Topic: Using determinants to find area

Question: Find the area of the parallelogram formed by $\vec{v}_1 = (2,3)$ and $\vec{v}_2 = (-1,4)$, if the two vectors form adjacent edges of the parallelogram.

Answer choices:

A $|A| = 5$

B $|A| = 6$

C $|A| = 11$

D $|A| = 13$

Solution: C

When two vectors form adjacent edges of a parallelogram, we can find the area of the parallelogram by taking the determinant of the matrix of the vectors as column vectors.

In other words, we'll put $\vec{v}_1 = (2,3)$ and $\vec{v}_2 = (-1,4)$ as column vectors into a matrix

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}$$

and then find the determinant of that matrix, which will be the area of the parallelogram.

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$|A| = (2)(4) - (-1)(3)$$

$$|A| = 8 + 3$$

$$|A| = 11$$

The area of the parallelogram is 11 square units.

Topic: Using determinants to find area

Question: The square S is defined by the vertices $(1,1)$, $(-1,1)$, $(-1,-1)$, and $(1,-1)$. If the transformation of S by T creates a transformed figure F, find the area of F.

$$T(\vec{x}) = \begin{bmatrix} -3 & 2 \\ -2 & 1 \end{bmatrix} \vec{x}$$

Answer choices:

A $\quad \text{Area}_F = 4$

B $\quad \text{Area}_F = -4$

C $\quad \text{Area}_F = 3$

D $\quad \text{Area}_F = -3$

Solution: A

The area of the transformed figure F can be found using just the area of the square S, and the determinant of the transformation T.

$$\text{Area}_F = |\text{Area}_S(\text{Det}(T))|$$

The square S is defined between $x = -1$ and $x = 1$, so its width is 2, and it's defined between $y = -1$ and $y = 1$, so its height is 2. Therefore, the area of the square is $\text{Area}_S = 2 \cdot 2 = 4$.

The determinant of the transformation matrix is

$$|T| = \begin{vmatrix} -3 & 2 \\ -2 & 1 \end{vmatrix}$$

$$|T| = (-3)(1) - (2)(-2)$$

$$|T| = -3 + 4$$

$$|T| = 1$$

Then the area of the transformed figure F is

$$\text{Area}_F = |\text{Area}_S(\text{Det}(T))|$$

$$\text{Area}_F = |(4)(1)|$$

$$\text{Area}_F = |4|$$

$$\text{Area}_F = 4$$

Topic: Using determinants to find area

Question: The rectangle R is defined by the vertices $(-6,2)$, $(1,2)$, $(1,-4)$, and $(-6,-4)$. If the transformation of R by T creates a transformed figure L, find the area of L.

$$T(\vec{x}) = \begin{bmatrix} 2 & 0 \\ -1 & 4 \end{bmatrix} \vec{x}$$

Answer choices:

A $\quad$ $\text{Area}_L = 123$

B $\quad$ $\text{Area}_L = 164$

C $\quad$ $\text{Area}_L = 271$

D $\quad$ $\text{Area}_L = 336$

Solution: D

The area of the transformed figure L can be found using just the area of the rectangle R, and the determinant of the transformation T.

$$\text{Area}_L = |\text{Area}_R(\text{Det}(T))|$$

The rectangle R is defined between $x = -6$ and $x = 1$, so its width is 7, and it's defined between $y = -4$ and $y = 2$, so its height is 6. Therefore, the area of the square is $\text{Area}_S = 7 \cdot 6 = 42$.

The determinant of the transformation matrix is

$$|T| = \begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix}$$

$$|T| = (2)(4) - (0)(-1)$$

$$|T| = 8 + 0$$

$$|T| = 8$$

Then the area of the transformed figure L is

$$\text{Area}_L = |\text{Area}_R(\text{Det}(T))|$$

$$\text{Area}_L = |(42)(8)|$$

$$\text{Area}_L = |336|$$

$$\text{Area}_L = 336$$