Linear Algebra Workbook

Orthonormal bases and Gram-Schmidt

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MATH

ORTHONORMAL BASES

■ 1. Verify that the vector set $V = \{\vec{v}_1, \vec{v}_2\}$ is orthonormal if $\vec{v}_1 = (1,0,0)$ and $\vec{v}_2 = (0,0,-1)$.

■ 2. Determine that the vector set $V = \{\vec{v}_1, \vec{v}_2\}$ is orthonormal.

$$\vec{v}_1 = \left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$$

$$\vec{v}_2 = \left(-\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$$

■ 3. Convert $\vec{x} = (-2,10)$ from the standard basis to the alternate basis $B = \{\vec{v}_1, \vec{v}_2\}$.

$$\vec{v}_1 = \begin{bmatrix} \frac{3}{4} \\ -\frac{\sqrt{7}}{4} \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} \frac{\sqrt{7}}{4} \\ \frac{3}{4} \end{bmatrix}$$

■ 4. Convert $\vec{x} = (-25,10)$ from the standard basis to the alternate basis $B = \{\vec{v}_1, \vec{v}_2\}$.

$$\vec{v}_1 = \begin{bmatrix} \frac{3}{5} \\ -\frac{4}{5} \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -\frac{4}{5} \\ -\frac{3}{5} \end{bmatrix}$$

■ 5. Convert $\vec{x} = (-6,3,12)$ from the standard basis to the alternate basis $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

$$\vec{v}_1 = \begin{bmatrix} \frac{2}{3} \\ -\frac{1}{3} \\ \frac{2}{3} \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \\ \frac{2}{3} \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} \frac{2}{3} \\ \frac{2}{3} \\ -\frac{1}{3} \end{bmatrix}$$

■ 6. Convert $\vec{x} = (2,0,-3)$ from the standard basis to the alternate basis $B = \{\vec{v}_1, \vec{v}_2, \vec{v}_3\}$.

$$\vec{v}_1 = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} -\frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \end{bmatrix}$$

PROJECTION ONTO AN ORTHONORMAL BASIS

■ 1. Find the projection of $\vec{x}=(-5,0,-2)$ onto the subspace V.

$$V=\text{Span}\left(\begin{bmatrix}\frac{2}{\sqrt{5}}\\ \frac{1}{\sqrt{5}}\\ 0\end{bmatrix},\begin{bmatrix}0\\ 0\\ -1\end{bmatrix}\right)$$

■ 2. Find the projection of $\vec{x}=(-66,33,11)$ onto the subspace V.

$$V=\text{Span}\left(\begin{bmatrix}\frac{1}{\sqrt{6}}\\ \frac{1}{\sqrt{6}}\\ -\frac{2}{\sqrt{6}}\end{bmatrix},\begin{bmatrix}-\frac{3}{\sqrt{11}}\\ \frac{1}{\sqrt{11}}\\ -\frac{1}{\sqrt{11}}\end{bmatrix}\right)$$

■ 3. Find the projection of $\vec{x}=(-6,-3,6)$ onto the subspace V.

$$V=\text{Span}\left(\begin{bmatrix}-\frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\\ \frac{1}{\sqrt{3}}\end{bmatrix},\begin{bmatrix}-\frac{1}{\sqrt{2}}\\ 0\\ -\frac{1}{\sqrt{2}}\end{bmatrix}\right)$$

4. Find the projection of $\vec{x}=(-2,3,5)$ onto the subspace V.

$$V=\text{Span}\left(\begin{bmatrix}0\\-1\\0\end{bmatrix},\begin{bmatrix}\frac{3}{\sqrt{10}}\\0\\\frac{1}{\sqrt{10}}\end{bmatrix}\right)$$

5. Find the projection of $\vec{x}=(0,-13,4)$ onto the subspace V.

$$V=\text{Span}\left(\begin{bmatrix}\frac{3}{\sqrt{13}}\\\frac{2}{\sqrt{13}}\\0\end{bmatrix},\begin{bmatrix}\frac{2}{\sqrt{17}}\\-\frac{3}{\sqrt{17}}\\\frac{2}{\sqrt{17}}\end{bmatrix}\right)$$

6. Find the projection of $\vec{x}=(-3,10,-10)$ onto the subspace V.

$$V=\text{Span}\left(\begin{bmatrix}\frac{3}{\sqrt{19}}\\-\frac{3}{\sqrt{19}}\\\frac{1}{\sqrt{19}}\end{bmatrix},\begin{bmatrix}0\\\frac{1}{\sqrt{10}}\\\frac{3}{\sqrt{10}}\end{bmatrix}\right)$$

GRAM-SCHMIDT PROCESS FOR CHANGE OF BASIS

■ 1. Use a Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix}0\\-4\\3\end{bmatrix}, \begin{bmatrix}-2\\3\\-1\end{bmatrix}\right)$$

■ 2. Use a Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix}1\\-1\\1\end{bmatrix}, \begin{bmatrix}-3\\5\\2\end{bmatrix}\right)$$

■ 3. Use a Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix}-2\\1\\-2\end{bmatrix}, \begin{bmatrix}-3\\-1\\4\end{bmatrix}, \begin{bmatrix}2\\-1\\5\end{bmatrix}\right)$$

■ 4. Use a Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix}-3\\0\\0\end{bmatrix}, \begin{bmatrix}-2\\1\\2\end{bmatrix}, \begin{bmatrix}-5\\5\\0\end{bmatrix}\right)$$

■ 5. Use a Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix}-3\\0\\4\\0\end{bmatrix}, \begin{bmatrix}-1\\2\\-2\\0\end{bmatrix}, \begin{bmatrix}5\\-1\\0\\2\end{bmatrix}\right)$$

■ 6. Use a Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix}-2\\-2\\2\\-2\end{bmatrix}, \begin{bmatrix}-2\\1\\0\\-1\end{bmatrix}, \begin{bmatrix}4\\0\\-1\\-1\end{bmatrix}\right)$$

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