Integration By Parts

Saturday, 24 August 2024 11:54 pm

7.1 Integration by Parts

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

$$\int [f(x)g'(x) + g(x)f'(x)]\,dx = f(x)g(x)$$

$$\int f(x)g'(x)\,dx + \int g(x)f'(x)\,dx = f(x)g(x)$$

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int g(x)f'(x)\,dx$$

Derive

Integration by Parts

u and v are Functions x

$$\int uv\,dx = u\int v\,dx - \int \left\{ \frac{du}{dx}\int v\,dx \right\} dx$$

Performing Integration By Parts

Integration By Parts

$$\int f(x)\,g'(x)dx = f(x)\,g(x) - \int g(x)\,f'(x)dx$$

$f(x) = u$ $\qquad$ $g(x) = v$

$f'(x)dx = du$ $\qquad$ $g'(x)dx = dv$

$$\int u\,dv = uv - \int v\,du$$

I - inverse trig (arc functions)

L - logarithmic functions

A - algebraic (polynomials)

T - trigonometric functions

E - exponential functions

EXAMPLE 1 Find $\int x \sin x\,dx$.

$$\int x \sin x\,dx = f(x)g(x) - \int g(x)\,f'(x)\,dx$$

$$= x(-\cos x) - \int (-\cos x)\,dx$$

$$= -x\cos x + \int \cos x\,dx$$

$$= -x\cos x + \sin x + C$$

V EXAMPLE 2 Evaluate $\int \ln x\,dx$.

SOLUTION Here we don't have much choice for u and dv. Let

$$u = \ln x \qquad dv = dx$$

Then $$du = \frac{1}{x}\,dx \qquad v = x$$

Integrating by parts, we get

$$\int \ln x\,dx = x \ln x - \int x \frac{dx}{x}$$

$$= x \ln x - \int dx$$

$$= x \ln x - x + C$$

V EXAMPLE 3 Find $\int t^2 e^t\,dt$.

$$u = t^2 \qquad dv = e^t\,dt$$

$$du = 2t\,dt \qquad v = e^t$$

$$\int t^2 e^t\,dt = t^2 e^t - 2\int te^t\,dt$$

$$\int te^t\,dt = te^t - \int e^t\,dt$$

$$= te^t - e^t + C$$

$$\int t^2 e^t\,dt = t^2 e^t - 2\int te^t\,dt$$

$$= t^2 e^t - 2(te^t - e^t + C)$$

$$= t^2 e^t - 2te^t + 2e^t + C_1 \qquad \text{where } C_1 = -2C$$

V EXAMPLE 4 Evaluate $\int e^x \sin x\,dx$.


When you integrate by parts then realize you need to integrate by parts again