, y) = x \sin y$, where $x = t^2$ and $y = 2t + 1$. Let $z = g(t)$. Compute $g'(t)$ directly and using the chain rule.

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Example 4 Let $w = x^2e^y$, $x = 4u$, and $y = 3u^2 - 2v$. Compute $\partial w/\partial u$ and $\partial w/\partial v$ using the chain rule.

Example 3 The capacity, C, of a communication channel, such as a telephone line, to carry information depends on the ratio of the signal strength, S, to the noise, N. For some positive constant k,

$$C = k \ln\left(1 + \frac{S}{N}\right).$$

Suppose that the signal and noise are given as a function of time, t in seconds, by

$$S(t) = 4 + \cos(4\pi t) \qquad N(t) = 2 + \sin(2\pi t).$$

What is dC/dt one second after transmission started? Is the capacity increasing or decreasing at that instant?

EXAMPLE: If $z = x^2 y + 3xy^4$, where $x = \sin 2t$ and $y = \cos t$, Find $\frac{dz}{dt}$ when $t = 0$.

EXAMPLE: $z = e^x \sin y$ where $x = st^2$ and $y = s^2 t$. Find dz/ds and dz/dt

Q $f(x,y) = e^{xy}$

when $x(u,v) = 3u \sin v$

and $y(u,v) = 4v^2 u$

Find $f_u(x,y)$ and $f_v(x,y)$