Topic: Angle between vectors

Question: Say whether or not the vectors are orthogonal.

$$\vec{u} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$$

$$\vec{v} = 2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$$

Answer choices:

A The vectors are orthogonal

B The vectors are not orthogonal

C It's impossible to say whether or not the vectors are orthogonal

Solution: B

We'll test to see whether or not the vectors $\overrightarrow{u} = \mathbf{i} + \mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{v} = 2\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ are orthogonal by calculating their dot product.

$$\overrightarrow{u} \cdot \overrightarrow{v} = (1)(2) + (1)(2) + (2)(4)$$

$$\overrightarrow{u} \cdot \overrightarrow{v} = 2 + 2 + 8$$

$$\overrightarrow{u} \cdot \overrightarrow{v} = 12$$

Since the dot product is not 0, the vectors are not orthogonal.

Topic: Angle between vectors

Question: Find the angle between the vectors.

$$\vec{a} = (2,0,-1)$$

$$\vec{b} = (-1,4,2)$$

Answer choices:

A $\theta \approx 113^\circ$

B $\theta \approx 247^\circ$

C $\theta \approx 293^\circ$

D $\theta \approx 67^\circ$

Solution: A

The angle between the vectors can be given by

$$\vec{a} \cdot \vec{b} = ||\vec{a}||\ ||\vec{b}||\cos\theta$$

Find the lengths of both vectors.

$$||\vec{a}|| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{2^2 + 0^2 + (-1)^2} = \sqrt{4 + 0 + 1} = \sqrt{5}$$

$$||\vec{b}|| = \sqrt{b_1^2 + b_2^2 + b_3^2} = \sqrt{(-1)^2 + 4^2 + 2^2} = \sqrt{1 + 16 + 4} = \sqrt{21}$$

Then find the dot product of the vectors.

$$\vec{a} \cdot \vec{b} = (2)(-1) + (0)(4) + (-1)(2)$$

$$\vec{a} \cdot \vec{b} = -2 + 0 - 2$$

$$\vec{a} \cdot \vec{b} = -4$$

Plug everything into the formula for the angle between vectors.

$$\vec{a} \cdot \vec{b} = ||\vec{a}||\ ||\vec{b}||\cos\theta$$

$$-4 = \sqrt{5}\sqrt{21}\cos\theta$$

$$-\frac{4}{\sqrt{105}} = \cos\theta$$

Take the inverse cosine of each side of the equation to solve for θ.

$$\theta = \arccos\left(-\frac{4}{\sqrt{105}}\right)$$

If we use a calculator to find this arccosine value, we find that the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is $\theta \approx 113^\circ$.

Topic: Angle between vectors

Question: Find the angle between the vectors.

$$\overrightarrow{a} = (1, -3, 1)$$

$$\overrightarrow{b} = (0, 6, -2)$$

Answer choices:

A $\theta \approx 12^\circ$

B $\theta \approx 72^\circ$

C $\theta \approx 102^\circ$

D $\theta \approx 162^\circ$

Solution: D

The angle between the vectors can be given by

$$\vec{a} \cdot \vec{b} = ||\vec{a}||\ ||\vec{b}||\cos\theta$$

Find the lengths of both vectors.

$$||\vec{a}|| = \sqrt{a_1^2 + a_2^2 + a_3^2} = \sqrt{1^2 + (-3)^2 + 1^2} = \sqrt{1 + 9 + 1} = \sqrt{11}$$

$$||\vec{b}|| = \sqrt{b_1^2 + b_2^2 + b_3^2} = \sqrt{0^2 + 6^2 + (-2)^2} = \sqrt{0 + 36 + 4} = \sqrt{40} = 2\sqrt{10}$$

Then find the dot product of the vectors.

$$\vec{a} \cdot \vec{b} = (1)(0) + (-3)(6) + (1)(-2)$$

$$\vec{a} \cdot \vec{b} = 0 - 18 - 2$$

$$\vec{a} \cdot \vec{b} = -20$$

Plug everything into the formula for the angle between vectors.

$$\vec{a} \cdot \vec{b} = ||\vec{a}||\ ||\vec{b}||\cos\theta$$

$$-20 = \sqrt{11}\left(2\sqrt{10}\right)\cos\theta$$

$$-10 = \sqrt{11}\sqrt{10}\cos\theta$$

$$-10 = \sqrt{110}\cos\theta$$

$$-\frac{10}{\sqrt{110}} = \cos\theta$$

Take the inverse cosine of each side of the equation to solve for θ.

$$\theta = \arccos \left(-\frac{10}{\sqrt{110}} \right)$$

If we use a calculator to find this arccosine value, we find that the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is $\theta \approx 162^\circ$.