Linear Algebra and Geometry 1

Systems of equations, matrices, vectors, and geometry

Cosine Rule

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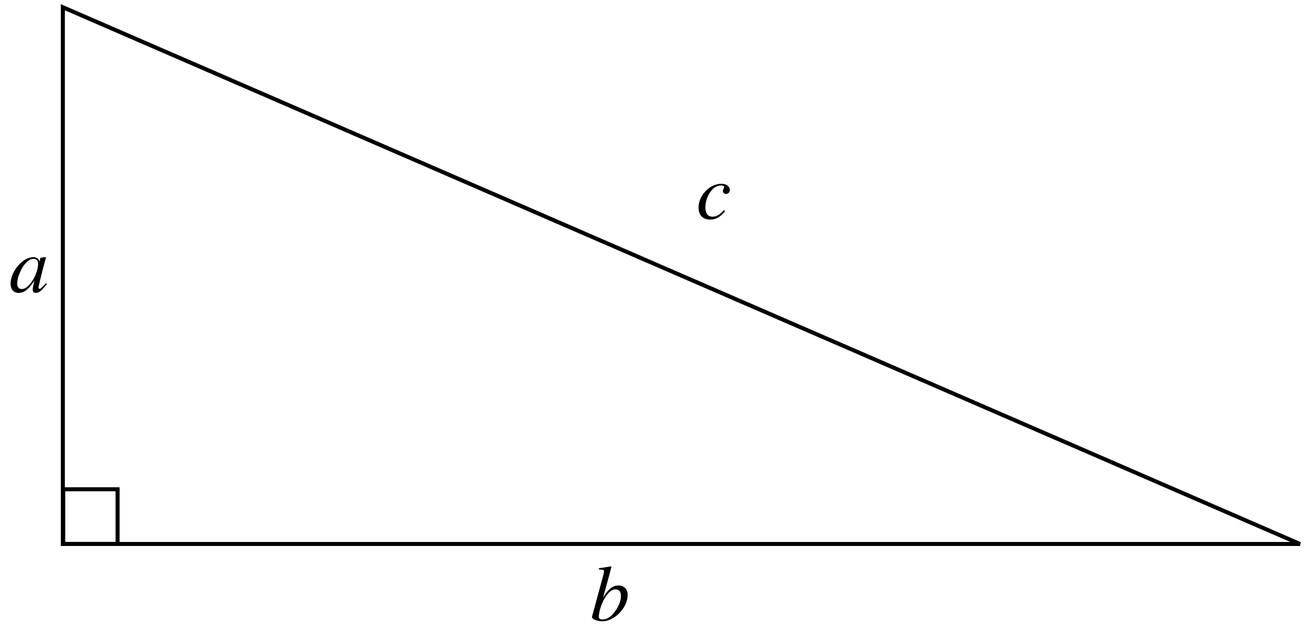
MiTM
Man in The Middle AB



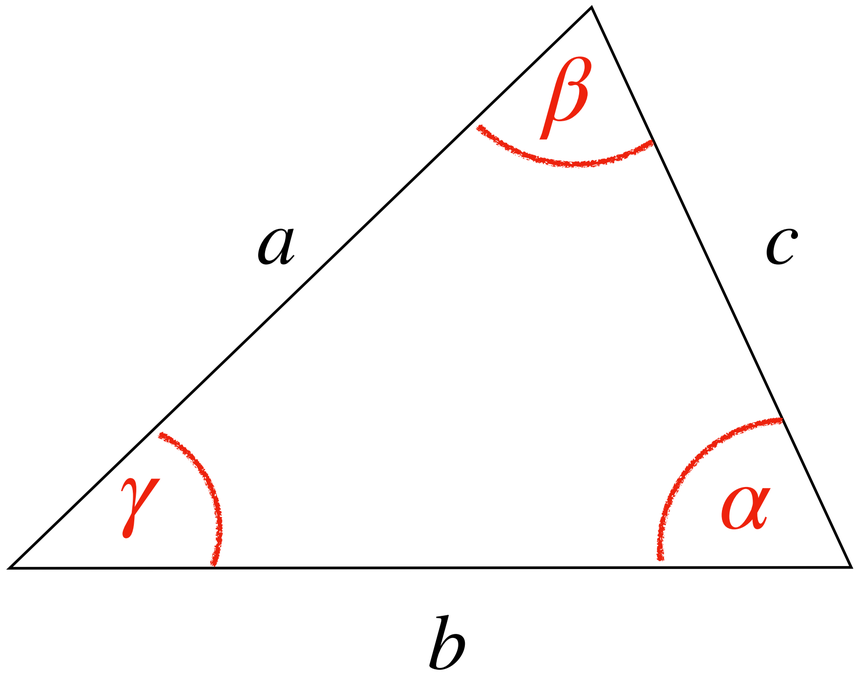
$$a^2 + b^2 = c^2$$

Pythagorean Theorem

$$a^2 + b^2 = c^2 + 2ab\cos\gamma$$

$$b^2 + c^2 = a^2 + 2bc\cos\alpha$$

$$a^2 + c^2 = b^2 + 2ac\cos\beta$$



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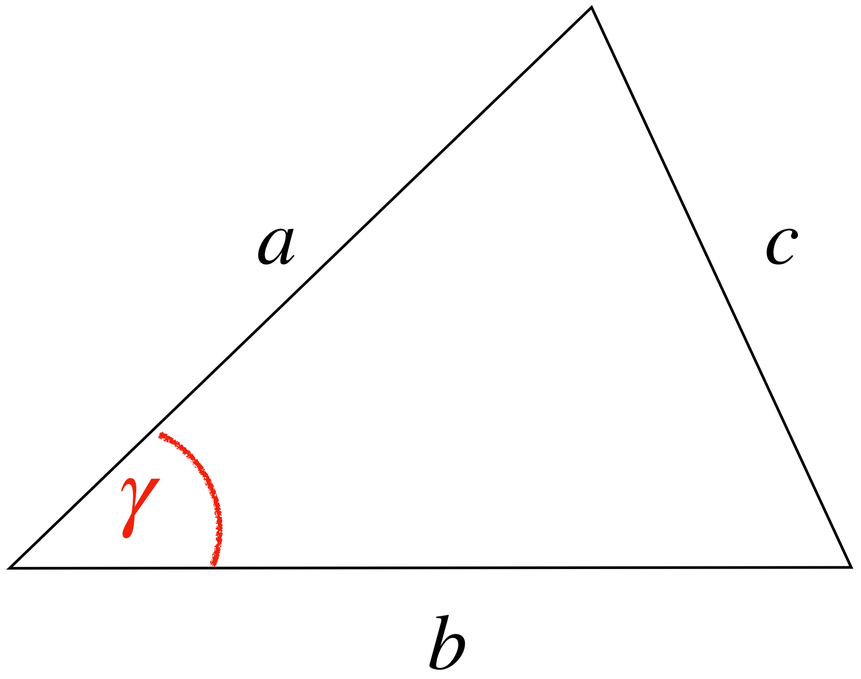

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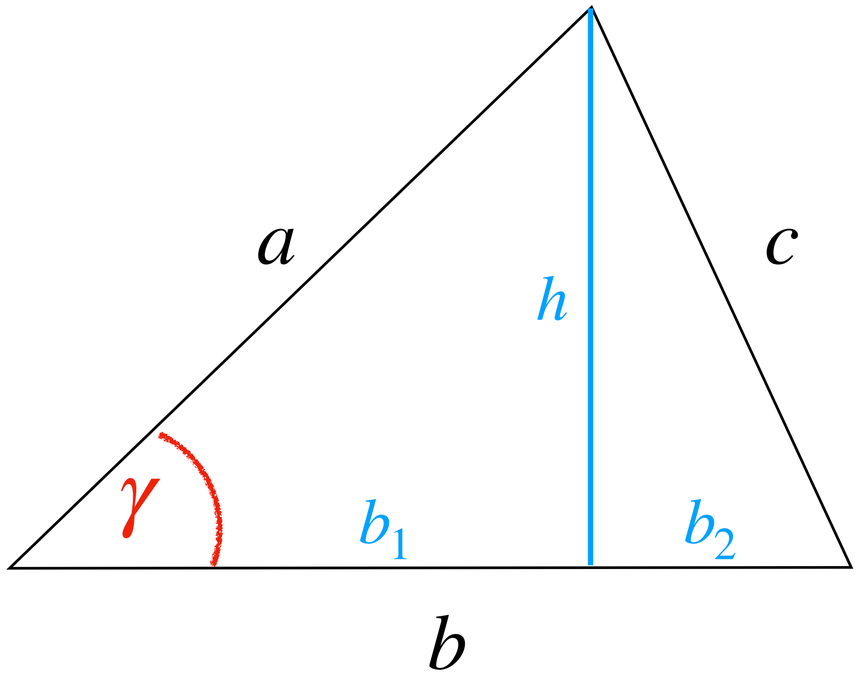

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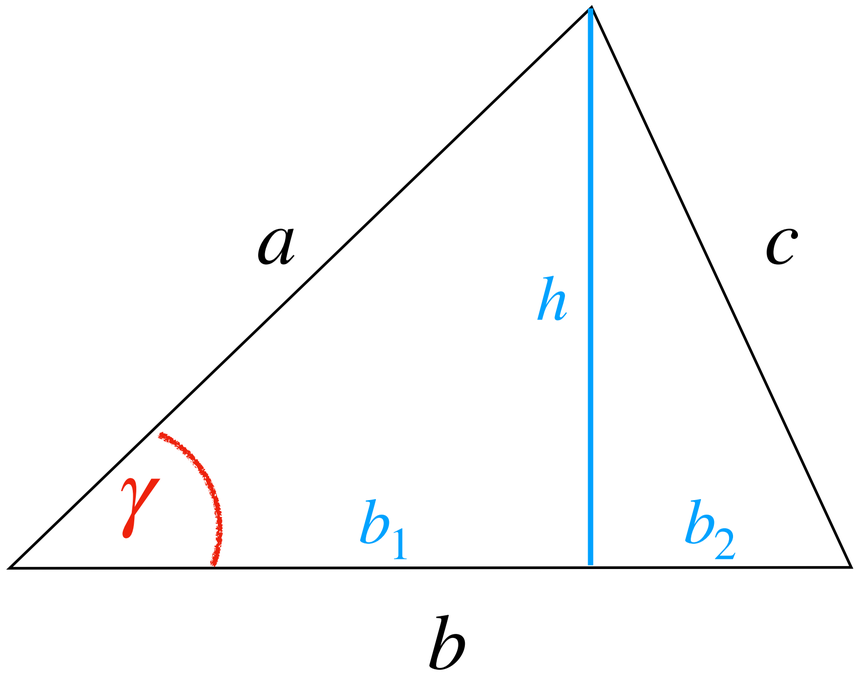
Pythagoras



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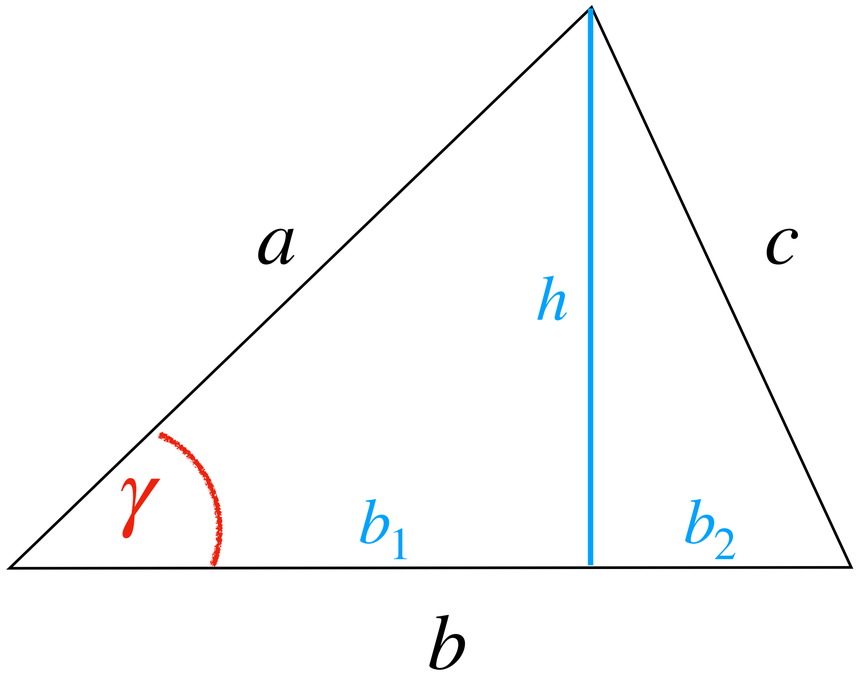

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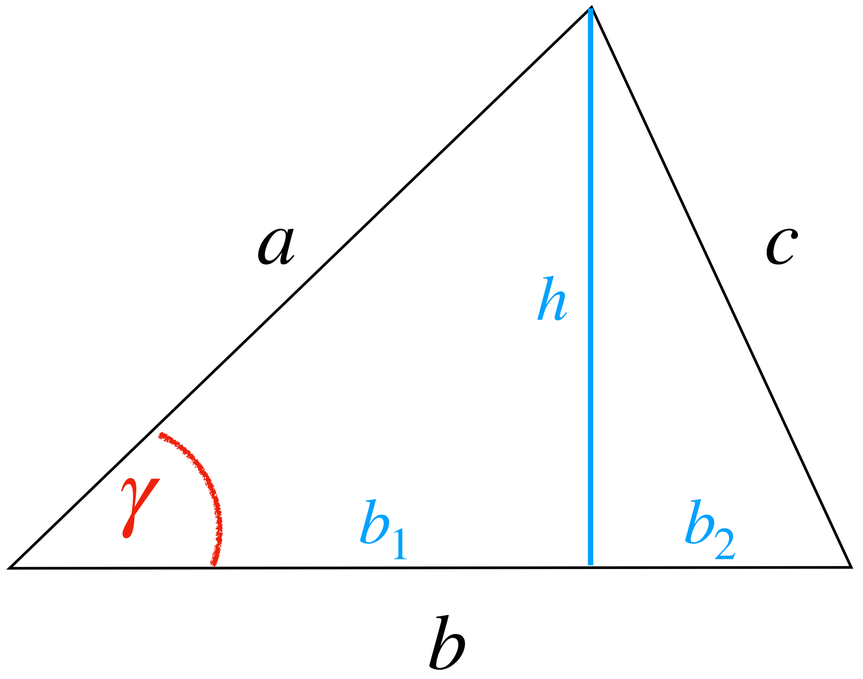

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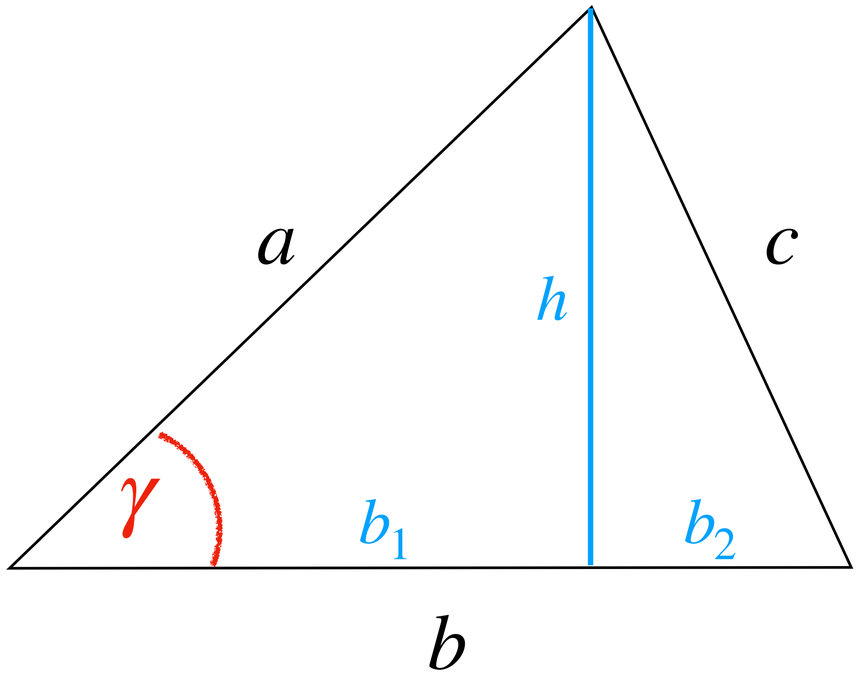

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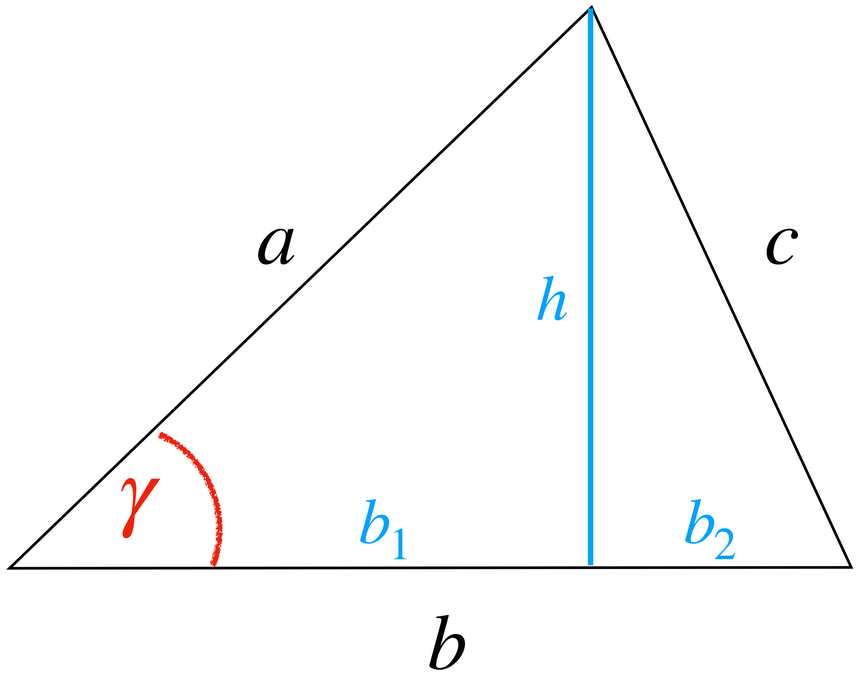

$$a^2 + b^2 = h^2 + b_1^2 + (b_1 + b_2)^2$$

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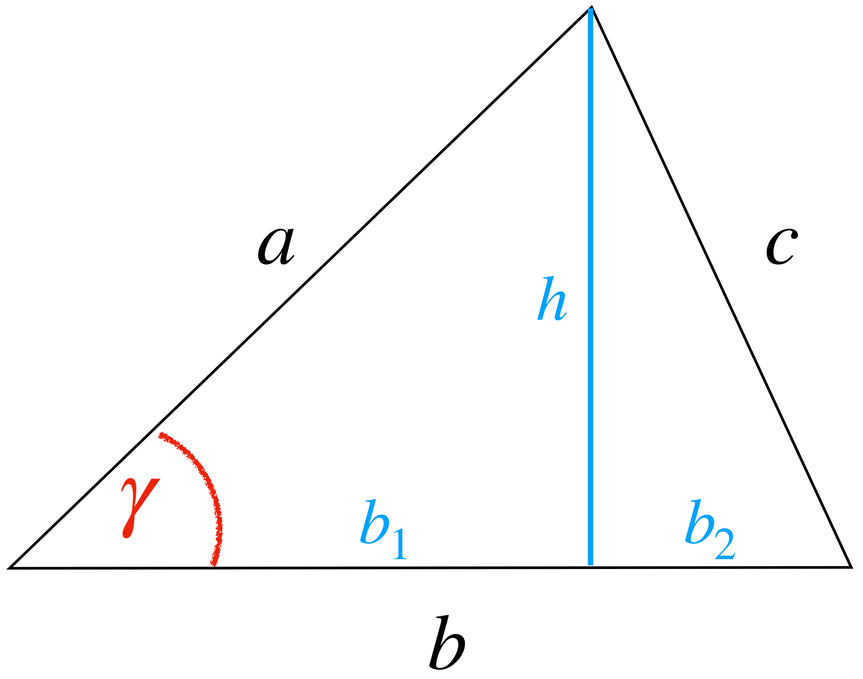

$$a^2 + b^2 = h^2 + b_1^2 + (b_1 + b_2)^2 = h^2 + b_1^2 + b_1^2 + 2b_1b_2 + b_2^2$$

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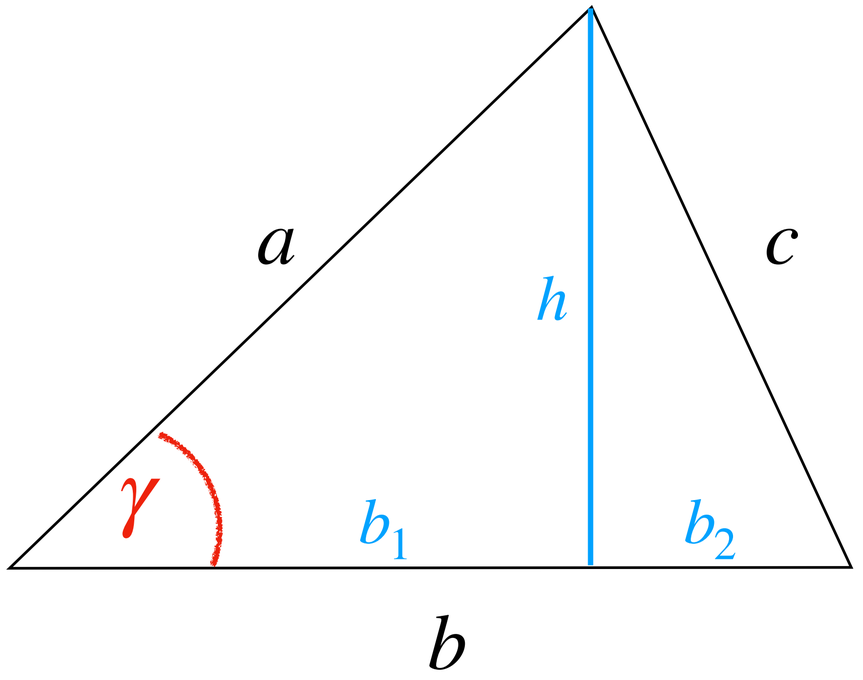

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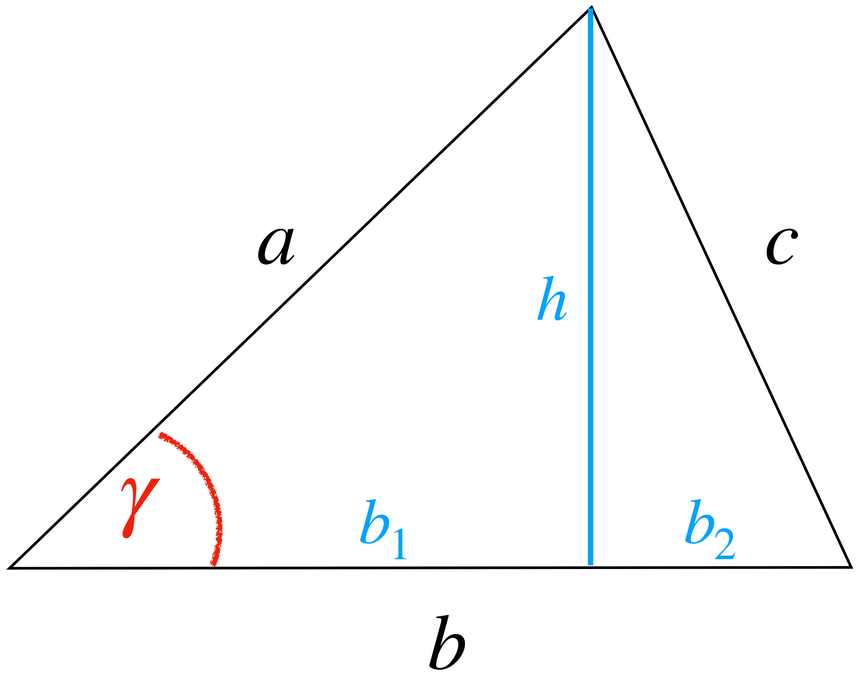

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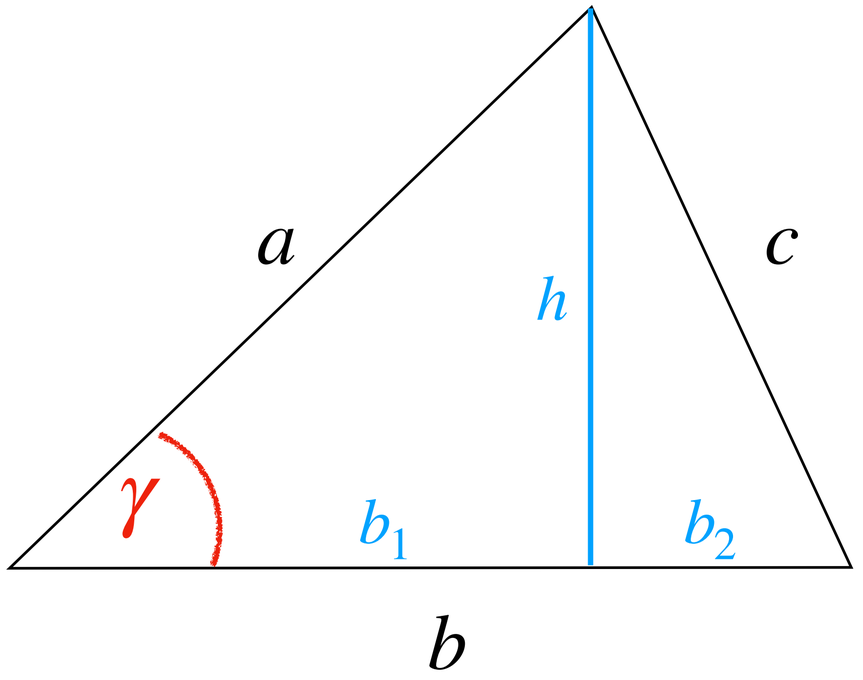

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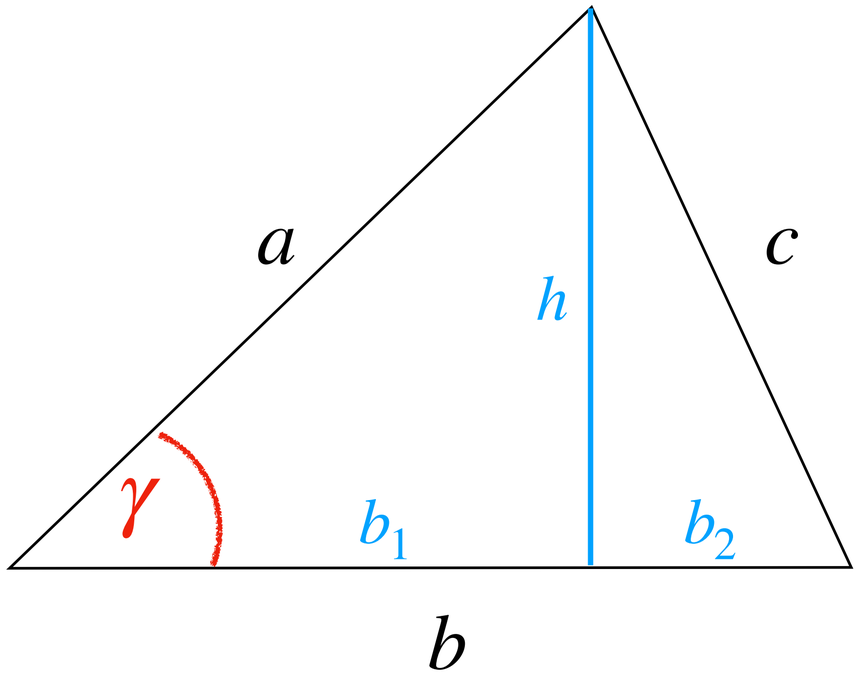

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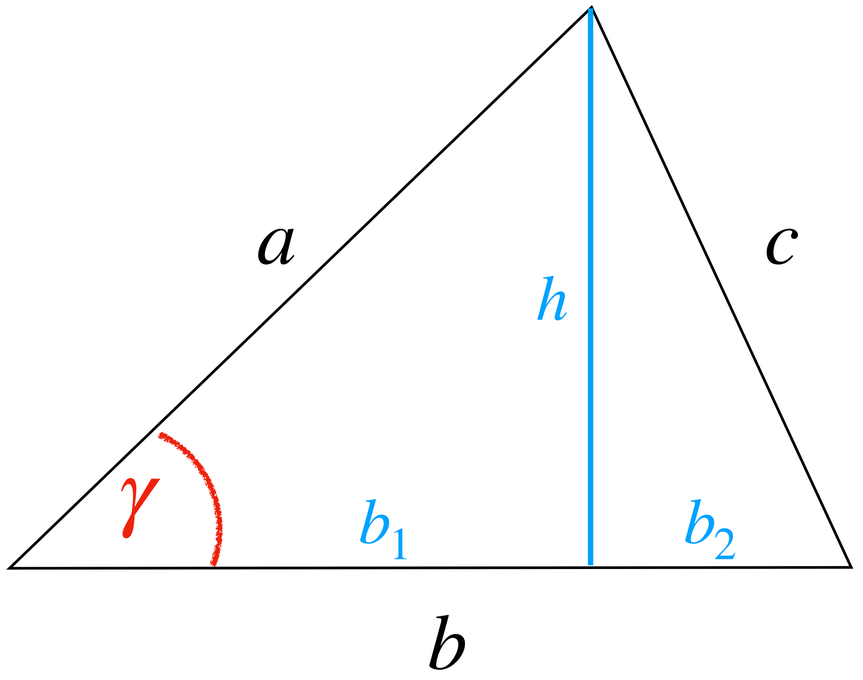

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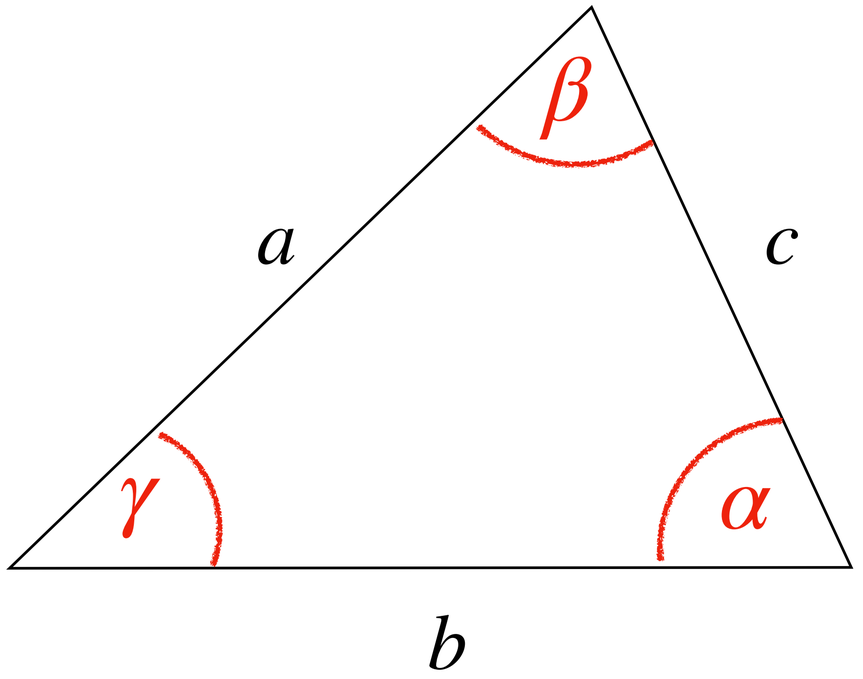

Two other formulas are proven in the same way.

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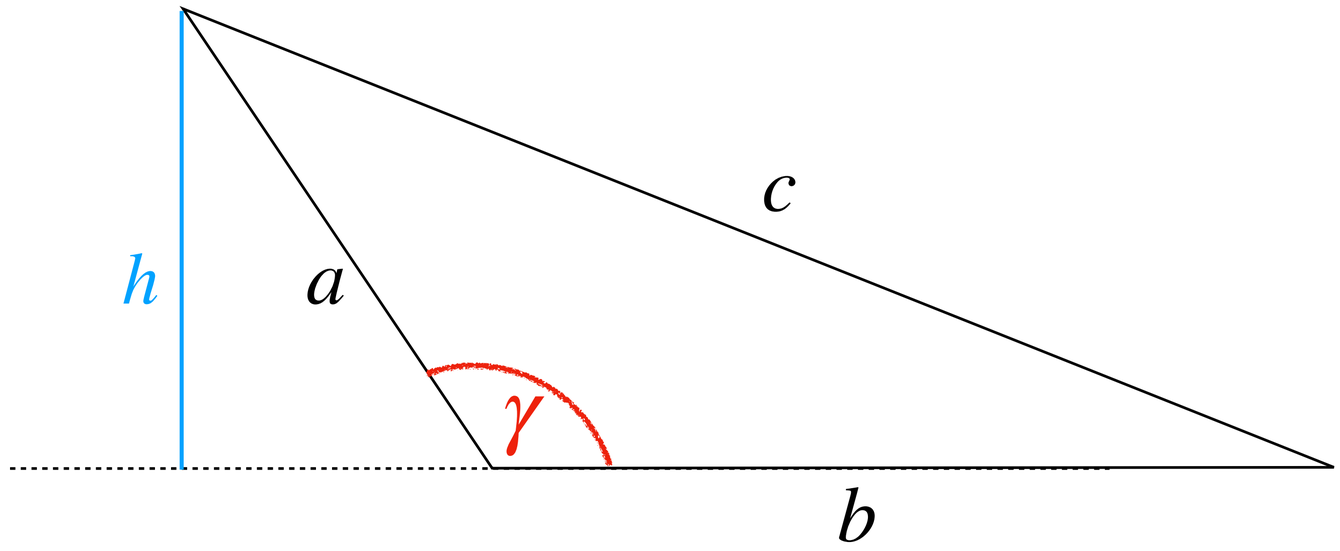

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If some angle is obtuse (greater than 90 degrees), the height falls outside the triangle.

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$$g(x) = \cos x$$