Mid Exam Summer 2024

Tuesday, 3 September 2024 3:21 pm

IQRA UNIVERSITY IU

Mid-Term Examination Summer-24

Subject:	**Multivariate Calculus**	**Date:**	**Thursday**
Instructor:	**Dr. Muhammad Sami Siddiqui**	**Day:**	**29/08/2024**
Program:	**BS (CS)**	**Time Slot:**	**15:30 to 17:00**
No. of Students:	**28**	**Duration:**	**1.5 Hours**
Section Code:	**032407029**	**Max. Marks**	**25**

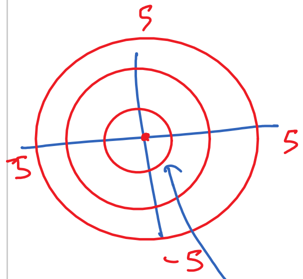
Instructions:

1. Attempt all questions in the answer sheet provided to you and return the question paper after the exam.
2. Please do not use pencils except for underlining or drawing diagrams.
3. Any attempt to use unfair means will disqualify you from the examination.
4. Students are not supposed to ask questions after the first fifteen minutes of the commencement of the exam.
5. The invigilator will not return the answer script to the candidate in any case once it is submitted.
6. All students must bring their own stationery and calculators. Borrowing in the examination hall is strictly prohibited.
7. All students shall comply with any other instruction, written or oral, given by the examiner/invigilator in the examination hall.
8. Marks of each question are mentioned at the end of each question.
9. Please follow any other instructions the invigilator/examiner provided in the question paper.

Instructor's Signature

Question 1 **[4+3+5 Marks]**

a) The temperature T (in °C) at any point in the region $-5 \le x \le 5$, $-5 \le y \le 5$ is given by the function $T(x,y) = 25 - x^2 - y^2$. **Picture graphically** the isothermal curves (curves of constant temperature) for $T = 25$°C, $T = 9$°C, $T = 16$°C, $T = 21$°C, and $T = 0$°C.



$25 = 25 - x^2 - y^2$

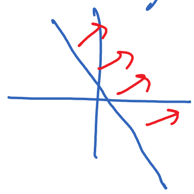
$x^2 + y^2 = 0$

$21 = 25 - x^2 - y^2 \Rightarrow x^2 + y^2 = 4 = 2^2$

b) **Describe** the domain and sketch.

$$f(x,y) = 1/(\ln(2x - 3y + 1)$$

b) $(2x - 3y + 1) > 0 \Rightarrow y < \dfrac{2x+1}{3}$



c) **Compute** the limit & investigate whether it is continuous or not.

i.

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2 + y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

Limit zero & continous

ii.

$$g(x,y) = \begin{cases} \dfrac{x^2}{x^2 + y^2}, & (x,y) \neq (0,0), \\ 0, & (x,y) = (0,0). \end{cases}$$

→ limit doesnot exist

not continous

[CLO 1]

Question 2 **[3+3+3+4 Marks]**

a) The length and width of the rectangle are 10 cm and 20 cm, with an error of 0.2 in length and 0.05 in width. Use differential to **analyze** the total error in a rectangle.

$\Delta A = y\,\Delta x + x\,\Delta y$

$= 20 \times (0.2) + 10 \times (0.05)$

b) Mango production (M) depends on annual rainfall (R) and average temperature (T), so $M = f(R, T)$. Global warming predicts that both rainfall and temperature depend on time. According to a particular global warming model, rainfall decreases at 0.2 cm per year, and temperature increases at 0.1°C per year. Use that at current production levels., $f_R = 3.3$ and $f_T = -5$, to **figure out** the current rate of change, dM/dt, using the chain rule.

$\dfrac{dM}{dt} = \dfrac{\partial M}{\partial R}\dfrac{dR}{dt} + \dfrac{\partial M}{\partial T}\dfrac{dT}{dt}$

$= 3.3\,(-0.2) + (-5) \times (0.1)$

c) **Figure out** $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for the following implicit function $x^2 + y^2 + z^2 + 3xyz = 2$.

$\dfrac{\partial z}{\partial x} = -\dfrac{(2x + 3yz)}{(2z + 3xy)}$

$\dfrac{\partial z}{\partial y} = -\dfrac{(2y + 3xz)}{(2z + 3yz)}$

d) Let $f(x,y,z) = x\sin(yz)$, **examine** $D_u f\,(1,3,1)$ in the direction of $v = i + 2j - k$.

[CLO 2]

CLO No.	Mapped GAs	BT (Domain-level)	Question No
1	2 (Knowledge for Solving Computing Problems)	C2	1
2	3 (Problem Analysis)	C4	2

$\nabla f = f_x \hat{i} + f_y \hat{j} + f_z \hat{k}$

$= \sin(yz)\hat{i} + xz\sin(yz)\hat{j} + xy\sin(yz)\hat{k}$

$= \sin(3)\hat{i} + \sin(3)\hat{j} + 3\sin(3)\hat{k}$

BT=Bloom's Taxonomy; C= Cognitive Domain

$\bar{u} = \dfrac{\hat{i}}{\sqrt{5}} + \dfrac{2}{\sqrt{5}}\hat{j} - \dfrac{1}{\sqrt{5}}\hat{k}$

$D_u f = \nabla f \cdot \bar{u} = \dfrac{\sin(3)}{\sqrt{3}} + 2\dfrac{\sin(3)}{\sqrt{5}} - 3\dfrac{\sin(3)}{\sqrt{5}}$

$= 0$