Multivariable Function, Sketching & Domain

Sunday, 6 October 2024 1:40 am

14.1 Functions of Several Variables

Equation y=c in 1D vs 2D vs 3D

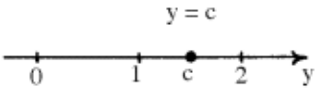


FIGURE 5

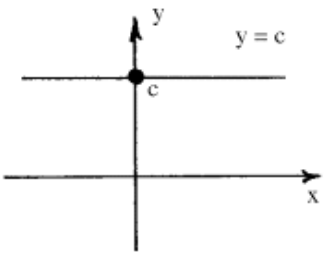


FIGURE 6

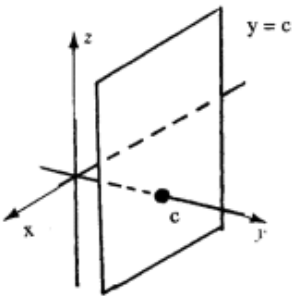


FIGURE 7

In this section we study functions of two or more variables from four points of view:

- verbally (by a description in words)
- numerically (by a table of values)
- algebraically (by an explicit formula)
- visually (by a graph or level curves)

Functions of Two Variables

The temperature T at a point on the surface of the earth at any given time depends on the longitude x and latitude y of the point. We can think of T as being a function of the two variables x and y, or as a function of the pair (x, y). We indicate this functional dependence by writing $T = f(x, y)$.

The volume V of a circular cylinder depends on its radius r and its height h. In fact, we know that $V = \pi r^2 h$. We say that V is a function of r and h, and we write $V(r, h) = \pi r^2 h$.

Definition A **function f of two variables** is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is, $\{f(x, y) \mid (x, y) \in D\}$.

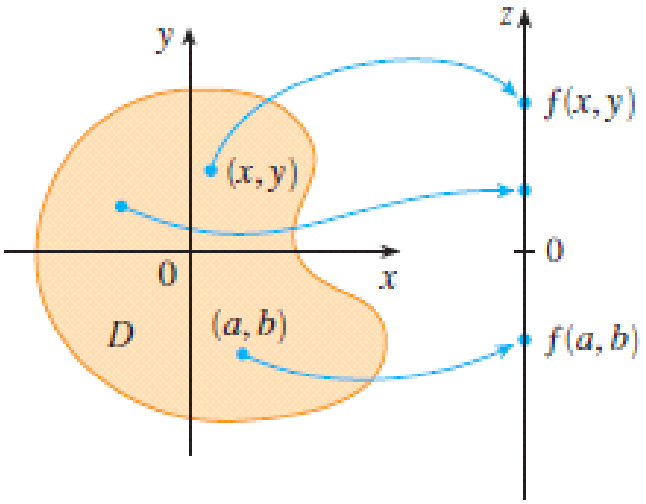


FIGURE 1

Numerical Example: Beef Consumption

Table 12.1 *Quantity of beef bought (pounds/household/week)*

Household income per year, I ($1000)	Price of beef ($/lb) 3.00	3.50	4.00	4.50
20	2.65	2.59	2.51	2.43
40	4.14	4.05	3.94	3.88
60	5.11	5.00	4.97	4.84
80	5.35	5.29	5.19	5.07
100	5.79	5.77	5.60	5.53

Algebraic Examples: Formulas

Example 2 Give a formula for the function $M = f(B, t)$ where M is the amount of money in a bank account t years after an initial investment of B dollars, if interest is accrued at a rate of 1.2% per year compounded annually.

Solution Annual compounding means that M increases by a factor of 1.012 every year, so

$$M = f(B, t) = B(1.012)^t.$$

Example 3 A cylinder with closed ends has radius r and height h. If its volume is V and its surface area is A, find formulas for the functions $V = f(r, h)$ and $A = g(r, h)$.

Solution Since the area of the circular base is πr^2, we have

$$V = f(r, h) = \text{Area of base} \cdot \text{Height} = \pi r^2 h.$$

The surface area of the side is the circumference of the bottom, $2\pi r$, times the height h, giving $2\pi rh$. Thus,

$$A = g(r, h) = 2 \cdot \text{Area of base} + \text{Area of side} = 2\pi r^2 + 2\pi rh.$$

A Tour of 3-Space

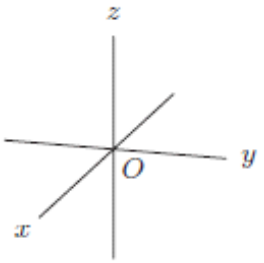


Figure 12.2: Coordinate axes in three-dimensional space

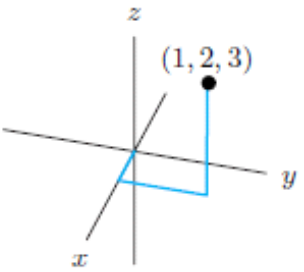


Figure 12.3: The point $(1, 2, 3)$ in 3-space

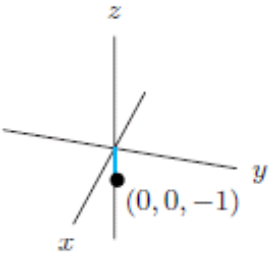


Figure 12.4: The point $(0, 0, -1)$ in 3-space

Graphing Equations in 3-Space

Example 6 What do the graphs of the equations $z = 0$, $z = 3$, and $z = -1$ look like?

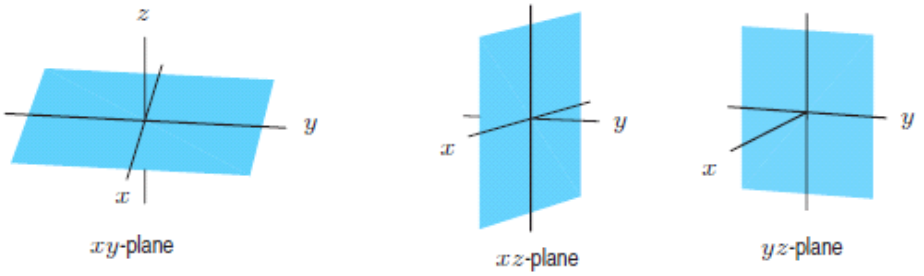


Figure 12.7: The three coordinate planes

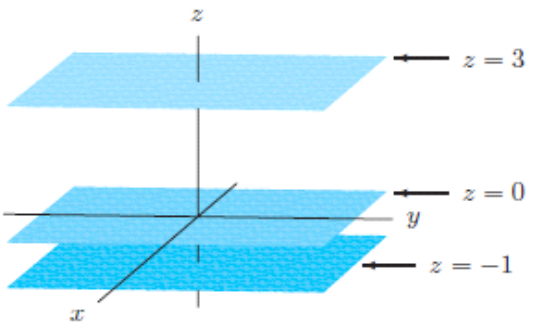


Figure 12.6: The planes $z = -1$, $z = 0$, and $z = 3$

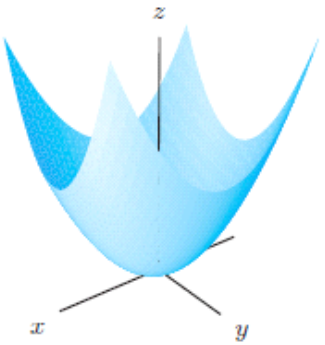


Figure 12.13: Graph of $f(x, y) = x^2 + y^2$ for $-3 \le x \le 3, -3 \le y \le 3$

Example 1 Let $f(x, y) = x^2 + y^2$. Describe in words the graphs of the following functions:
(a) $g(x, y) = x^2 + y^2 + 3$, (b) $h(x, y) = 5 - x^2 - y^2$, (c) $k(x, y) = x^2 + (y - 1)^2$.

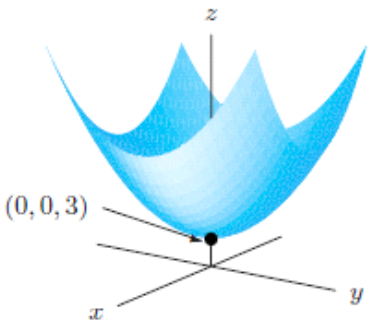


Figure 12.14: Graph of $g(x, y) = x^2 + y^2 + 3$

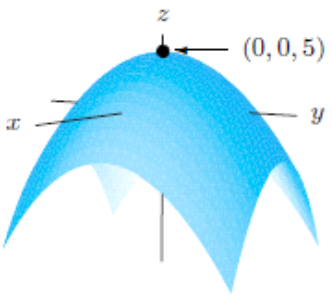


Figure 12.15: Graph of $h(x, y) = 5 - x^2 - y^2$

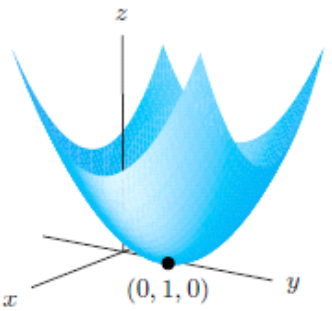


Figure 12.16: Graph of $k(x, y) = x^2 + (y - 1)^2$

EXAMPLE 5 Sketch the graph of the function $f(x, y) = 6 - 3x - 2y$.

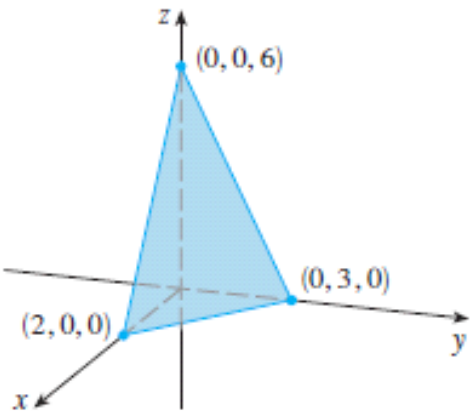


FIGURE 6

V EXAMPLE 6 Sketch the graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$.

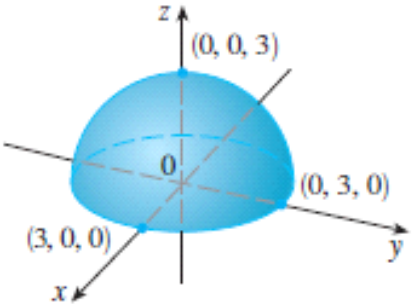


FIGURE 7
Graph of $g(x, y) = \sqrt{9 - x^2 - y^2}$

EXAMPLE 1 For each of the following functions, evaluate $f(3, 2)$ and find and sketch the domain.

(a) $f(x, y) = \dfrac{\sqrt{x + y + 1}}{x - 1}$ (b) $f(x, y) = x \ln(y^2 - x)$

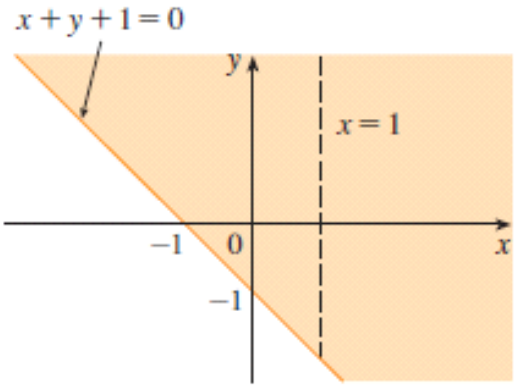


FIGURE 2
Domain of $f(x, y) = \dfrac{\sqrt{x + y + 1}}{x - 1}$

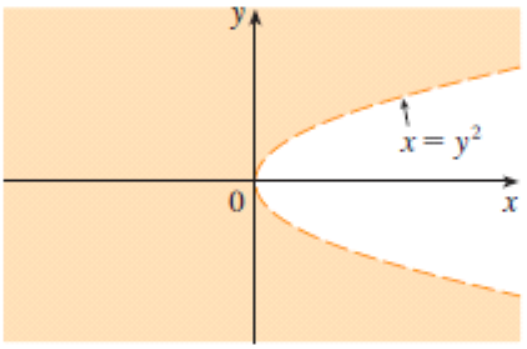
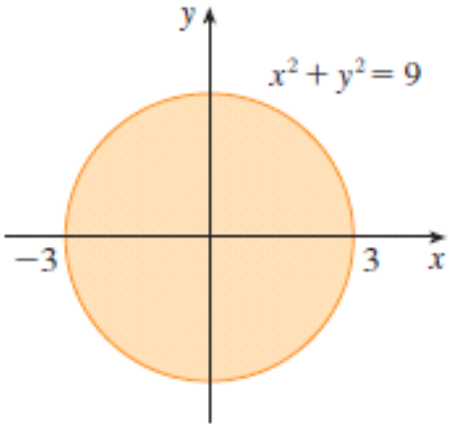


FIGURE 3
Domain of $f(x, y) = x \ln(y^2 - x)$

EXAMPLE 4 Find the domain and range of $g(x, y) = \sqrt{9 - x^2 - y^2}$.



Graphs

Another way of visualizing the behavior of a function of two variables is to consider its graph.

Definition If f is a function of two variables with domain D, then the **graph** of f is the set of all points (x, y, z) in $\mathbb{R}^3$ such that $z = f(x, y)$ and (x, y) is in D.

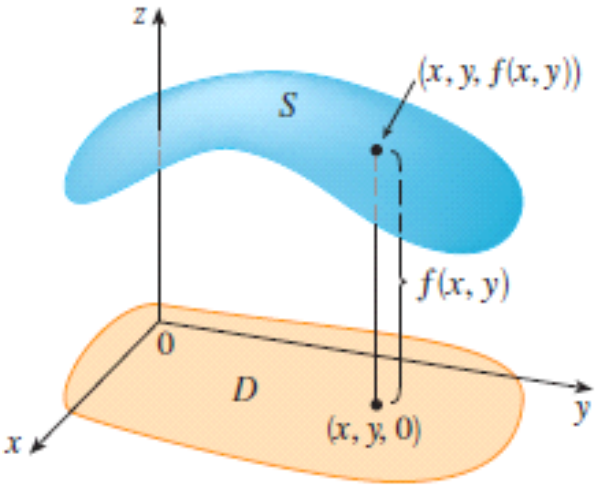


FIGURE 5

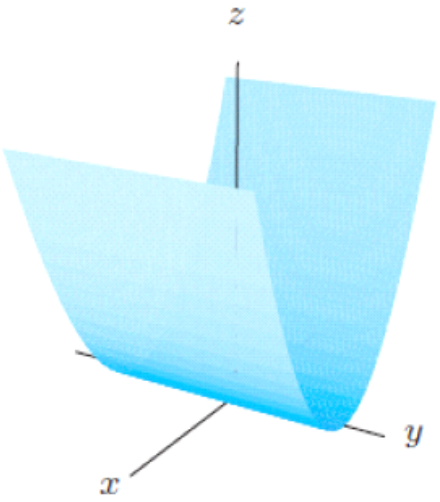


Figure 12.25: A parabolic cylinder $z = x^2$

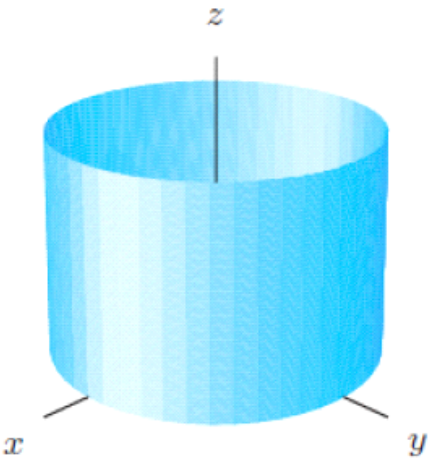


Figure 12.26: Circular cylinder $x^2 + y^2 = 1$

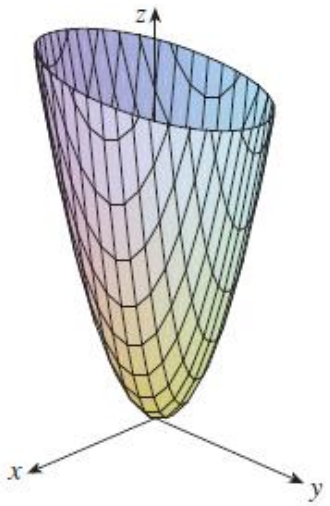
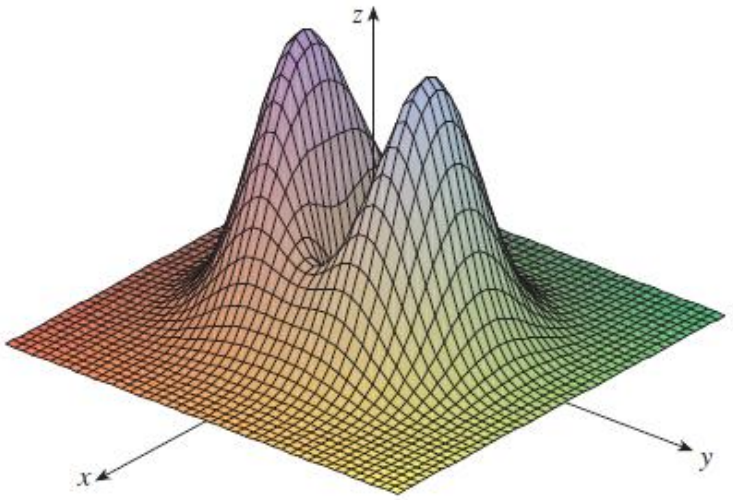
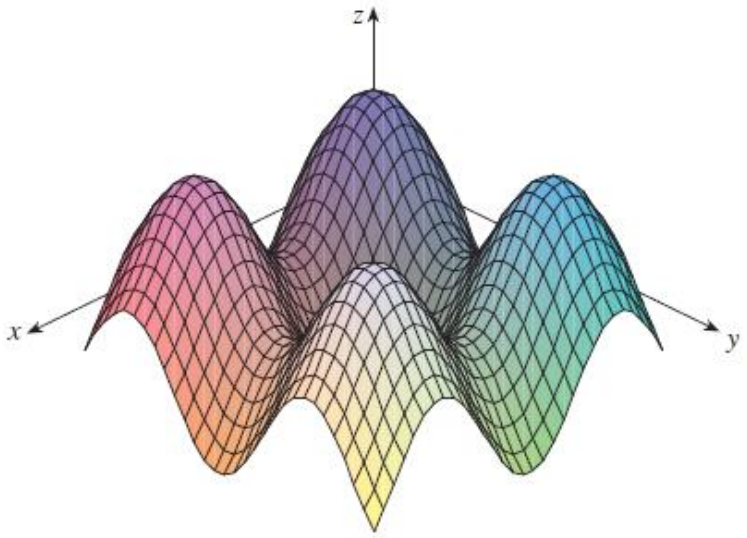


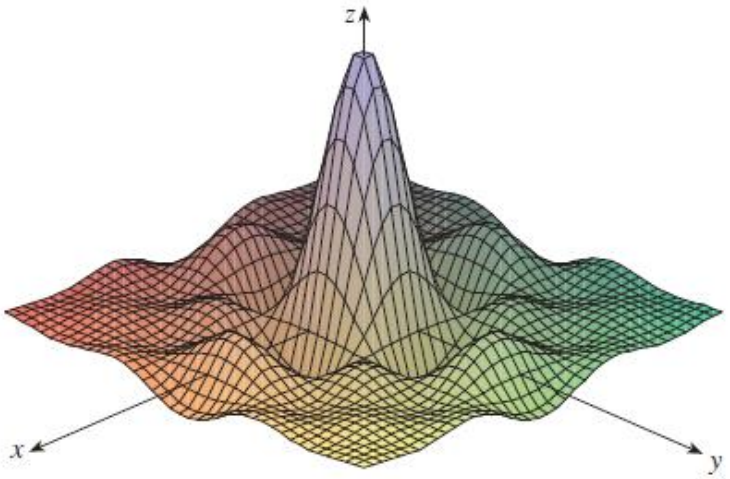
FIGURE 9
Graph of $h(x, y) = 4x^2 + y^2$



(a) $f(x, y) = (x^2 + 3y^2)e^{-x^2-y^2}$



(c) $f(x, y) = \sin x + \sin y$



(d) $f(x, y) = \dfrac{\sin x \, \sin y}{xy}$

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