Linear Algebra and Geometry 1

Systems of equations, matrices, vectors, and geometry

Vector addition and vector scaling

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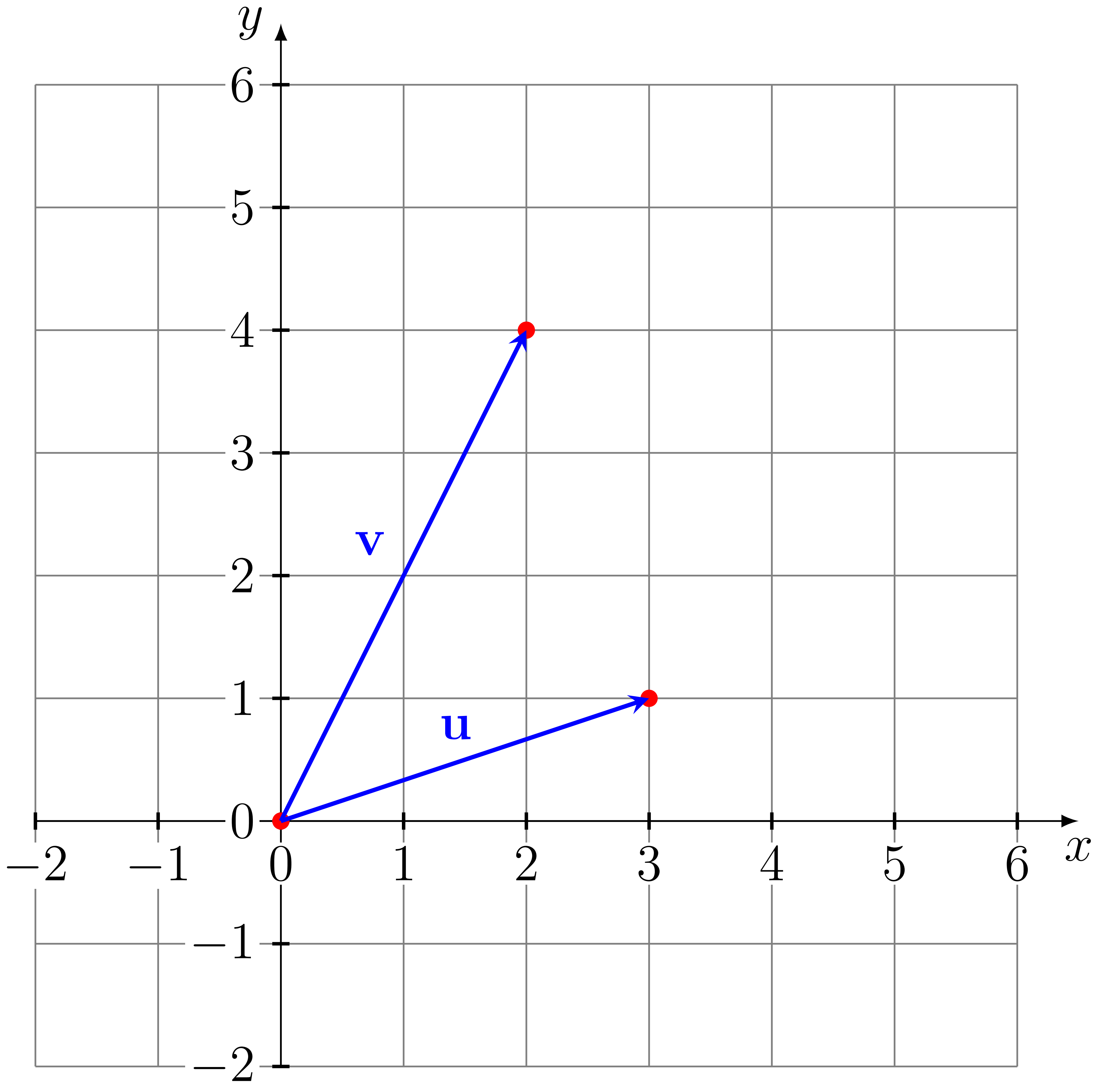
Man in The Middle AB

Vector addition and Vector scaling (scalar multiplication)

Example. If $\mathbf{u} = (3,1)$ and $\mathbf{v} = (2,4)$ are two vectors.



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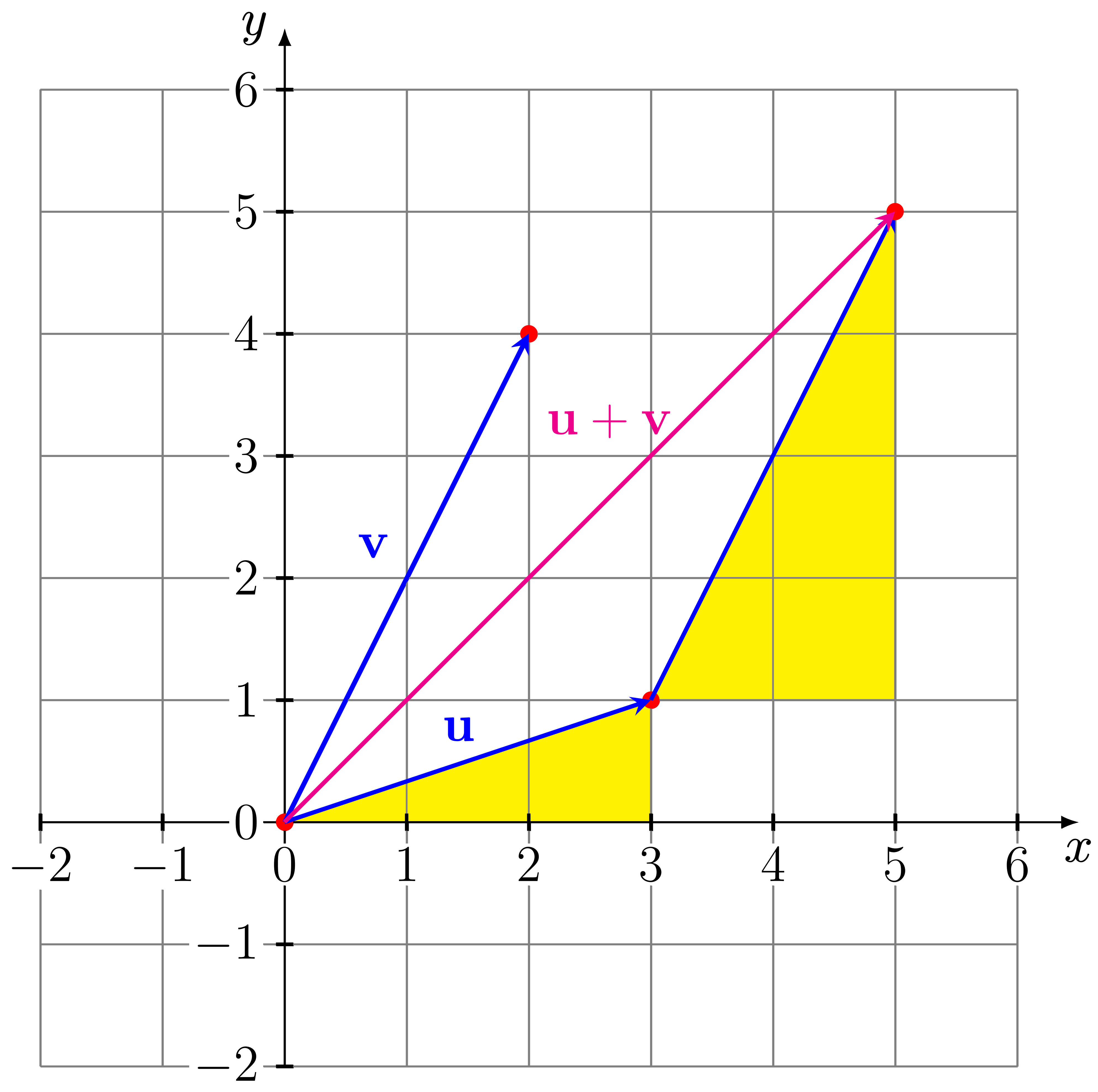
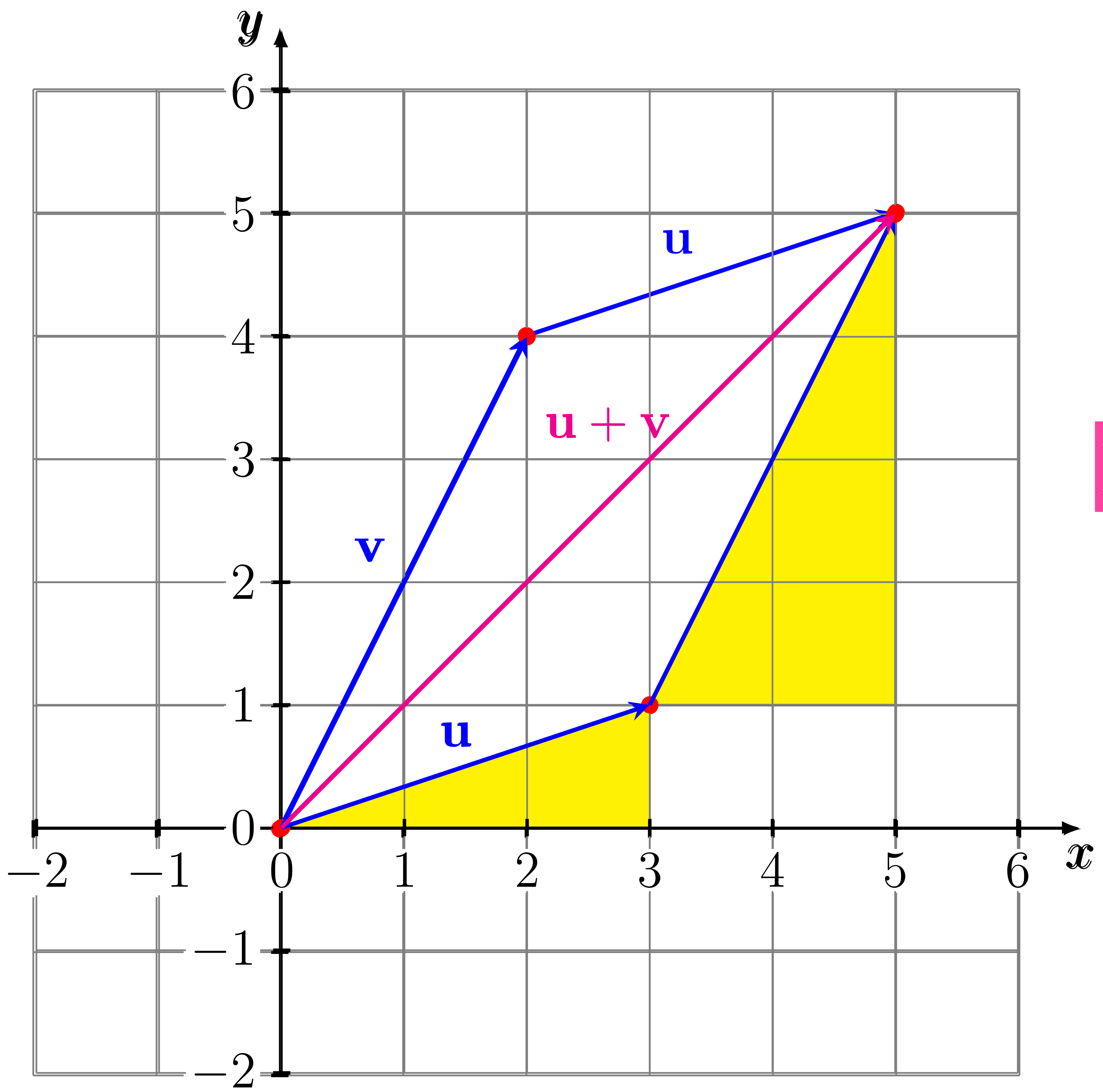


Triangle method

Vector addition

Example. If $\mathbf{u} = (3,1)$ and $\mathbf{v} = (2,4)$ are two vectors then

$$\mathbf{u} + \mathbf{v} = (5,5).$$



Parallelogram method

Vector addition and Vector scaling (scalar multiplication)

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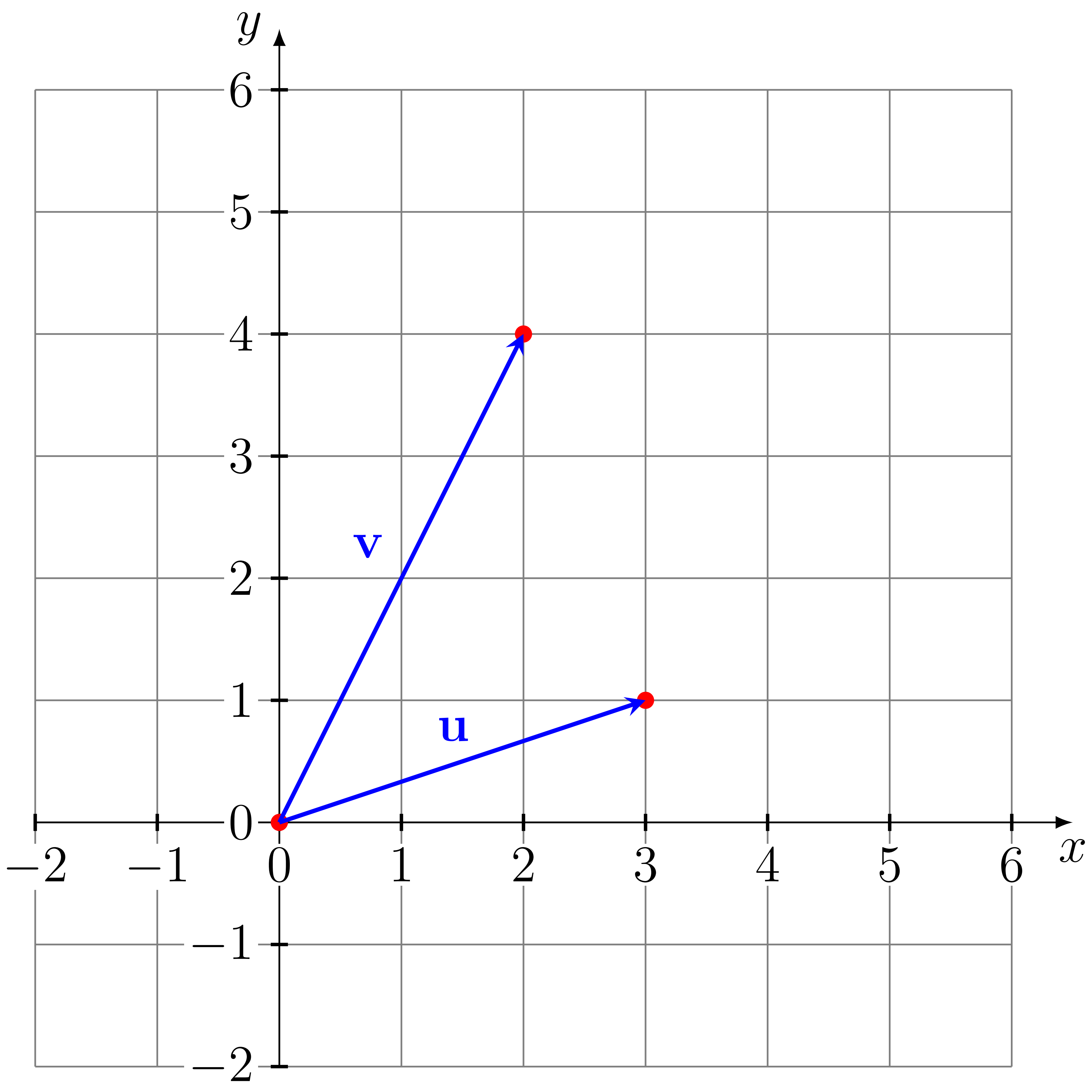

$$2\mathbf{u} = (2 \cdot 3, 2 \cdot 1) = (6, 2)$$

Vector scaling

Vector addition and Vector scaling (scalar multiplication)

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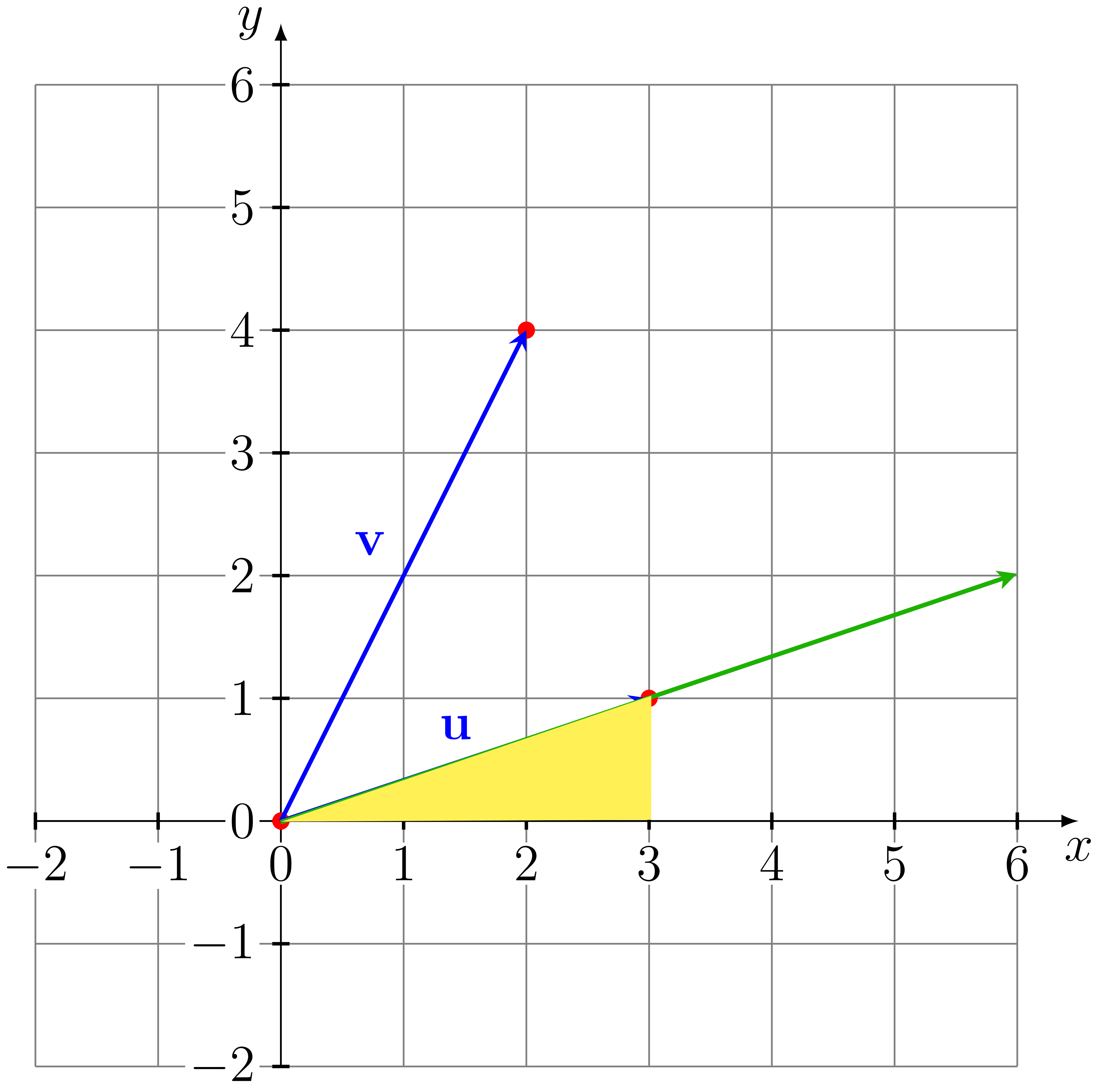

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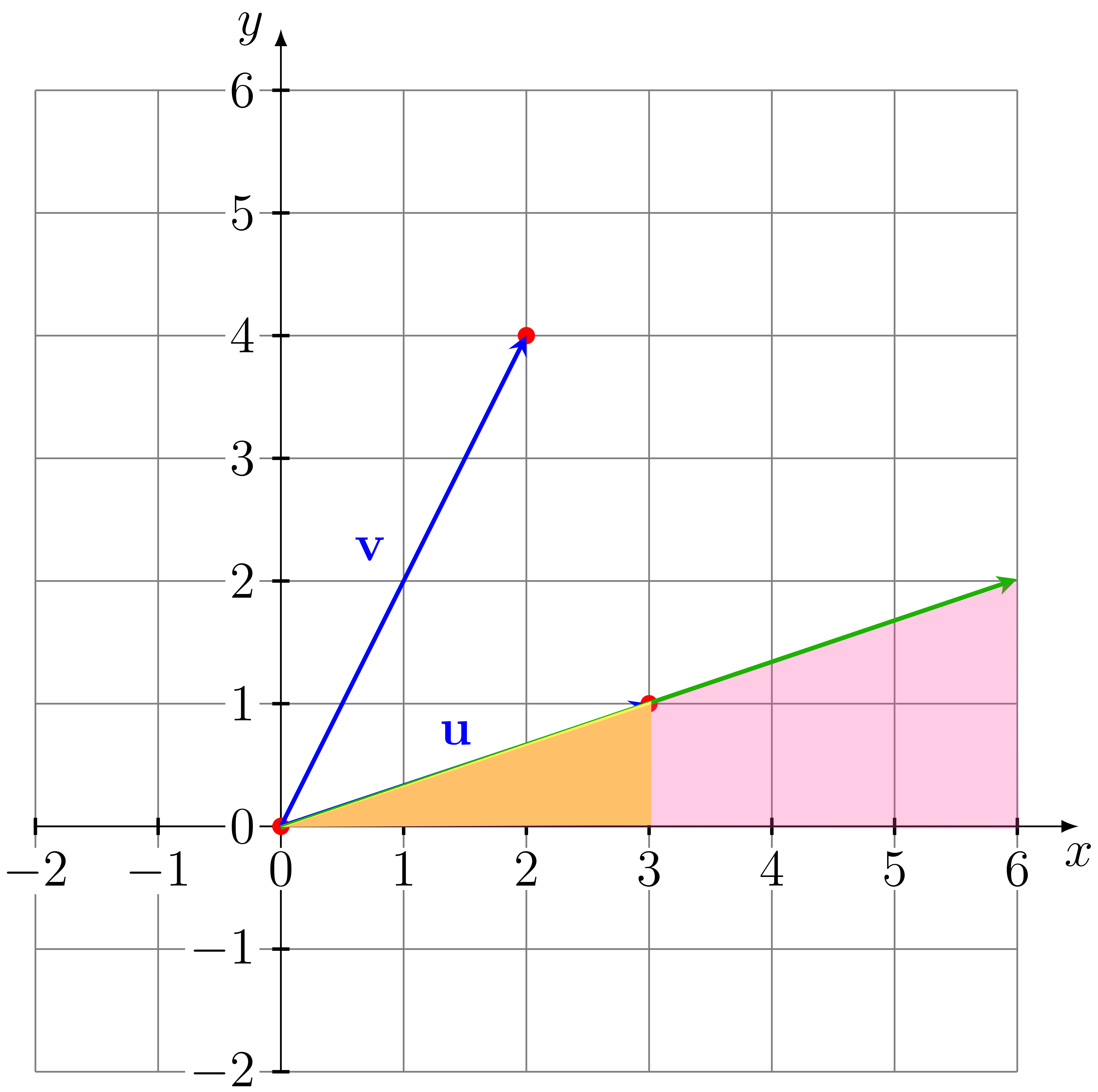

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Vector scaling

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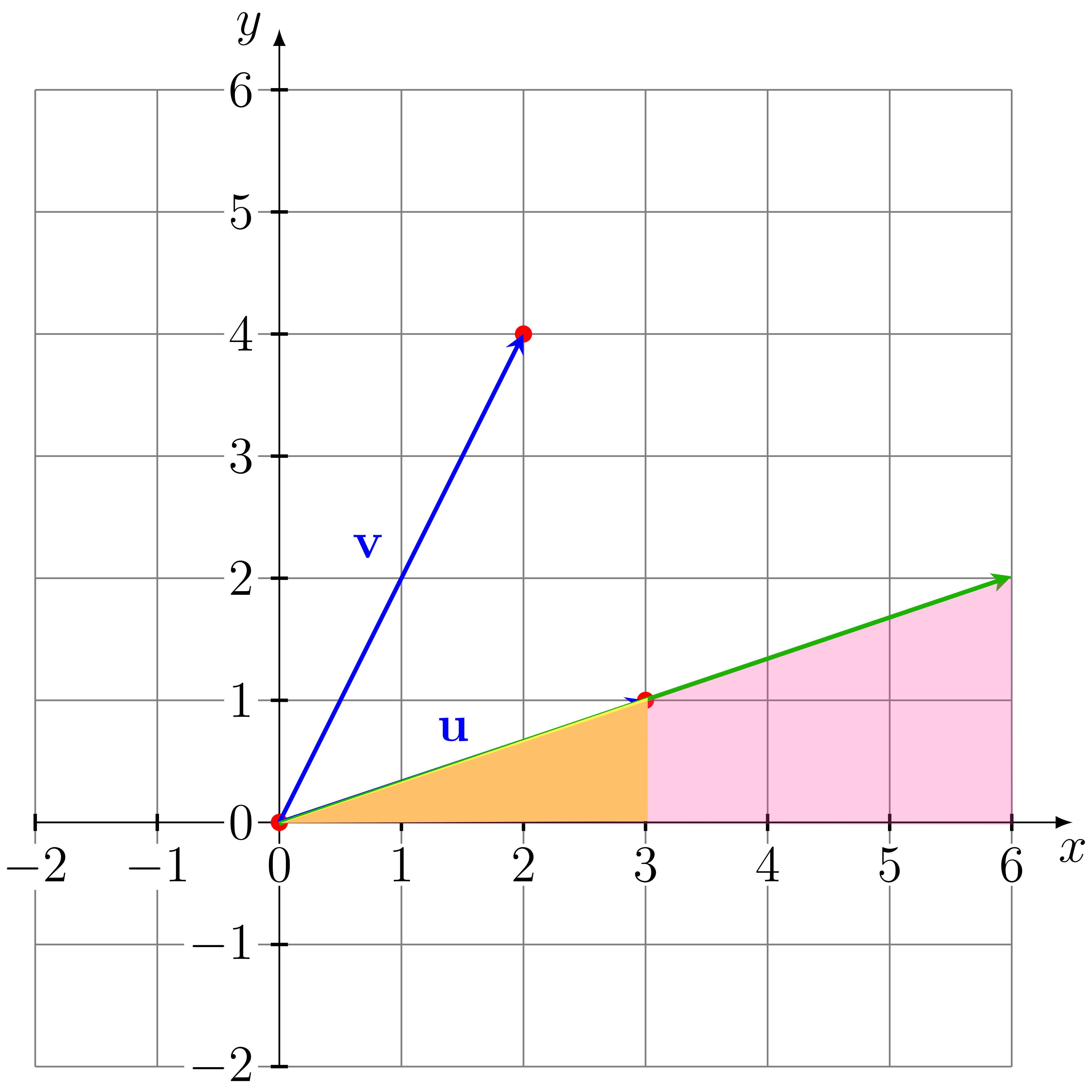

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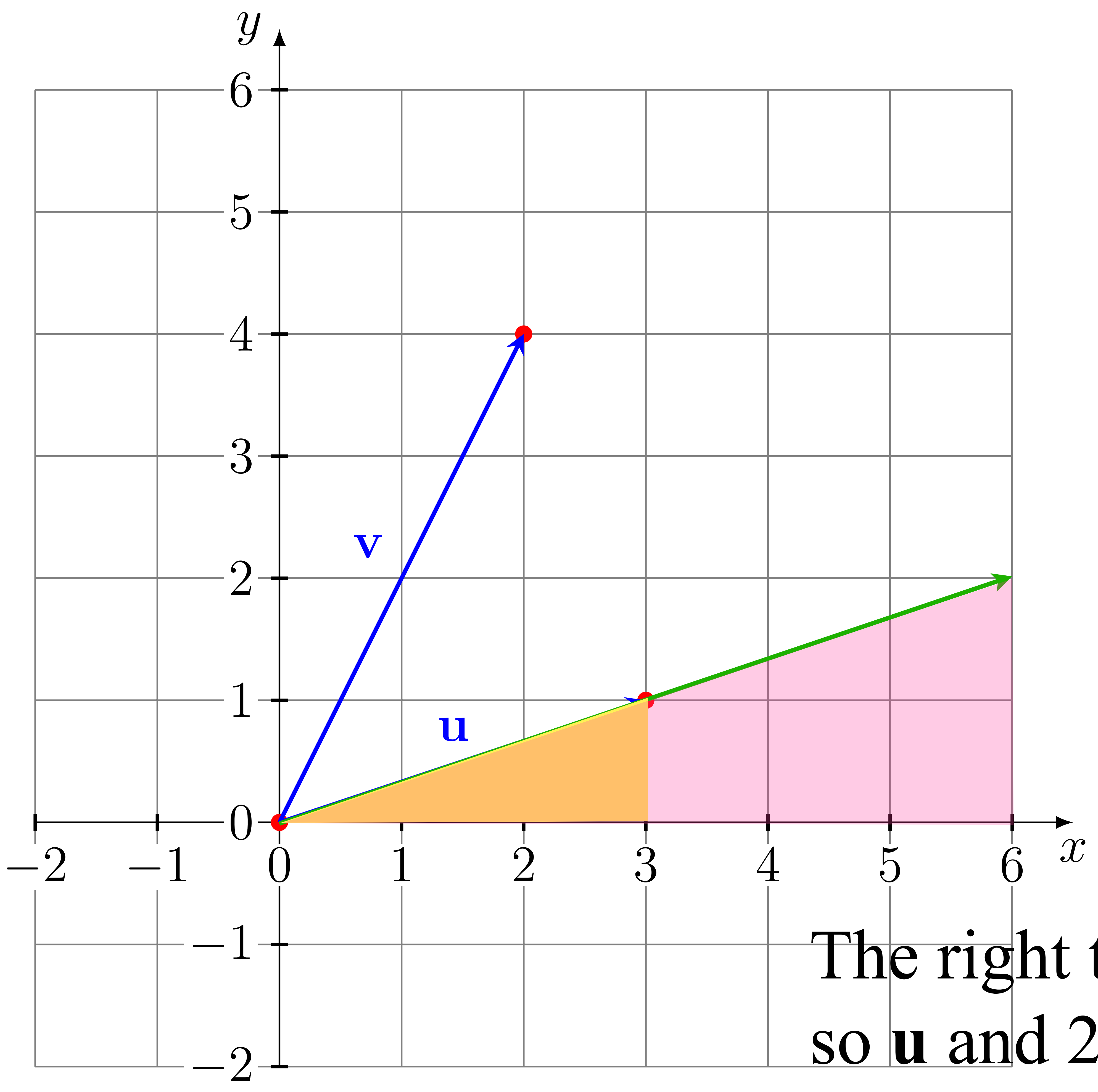

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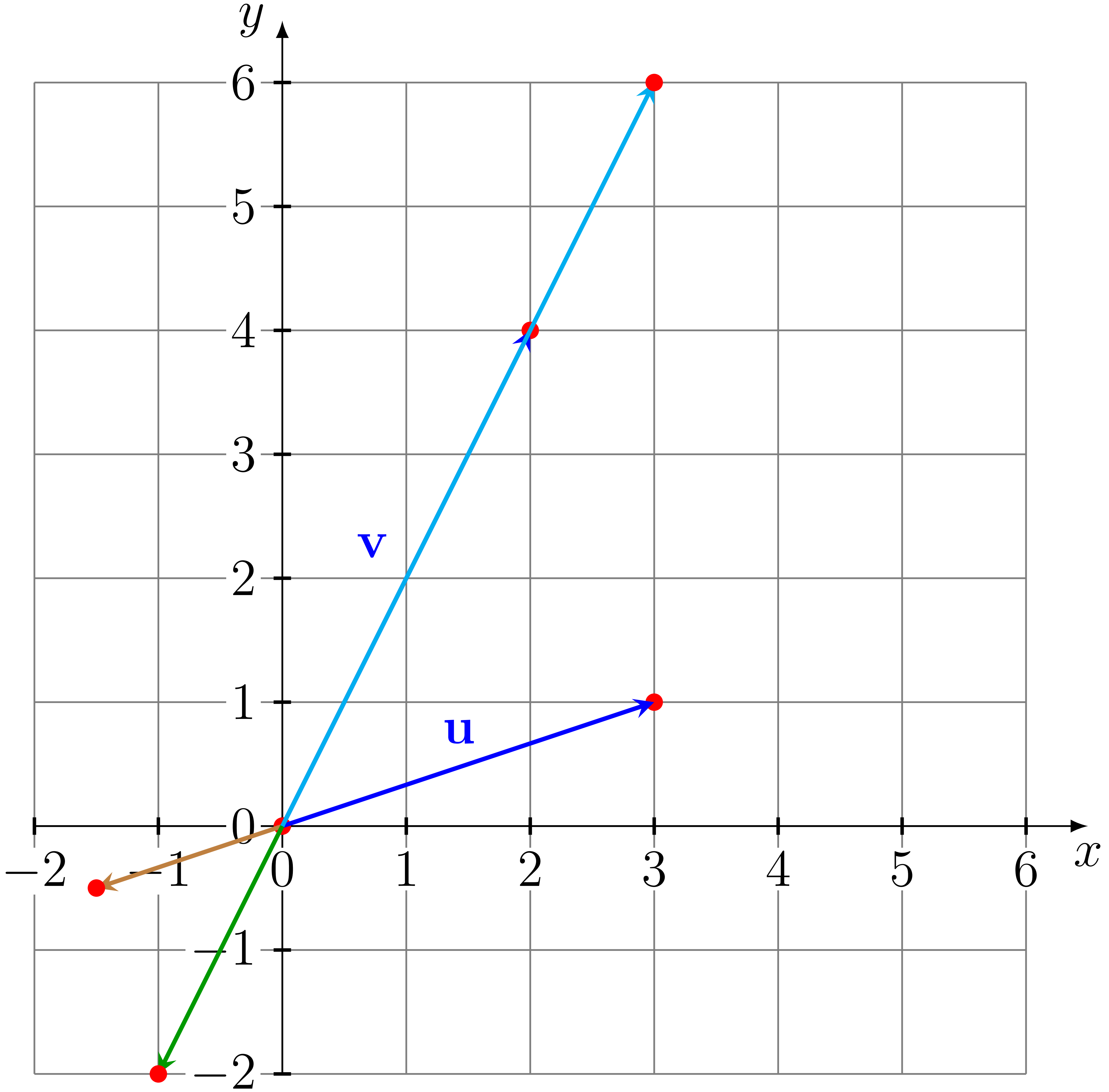
The right triangles are similar,
so **u** and 2**u** lie on the same straight line.

Vector scaling

Example. If $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (2, 4)$ are two vectors then

$$-\frac{1}{2}\mathbf{u} = (-\frac{3}{2}, -\frac{1}{2}), \qquad -\frac{1}{2}\mathbf{v} = (-1, -2), \qquad \frac{3}{2}\mathbf{v} = (3, 6).$$

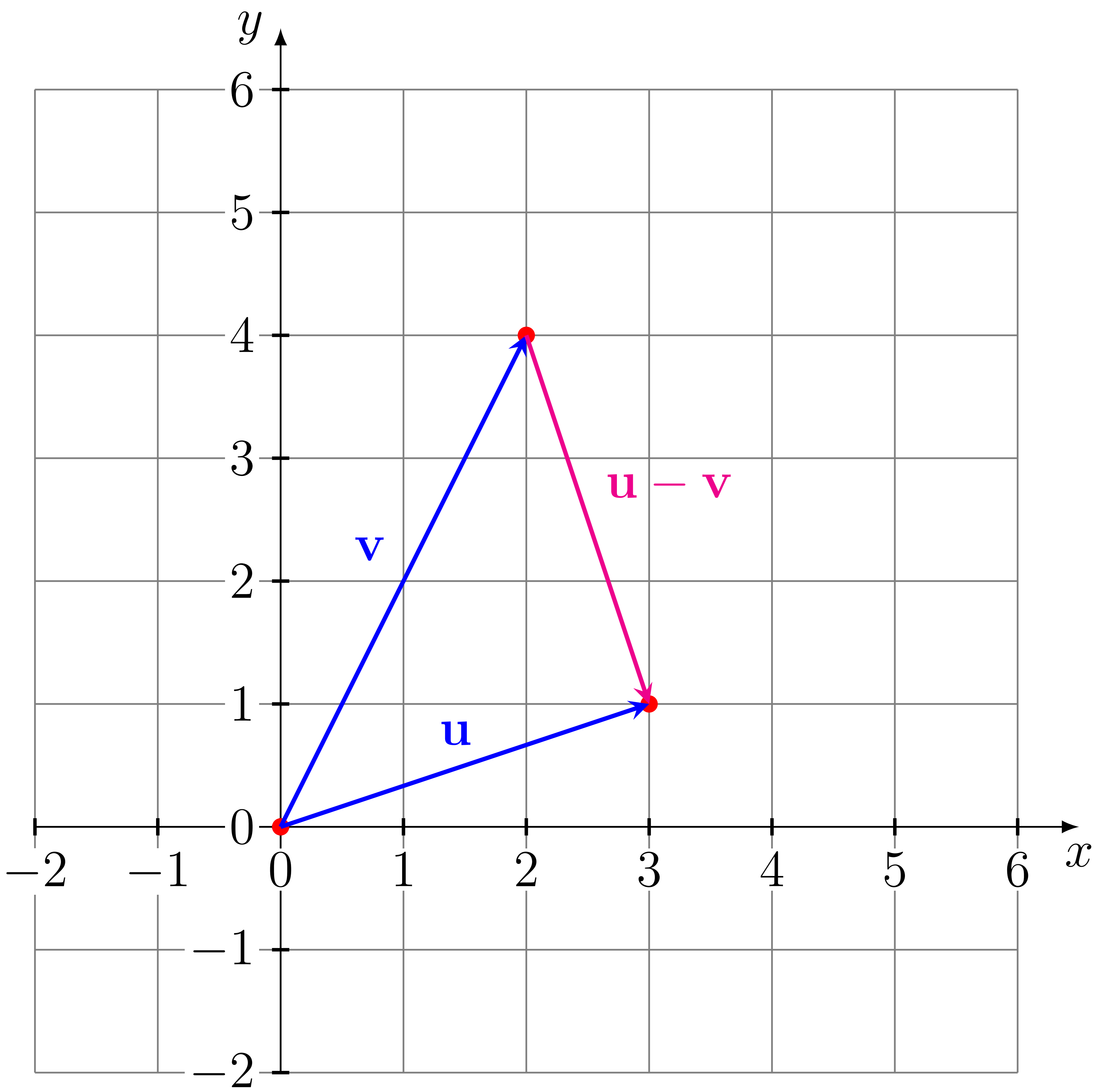


Vector scaling
(scalar multiplication)

Vector subtraction

Example. If $\mathbf{u} = (3, 1)$ and $\mathbf{v} = (2, 4)$ are two vectors then

$$\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v}) = (1, -3).$$



$$\mathbb{R}^n$$

$$\overrightarrow{v} = (v_1, v_2, \ \dots, v_n)$$

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$$\overrightarrow{u} = (u_1, u_2, \ \ldots, u_n)$$

$$\overrightarrow{v} + \overrightarrow{u} = (v_1 + u_1, v_2 + u_2, \ \ldots, v_n + u_n)$$