Topic: Projections as linear transformations

Question: Find the projection of $\vec{v}$ onto L.

$$L = \left\{ c \begin{bmatrix} -2 \\ 0 \end{bmatrix} \;\middle|\; c \in \mathbb{R} \right\}$$

$$\vec{v} = \begin{bmatrix} 1 \\ 5 \end{bmatrix}$$

Answer choices:

A $\text{Proj}_L(\vec{v}) = (1,0)$

B $\text{Proj}_L(\vec{v}) = (0,1)$

C $\text{Proj}_L(\vec{v}) = (-1,0)$

D $\text{Proj}_L(\vec{v}) = (0,-1)$

Solution: A

The line L is given as all the scaled versions of the vector $\vec{x} = (-2,0)$. Then the projection of $\vec{v}$ onto L is given by

$$\text{Proj}_L(\vec{v}) = \left(\frac{\vec{v} \cdot \vec{x}}{\vec{x} \cdot \vec{x}}\right)\vec{x}$$

$$\text{Proj}_L(\vec{v}) = \frac{\begin{bmatrix}1 & 5\end{bmatrix} \cdot \begin{bmatrix}-2\\0\end{bmatrix}}{\begin{bmatrix}-2 & 0\end{bmatrix} \cdot \begin{bmatrix}-2\\0\end{bmatrix}} \cdot \begin{bmatrix}-2\\0\end{bmatrix} = \frac{1(-2)+5(0)}{-2(-2)+0(0)} \cdot \begin{bmatrix}-2\\0\end{bmatrix}$$

$$\text{Proj}_L(\vec{v}) = \frac{-2}{4} \cdot \begin{bmatrix}-2\\0\end{bmatrix} = -\frac{1}{2} \cdot \begin{bmatrix}-2\\0\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

Topic: Projections as linear transformations

Question: Find the projection of $\vec{v}$ onto L.

$$L = \left\{ c \begin{bmatrix} 3 \\ -4 \end{bmatrix} \;\middle|\; c \in \mathbb{R} \right\}$$

$$\vec{v} = \begin{bmatrix} -2 \\ -1 \end{bmatrix}$$

Answer choices:

A $\text{Proj}_L(\vec{v}) = \left(\frac{6}{25}, \frac{8}{25} \right)$

B $\text{Proj}_L(\vec{v}) = \left(-\frac{6}{25}, -\frac{8}{25} \right)$

C $\text{Proj}_L(\vec{v}) = \left(\frac{6}{25}, -\frac{8}{25} \right)$

D $\text{Proj}_L(\vec{v}) = \left(-\frac{6}{25}, \frac{8}{25} \right)$

Solution: D

The line L is given as all the scaled versions of the vector $\vec{x} = (3, -4)$. Then the projection of $\vec{v}$ onto L is given by

$$\text{Proj}_L(\vec{v}) = \left(\frac{\vec{v} \cdot \vec{x}}{\vec{x} \cdot \vec{x}} \right) \vec{x}$$

$$\text{Proj}_L(\vec{v}) = \frac{\begin{bmatrix} -2 & -1 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -4 \end{bmatrix}}{\begin{bmatrix} 3 & -4 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -4 \end{bmatrix}} \cdot \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \frac{-2(3) - 1(-4)}{3(3) - 4(-4)} \cdot \begin{bmatrix} 3 \\ -4 \end{bmatrix}$$

$$\text{Proj}_L(\vec{v}) = \frac{-2}{25} \cdot \begin{bmatrix} 3 \\ -4 \end{bmatrix} = -\frac{2}{25} \cdot \begin{bmatrix} 3 \\ -4 \end{bmatrix} = \begin{bmatrix} -\frac{6}{25} \\ \frac{8}{25} \end{bmatrix}$$

Topic: Projections as linear transformations

Question: Find the projection of $\vec{v}$ onto L.

$$L = \left\{ c \begin{bmatrix} -6 \\ 2 \end{bmatrix} \;\middle|\; c \in \mathbb{R} \right\}$$

$$\vec{v} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

Answer choices:

A $\text{Proj}_L(\vec{v}) = \left(\frac{3}{5}, \frac{1}{5}\right)$

B $\text{Proj}_L(\vec{v}) = \left(\frac{3}{5}, -\frac{1}{5}\right)$

C $\text{Proj}_L(\vec{v}) = \left(\frac{3}{10}, -\frac{2}{5}\right)$

D $\text{Proj}_L(\vec{v}) = \left(-\frac{3}{10}, -\frac{2}{5}\right)$

Solution: B

The line L is given as all the scaled versions of the vector $\vec{x} = (-6,2)$. Then the projection of $\vec{v}$ onto L is given by

$$\text{Proj}_L(\vec{v}) = \left(\frac{\vec{v} \cdot \vec{x}}{\vec{x} \cdot \vec{x}}\right)\vec{x}$$

$$\text{Proj}_L(\vec{v}) = \frac{\begin{bmatrix}2 & 4\end{bmatrix} \cdot \begin{bmatrix}-6\\2\end{bmatrix}}{\begin{bmatrix}-6 & 2\end{bmatrix} \cdot \begin{bmatrix}-6\\2\end{bmatrix}} \cdot \begin{bmatrix}-6\\2\end{bmatrix} = \frac{2(-6)+4(2)}{-6(-6)+2(2)} \cdot \begin{bmatrix}-6\\2\end{bmatrix}$$

$$\text{Proj}_L(\vec{v}) = \frac{-4}{40} \cdot \begin{bmatrix}-6\\2\end{bmatrix} = -\frac{1}{10} \cdot \begin{bmatrix}-6\\2\end{bmatrix} = \begin{bmatrix}\frac{3}{5}\\-\frac{1}{5}\end{bmatrix}$$