Topic: Dot and cross products as opposite ideas

Question: When is the dot product of two vectors maximized?

Answer choices:

A When the vectors point in exactly opposite directions

B When the vectors are orthogonal

C When the vectors point in exactly the same direction

D When the vectors are the same length

Solution: C

For a given pair of vectors, the dot product will be minimized when the vectors point in exactly opposite directions, and the dot product will be 0 when the vectors are orthogonal.

The dot product will be maximized when the vectors point in the same direction. If the do point in exactly the same direction, then their dot product will be equal to the product of their lengths.

Topic: Dot and cross products as opposite ideas

Question: When two vectors point in exactly the same direction...

Answer choices:

A The length of the cross product is maximized.

B The length of the cross product is 0.

C The length of the cross product is given by the product of the lengths of the individual vectors.

D The length of the cross product is given by the reciprocal of the dot product.

Solution: B

When two vectors are collinear, whether they point in exactly the same direction or in exactly the opposite direction, the length of their cross product is 0. The length of the cross product of two vectors is maximized when the vectors are orthogonal.

Topic: Dot and cross products as opposite ideas

Question: Describe the dot product and length of the cross product of the vector pair.

$$\vec{v} = (3,4)$$

$$\vec{w} = (6,8)$$

Answer choices:

A The dot product is the product of the lengths of the vectors, $\vec{v} \cdot \vec{w} = 50$, and the length of the cross product is $||\vec{v} \times \vec{w}|| = 0$, because the vectors point in exactly the same direction.

B The dot product is the product of the lengths of the vectors, $\vec{v} \cdot \vec{w} = 50$, and the length of the cross product is $||\vec{v} \times \vec{w}|| = 0$, because the vectors point in exactly opposite directions.

C The dot product is $\vec{v} \cdot \vec{w} = 0$, and the length of the cross product is the product of the lengths of the vectors, $||\vec{v} \times \vec{w}|| = 50$, because the vectors point in the same direction.

D The dot product is $\vec{v} \cdot \vec{w} = 0$, and the length of the cross product is the product of the lengths of the vectors, $||\vec{v} \times \vec{w}|| = 50$, because the vectors point in exactly opposite directions.

Solution: A

The vector $\overrightarrow{v} = (3,4)$ points into the first quadrant, and the vector $\overrightarrow{w} = (6,8)$ points in exactly the same direction, since $\overrightarrow{w} = (6,8)$ is just a scalar multiple of $\overrightarrow{v} = (3,4)$. Which means the angle between them is $\theta = 0^\circ$. The length of $\overrightarrow{v} = (3,4)$ is 5, and the length of $\overrightarrow{w} = (6,8)$ is 10.

The dot product of $\overrightarrow{v} = (3,4)$ and $\overrightarrow{u} = (6,8)$ is

$$\overrightarrow{v} \cdot \overrightarrow{w} = \begin{bmatrix} 3 & 4 \end{bmatrix} \cdot \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

$$\overrightarrow{v} \cdot \overrightarrow{w} = 3(6) + 4(8)$$

$$\overrightarrow{v} \cdot \overrightarrow{w} = 18 + 32$$

$$\overrightarrow{v} \cdot \overrightarrow{w} = 50$$

And the length of the cross product is

$$||\overrightarrow{v} \times \overrightarrow{w}|| = ||\overrightarrow{v}|| \ ||\overrightarrow{w}|| \sin\theta$$

$$||\overrightarrow{v} \times \overrightarrow{w}|| = (5)(10)\sin(0^\circ)$$

$$||\overrightarrow{v} \times \overrightarrow{w}|| = 50(0)$$

$$||\overrightarrow{v} \times \overrightarrow{w}|| = 0$$

Because the length of the cross product is 0, we know that the vectors are collinear. For collinear vectors, the dot product is just the product of the lengths of the vectors, which we see in $\overrightarrow{v} \cdot \overrightarrow{w} = 50$. The fact that the dot product is positive tells us that the vectors point in exactly the same direction along the same line.