Domain & Range

Monday, 24 February 2025 10:33 am

Example 1

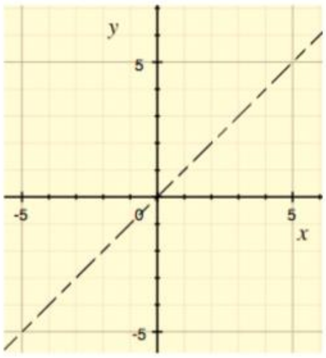
Determine and illustrate the domain of the function $f(x, y) = x^2y^2 + 2x + 2y$.

We note that for any $(x, y) \in \mathbb{R}^2$, $x^2y^2 + 2x + 2y$ is defined. In other words, there is no point (x, y) for which $f(x, y)$ is undefined. Therefore, $D(f) = \mathbb{R}^2$.

Example 2

Determine and illustrate the domain of the function $f(x, y) = \frac{x^2+y^2}{x-y}$.

We note that both the numerator and denominator of f is defined for all $(x, y) \in \mathbb{R}^2$. However, $x - y \neq 0$, otherwise the denominator would be zero. Therefore the domain of f contains all of $\mathbb{R}^2$ except for the line $y = x$, thus, $D(f) = \mathbb{R}^2 \setminus \{(x, y) : x = y\}$. The domain of this function is depicted below.

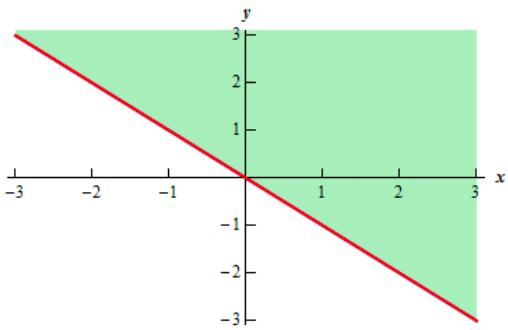


(a) $f(x, y) = \sqrt{x + y}$ ***Hide Solution*** ▾

In this case we know that we can't take the square root of a negative number so this means that we must require,

$$x + y \geq 0$$

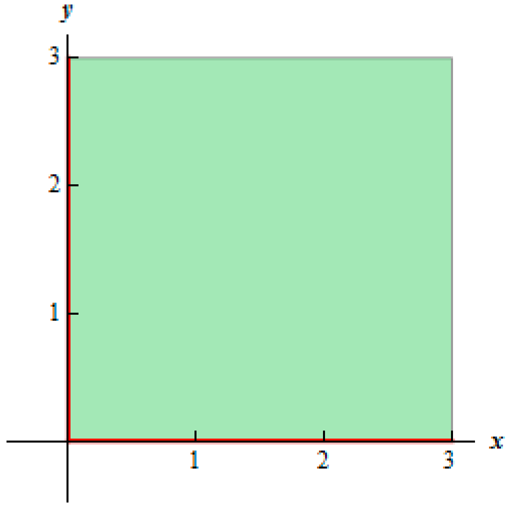
Here is a sketch of the graph of this region.



(b) $f(x,y) = \sqrt{x} + \sqrt{y}$ *Hide Solution* ▾

This function is different from the function in the previous part. Here we must require that,

$$x \ge 0 \quad \text{and} \quad y \ge 0$$



(c) $f(x,y) = \ln\left(9 - x^2 - 9y^2\right)$ *Hide Solution* ▾

In this final part we know that we can't take the logarithm of a negative number or zero. Therefore, we need to require that,

$$9 - x^2 - 9y^2 > 0 \quad \Rightarrow \quad \frac{x^2}{9} + y^2 < 1$$