Summary

Monday, 9 June 2025 4:56 pm

Summary

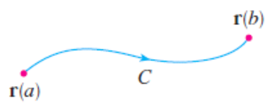
The main results of this chapter are all higher-dimensional versions of the Fundamental Theorem of Calculus. To help you remember them, we collect them together here (without hypotheses) so that you can see more easily their essential similarity. Notice that in each case we have an integral of a "derivative" over a region on the left side, and the right side involves the values of the original function only on the *boundary* of the region.

Fundamental Theorem of Calculus

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

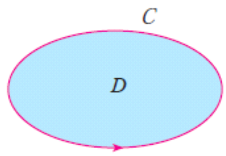
Fundamental Theorem for Line Integrals

$$\int_C \nabla f \cdot d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a))$$



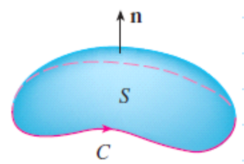
Green's Theorem

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA = \int_C P\,dx + Q\,dy$$



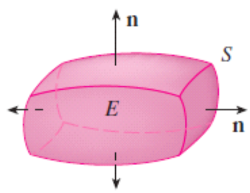
Stokes' Theorem

$$\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S} = \int_C \mathbf{F} \cdot d\mathbf{r}$$



Divergence Theorem

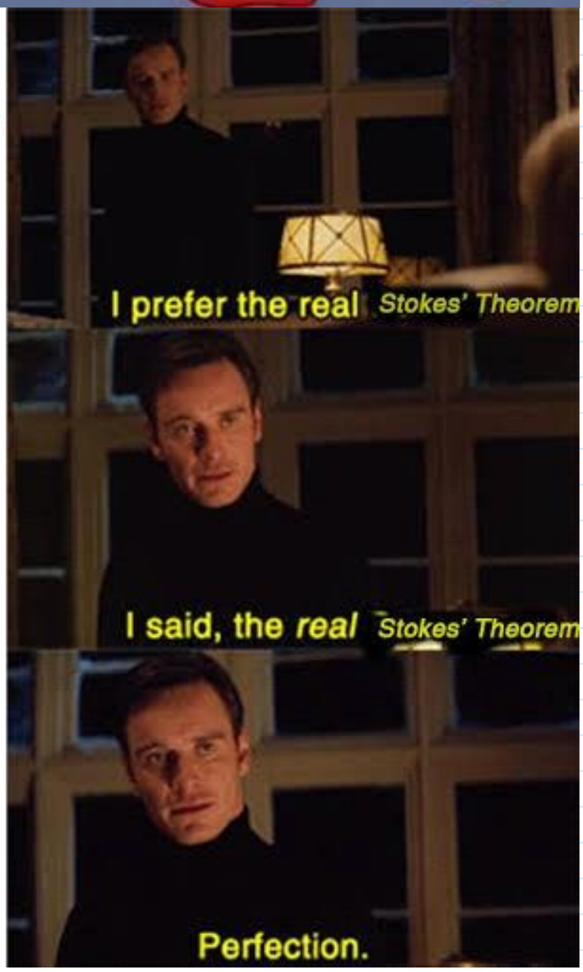
$$\iiint_E \operatorname{div} \mathbf{F}\,dV = \iint_S \mathbf{F} \cdot d\mathbf{S}$$

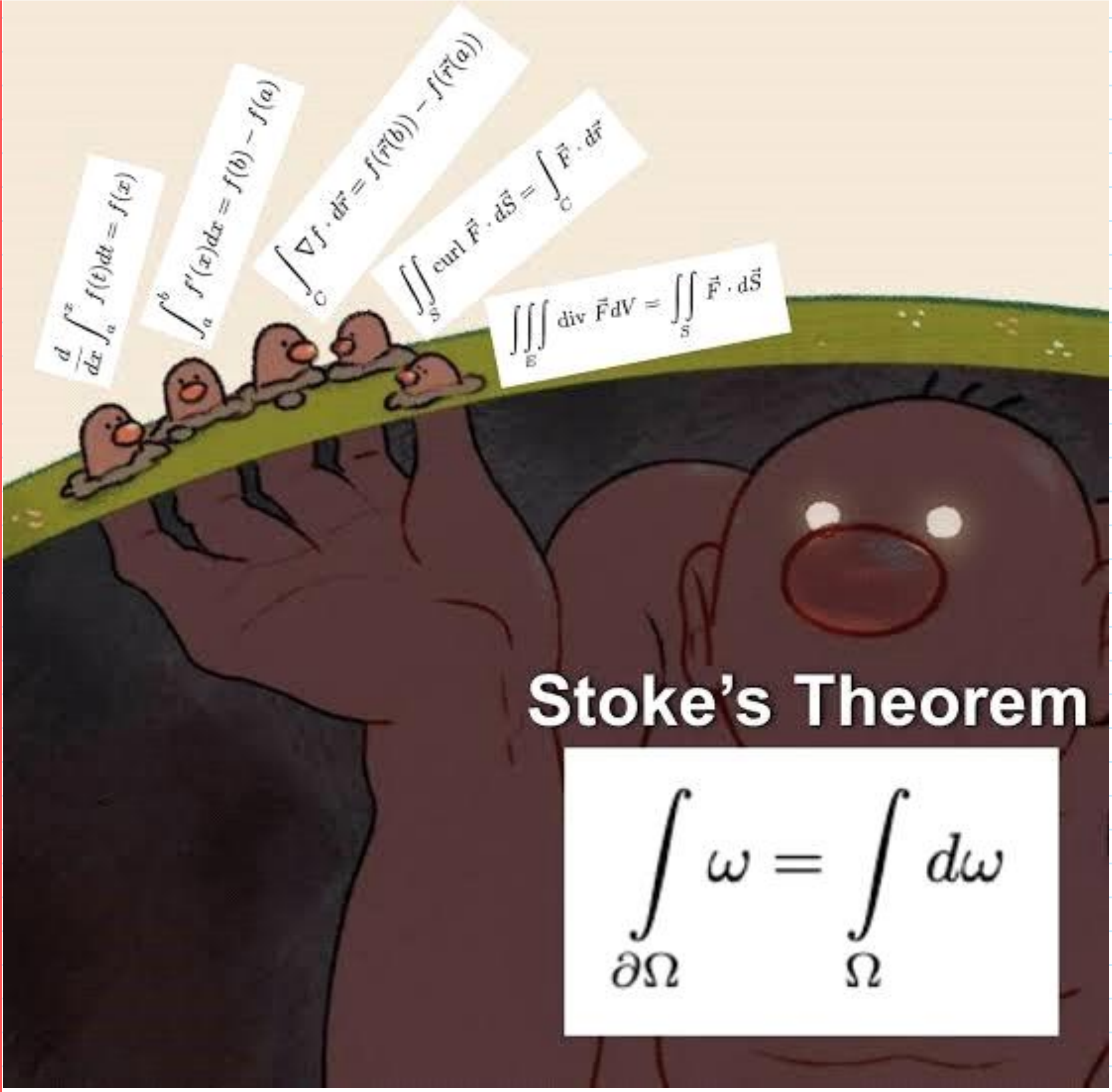


$$\oint_C (L\,dx + M\,dy) = \iint_D \left(\frac{\partial M}{\partial x} - \frac{\partial L}{\partial y}\right) dx\,dy$$

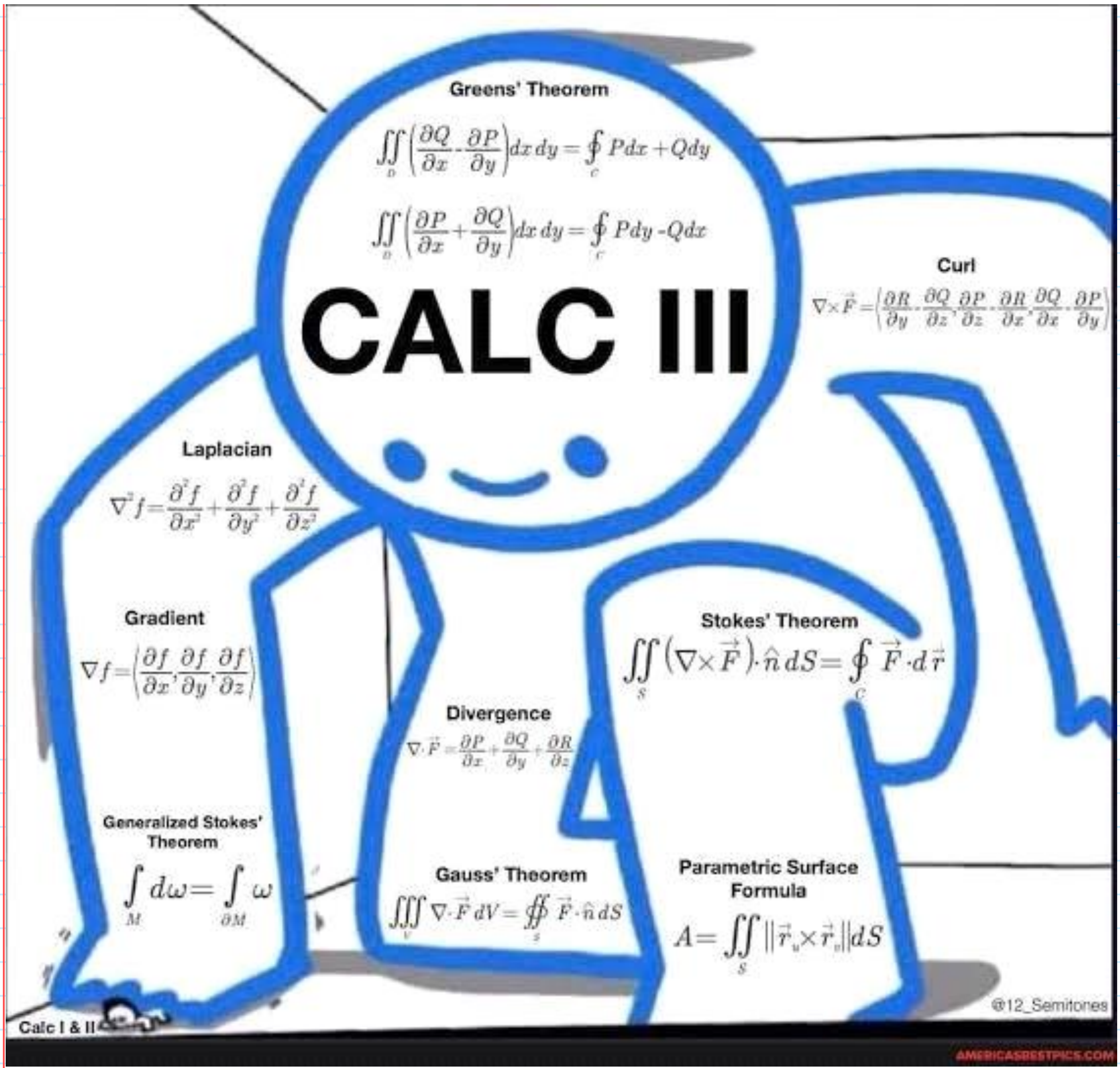
$$\oint_\Gamma \mathbf{F} \cdot d\mathbf{\Gamma} = \iint_S \nabla \times \mathbf{F} \cdot d\mathbf{S}.$$

$$\int_{\partial\Omega} \omega = \int_\Omega d\omega$$





$\frac{d}{dx}\int_a^x f(t)dt = f(x)$
$\int_a^b f'(x)dx = f(b) - f(a)$
$\int_C \nabla f \cdot d\vec{r} = f(\vec{r}(b)) - f(\vec{r}(a))$
$\iint_S \text{curl } \vec{F} \cdot d\vec{S} = \int_C \vec{F} \cdot d\vec{r}$
$\iiint_E \text{div } \vec{F} dV = \iint_S \vec{F} \cdot d\vec{S}$
Stoke's Theorem
$\int_{\partial\Omega} \omega = \int_{\Omega} d\omega$

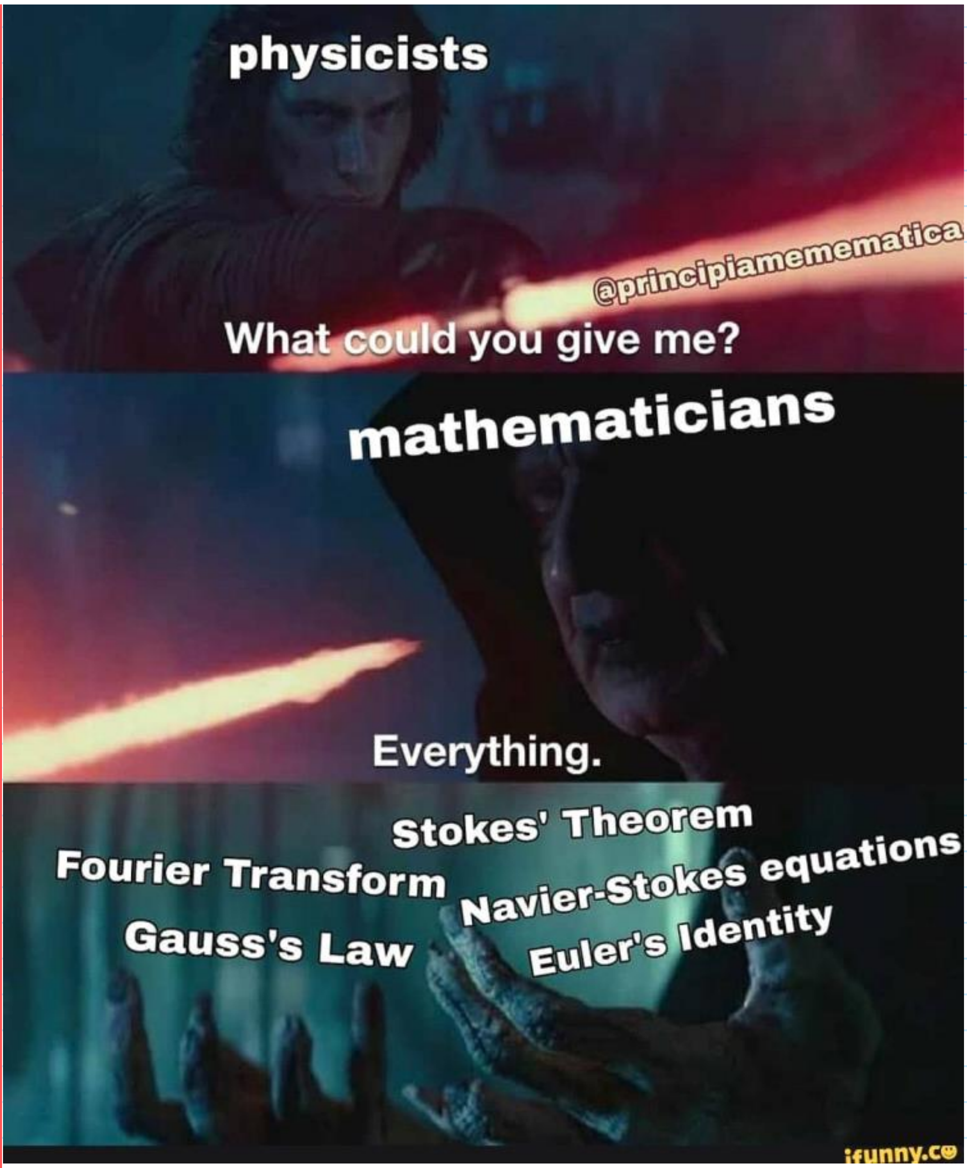


Green's Theorem: $$\iint_D \left(\frac{\partial M}{\partial x}-\frac{\partial L}{\partial y}\right)dxdy=\oint_C L\,dx+M\,dy$$

Kelvin-Stokes Theorem: $$\iint_S \left(\vec{\nabla}\times\vec{F}\right)\cdot\hat{n}dS=\oint_C \vec{F}\cdot d\vec{r}$$

Divergence Theorem: $$\iiint_V \left(\vec{\nabla}\cdot\vec{F}\right)dV=\oiint_S \left(\vec{F}\cdot\hat{n}\right)dS$$

Generalized Stokes Theorem: $$\int_C d\omega=\int_{\partial C}\omega$$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