Linear Algebra Workbook

Operations on one matrix

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LINEAR SYSTEMS IN TWO UNKNOWNS

1. Find the unique solution to the system of equations.

$$-x + 2y = 6$$

$$3x = y - 10$$

2. Find the unique solution to the system of equations.

$$-5x + y = 8$$

$$y = 3x - 8$$

3. Find the unique solution to the system of equations.

$$2x - y = 5$$

$$-3x + y = 7$$

4. Find the unique solution to the system of equations.

$$x = 2y - 5$$

$$-3x + 6y = 15$$

■ 5. Find the unique solution to the system of equations using the graphing method.

$$y - 2 = -(x + 1)$$

$$y = x + 1$$

■ 6. Find the unique solution to the system of equations using the substitution method.

$$5y + x = 4$$

$$3y - 3x = 6$$

LINEAR SYSTEMS IN THREE UNKNOWNS

1. Find the unique solution to the system of equations.

$$2x + y - z = 3$$

$$x - y + z = 0$$

$$x - 2y - 3z = 4$$

2. Find the unique solution to the system of equations.

$$3x + y - z = -2$$

$$x - 2y + 3z = 23$$

$$2x + 3y + 2z = 5$$

3. Find the unique solution to the system of equations.

$$5x - 3y + z = -8$$

$$2x + y - 2z = -6$$

$$-3x + 2y + 4z = 19$$

4. Find the unique solution to the system of equations.

$$-2x + 3y - 4z = 10$$

$$4x + 3y + 2z = 4$$

$$x - 6y + 4z = -19$$

5. Find the unique solution to the system of equations.

$$2x - y + z = 9$$

$$4x - 2y + 2z = 18$$

$$-2x + y - z = -9$$

6. Find the unique solution to the system of equations.

$$x + 2y - z = 9$$

$$3x + y - z = 5$$

$$-x - 4y + z = 2$$

MATRIX DIMENSIONS AND ENTRIES

1. Give the dimensions of the matrix.

$$D = \begin{bmatrix} 11 & 9 \\ -4 & 8 \end{bmatrix}$$

2. Give the dimensions of the matrix.

$$A = [3 \quad 5 \quad -2 \quad 1 \quad 8]$$

3. Given matrix J, find $J_{4,1}$.

$$J = \begin{bmatrix} 6 \\ 2 \\ 7 \\ 1 \end{bmatrix}$$

4. Given matrix C, find $C_{1,2}$.

$$C = \begin{bmatrix} 3 & 12 \\ 1 & 4 \\ 9 & 5 \\ -3 & 2 \end{bmatrix}$$

5. Given matrix N, state the dimensions and find $N_{1,3}$.

$$N = \begin{bmatrix} 1 & 5 & 9 \\ 14 & -8 & 6 \end{bmatrix}$$

6. Given matrix S, state the dimensions and find $S_{3,4}$.

$$S = \begin{bmatrix} 3 & 6 & -7 & 1 & 0 \\ 0 & 9 & 15 & 3 & 4 \\ 4 & 0 & 2 & 11 & 8 \\ -5 & 8 & 7 & 9 & 2 \end{bmatrix}$$

REPRESENTING SYSTEMS WITH MATRICES

1. Represent the system with a matrix called A.

$$-2x + 5y = 12$$

$$6x - 2y = 4$$

2. Represent the system with a matrix called D.

$$9y - 3x + 12 = 0$$

$$8 - 4x = 11y$$

3. Represent the system with an augmented matrix called H.

$$4a + 7b - 5c + 13d = 6$$

$$3a - 8b = -2c + 1$$

4. Represent the system with a matrix called M.

$$-2x + 4y = 9 - 6z$$

$$7y + 2z - 3 = -3t - 9x$$

5. Represent the system with a matrix called A.

$$3x - 8y + z = 7$$

$$2z = 3y - 2x + 4$$

$$5y = 12 - 9x$$

6. Represent the system with a matrix called K.

$$-4b + 2c = 3 - 7a$$

$$9c = 4 - 2b$$

$$8a - 2c = 5b$$

SIMPLE ROW OPERATIONS

1. Write the new matrix after $R_1 \leftrightarrow R_2$.

$$\begin{bmatrix} 2 & 6 & -4 & 1 \\ 8 & 2 & 1 & -5 \end{bmatrix}$$

2. Write the new matrix after $R_2 \leftrightarrow R_4$.

$$\begin{bmatrix} 1 & 2 & 7 & -3 \\ 6 & 1 & 5 & -4 \\ -7 & 7 & 0 & 3 \\ 9 & 2 & 8 & 3 \end{bmatrix}$$

3. Write the new matrix after $R_1 \leftrightarrow 3R_2$.

$$\begin{bmatrix} 9 & 2 & -7 \\ 1 & 6 & 4 \end{bmatrix}$$

4. Write the new matrix after $3R_2 \leftrightarrow 3R_4$.

$$\begin{bmatrix} 0 & 11 & 6 \\ 7 & -3 & 9 \\ 8 & 8 & 1 \\ 6 & 2 & 4 \end{bmatrix}$$

5. Write the new matrix after $R_1 + 2R_2 \to R_1$.

$$\begin{bmatrix} 6 & 2 & 7 \\ 1 & -5 & 15 \end{bmatrix}$$

6. Write the new matrix after $4R_2 + R_3 \to R_3$.

$$\begin{bmatrix} 13 & 5 & -2 & 9 \\ 8 & 2 & 0 & 6 \\ 4 & 1 & 7 & -3 \end{bmatrix}$$

PIVOT ENTRIES AND ROW-ECHELON FORMS

1. Use row operations to put the matrix into row-echelon form.

$$\begin{bmatrix} 3 & 6 & -7 \\ 1 & 2 & -1 \\ 1 & 2 & 1 \end{bmatrix}$$

2. Use row operations to put the matrix into reduced row-echelon form.

$$\begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 \\ 3 & 0 & -6 & 0 \end{bmatrix}$$

3. Use row operations to put the matrix into reduced row-echelon form.

$$\begin{bmatrix} 1 & 5 & 2 \\ 0 & -3 & 9 \\ 0 & 0 & 7 \end{bmatrix}$$

4. Use row operations to put the matrix into row-echelon form.

$$\begin{bmatrix} 3 & 2 & 0 & 9 \\ 2 & 4 & -3 & -1 \\ 2 & 12 & -12 & 1 \end{bmatrix}$$

5. Use row operations to put the matrix into reduced row-echelon form.

$$\begin{bmatrix} 1 & -2 \\ 3 & 1 \\ -3 & 0 \\ 2 & -3 \end{bmatrix}$$

6. Use row operations to put the matrix into row-echelon form.

$$\begin{bmatrix} 1 & 0 & -3 & 7 \\ 0 & 1 & -2 & 3 \\ -1 & 3 & -6 & -13 \\ -5 & -2 & 22 & -28 \end{bmatrix}$$

GAUSS-JORDAN ELIMINATION

1. Use Gauss-Jordan elimination to find the solution to the linear system from the rref matrix.

$$x + 2y = -2$$

$$3x + 2y = 6$$

2. Use Gauss-Jordan elimination to find the solution to the linear system from the rref matrix.

$$2x + 4y = 22$$

$$3x + 3y = 15$$

3. Use Gauss-Jordan elimination to find the solution to the linear system from the rref matrix.

$$x - 3y - 6z = 4$$

$$y + 2z = -2$$

$$-4x + 12y + 21z = -4$$

■ 4. Use Gauss-Jordan elimination to find the solution to the linear system from the rref matrix.

$$2y + 4z = 4$$

$$x + 3y + 3z = 5$$

$$2x + 7y + 6z = 10$$

■ 5. Use Gauss-Jordan elimination to find the solution to the linear system from the rref matrix.

$$3x + 12y + 42z = -27$$

$$x + 2y + 8z = -5$$

$$2x + 5y + 16z = -6$$

■ 6. Use Gauss-Jordan elimination to find the solution to the linear system from the rref matrix.

$$4x + 8y + 4z = 20$$

$$4x + 6y = 4$$

$$3x + 3y - z = 1$$

NUMBER OF SOLUTIONS TO THE LINEAR SYSTEM

■ 1. Determine whether the system has one solution, no solutions, or infinitely many solutions.

$$2x - 8y = 18$$

$$-7x + 2y - 5z = -6$$

$$3x + 2z = 1$$

■ 2. Determine whether the system has one solution, no solutions, or infinitely many solutions.

$$-x + 3y - 5z - 8w = 2$$

$$4x - 8y + 4z + 4w = -44$$

$$3x + 5y - 16z + w = 18$$

$$-x + y - 3z - w = 6$$

■ 3. How many solutions does the linear system have?

$$3x - 3y + 5z = -11$$

$$-2x + y - 2z = 5$$

$$x + y - z = 9$$

4. How many solutions does the linear system have?

$$-x + 6y + 4z = -22$$

$$4x - 22y - 2z + 2w = 0$$

$$x - 6y - 5z + 3w = 5$$

$$-3y - 22z = 6$$

5. Determine whether the system has one solution, no solutions, or infinitely many solutions.

$$2x + 2y - 8z = 4$$

$$-3x - 5y + 6z = -4$$

$$5x - y - 38z = 16$$

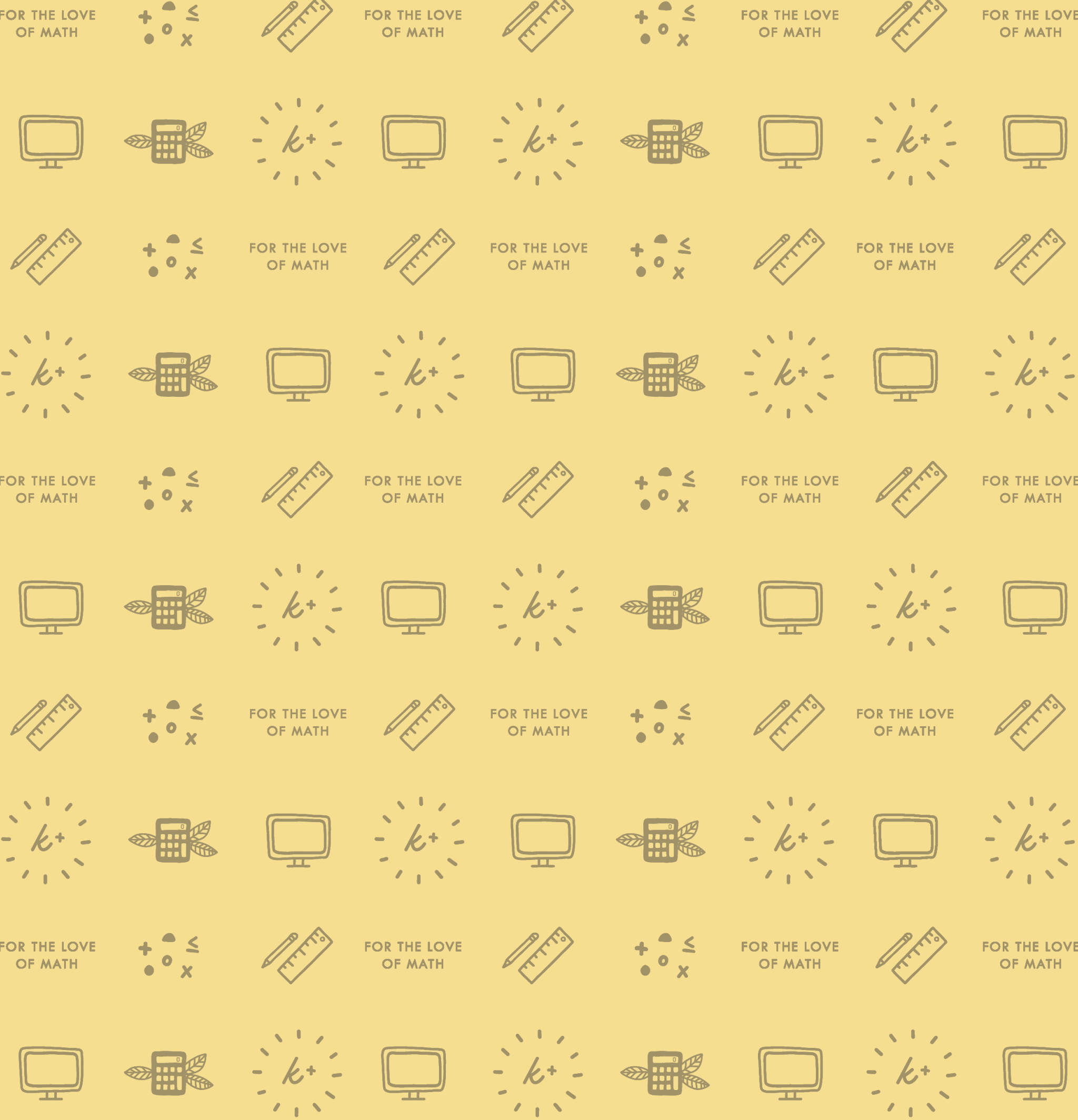
6. For the linear system below, determine whether it has one solution, no solutions, or infinitely many solutions.

$$x + y - z + 2w = 7$$

$$4x + 2y - 6z + 2w = 16$$

$$-3x + y + 7z + 6w = 3$$

$$-x - y + 4z + 3w = 8$$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