# Linear Algebra Workbook

Dot products and cross products

krista king
MATH

## DOT PRODUCTS

1. Find the dot product.

$$\vec{a} = (-2,5)$$

$$\vec{b} = (3,4)$$

2. Find the dot product.

$$\vec{x} = (1, -2,0)$$

$$\vec{y} = (5, -1, -3)$$

3. Use the dot product to find the length of the vector $\vec{u} = (-5,2, -4, -2)$.

4. Simplify the expression if $\vec{x} = (-2,4)$, $\vec{y} = (0, -1)$, and $\vec{z} = (4,7)$.

$$4\vec{x} \cdot (3\vec{y} - \vec{z})$$

5. Use the dot product to find $-\vec{a} \cdot (5\vec{b} + 3\vec{c})$.

$$\vec{a} = (-2,0,4)$$

$$\vec{b} = (1,5,3)$$

$\vec{c} = (-1, -4,0)$

■ 6. Use the dot product to find $\vec{w}(2\vec{x} + \vec{y}) - 3\vec{y}(\vec{w} + 4\vec{x} - \vec{z})$.

$\vec{x} = (4, -3,0,7)$

$\vec{y} = (-1,5,2, -1)$

$\vec{z} = (0,6, -1,9)$

$\vec{w} = (1,0,5,0)$

## CAUCHY-SCHWARZ INEQUALITY

1. Use the Cauchy-Schwarz inequality to say whether or not the vectors are linearly independent.

$$\vec{u} = (-1,2)$$

$$\vec{v} = (-5,10)$$

2. Use the Cauchy-Schwarz inequality to say whether or not the vectors are linearly independent.

$$\vec{u} = (-5,2)$$

$$\vec{v} = (3,-7)$$

3. Use the Cauchy-Schwarz inequality to say whether or not the vectors are linearly independent.

$$\vec{u} = (-2,4,0)$$

$$\vec{v} = (1,-5,3)$$

4. Use the Cauchy-Schwarz inequality to say whether or not the vectors are linearly independent.

$\vec{u} = (6,3,6)$

$\vec{v} = (-2,-1,-2)$

5. Use the Cauchy-Schwarz inequality to say whether or not the vectors are linearly independent.

$\vec{u} = (-13,5,7)$

$\vec{v} = (1,-1,-1)$

6. Use the Cauchy-Schwarz inequality to say whether or not the vectors are linearly independent.

$\vec{u} = (-2,0,2)$

$\vec{v} = (8,0,-8)$

## VECTOR TRIANGLE INEQUALITY

1. Use the vector triangle inequality to say whether $\vec{u}$ and $\vec{v}$ are linearly independent.

$\vec{u}=(\sqrt{3},3)$ and $\vec{v}=(2\sqrt{3},0)$

2. Use the vector triangle inequality to say whether $\vec{u}$ and $\vec{v}$ span $\mathbb{R}^2$.

$\vec{u}=(5,-7)$ and $\vec{v}=(-4,-3)$

3. Use the vector triangle inequality to say whether $\vec{u}$ and $\vec{v}$ are linearly independent.

$\vec{u}=(-2,5)$ and $\vec{v}=(2,-5)$

4. Use the vector triangle inequality to say whether $\vec{u}$ and $\vec{v}$ are linearly independent.

$\vec{u}=(-3,12,-15)$ and $\vec{v}=(-1,4,-5)$

5. Use the vector triangle inequality to say whether $\vec{u}$ and $\vec{v}$ are linearly independent.

$\vec{u} = (1,2,0)$ and $\vec{v} = (-5,1,-6)$

■ 6. Use the vector triangle inequality to say whether $\vec{u}$ and $\vec{v}$ are linearly independent.

$\vec{u} = (2,-5,4)$ and $\vec{v} = (6,-15,12)$

## ANGLE BETWEEN VECTORS

1. Say whether or not the vectors are orthogonal.

$$\overrightarrow{a}=(-1,3)$$

$$\overrightarrow{b}=(6,2)$$

2. Say whether or not the vectors are orthogonal.

$$\overrightarrow{u}=2i-j+3k$$

$$\overrightarrow{v}=-i-3j+2k$$

3. Find the angle between the vectors.

$$\overrightarrow{x}=(0,2)$$

$$\overrightarrow{y}=(1,1)$$

4. Find the angle between the vectors.

$$\overrightarrow{a}=(-5,7,3)$$

$$\overrightarrow{b}=(1,2,-3)$$

5. Find the angle between the vectors.

$$\vec{a} = (-1, 3, -4)$$

$$\vec{b} = (2, 1, 0)$$

6. Find the angle between the vectors.

$$\vec{a} = (1, -2, 5)$$

$$\vec{b} = (8, 6, 3)$$

## EQUATION OF A PLANE, AND NORMAL VECTORS

■ 1. What is the normal vector to the plane?

$$-2x + 5y - 7z = 0$$

■ 2. What is the normal vector to the plane?

$$10y - 5z + 6 = 0$$

■ 3. Find the equation of a plane with normal vector $\vec{n} = (-1,0,4)$ that passes through $(1, -3,0)$.

■ 4. Find the equation of a plane with normal vector $\vec{n} = (4, -7,3)$ that passes through $(-2,1,6)$.

■ 5. Find the equation of a plane with normal vector $\vec{n} = -3i + 4j - z$ that passes through $(-2,0, -7)$.

■ 6. Find the equation of the plane passing through $P$ and perpendicular to $\overrightarrow{PQ}$.

$P(1,-5,4)$

$Q(0,3,-1)$

## CROSS PRODUCTS

1. Find the cross product of $\overrightarrow{a} = (1, -3, -1)$ and $\overrightarrow{b} = (5, 6, -2)$.

2. Find a vector orthogonal to both $\overrightarrow{a} = (-3, -5, 2)$ and $\overrightarrow{b} = (-2, 4, -7)$.

3. Find the length of the cross product of $\overrightarrow{a} = (-1, -2, 0)$ and $\overrightarrow{b} = (1, 1, -2)$.

4. Find the length of the cross product of $\overrightarrow{a} = (6, -3, 3)$ and $\overrightarrow{b} = (3, 0, 3)$ when the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$ is $\theta = 30^\circ$.

5. Find the length of the cross product of the vectors $\overrightarrow{a} = (2, -5, 3)$ and $\overrightarrow{b} = (4, 6, -1)$, and find the sine of the angle between them.

6. Find the angle between the vectors $\overrightarrow{a} = (2, -2, 1)$ and $\overrightarrow{b} = (1, 0, 1)$, and find the length of their cross product.

## DOT AND CROSS PRODUCTS AS OPPOSITE IDEAS

■ 1. Find the maximum value of the dot product, if $||\vec{u}||=4$ and $||\vec{v}||=5$.

■ 2. Find the minimum value of the dot product of two vectors, if $||\vec{u}||=\sqrt{56}$ and $||\vec{v}||=\sqrt{126}$.

■ 3. Find the maximum value of the length of the cross product of $\vec{u}$ and $\vec{v}$, if $||\vec{u}||=\sqrt{50}$ and $||\vec{v}||=\sqrt{128}$.

■ 4. Find the dot product and the length of the cross product of $\vec{u}=(2,1)$ and $\vec{v}=(-6,-3)$. Then interpret the results based on what the dot and cross products indicate.

■ 5. Find the dot product and the length of the cross product of $\vec{u}=(2,-3,-1)$ and $\vec{v}=(4,-6,-2)$. Then interpret the results based on what the dot and cross products indicate.

■ 6. Find the dot product and the length of the cross product of $\vec{u}=(-2,4,3)$ and $\vec{v}=(2,1,0)$. Then interpret the results based on what the dot and cross products indicate.

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