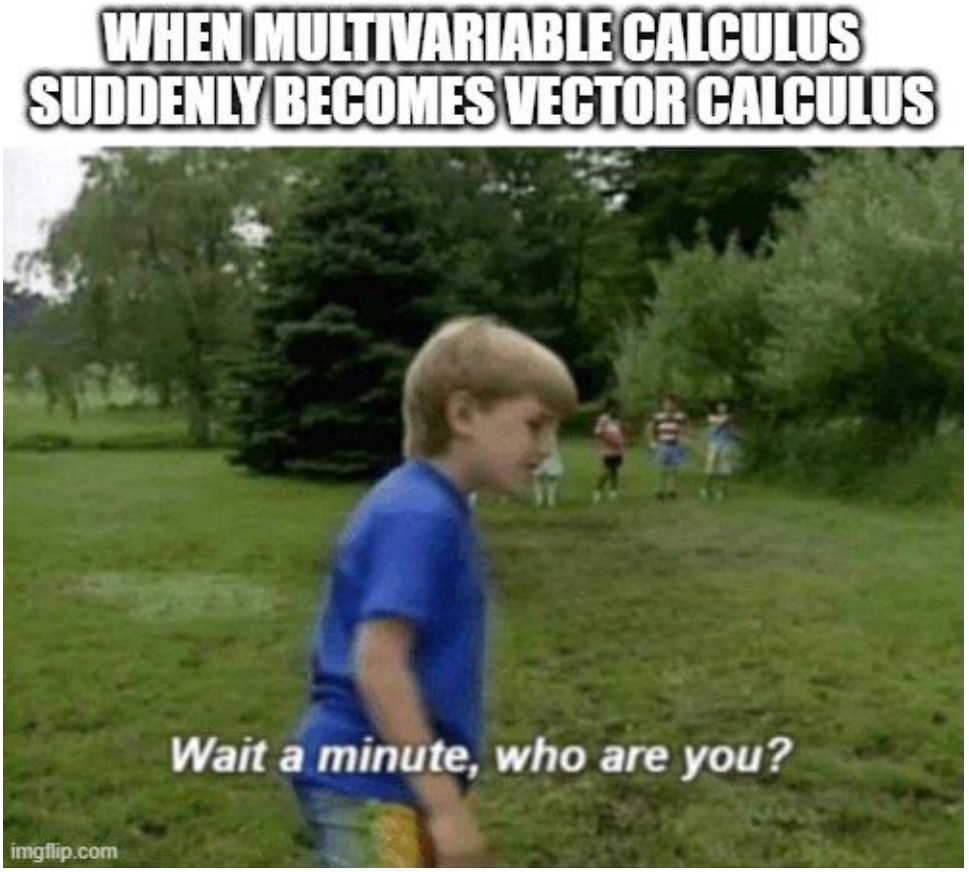
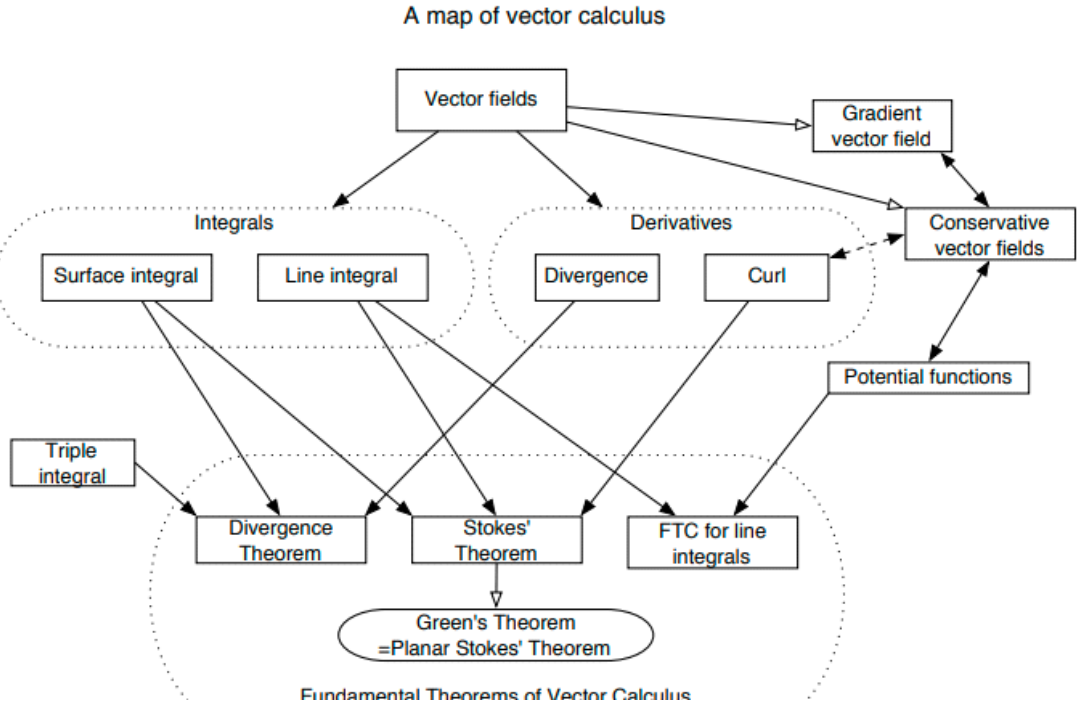
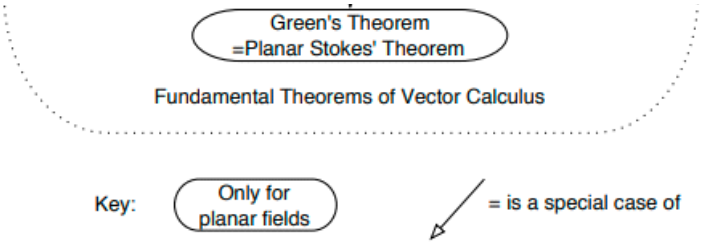
Vector Calculus & Vector Field Intro

Friday, 16 May 2025 6:09 pm



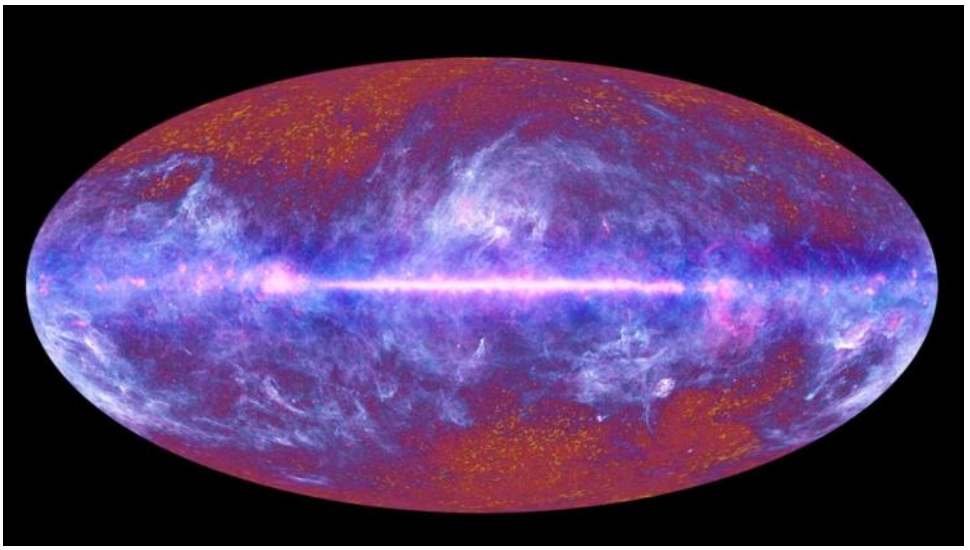
Let's first overview vector calculus





What is a Vector? And Why Field?

Universe!!!



Gravitational and electric forces have both a direction and a magnitude. They are represented by a vector at each point in their domain, producing a vector field.

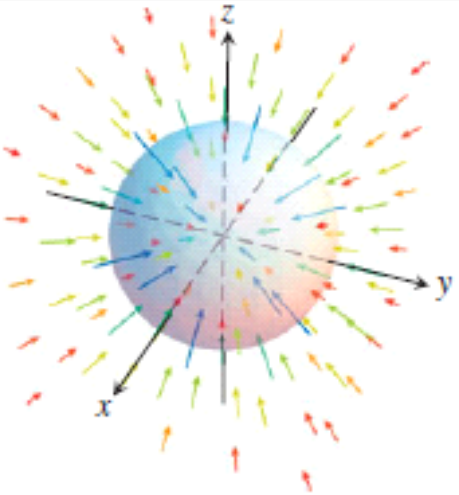


FIGURE 16.8 Vectors in a gravitational field point toward the center of mass that gives the source of the field.

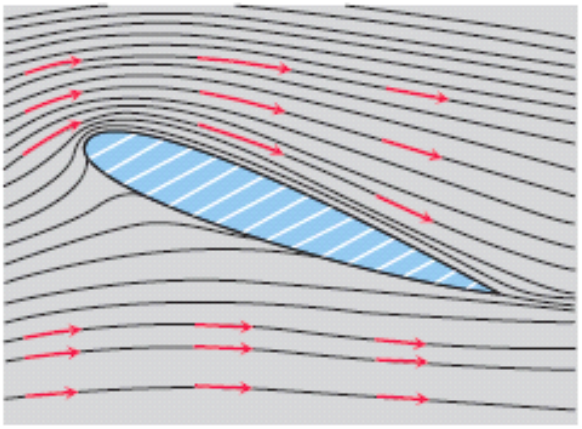


FIGURE 16.6 Velocity vectors of a flow around an airfoil in a wind tunnel.

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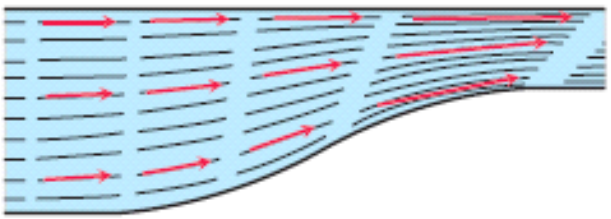


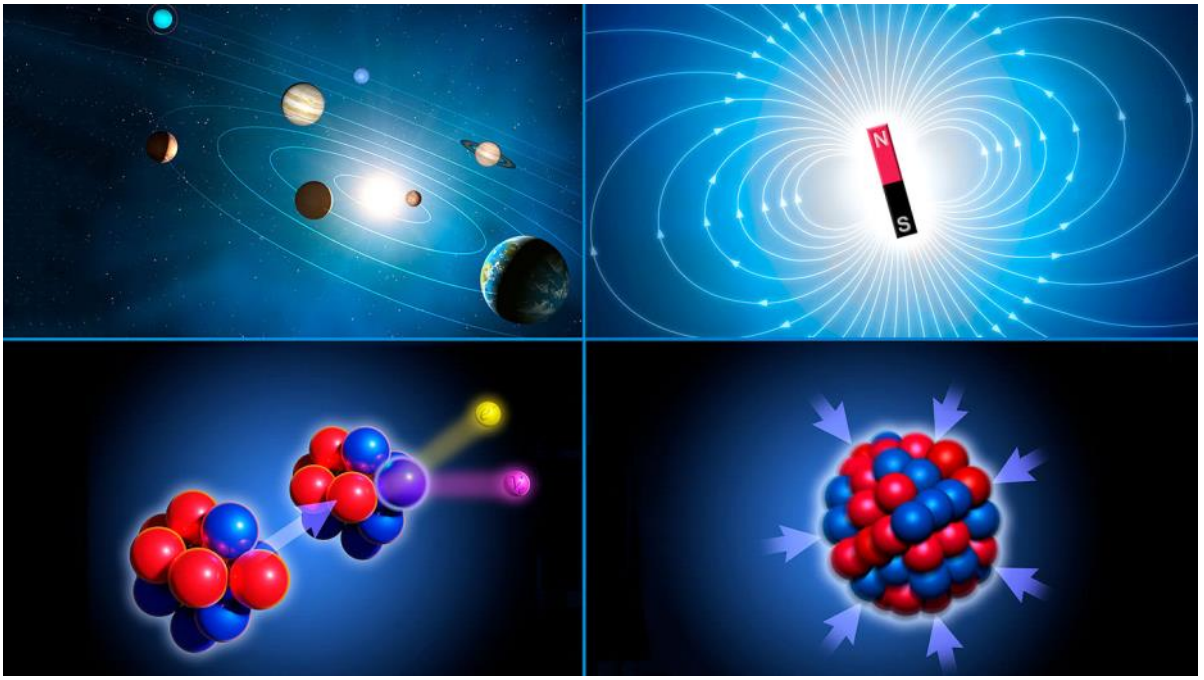
FIGURE 16.7 Streamlines in a contracting channel. The water speeds up as the channel narrows and the velocity vectors increase in length.

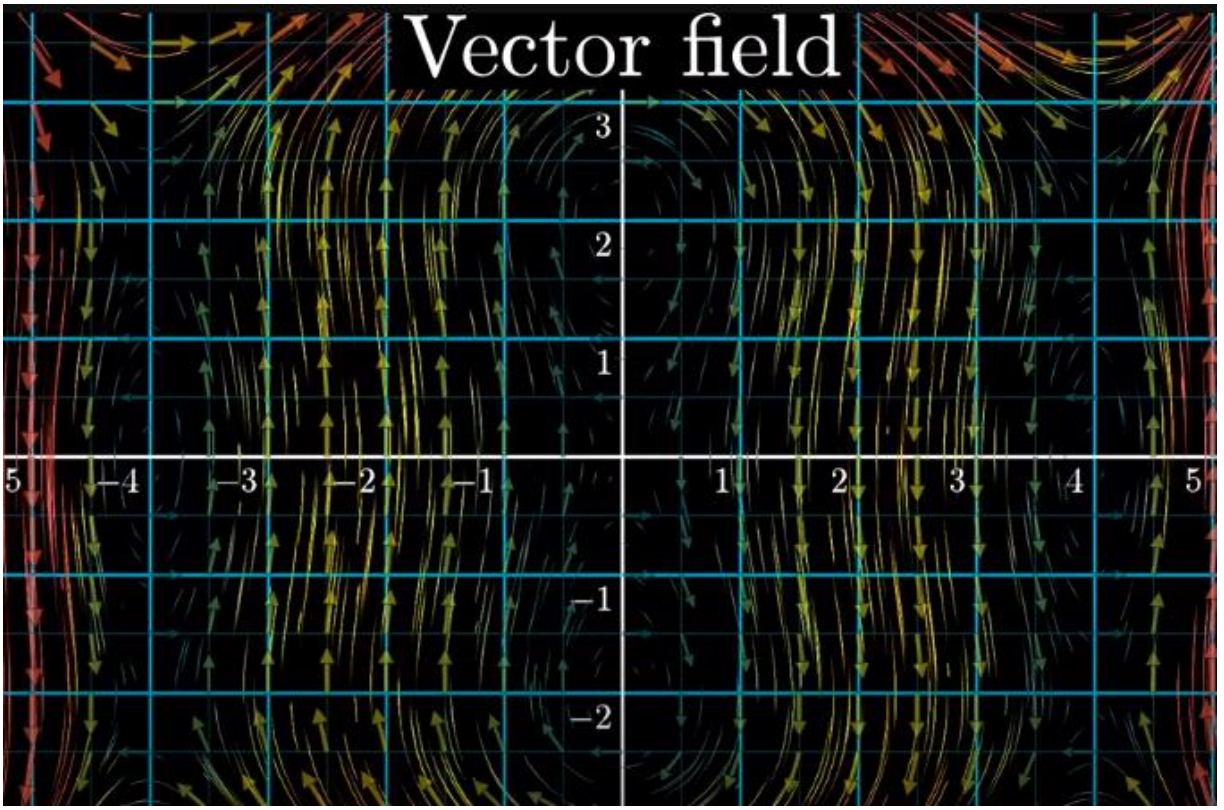
Generally, a **vector field** is a function that assigns a vector to each point in its domain. A vector field on a three-dimensional domain in space might have a formula like

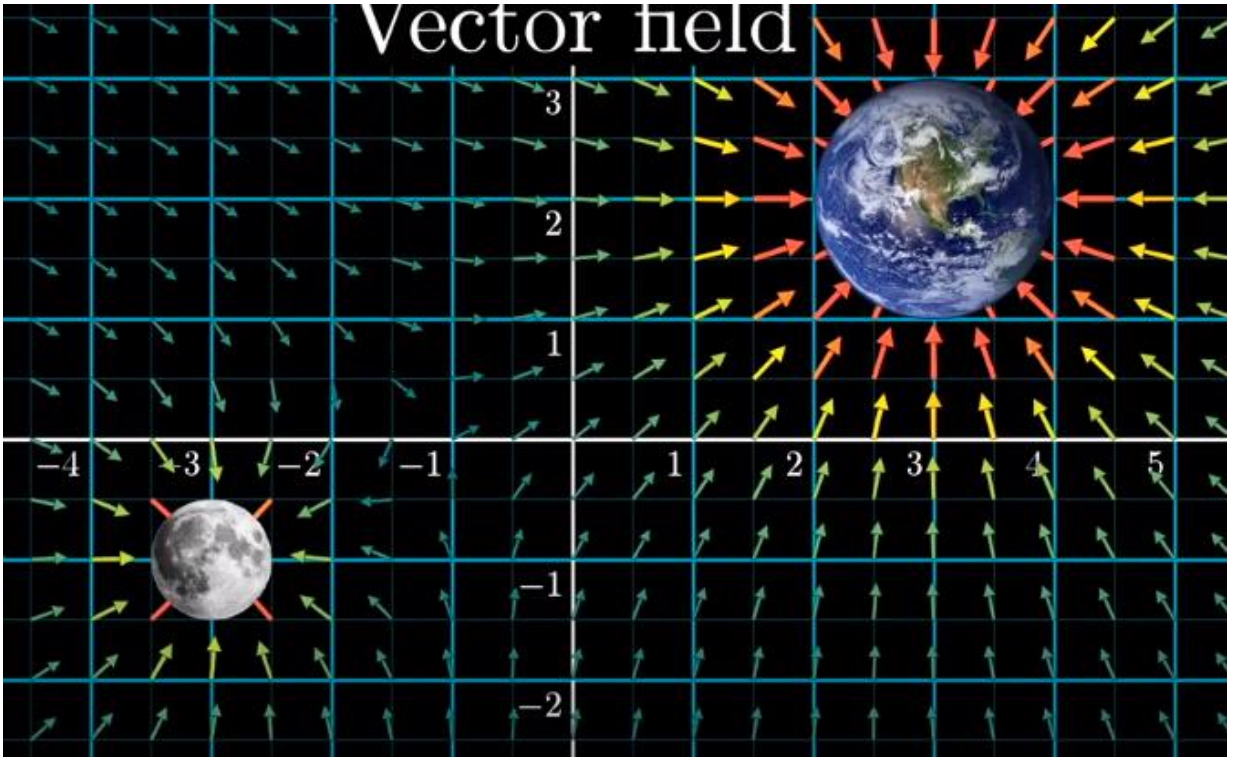
$$\mathbf{F}(x, y, z) = M(x, y, z)\mathbf{i} + N(x, y, z)\mathbf{j} + P(x, y, z)\mathbf{k}.$$

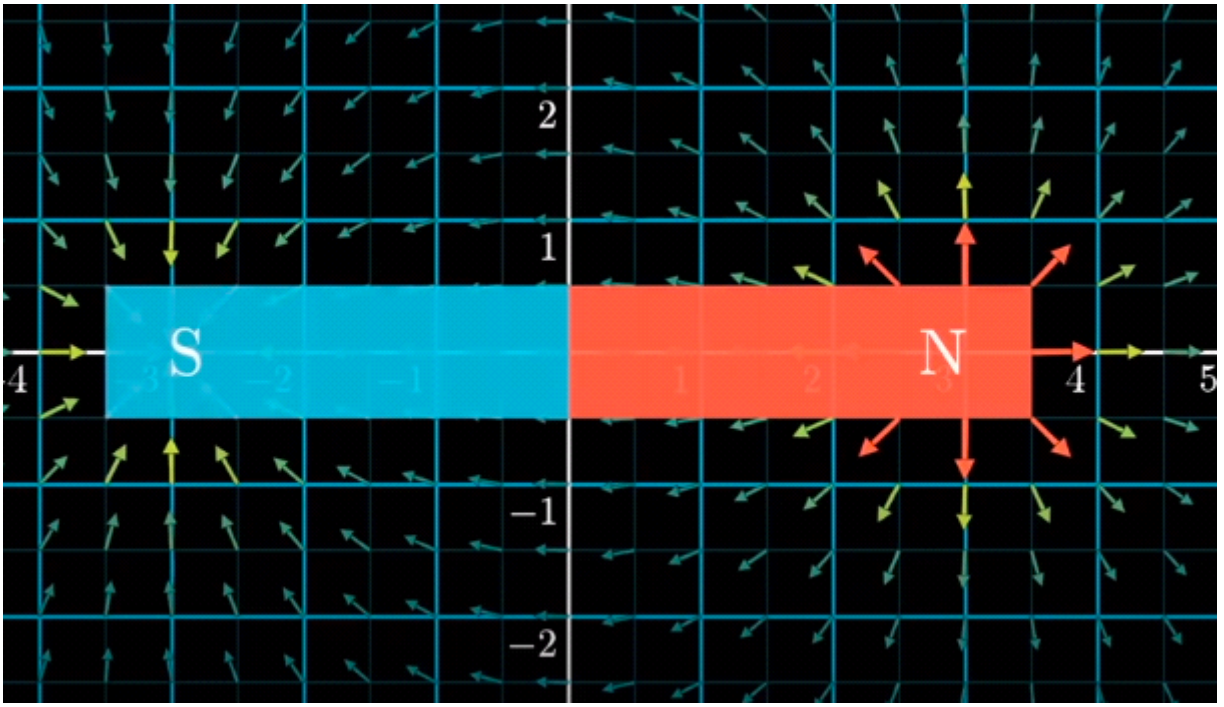
The field is **continuous** if the **component functions** M, N, and P are continuous; it is **differentiable** if each of the component functions is differentiable. The formula for a field of two-dimensional vectors could look like

$$\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}.$$

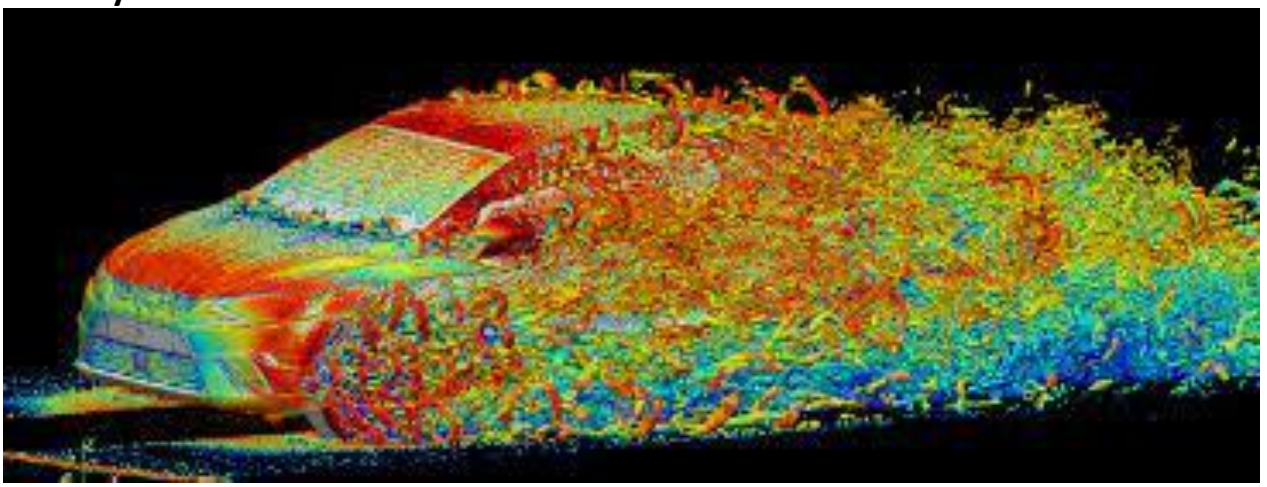

N
S

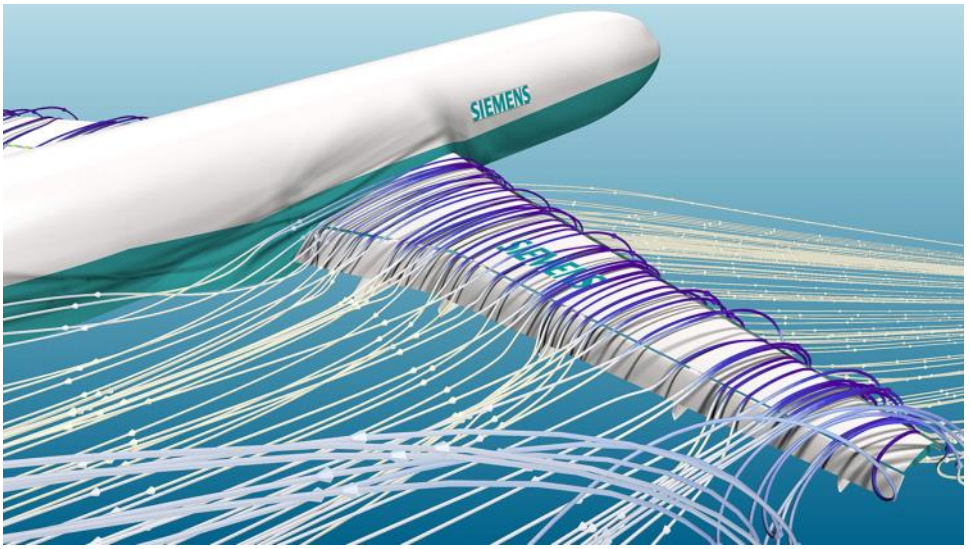

Vector field
3
2
1
−1
−2
5
−4
−3
−2
−1
1
2
3
4
5





Aero Dynamics





Fluid Dynamics