Topic: Modifying determinants

Question: If the determinant of matrix A is $|A| = -4$, find the determinant of matrix B, if B is identical to matrix A, except that the second and third rows have been swapped.

Answer choices:

A $|B| = -4$

B $|B| = 0$

C $|B| = 1$

D $|B| = 4$

Solution: D

Every time you swap a row in a matrix, the determinant gets multiplied by -1. Because B is the same as A, just with one row swap, $R_2 \leftrightarrow R_3$, then the determinant of B is

$$|B| = -1|A|$$

$$|B| = -1(-4)$$

$$|B| = 4$$

Topic: Modifying determinants

Question: Find the determinant. Hint: Consider the rows of A.

$$A = \begin{bmatrix} 1 & 5 & 0 & -1 & 6 & -1 \\ 3 & -2 & -1 & 2 & 0 & 2 \\ 3 & -2 & -1 & 2 & 0 & 2 \\ -1 & 1 & 0 & 3 & -2 & 3 \\ 1 & 3 & 2 & -2 & 1 & 0 \\ 0 & 4 & 1 & -3 & 5 & -1 \end{bmatrix}$$

Answer choices:

A $|A| = 0$

B $|A| = 1$

C $|A| = 2$

D $|A| = 5$

Solution: A

If a matrix contains duplicate rows, by definition its determinant must be 0. In this case, the second and third rows of A are identical. And because those rows are identical, we know the determinant of A is $|A| = 0$, and we don't have to calculate $|A|$ to figure this out.

Topic: Modifying determinants

Question: Find the determinant of C, using only the determinants of A and B.

$$A = \begin{bmatrix} -2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -2 & 1 \\ -2 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} -2 & 1 \\ -1 & 5 \end{bmatrix}$$

Answer choices:

A $|C| = 9$

B $|C| = 11$

C $|C| = -11$

D $|C| = -9$

Solution: D

Notice that all three of these matrices have identical first rows. Furthermore, the second row of C is the sum of the second rows of A and B. When this is the case, the determinants have the relationship

$$|C| = |A| + |B|$$

So we'll just find the determinants of A and B, and that'll give us the determinant of C.

$$|A| = \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} = (-2)(1) - (1)(1) = -2 - 1 = -3$$

$$|B| = \begin{vmatrix} -2 & 1 \\ -2 & 4 \end{vmatrix} = (-2)(4) - (1)(-2) = -8 + 2 = -6$$

Then the determinant of C is

$$|C| = -3 + (-6)$$

$$|C| = -3 - 6$$

$$|C| = -9$$