

1. Evaluate each expression without using a calculator.

(a) $(-3)^4$ (b) -3^4 (c) 3^{-4}
 (d) $\frac{5^{23}}{5^{21}}$ (e) $\left(\frac{2}{3}\right)^{-2}$ (f) $16^{-3/4}$

2. Simplify each expression. Write your answer without negative exponents.

(a) $\sqrt{200} - \sqrt{32}$
 (b) $(3a^3b^3)(4ab^2)^2$
 (c) $\left(\frac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

3. Expand and simplify.

(a) $3(x + 6) + 4(2x - 5)$ (b) $(x + 3)(4x - 5)$
 (c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$ (d) $(2x + 3)^2$
 (e) $(x + 2)^3$

4. Factor each expression.

(a) $4x^2 - 25$ (b) $2x^2 + 5x - 12$
 (c) $x^3 - 3x^2 - 4x + 12$ (d) $x^4 + 27x$
 (e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$ (f) $x^3y - 4xy$

5. Simplify the rational expression.

(a) $\frac{x^2 + 3x + 2}{x^2 - x - 2}$ (b) $\frac{2x^2 - x - 1}{x^2 - 9} \cdot \frac{x + 3}{2x + 1}$
 (c) $\frac{x^2}{x^2 - 4} - \frac{x + 1}{x + 2}$ (d) $\frac{\frac{y}{x} - \frac{x}{y}}{\frac{1}{y} - \frac{1}{x}}$

Screen clipping taken: 24/02/2025 8:12 pm

6. Rationalize the expression and simplify.

(a) $\frac{\sqrt{10}}{\sqrt{5} - 2}$ (b) $\frac{\sqrt{4 + h} - 2}{h}$

7. Rewrite by completing the square.

(a) $x^2 + x + 1$ (b) $2x^2 - 12x + 11$

8. Solve the equation. (Find only the real solutions.)

(a) $x + 5 = 14 - \frac{1}{2}x$ (b) $\frac{2x}{x + 1} = \frac{2x - 1}{x}$
 (c) $x^2 - x - 12 = 0$ (d) $2x^2 + 4x + 1 = 0$
 (e) $x^4 - 3x^2 + 2 = 0$ (f) $3|x - 4| = 10$
 (g) $2x(4 - x)^{-1/2} - 3\sqrt{4 - x} = 0$

9. Solve each inequality. Write your answer using interval notation.

(a) $-4 < 5 - 3x \leq 17$ (b) $x^2 < 2x + 8$
 (c) $x(x - 1)(x + 2) > 0$ (d) $|x - 4| < 3$
 (e) $\frac{2x - 3}{x + 1} \leq 1$

10. State whether each equation is true or false.

(a) $(p + q)^2 = p^2 + q^2$ (b) $\sqrt{ab} = \sqrt{a}\sqrt{b}$
 (c) $\sqrt{a^2 + b^2} = a + b$ (d) $\frac{1 + TC}{C} = 1 + T$
 (e) $\frac{1}{x - y} = \frac{1}{x} - \frac{1}{y}$ (f) $\frac{1/x}{a/x - b/x} = \frac{1}{a - b}$

Screen clipping taken: 24/02/2025 8:12 pm

Diagnostic Test: Analytic Geometry

- Find an equation for the line that passes through the point $(2, -5)$ and
 - has slope -3
 - is parallel to the x -axis
 - is parallel to the y -axis
 - is parallel to the line $2x - 4y = 3$
- Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.
- Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.
- Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.
 - Find the slope of the line that contains A and B .
 - Find an equation of the line that passes through A and B . What are the intercepts?
 - Find the midpoint of the segment AB .
 - Find the length of the segment AB .
 - Find an equation of the perpendicular bisector of AB .
 - Find an equation of the circle for which AB is a diameter.
- Sketch the region in the xy -plane defined by the equation or inequalities.
 - $-1 \leq y \leq 3$
 - $|x| < 4$ and $|y| < 2$
 - $y < 1 - \frac{1}{2}x$
 - $y \geq x^2 - 1$
 - $x^2 + y^2 < 4$
 - $9x^2 + 16y^2 = 144$

Screen clipping taken: 24/02/2025 8:13 pm

C Diagnostic Test: Functions

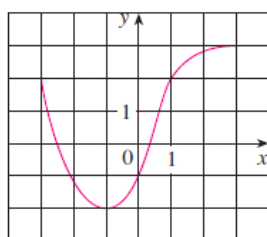


FIGURE FOR PROBLEM 1

- The graph of a function f is given at the left.
 - State the value of $f(-1)$.
 - Estimate the value of $f(2)$.
 - For what values of x is $f(x) = 2$?
 - Estimate the values of x such that $f(x) = 0$.
 - State the domain and range of f .
- If $f(x) = x^3$, evaluate the difference quotient $\frac{f(2+h) - f(2)}{h}$ and simplify your answer.
- Find the domain of the function.
 - $f(x) = \frac{2x+1}{x^2+x-2}$
 - $g(x) = \frac{\sqrt[3]{x}}{x^2+1}$
 - $h(x) = \sqrt{4-x} + \sqrt{x^2-1}$
- How are graphs of the functions obtained from the graph of f ?
 - $y = -f(x)$
 - $y = 2f(x) - 1$
 - $y = f(x-3) + 2$
- Without using a calculator, make a rough sketch of the graph.
 - $y = x^3$
 - $y = (x+1)^3$
 - $y = (x-2)^3 + 3$
 - $y = 4 - x^2$
 - $y = \sqrt{x}$
 - $y = 2\sqrt{x}$
 - $y = -2^x$
 - $y = 1 + x^{-1}$
- Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leq 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$
 - Evaluate $f(-2)$ and $f(1)$.
 - Sketch the graph of f .
- If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.
 - $f \circ g$
 - $g \circ f$
 - $g \circ g \circ g$

Screen clipping taken: 24/02/2025 8:13 pm

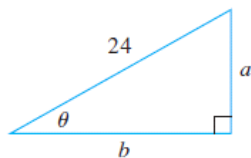


FIGURE FOR PROBLEM 5

- Convert from degrees to radians.
(a) 300° (b) -18°
- Convert from radians to degrees.
(a) $5\pi/6$ (b) 2
- Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of 30° .
- Find the exact values.
(a) $\tan(\pi/3)$ (b) $\sin(7\pi/6)$ (c) $\sec(5\pi/3)$
- Express the lengths a and b in the figure in terms of θ .
- If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.
- Prove the identities.
(a) $\tan \theta \sin \theta + \cos \theta = \sec \theta$
(b) $\frac{2 \tan x}{1 + \tan^2 x} = \sin 2x$
- Find all values of x such that $\sin 2x = \sin x$ and $0 \leq x \leq 2\pi$.
- Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

Screen clipping taken: 24/02/2025 8:13 pm