Linear Algebra Workbook Solutions

Inverses

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INVERSE OF A TRANSFORMATION

1. Given a vector $\vec{v}$ in $\mathbb{R}^3$, what would the identity transformation be?

Solution:

In $\mathbb{R}^3$, the identity transformation would be written as $I: \mathbb{R}^3 \to \mathbb{R}^3$, or maybe as $I_{\mathbb{R}^3}(\vec{v}) = \vec{v}$.

2. If a transformation T is invertible, what are the three conclusions that we can make about it?

Solution:

If a transformation T is invertible, we can conclude that

1. its inverse transformation is unique,
2. T is injective (or one-to-one), and
3. T is surjective (or onto).

3. If you can prove that a transformation T is both injective and surjective, and if you know that its inverse is unique, then what can you say about the transformation?

Solution:

You know the transformation is invertible.

4. Is the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ invertible?

$$T(x) = x^2$$

Solution:

The transform says that, given a value of x, the transform will return a value of x^2. So for instance, if we put in $x = 2$, the transform will return $x^2 = 2^2 = 4$. But if we put in $x = -2$, the transform will also return $x^2 = (-2)^2 = 4$. In other words, the transform can use two different values of x and return the same value for both, which means the transformation isn't one-to-one, and therefore can't be invertible.

5. Prove that $(T^{-1})^{-1} = T$.

Solution:

We know that any transformation multiplied by the identity transformation will simply give us back the original transformation.

$$(T^{-1})^{-1} = (T^{-1})^{-1}I$$

We also know that the identity transformation is equal to an inverse transformation multiplied by itself, $I = T^{-1}T$, so we can write

$$(T^{-1})^{-1} = (T^{-1})^{-1}(T^{-1}T)$$

Rearrange the parentheses.

$$(T^{-1})^{-1} = [(T^{-1})^{-1}T^{-1}]T$$

Pull out the inverse, switching the order.

$$(T^{-1})^{-1} = [T(T^{-1})]^{-1}T$$

$$(T^{-1})^{-1} = [TT^{-1}]^{-1}T$$

Then the result in the parentheses is just the identity transformation.

$$(T^{-1})^{-1} = [I]^{-1}T$$

$$(T^{-1})^{-1} = IT$$

$$(T^{-1})^{-1} = T$$

■ 6. Prove that the inverse of a transformation is unique.

Solution:

Let's assume that the inverse of a transformation is actually *not* unique, such that there's an invertible transformation T that has two unique inverses T_1^{-1} and T_2^{-1}, and $T_1^{-1} \neq T_2^{-1}$.

If this were true, it means that $TT_1^{-1} = T_1^{-1}T = I$ and $TT_2^{-1} = T_2^{-1}T = I$, because a transformation multiplied by its inverse will give you the identity transformation.

What we want to show is that, in fact, $T_1^{-1} = T_2^{-1}$, which will mean that our initial assumption that they're two *unique* inverses will be wrong. Let's start with T_1^{-1}:

$$T_1^{-1} = T_1^{-1}I$$

$$T_1^{-1} = T_1^{-1}(TT_2^{-1})$$

$$T_1^{-1} = (T_1^{-1}T)T_2^{-1}$$

$$T_1^{-1} = IT_2^{-1}$$

$$T_1^{-1} = T_2^{-1}$$

We've shown that $T_1^{-1} = T_2^{-1}$, even though our initial assumption was that $T_1^{-1} \neq T_2^{-1}$. Therefore, we know that the inverse of a transformation must always be unique.

INVERTIBILITY FROM THE MATRIX-VECTOR PRODUCT

■ 1. Is the matrix invertible?

$$\begin{bmatrix} 1 & 2 & 0 \\ -3 & 5 & -1 \end{bmatrix}$$

Solution:

The matrix isn't square, so it can't be invertible.

■ 2. Is the matrix invertible?

$$\begin{bmatrix} \pi & -\pi \\ -\pi & \pi \end{bmatrix}$$

Solution:

Divide through the matrix by π,

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

then put it into reduced row-echelon form.

$$\begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}$$

Because we don't have a pivot entry in every row, the matrix is not invertible.

■ 3. Is the matrix invertible?

$$\begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix}$$

Solution:

Divide through the matrix by π,

$$\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$$

then put it into reduced row-echelon form.

$$\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} \to \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

Because we were able to get the matrix into reduced row-echelon form, the matrix is invertible.

■ 4. Find the dimensions of the transformation matrix for each transformation, if each transformation were written as a matrix-vector product, $T(\vec{x}) = M\vec{x}$.

$$T: \mathbb{R}^3 \to \mathbb{R}^6$$

$$T : \mathbb{R}^4 \to \mathbb{R}^2$$

$$T : \mathbb{R}^u \to \mathbb{R}^w$$

Solution:

If $T : \mathbb{R}^3 \to \mathbb{R}^6$ were written as $T(\vec{x}) = M\vec{x}$, then M would be a 6×3 matrix.

If $T : \mathbb{R}^4 \to \mathbb{R}^2$ were written as $T(\vec{x}) = M\vec{x}$, then M would be a 2×4 matrix.

If $T : \mathbb{R}^u \to \mathbb{R}^w$ were written as $T(\vec{x}) = M\vec{x}$, then M would be a $w \times u$ matrix.

■ 5. Using the transformations from the previous question, state the dimensions of $\vec{x}$, and then state the dimensions of $T(\vec{x})$.

Solution:

For the transformation $\mathbb{R}^3 \to \mathbb{R}^6$, $\vec{x}$ must be a 3×1 vector. We know that a 6×3 matrix multiplied by a 3×1 vector will return a 6×1 vector, so $T(\vec{x})$ is a 6×1 vector.

For the transformation $T : \mathbb{R}^4 \to \mathbb{R}^2$, $\vec{x}$ must be a 4×1 vector. We know that a 2×4 matrix multiplied by a 4×1 vector will return a 2×1 vector, so $T(\vec{x})$ is a 2×1 vector.

For the transformation $T:\mathbb{R}^u \to \mathbb{R}^w$, $\vec{x}$ must be a $u \times 1$ vector. We know that a $w \times u$ matrix multiplied by a $u \times 1$ vector will return a $w \times 1$ vector, so $T(\vec{x})$ is a $w \times 1$ vector.

6. What can we say about the invertibility of the transformation $T:\mathbb{R}^u \to \mathbb{R}^w$ from the last two questions?

Solution:

We actually can't tell whether $T:\mathbb{R}^u \to \mathbb{R}^w$ is invertible. If $w = u$, then the transformation matrix is square, and there's then a possibility that the matrix, and therefore the transformation, is invertible. If $u \neq w$, then the transformation matrix isn't square, and the transformation is definitely not invertible.

INVERSE TRANSFORMATIONS ARE LINEAR

■ 1. Given two $n \times n$ matrices, A and B, if we know that $AB = I$ and $BA = I$, where I is the $n \times n$ identity matrix, then what else do we know about A and B?

Solution:

We know that A and B must be inverses of one another.

■ 2. Find the inverse of the matrix.

$$\begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix}$$

Solution:

First, set up the augmented matrix,

$$\begin{bmatrix} \pi & -\pi & | & 1 & 0 \\ \pi & \pi & | & 0 & 1 \end{bmatrix}$$

then put it into reduced row-echelon form.

$$\begin{bmatrix} 1 & -1 & | & \frac{1}{\pi} & 0 \\ 1 & 1 & | & 0 & \frac{1}{\pi} \end{bmatrix} \to \begin{bmatrix} 1 & -1 & | & \frac{1}{\pi} & 0 \\ 0 & 2 & | & -\frac{1}{\pi} & \frac{1}{\pi} \end{bmatrix} \to \begin{bmatrix} 1 & -1 & | & \frac{1}{\pi} & 0 \\ 0 & 1 & | & -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & \frac{1}{2\pi} & \frac{1}{2\pi} \\ 0 & 1 & | & -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix}$$

So the inverse matrix is

$$\begin{bmatrix} \frac{1}{2\pi} & \frac{1}{2\pi} \\ -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix}$$

■ 3. Prove that the matrix found in the previous question is actually the inverse of the original matrix.

Solution:

To prove that the matrices are inverses of one another, multiply them to show that we get the identity matrix. The product of the original matrix by the inverse gives

$$\begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix} \begin{bmatrix} \frac{1}{2\pi} & \frac{1}{2\pi} \\ -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix} = \begin{bmatrix} \pi\left(\frac{1}{2\pi}\right) - \pi\left(-\frac{1}{2\pi}\right) & \pi\left(\frac{1}{2\pi}\right) - \pi\left(\frac{1}{2\pi}\right) \\ \pi\left(\frac{1}{2\pi}\right) + \pi\left(-\frac{1}{2\pi}\right) & \pi\left(\frac{1}{2\pi}\right) + \pi\left(\frac{1}{2\pi}\right) \end{bmatrix}$$

$$\begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix} \begin{bmatrix} \frac{1}{2\pi} & \frac{1}{2\pi} \\ -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} & \frac{1}{2} - \frac{1}{2} \\ \frac{1}{2} - \frac{1}{2} & \frac{1}{2} + \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix} \begin{bmatrix} \frac{1}{2\pi} & \frac{1}{2\pi} \\ -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

And the product of the inverse matrix by the original gives

$$\begin{bmatrix} \frac{1}{2\pi} & \frac{1}{2\pi} \\ -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix} \begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix} = \begin{bmatrix} \frac{1}{2\pi}\pi + \frac{1}{2\pi}\pi & \frac{1}{2\pi}(-\pi) + \frac{1}{2\pi}\pi \\ -\frac{1}{2\pi}\pi + \frac{1}{2\pi}\pi & -\frac{1}{2\pi}(-\pi) + \frac{1}{2\pi}\pi \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2\pi} & \frac{1}{2\pi} \\ -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix} \begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix} = \begin{bmatrix} \frac{1}{2} + \frac{1}{2} & -\frac{1}{2} + \frac{1}{2} \\ -\frac{1}{2} + \frac{1}{2} & \frac{1}{2} + \frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} \frac{1}{2\pi} & \frac{1}{2\pi} \\ -\frac{1}{2\pi} & \frac{1}{2\pi} \end{bmatrix} \begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

4. Find the inverse of the matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix}$$

Solution:

Set up the augmented matrix,

$$\begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 5 & | & 0 & 1 & 0 \\ 5 & 6 & 0 & | & 0 & 0 & 1 \end{bmatrix}$$

then put it into reduced row-echelon form.

$$\begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 5 & | & 0 & 1 & 0 \\ 5 & 6 & 0 & | & 0 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 2 & 3 & | & 1 & 0 & 0 \\ 0 & 1 & 5 & | & 0 & 1 & 0 \\ 0 & -4 & -15 & | & -5 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -7 & | & 1 & -2 & 0 \\ 0 & 1 & 5 & | & 0 & 1 & 0 \\ 0 & -4 & -15 & | & -5 & 0 & 1 \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -7 & | & 1 & -2 & 0 \\ 0 & 1 & 5 & | & 0 & 1 & 0 \\ 0 & 0 & 5 & | & -5 & 4 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -7 & | & 1 & -2 & 0 \\ 0 & 1 & 5 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -1 & \frac{4}{5} & \frac{1}{5} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & | & -6 & \frac{18}{5} & \frac{7}{5} \\ 0 & 1 & 5 & | & 0 & 1 & 0 \\ 0 & 0 & 1 & | & -1 & \frac{4}{5} & \frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & -6 & \frac{18}{5} & \frac{7}{5} \\ 0 & 1 & 0 & | & 5 & -3 & -1 \\ 0 & 0 & 1 & | & -1 & \frac{4}{5} & \frac{1}{5} \end{bmatrix}$$

Then the inverse matrix is

$$\begin{bmatrix} -6 & \frac{18}{5} & \frac{7}{5} \\ 5 & -3 & -1 \\ -1 & \frac{4}{5} & \frac{1}{5} \end{bmatrix}$$

■ 5. Prove that the matrix we found in the previous question is actually the inverse of the original matrix.

Solution:

Multiply the original matrix by its inverse.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix} \begin{bmatrix} -6 & \frac{18}{5} & \frac{7}{5} \\ 5 & -3 & -1 \\ -1 & \frac{4}{5} & \frac{1}{5} \end{bmatrix}$$

$$\begin{bmatrix} 1(-6)+2(5)+3(-1) & 1\left(\frac{18}{5}\right)+2(-3)+3\left(\frac{4}{5}\right) & 1\left(\frac{7}{5}\right)+2(-1)+3\left(\frac{1}{5}\right) \\ 0(-6)+1(5)+5(-1) & 0\left(\frac{18}{5}\right)+1(-3)+5\left(\frac{4}{5}\right) & 0\left(\frac{7}{5}\right)+1(-1)+5\left(\frac{1}{5}\right) \\ 5(-6)+6(5)+0(-1) & 5\left(\frac{18}{5}\right)+6(-3)+0\left(\frac{4}{5}\right) & 5\left(\frac{7}{5}\right)+6(-1)+0\left(\frac{1}{5}\right) \end{bmatrix}$$

$$\begin{bmatrix} -6+10-3 & \frac{18}{5}-6+\frac{12}{5} & \frac{7}{5}-2+\frac{3}{5} \\ 0+5-5 & 0-3+4 & 0-1+1 \\ -30+30+0 & 18-18+0 & 7-6+0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Because we get the identity matrix, we know that the two matrices we multiplied together must be inverses of one another.

■ 6. Prove that the inverse of an invertible linear transformation T is also a linear transformation.

Solution:

We know that T is linear, which means that for vectors $\vec{x}$ and $\vec{y}$ and a constant c, we know $T(\vec{x}+\vec{y})=T(\vec{x})+T(\vec{y})$ and $T(c\vec{x})=cT(\vec{x})$. We also know that $(T^{-1}\circ T)(\vec{x})=(T\circ T^{-1})(\vec{x})=I(\vec{x})$, and we want to prove that $T^{-1}(\vec{x}+\vec{y})=T^{-1}(\vec{x})+T^{-1}(\vec{y})$ and $T^{-1}(c\vec{x})=cT^{-1}(\vec{x})$.

We know that $(T\circ T^{-1})(\vec{x}+\vec{y})$ is the same as saying $I(\vec{x}+\vec{y})$.

$$(T\circ T^{-1})(\vec{x}+\vec{y})=I(\vec{x}+\vec{y})$$

$$(T\circ T^{-1})(\vec{x}+\vec{y})=\vec{x}+\vec{y}$$

Using that same logic, we can also say that $\vec{x}+\vec{y}$ is the same as saying $I\vec{x}+I\vec{y}$, or $(T\circ T^{-1})\vec{x}+(T\circ T^{-1})\vec{y}$, which means that we now have

$$(T\circ T^{-1})(\vec{x}+\vec{y})=I(\vec{x}+\vec{y})$$

$$(T\circ T^{-1})(\vec{x}+\vec{y})=I\vec{x}+I\vec{y}$$

$$(T \circ T^{-1})(\overrightarrow{x} + \overrightarrow{y}) = (T \circ T^{-1})\overrightarrow{x} + (T \circ T^{-1})\overrightarrow{y}$$

If we rearrange this, we get

$$T[T^{-1}(\overrightarrow{x} + \overrightarrow{y})] = T[T^{-1}(\overrightarrow{x}) + T^{-1}(\overrightarrow{y})]$$

and if we apply T^{-1} to both sides, we get

$$T^{-1}(T[T^{-1}(\overrightarrow{x} + \overrightarrow{y})]) = T^{-1}(T[T^{-1}(\overrightarrow{x}) + T^{-1}(\overrightarrow{y})])$$

$$(I)[T^{-1}(\overrightarrow{x} + \overrightarrow{y})] = (I)[T^{-1}(\overrightarrow{x}) + T^{-1}(\overrightarrow{y})]$$

$$T^{-1}(\overrightarrow{x} + \overrightarrow{y}) = T^{-1}(\overrightarrow{x}) + T^{-1}(\overrightarrow{y})$$

MATRIX INVERSES, AND INVERTIBLE AND SINGULAR MATRICES

■ 1. Find the inverse of matrix G.

$$G = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

Solution:

Plug into the formula for the inverse matrix.

$$G^{-1} = \frac{1}{|G|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$G^{-1} = \frac{1}{\begin{vmatrix} -3 & 8 \\ 0 & -2 \end{vmatrix}} \begin{bmatrix} -2 & -8 \\ 0 & -3 \end{bmatrix}$$

$$G^{-1} = \frac{1}{(-3)(-2) - (8)(0)} \begin{bmatrix} -2 & -8 \\ 0 & -3 \end{bmatrix}$$

$$G^{-1} = \frac{1}{6} \begin{bmatrix} -2 & -8 \\ 0 & -3 \end{bmatrix}$$

$$G^{-1} = \begin{bmatrix} -\frac{1}{3} & -\frac{4}{3} \\ 0 & -\frac{1}{2} \end{bmatrix}$$

2. Find the inverse of matrix N.

$$N = \begin{bmatrix} 11 & -4 \\ 5 & -3 \end{bmatrix}$$

Solution:

Plug into the formula for the inverse matrix.

$$N^{-1} = \frac{1}{|N|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$N^{-1} = \frac{1}{\begin{vmatrix} 11 & -4 \\ 5 & -3 \end{vmatrix}} \begin{bmatrix} -3 & 4 \\ -5 & 11 \end{bmatrix}$$

$$N^{-1} = \frac{1}{(11)(-3) - (-4)(5)} \begin{bmatrix} -3 & 4 \\ -5 & 11 \end{bmatrix}$$

$$N^{-1} = \frac{1}{-33 + 20} \begin{bmatrix} -3 & 4 \\ -5 & 11 \end{bmatrix}$$

$$N^{-1} = -\frac{1}{13} \begin{bmatrix} -3 & 4 \\ -5 & 11 \end{bmatrix}$$

$$N^{-1} = \begin{bmatrix} \frac{3}{13} & -\frac{4}{13} \\ \frac{5}{13} & -\frac{11}{13} \end{bmatrix}$$

■ 3. What is the inverse of matrix K?

$$K = \begin{bmatrix} 3 & 3 \\ -6 & 0 \end{bmatrix}$$

Solution:

Plug into the formula for the inverse matrix.

$$K^{-1} = \frac{1}{|K|} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$K^{-1} = \frac{1}{\begin{vmatrix} 3 & 3 \\ -6 & 0 \end{vmatrix}} \begin{bmatrix} 0 & -3 \\ 6 & 3 \end{bmatrix}$$

$$K^{-1} = \frac{1}{(3)(0) - (3)(-6)} \begin{bmatrix} 0 & -3 \\ 6 & 3 \end{bmatrix}$$

$$K^{-1} = \frac{1}{0 + 18} \begin{bmatrix} 0 & -3 \\ 6 & 3 \end{bmatrix}$$

$$K^{-1} = \frac{1}{18} \begin{bmatrix} 0 & -3 \\ 6 & 3 \end{bmatrix}$$

$$K^{-1} = \begin{bmatrix} 0 & -\frac{1}{6} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$$

4. Is the matrix invertible or singular?

$$Z = \begin{bmatrix} 4 & 2 \\ -2 & -1 \end{bmatrix}$$

Solution:

Find the determinant of the matrix.

$$|Z| = \begin{vmatrix} 4 & 2 \\ -2 & -1 \end{vmatrix}$$

$$|Z| = (4)(-1) - (2)(-2)$$

$$|Z| = -4 + 4$$

$$|Z| = 0$$

Because the determinant is 0, Z is a singular matrix that has no inverse.

5. Is the matrix invertible or singular?

$$Y = \begin{bmatrix} 0 & 6 \\ 2 & -1 \end{bmatrix}$$

Solution:

Find the determinant of the matrix.

$$|Y| = \begin{vmatrix} 0 & 6 \\ 2 & -1 \end{vmatrix}$$

$$|Y| = (0)(-1) - (6)(2)$$

$$|Y| = 0 - 12$$

$$|Y| = -12$$

Because the determinant is non-zero, Y is an invertible matrix with a defined inverse.

6. Is B invertible?

$$B = \begin{bmatrix} -4 & 1 \\ -5 & 0 \end{bmatrix}$$

Solution:

Find the determinant of the matrix.

$$|B| = \begin{vmatrix} -4 & 1 \\ -5 & 0 \end{vmatrix}$$

$$|B| = (-4)(0) - (1)(-5)$$

$$|B| = 0 + 5$$

$$|B| = 5$$

Because the determinant is non-zero, B is an invertible matrix with a defined inverse.

SOLVING SYSTEMS WITH INVERSE MATRICES

1. Use an inverse matrix to solve the system.

$$-4x + 3y = -14$$

$$7x - 4y = 32$$

Solution:

Transfer the system into a matrix equation.

$$\begin{bmatrix} -4 & 3 \\ 7 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -14 \\ 32 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M^{-1} = \frac{1}{(-4)(-4) - (3)(7)} \begin{bmatrix} -4 & -3 \\ -7 & -4 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{5} \begin{bmatrix} -4 & -3 \\ -7 & -4 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} \frac{4}{5} & \frac{3}{5} \\ \frac{7}{5} & \frac{4}{5} \end{bmatrix}$$

The solution to the system is

$$\vec{a}=\begin{bmatrix}\frac{4}{5} & \frac{3}{5}\\ \frac{7}{5} & \frac{4}{5}\end{bmatrix}\begin{bmatrix}-14\\ 32\end{bmatrix}$$

$$\vec{a}=\begin{bmatrix}\frac{4}{5}(-14)+\frac{3}{5}(32)\\ \frac{7}{5}(-14)+\frac{4}{5}(32)\end{bmatrix}$$

$$\vec{a}=\begin{bmatrix}-\frac{56}{5}+\frac{96}{5}\\ -\frac{98}{5}+\frac{128}{5}\end{bmatrix}$$

$$\vec{a}=\begin{bmatrix}\frac{40}{5}\\ \frac{30}{5}\end{bmatrix}$$

$$\vec{a}=\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}8\\ 6\end{bmatrix}$$

■ 2. Use an inverse matrix to solve the system.

$$6x-11y=2$$

$$-10x+7y=-26$$

Solution:

Transfer the system into a matrix equation.

$$\begin{bmatrix} 6 & -11 \\ -10 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -26 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M^{-1} = \frac{1}{(6)(7) - (-11)(-10)} \begin{bmatrix} 7 & 11 \\ 10 & 6 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{68} \begin{bmatrix} 7 & 11 \\ 10 & 6 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} -\frac{7}{68} & -\frac{11}{68} \\ -\frac{10}{68} & -\frac{6}{68} \end{bmatrix}$$

The solution to the system is

$$\vec{a} = \begin{bmatrix} -\frac{7}{68} & -\frac{11}{8} \\ -\frac{10}{68} & -\frac{6}{68} \end{bmatrix} \begin{bmatrix} 2 \\ -26 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\frac{7}{68}(2) - \frac{11}{68}(-26) \\ -\frac{10}{68}(2) - \frac{6}{68}(-26) \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\frac{14}{68} + \frac{286}{68} \\ -\frac{20}{68} + \frac{156}{68} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{272}{68} \\ \frac{136}{68} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$$

3. Use an inverse matrix to solve the system.

$$13y - 6x = -81$$

$$7x + 17 = -22y$$

Solution:

Transfer the system into a matrix equation.

$$\begin{bmatrix} -6 & 13 \\ 7 & 22 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -81 \\ -17 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M^{-1} = \frac{1}{(-6)(22) - (13)(7)} \begin{bmatrix} 22 & -13 \\ -7 & -6 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{223} \begin{bmatrix} 22 & -13 \\ -7 & -6 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} -\frac{22}{223} & \frac{13}{223} \\ \frac{7}{223} & \frac{6}{223} \end{bmatrix}$$

The solution to the system is

$$\vec{a} = \begin{bmatrix} -\frac{22}{223} & \frac{13}{223} \\ \frac{7}{223} & \frac{6}{223} \end{bmatrix} \begin{bmatrix} -81 \\ -17 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\frac{22}{223}(-81) + \frac{13}{223}(-17) \\ \frac{7}{223}(-81) + \frac{6}{223}(-17) \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{1,782}{223} - \frac{221}{223} \\ -\frac{567}{223} - \frac{102}{223} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{1,561}{223} \\ -\frac{669}{223} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$$

■ 4. Sketch a graph of vectors to visually find the solution to the system.

$$3x = 3$$

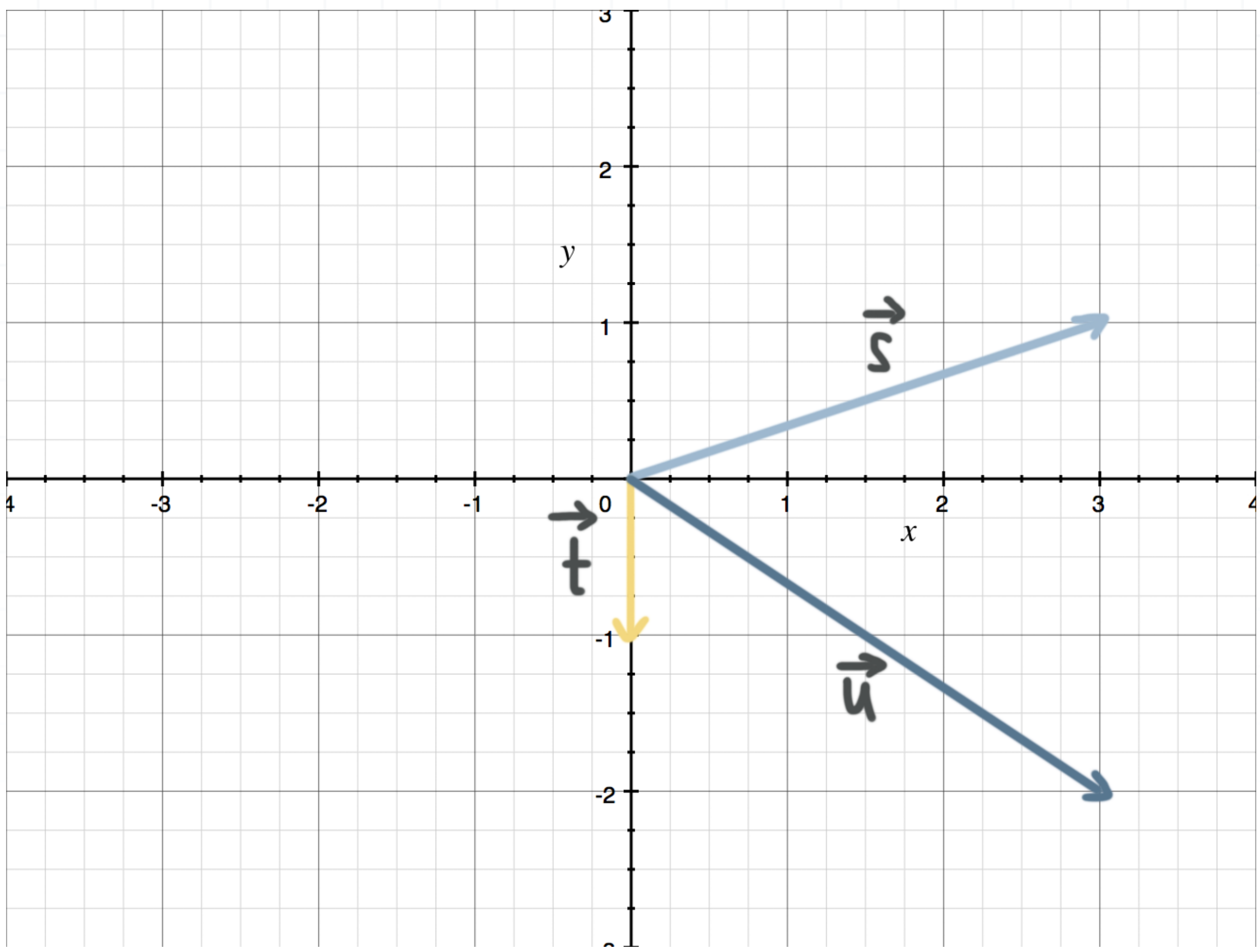
$$x - y = -2$$

Solution:

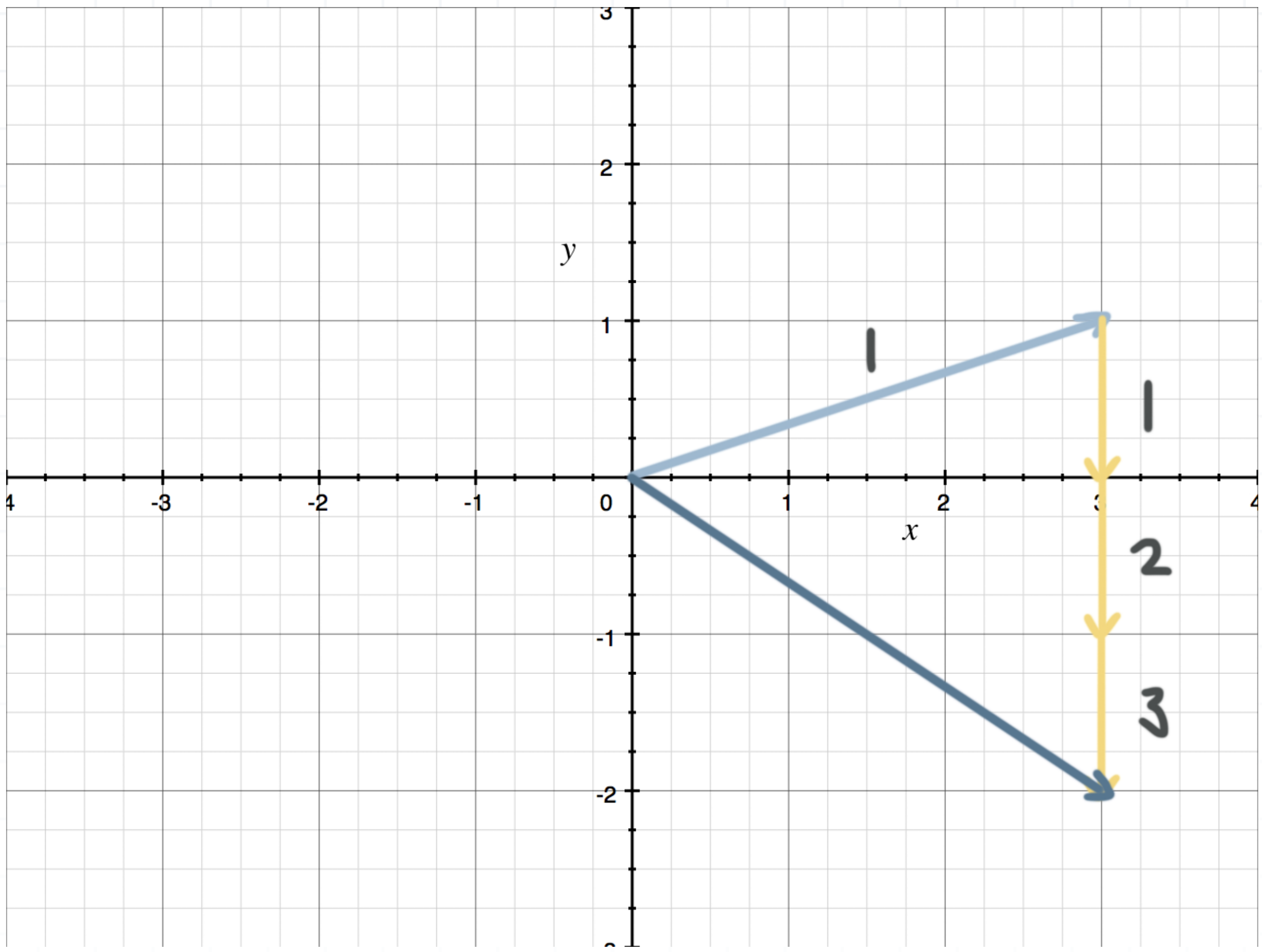
Put the system into a matrix equation.

$$\begin{bmatrix} 3 \\ 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ -1 \end{bmatrix} y = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

From the vector equation, we can sketch the vectors $\vec{s} = (3,1)$ for x, $\vec{t} = (0, -1)$ for y, and the resulting vector $\vec{u} = (3, -2)$.



If we play around a little bit with the vectors in the graph, we can see that putting one $\vec{s}$ and three $\vec{t}$s together will get us back to the terminal point of $\vec{u}$, so $x = 1$ and $y = 3$.



■ 5. Sketch a graph of vectors to visually find the solution to the system.

$$-y = -4$$

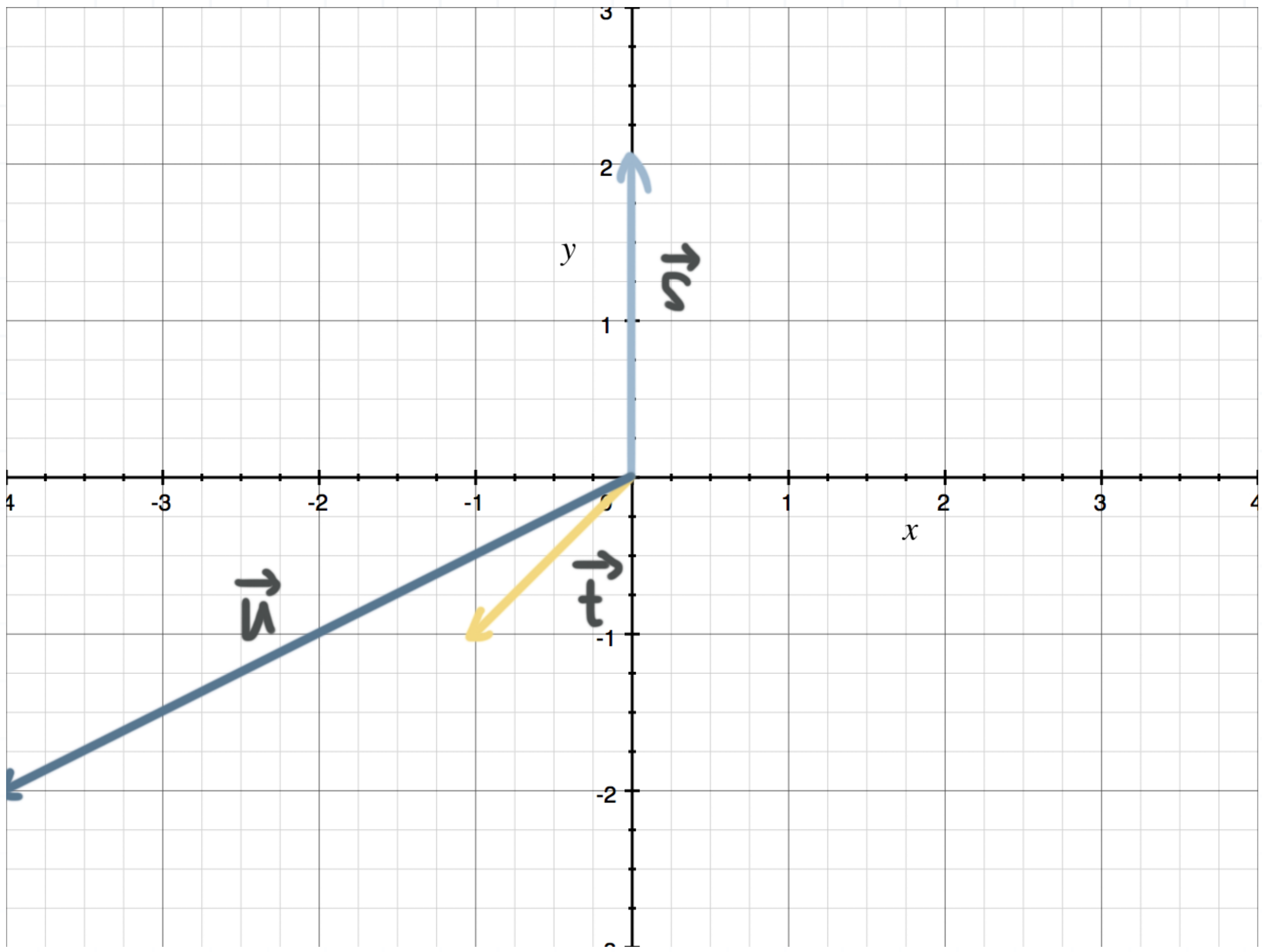
$$2x - y = -2$$

Solution:

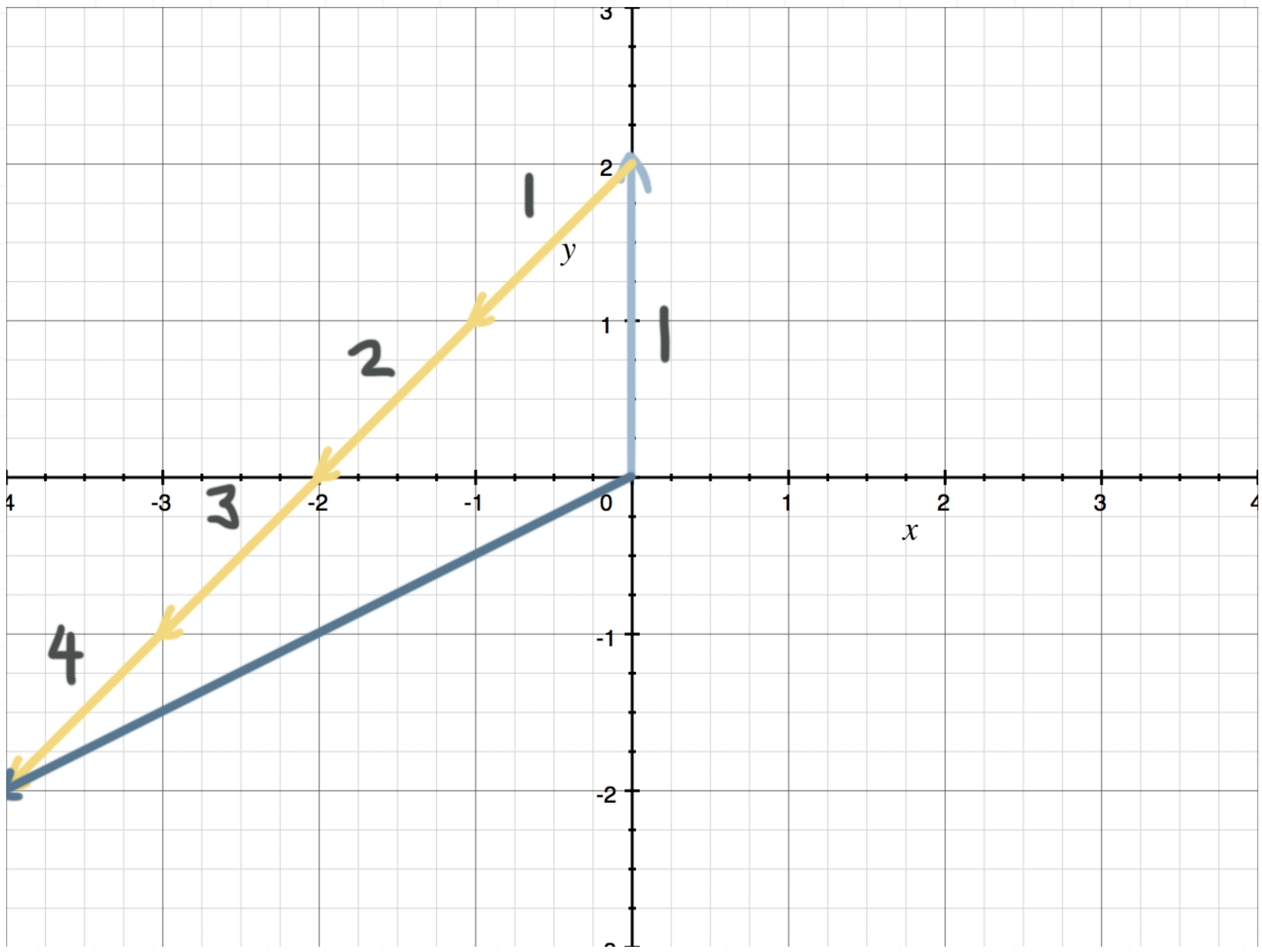
Put the system into a matrix equation.

$$\begin{bmatrix} 0 \\ 2 \end{bmatrix} x + \begin{bmatrix} -1 \\ -1 \end{bmatrix} y = \begin{bmatrix} -4 \\ -2 \end{bmatrix}$$

From the vector equation, we can sketch the vectors $\vec{s}=(0,2)$ for x, $\vec{t}=(-1,-1)$ for y, and the resulting vector $\vec{u}=(-4,-2)$.



If we play around a little bit with the vectors in the graph, we can see that putting one $\vec{s}$ and four $\vec{t}$s together will get us back to the terminal point of $\vec{u}$, so $x=1$ and $y=4$.



6. Sketch a graph of vectors to visually find the solution to the system.

$$x - y = 0$$

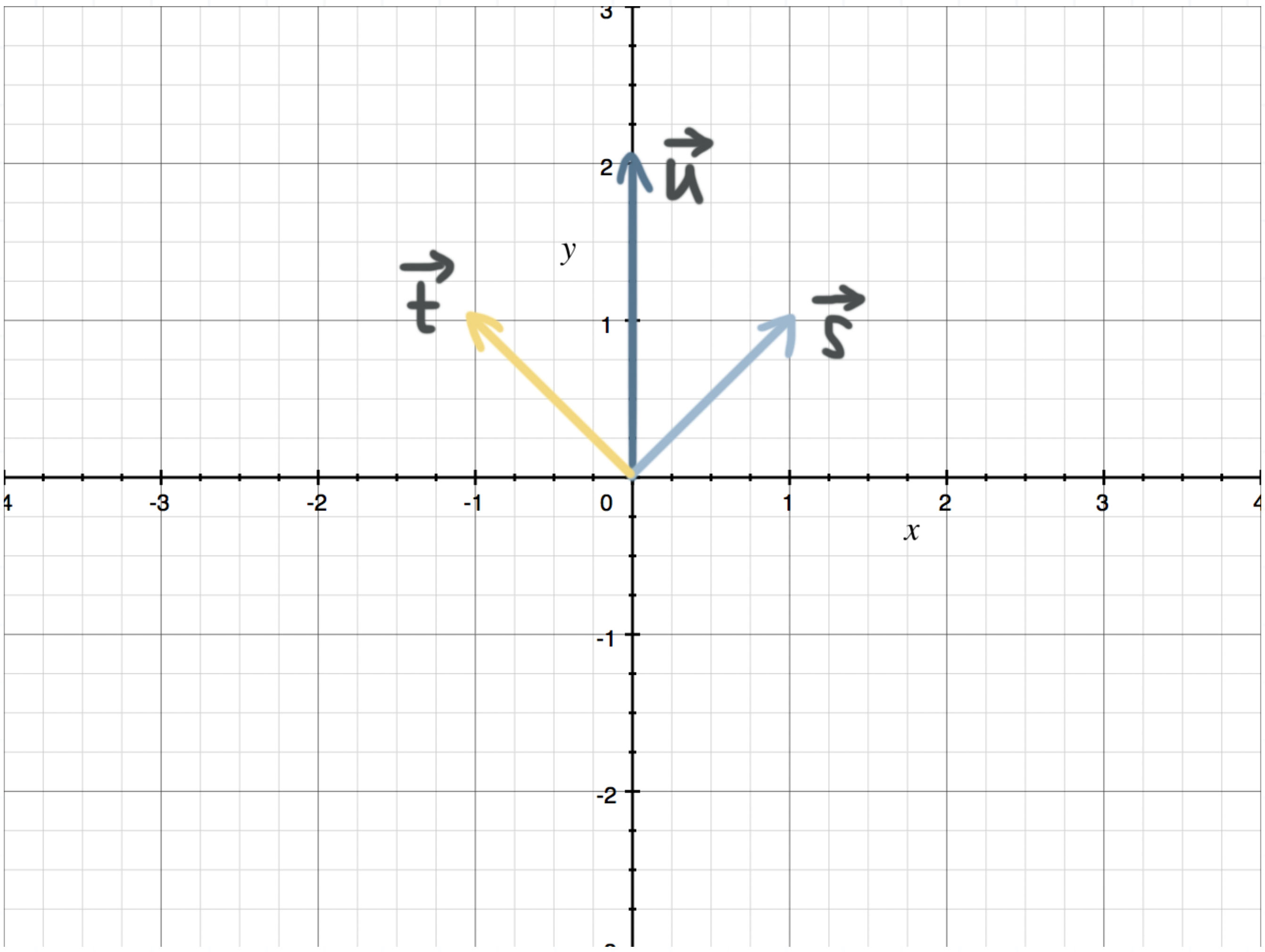
$$x + y = 2$$

Solution:

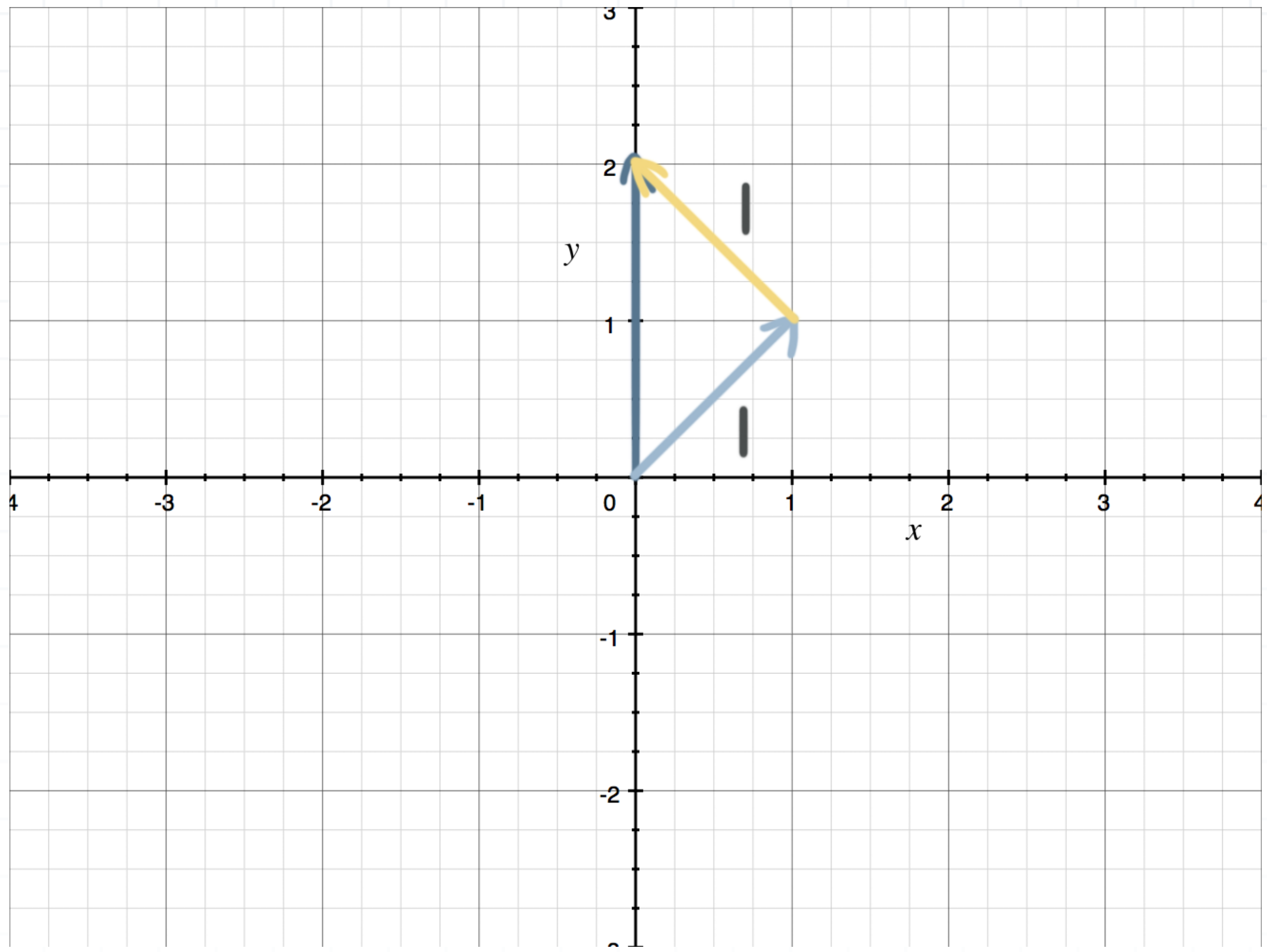
Put the system into a matrix equation.

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix} x + \begin{bmatrix} -1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

From the vector equation, we can sketch the vectors $\vec{s}=(1,1)$ for x, $\vec{t}=(-1,1)$ for y, and the resulting vector $\vec{u}=(0,2)$.



If we play around a little bit with the vectors in the graph, we can see that putting one $\vec{s}$ and one $\vec{t}$ together will get us back to the terminal point of $\vec{u}$, so $x=1$ and $y=1$.



3
2
1
0
-1
-2
-3
-4
4
y
x

W W W . K R I S T A K I N G M A T H . C O M