Linear Algebra and Geometry 1

Systems of equations, matrices, vectors, and geometry

Linear combinations

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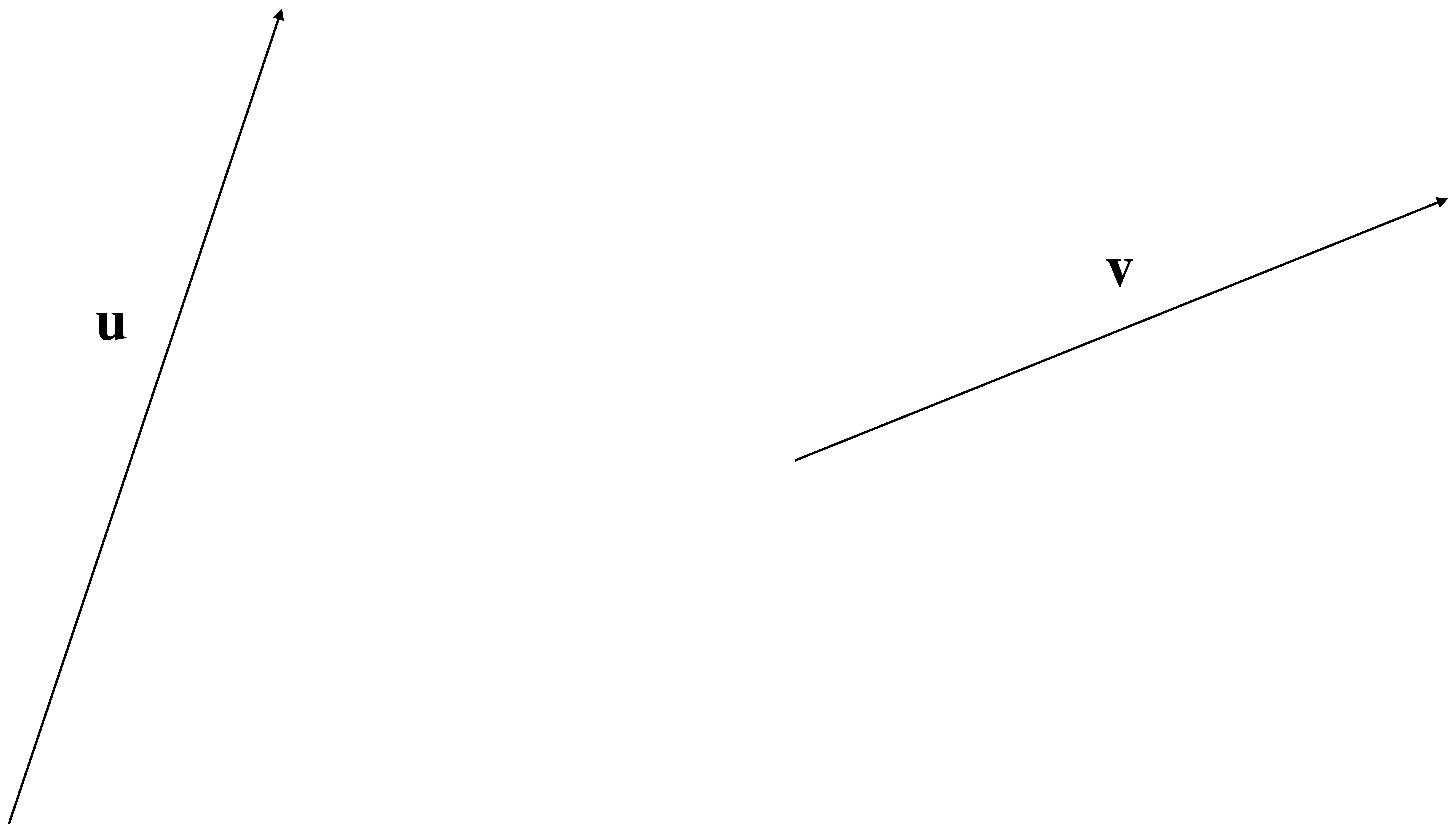
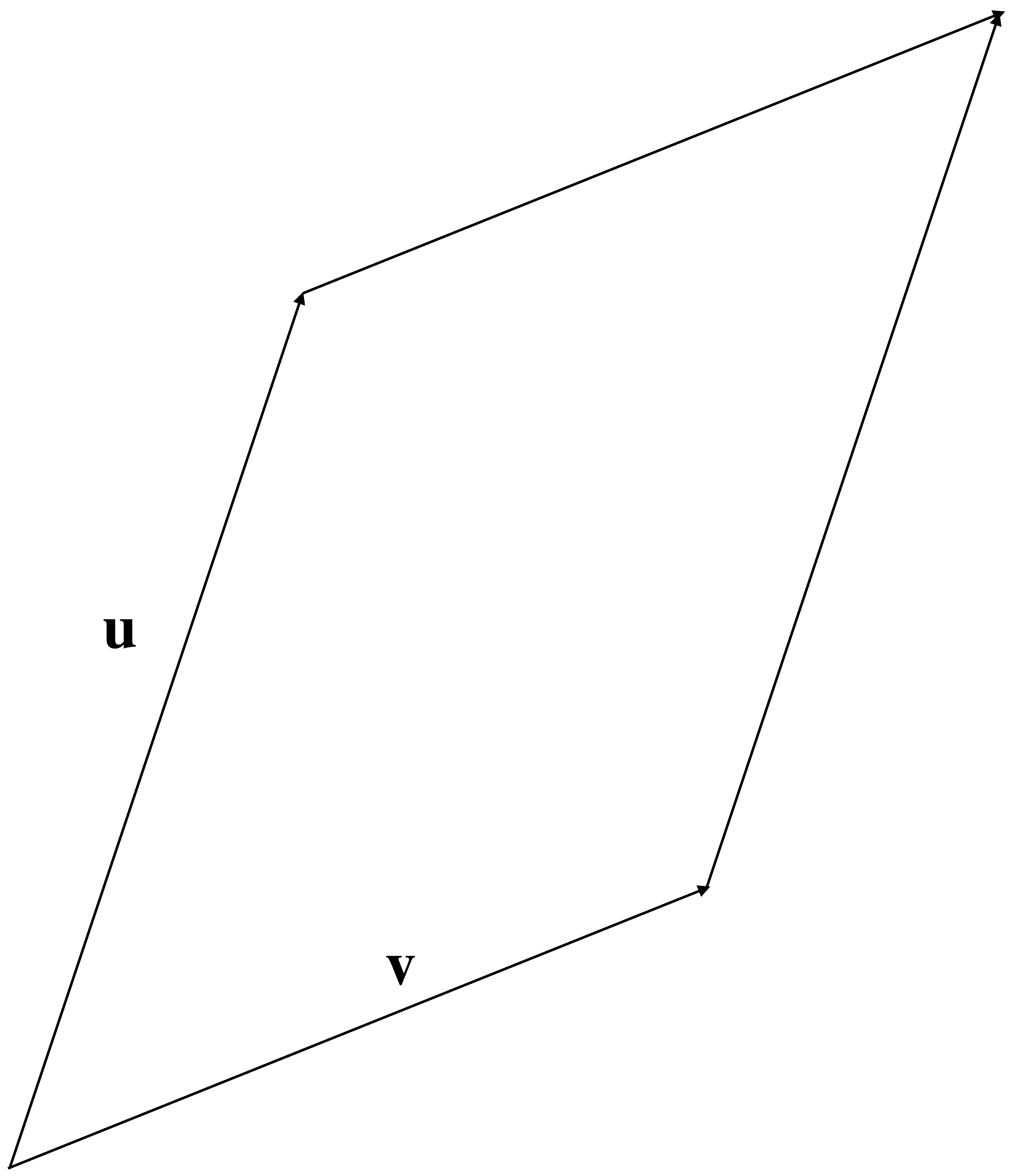
University teacher in mathematics (Associate Professor / Senior Lecturer) at Mälardalen University, Sweden

MiTM
Man in The Middle AB


u
v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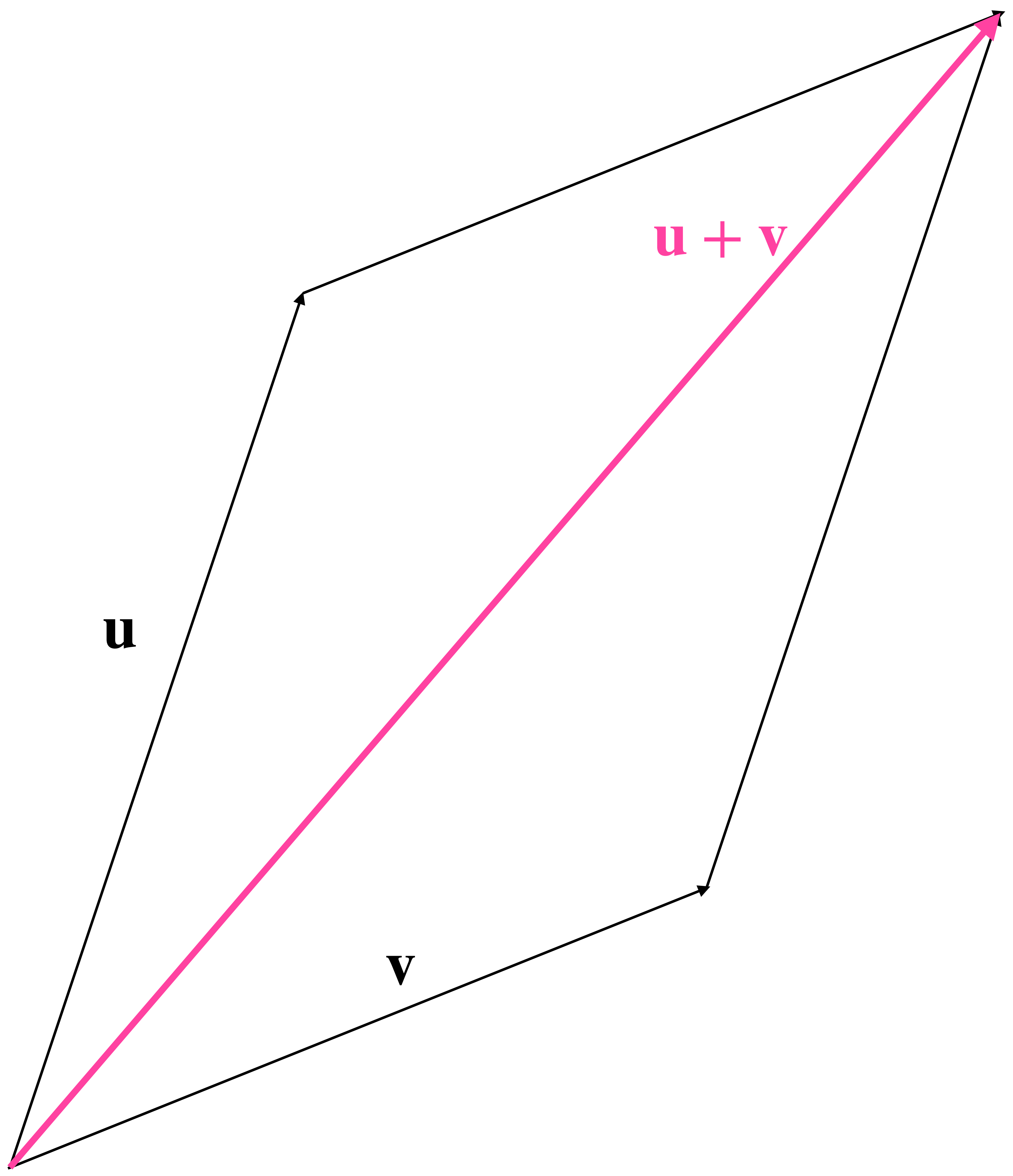
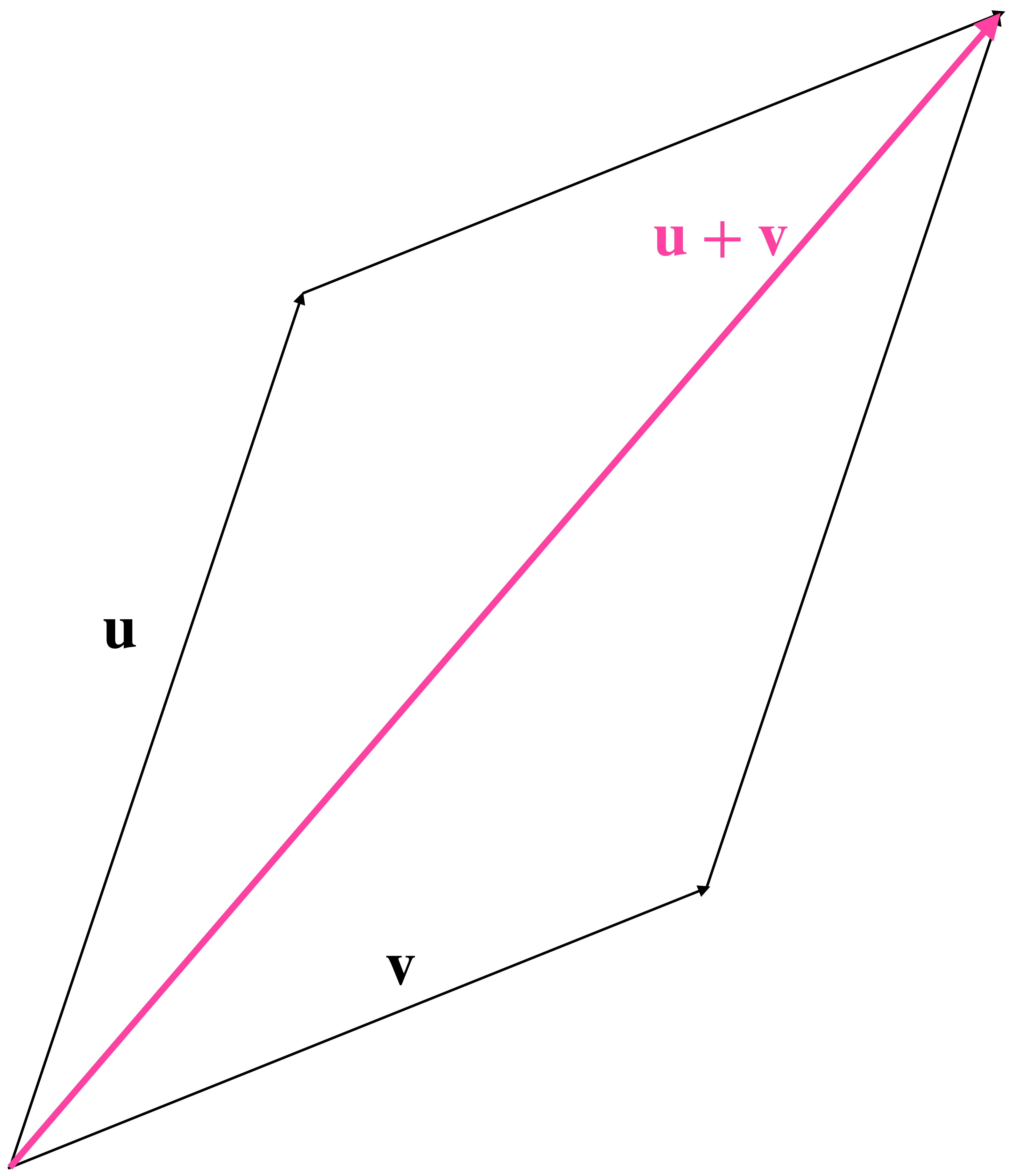

u + v
u
v

$$c_1 = \frac{1}{2}, \quad c_2 = \frac{1}{3}$$



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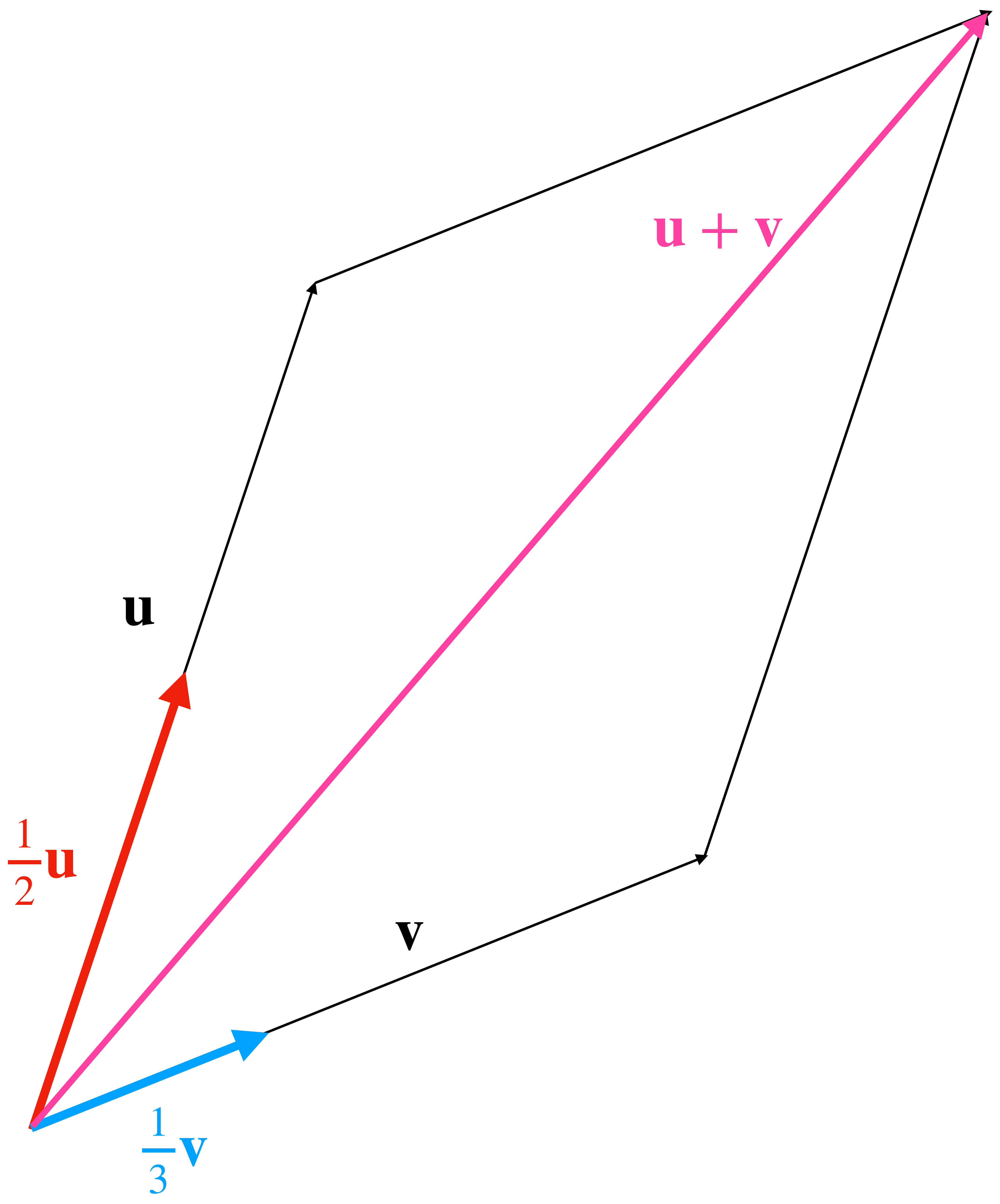

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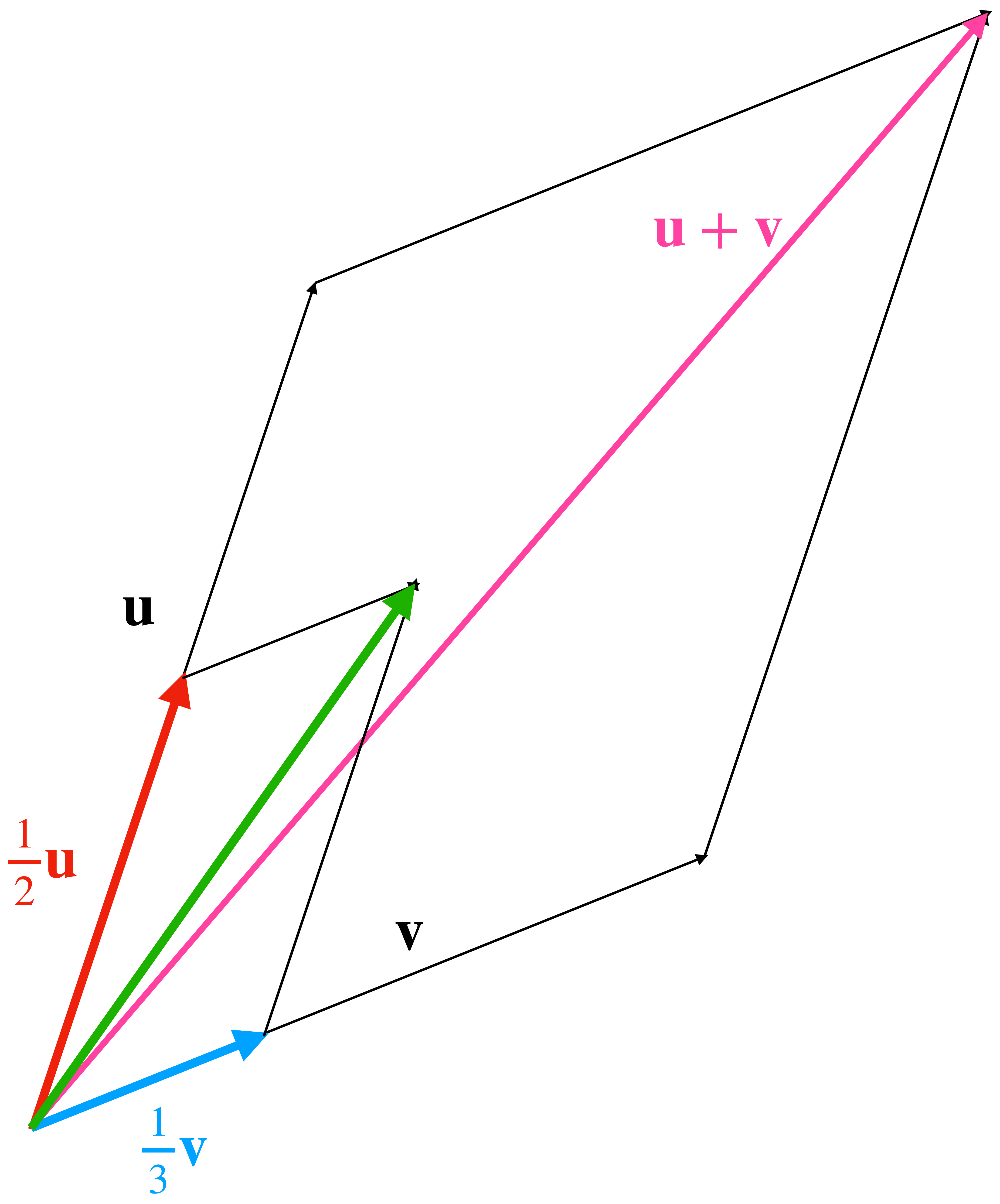

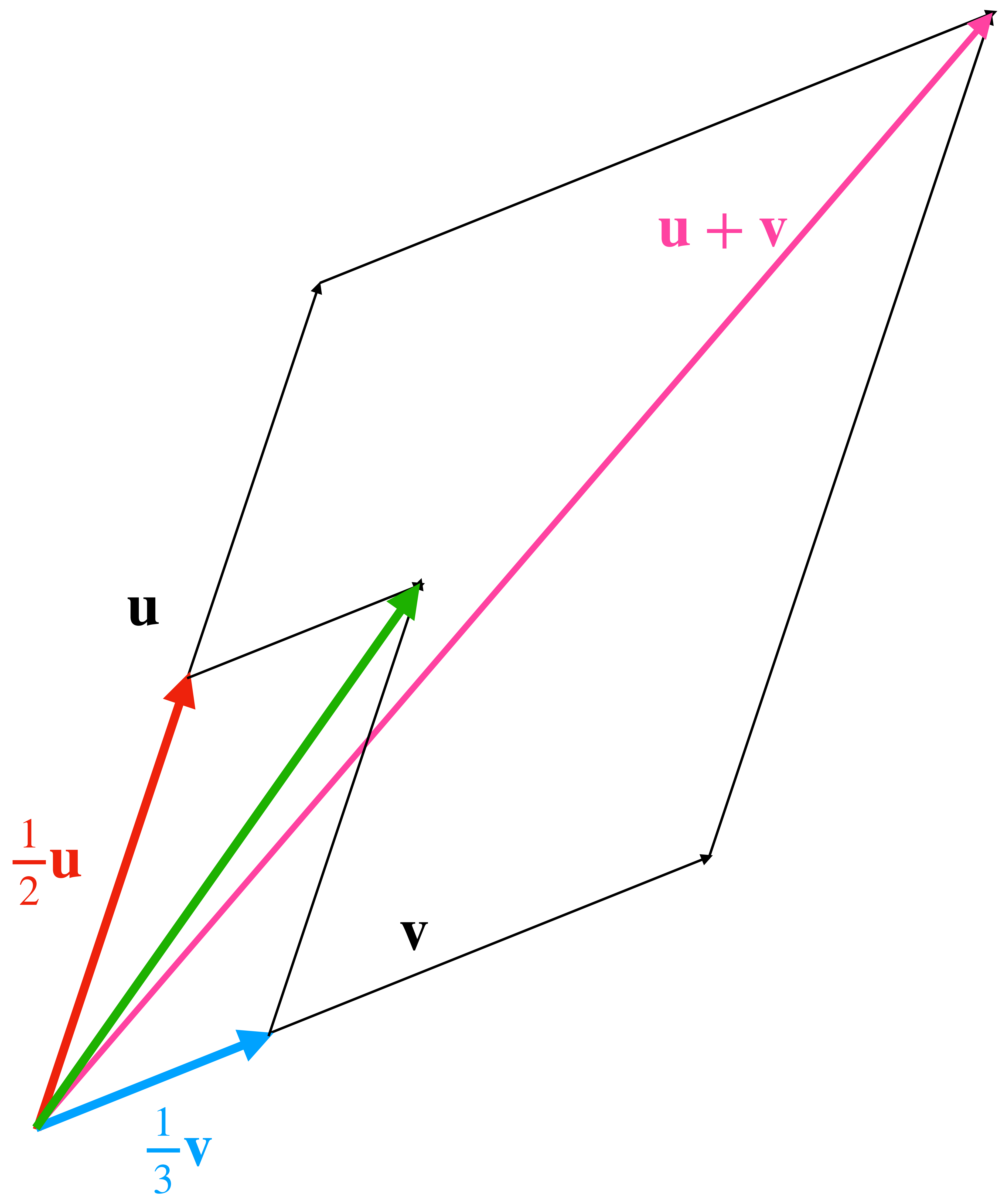

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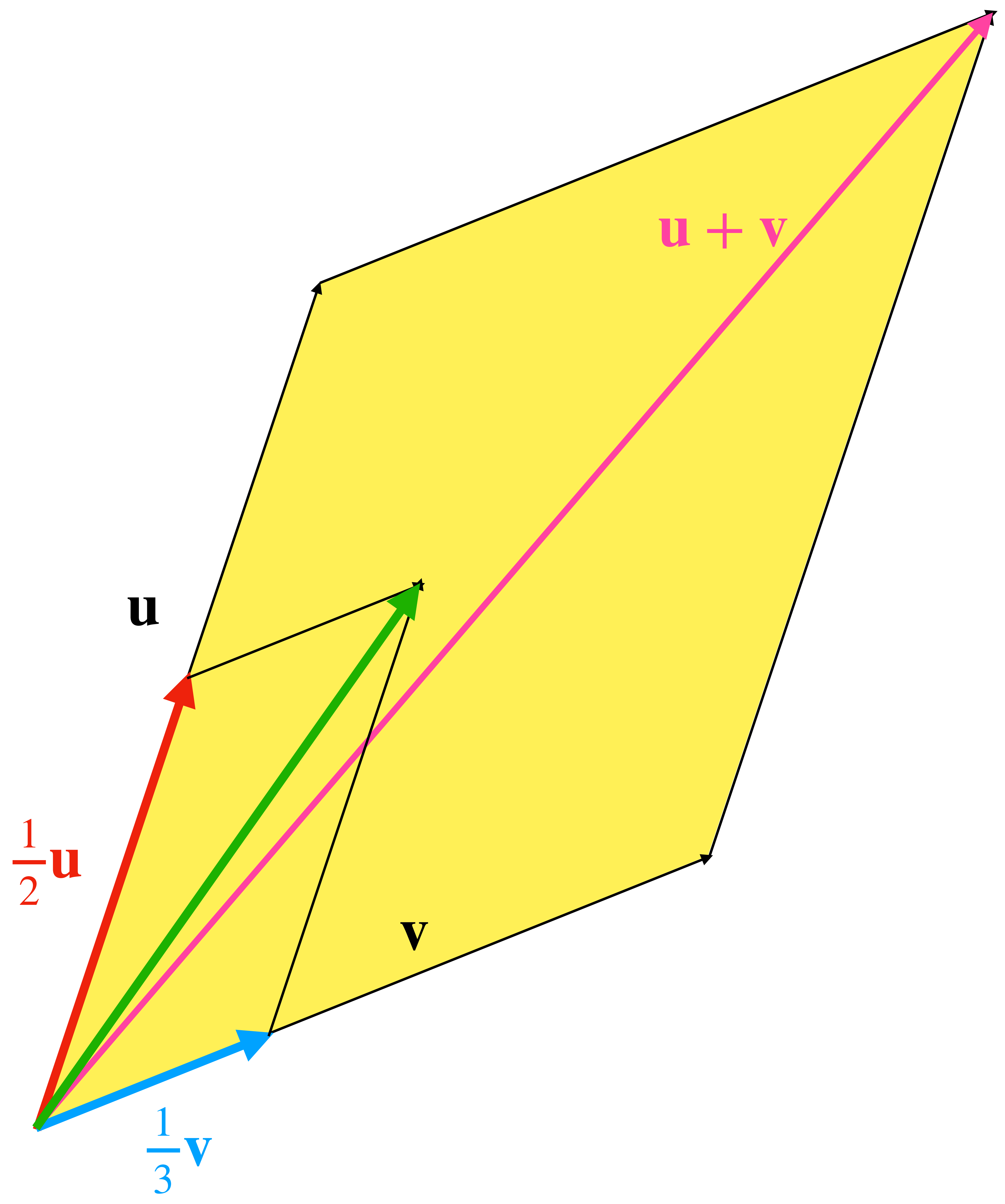
$$\alpha_1\mathbf{u} + \alpha_2\mathbf{v}$$

A linear combination of vectors **u** and **v**



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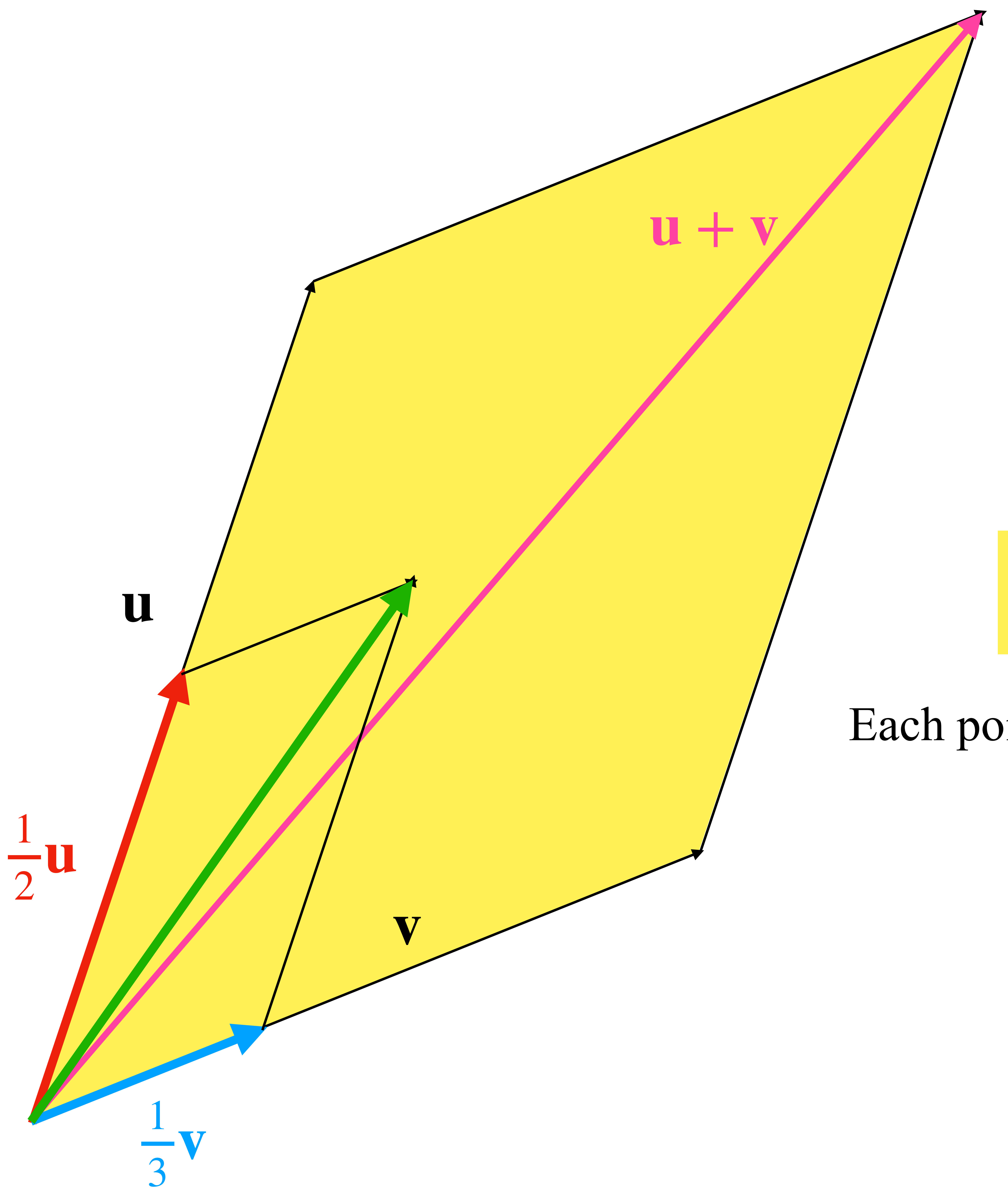

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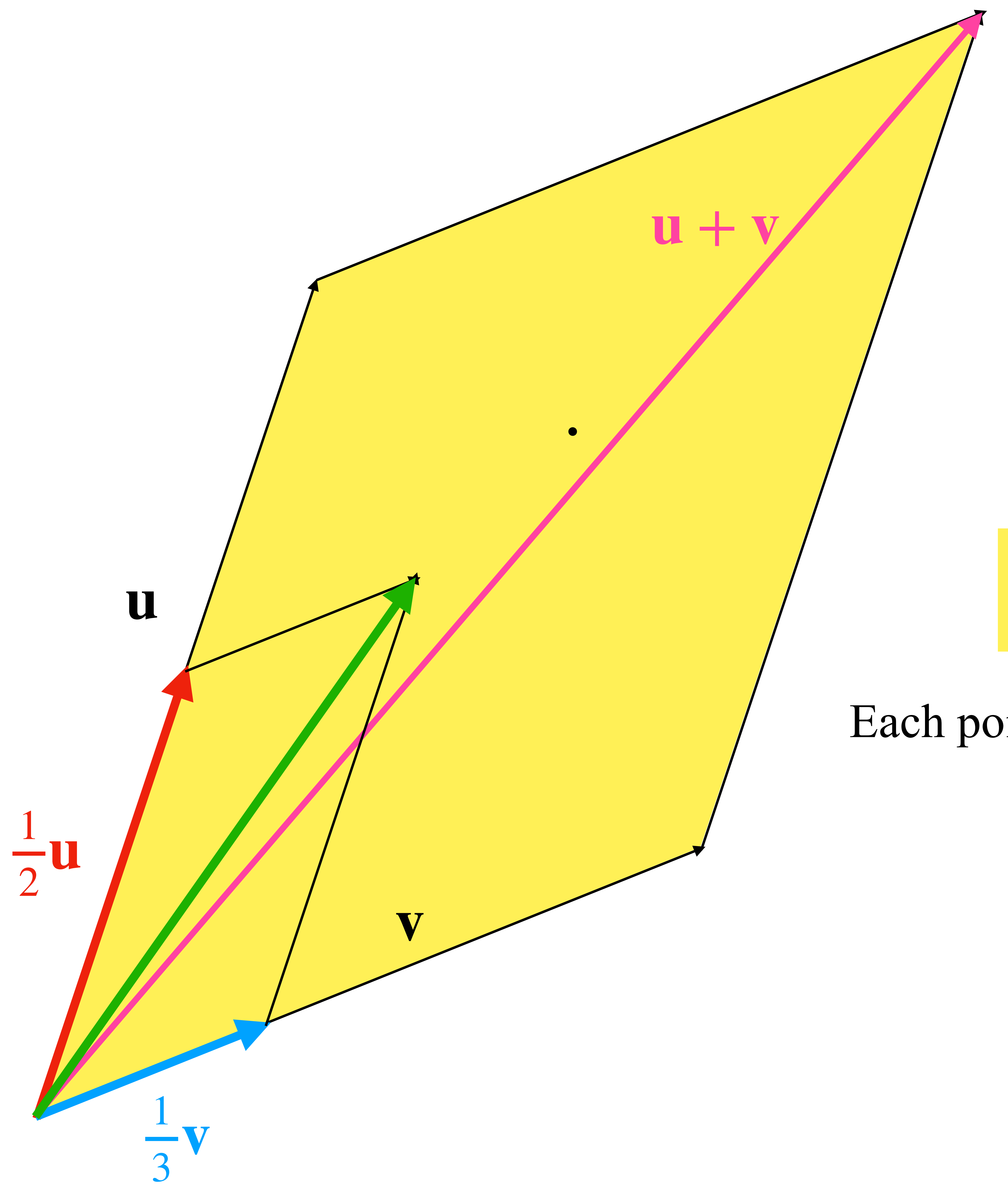
Each point of the parallelogram can be obtained in this way



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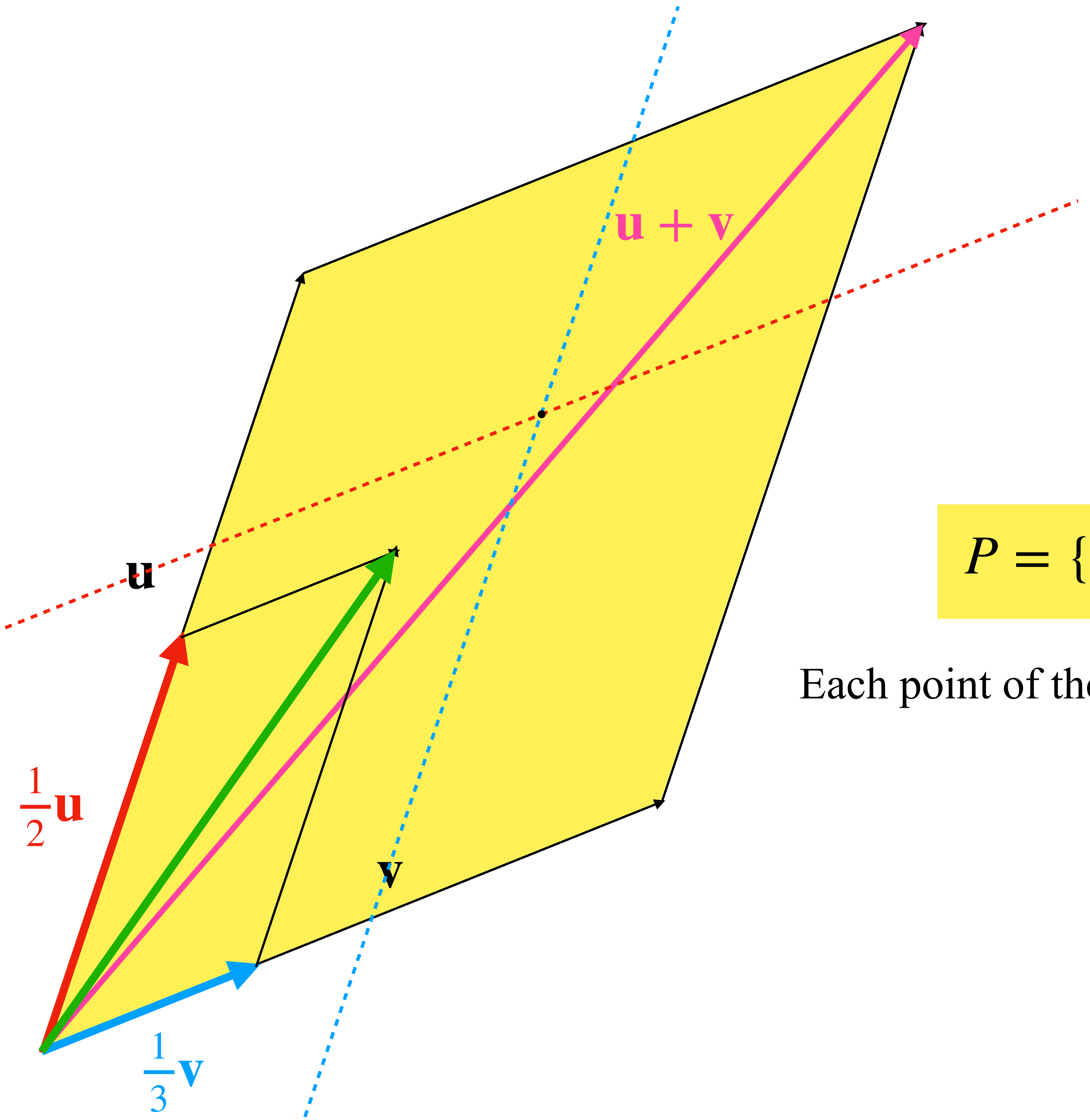

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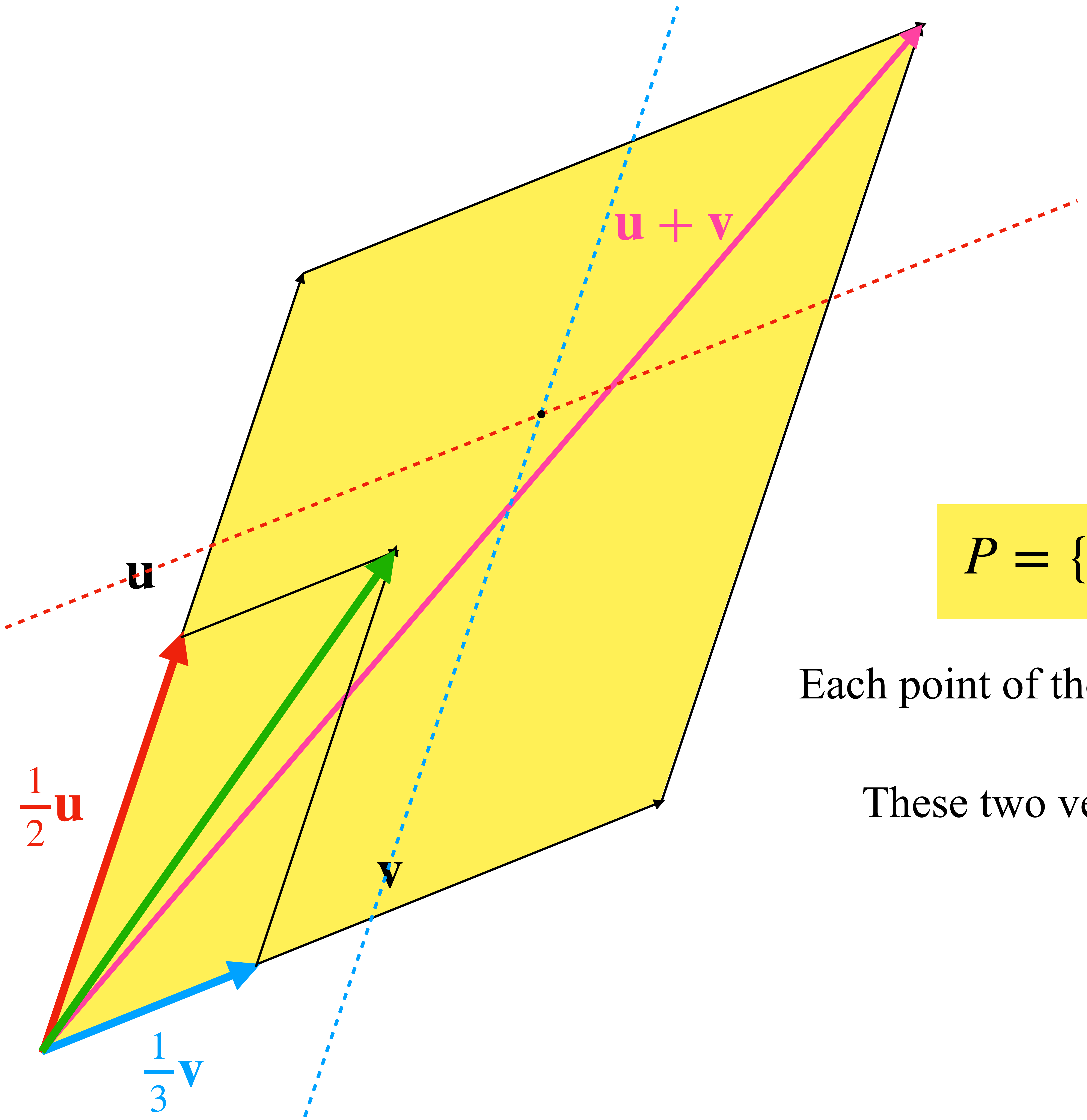

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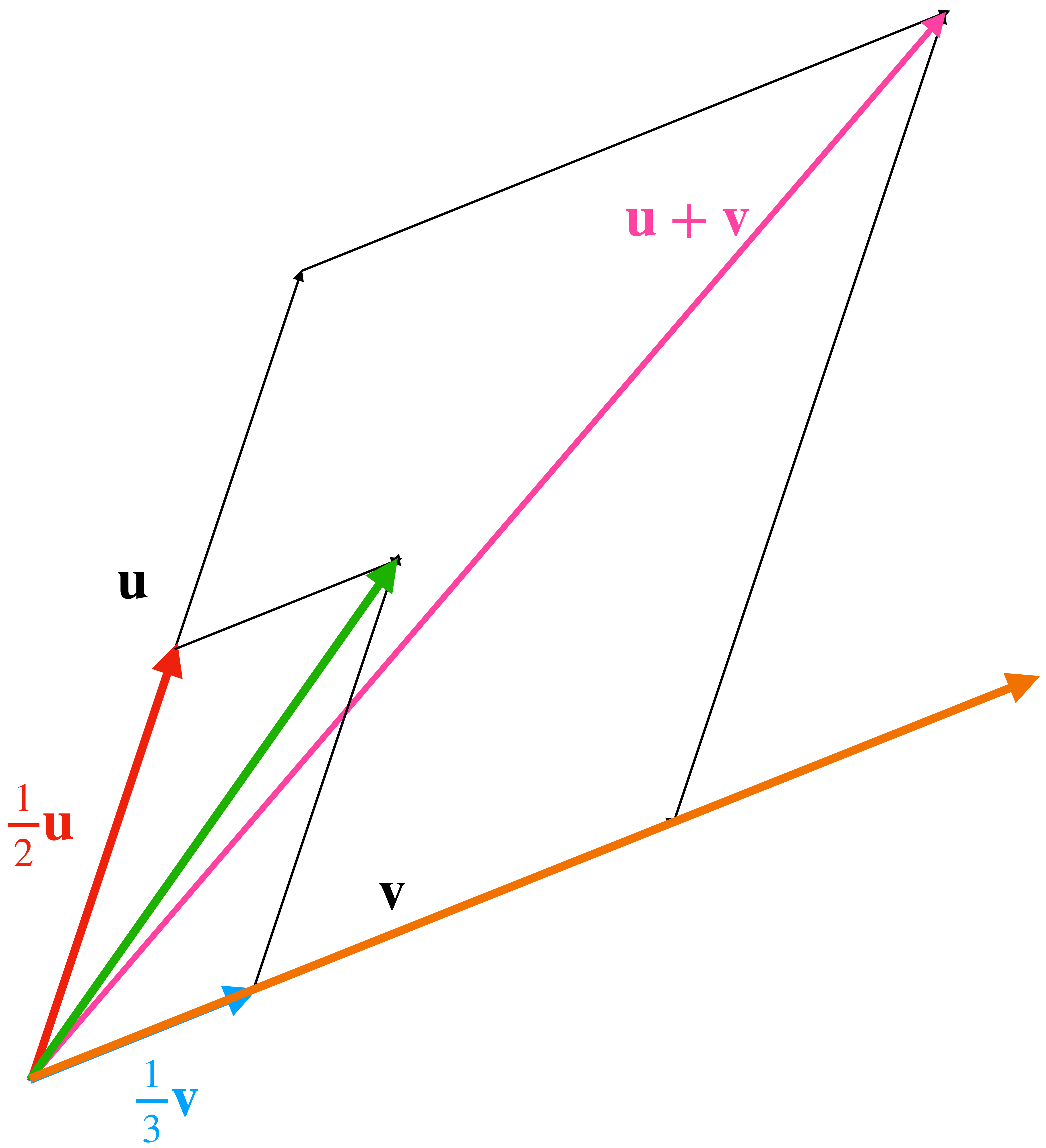
These two vectors **generate** the entire parallelogram



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$$c_1 = 0, \quad c_2 = \frac{3}{2}$$

$$c_1\mathbf{u} + c_2\mathbf{v} = 0\mathbf{u} + \frac{3}{2}\mathbf{v}$$

$$c_1 = \frac{1}{2}, \quad c_2 = -\frac{1}{3}$$

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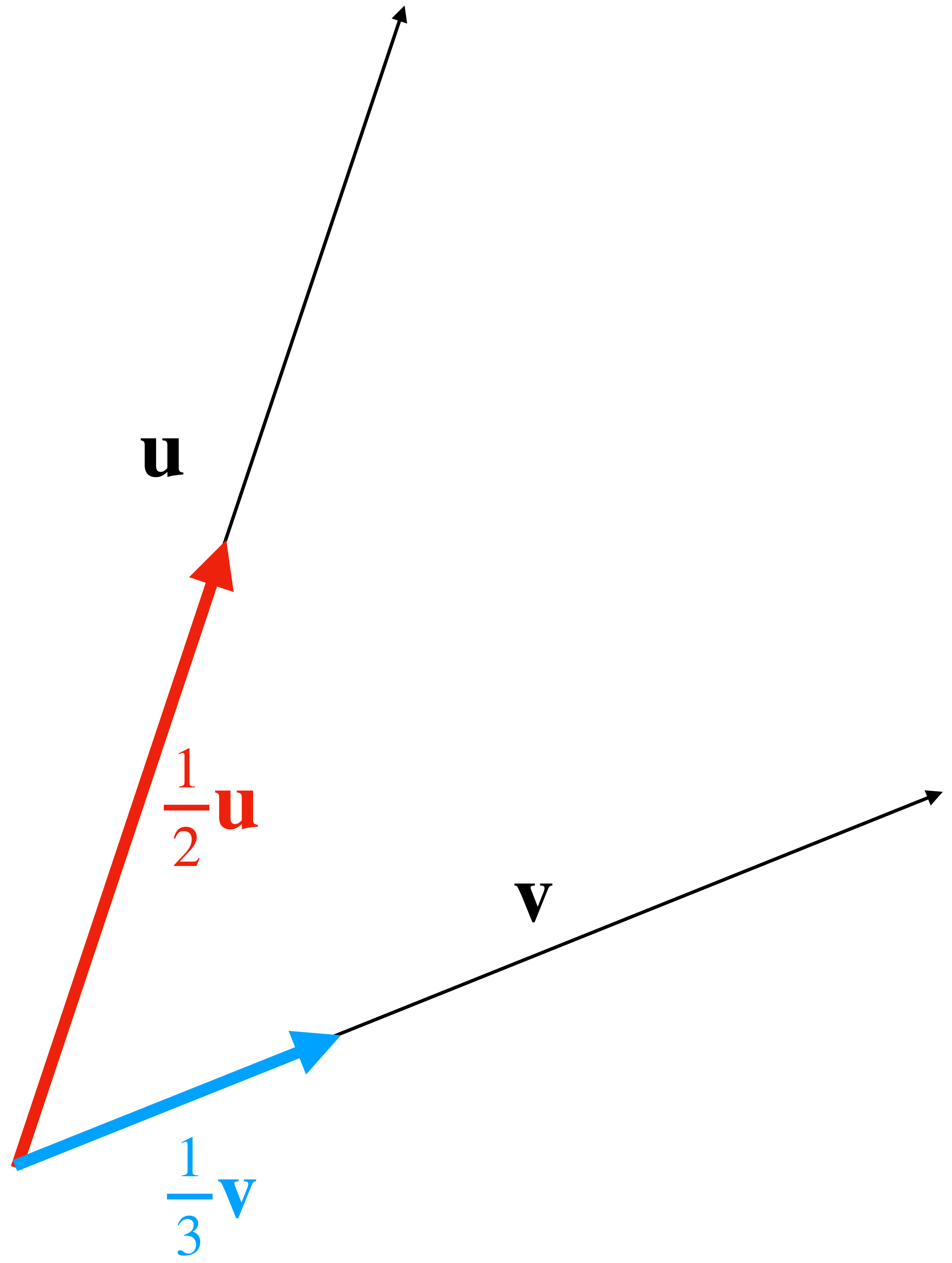

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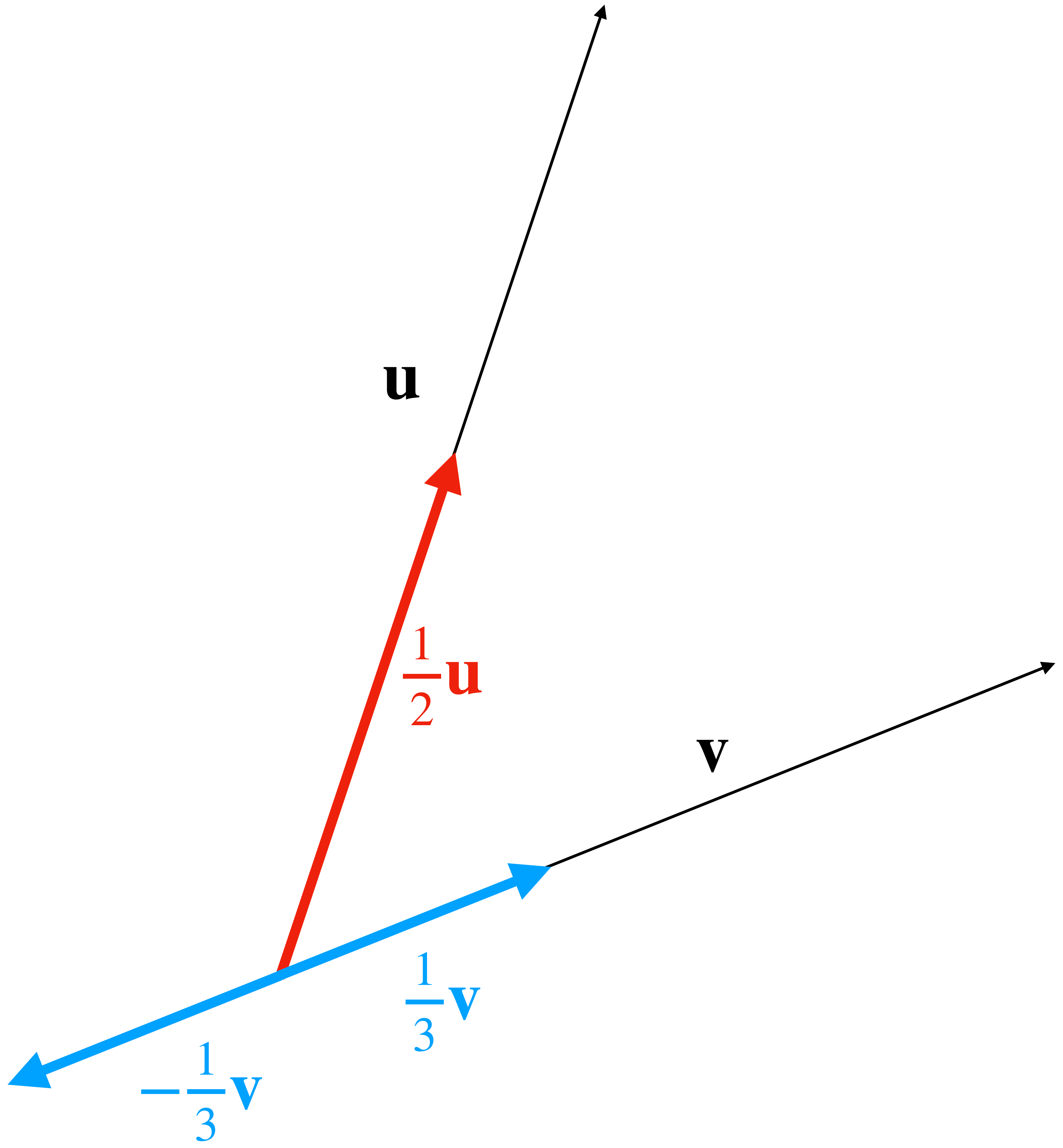

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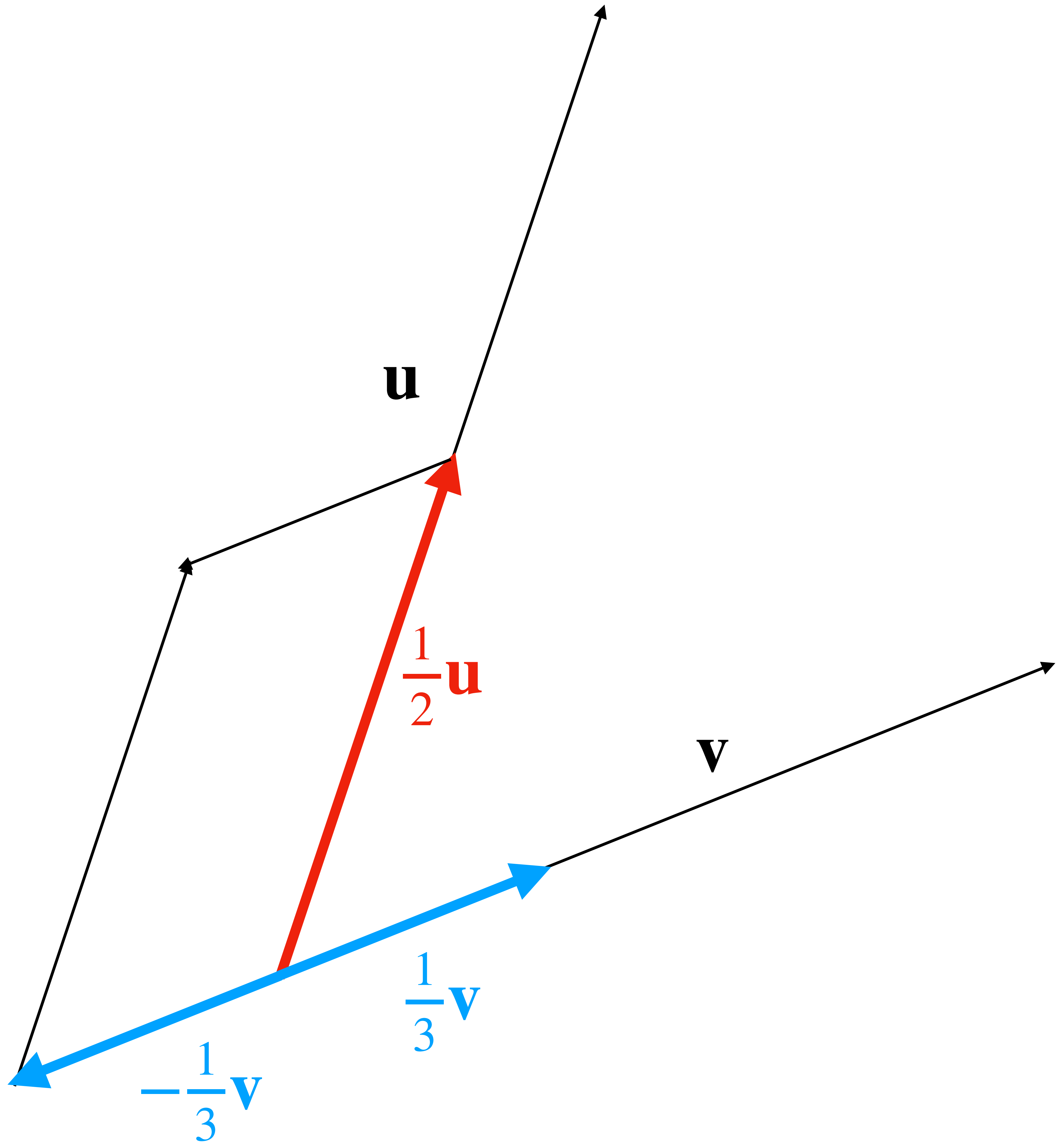

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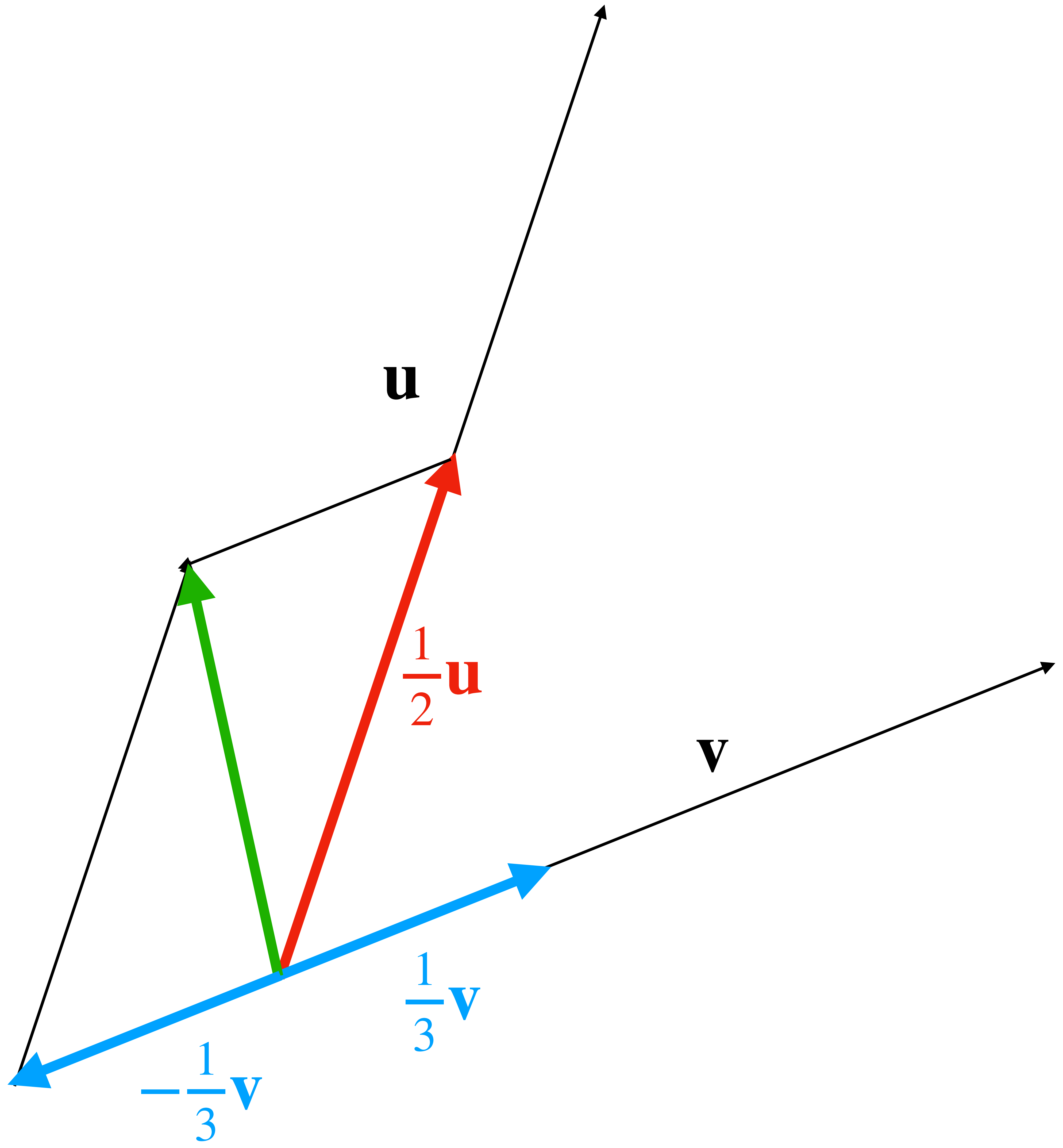

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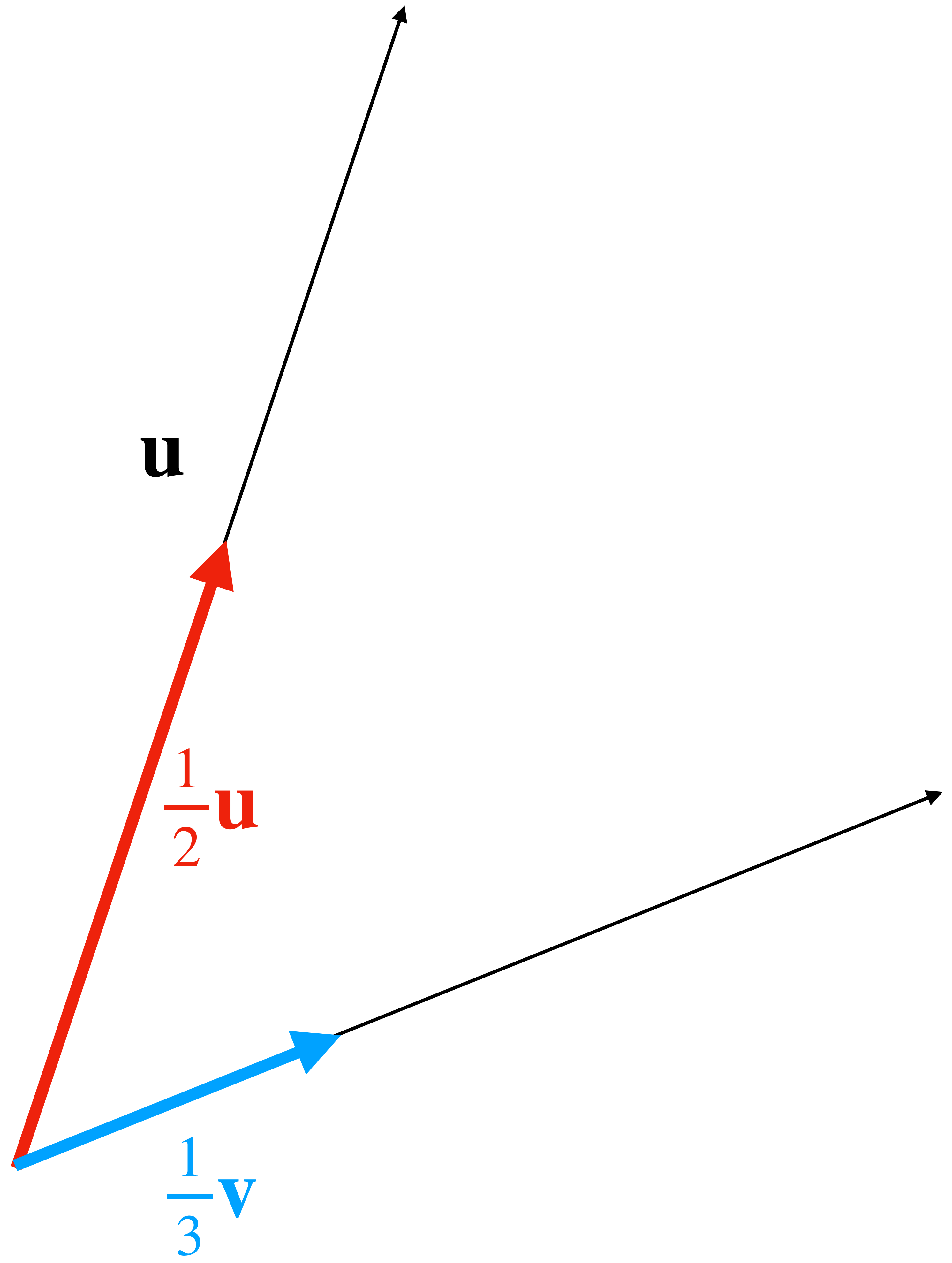

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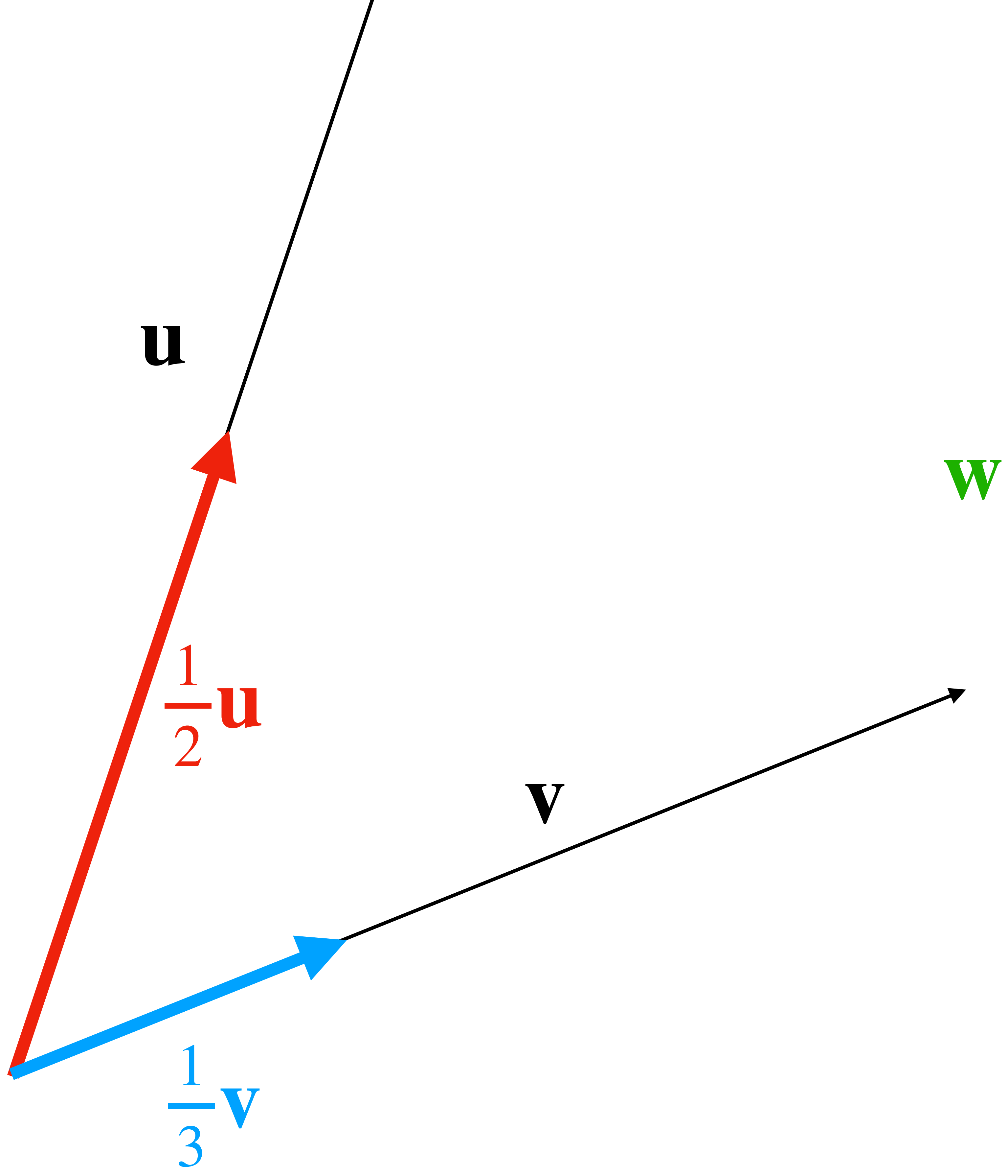

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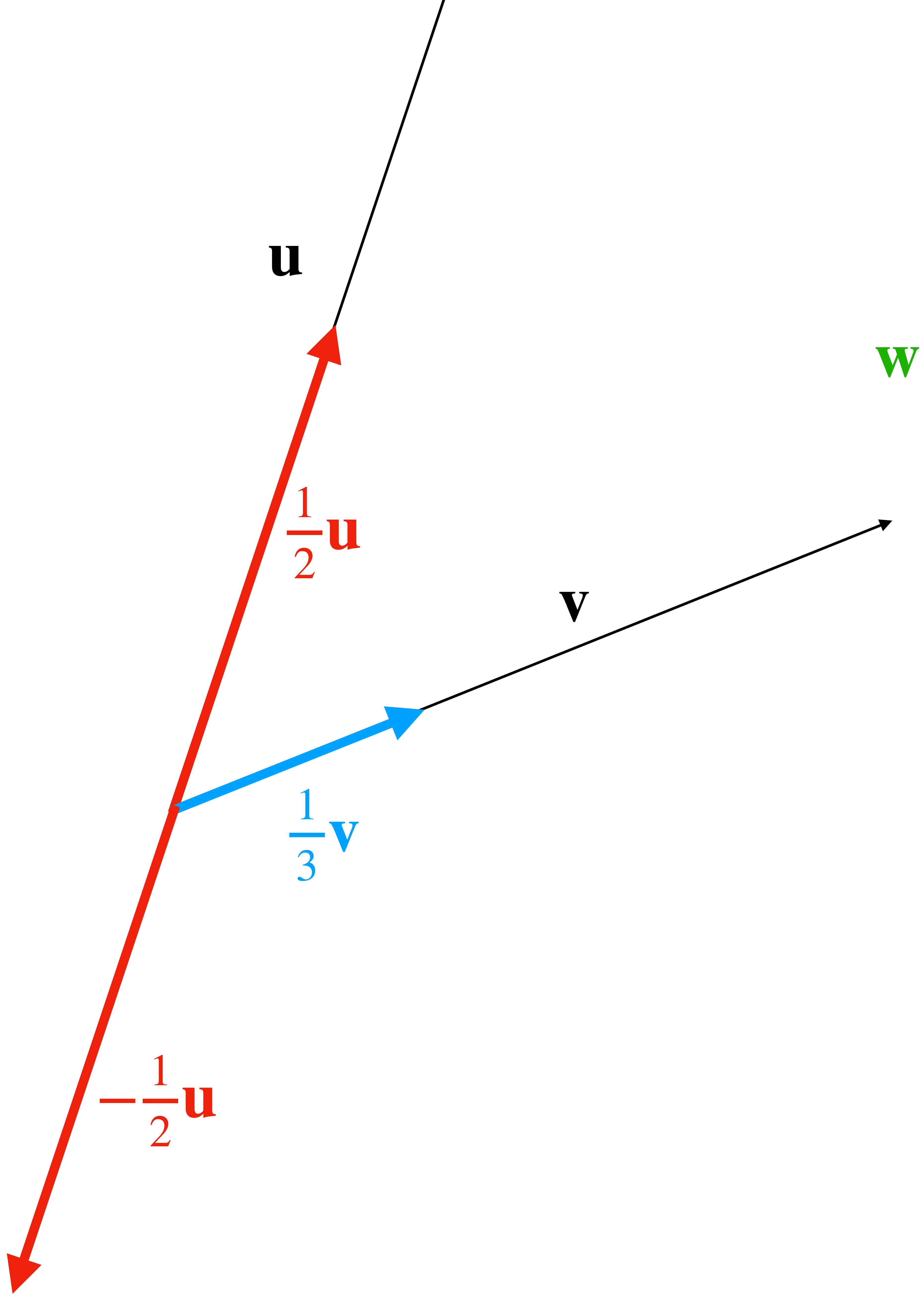

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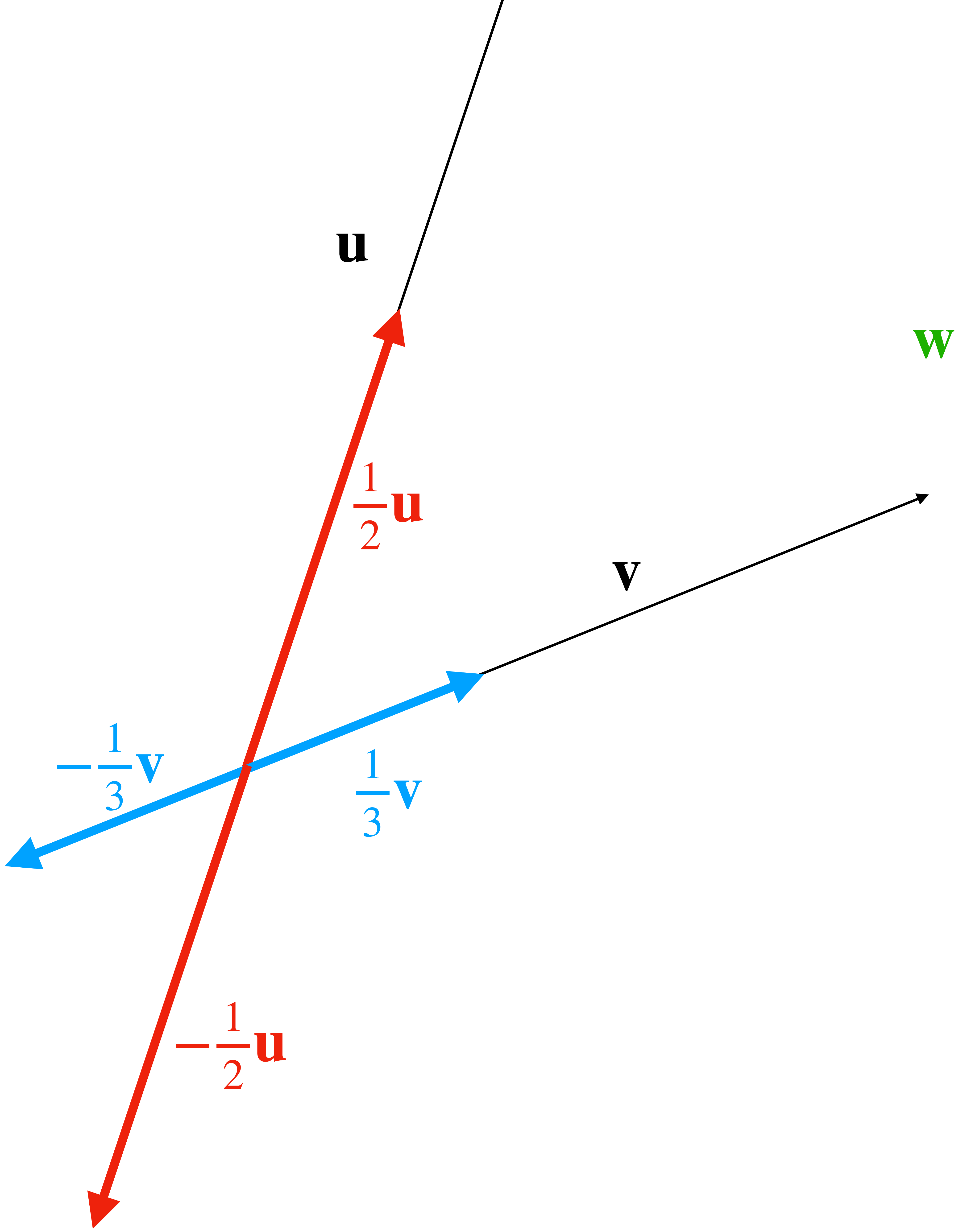

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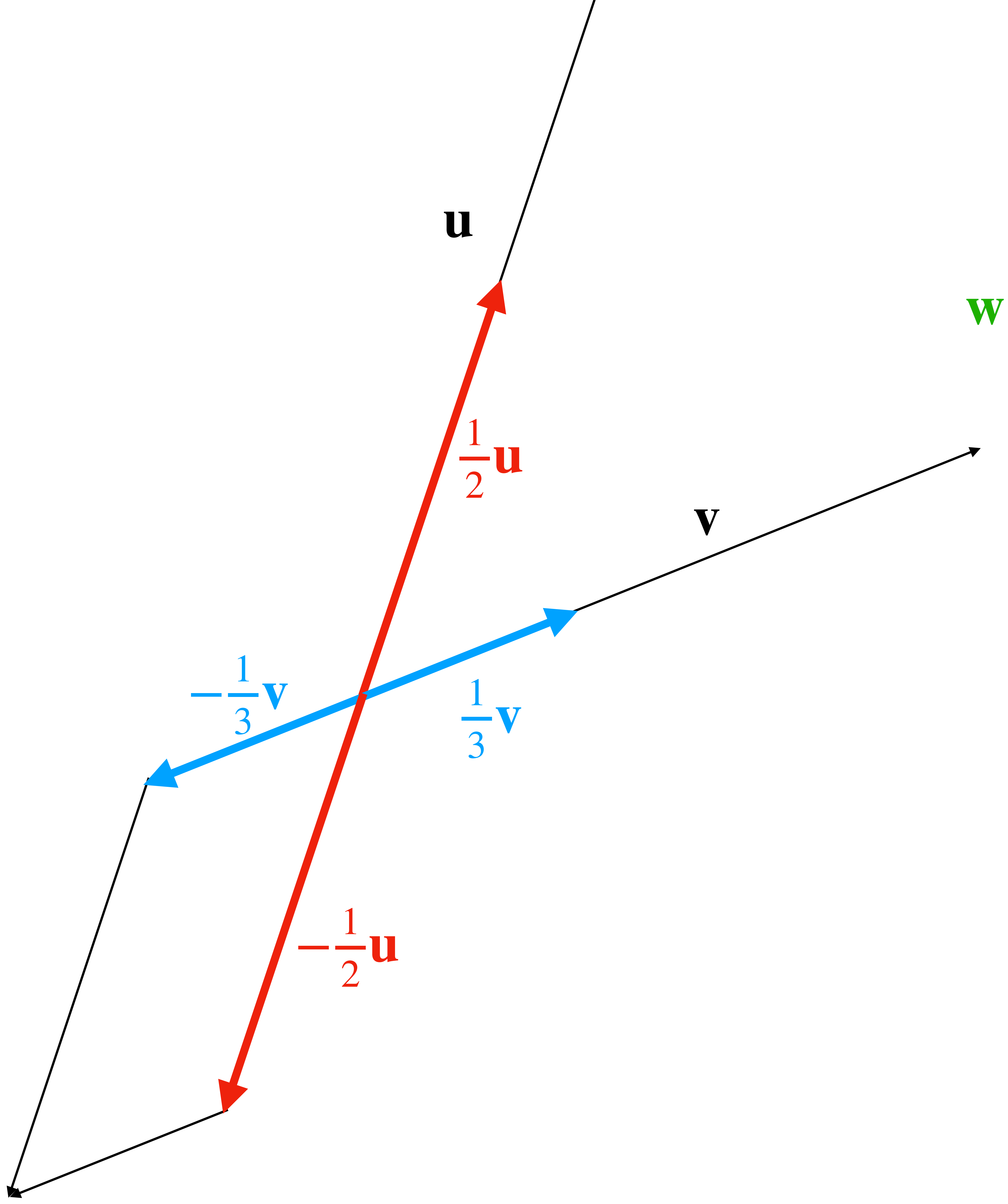

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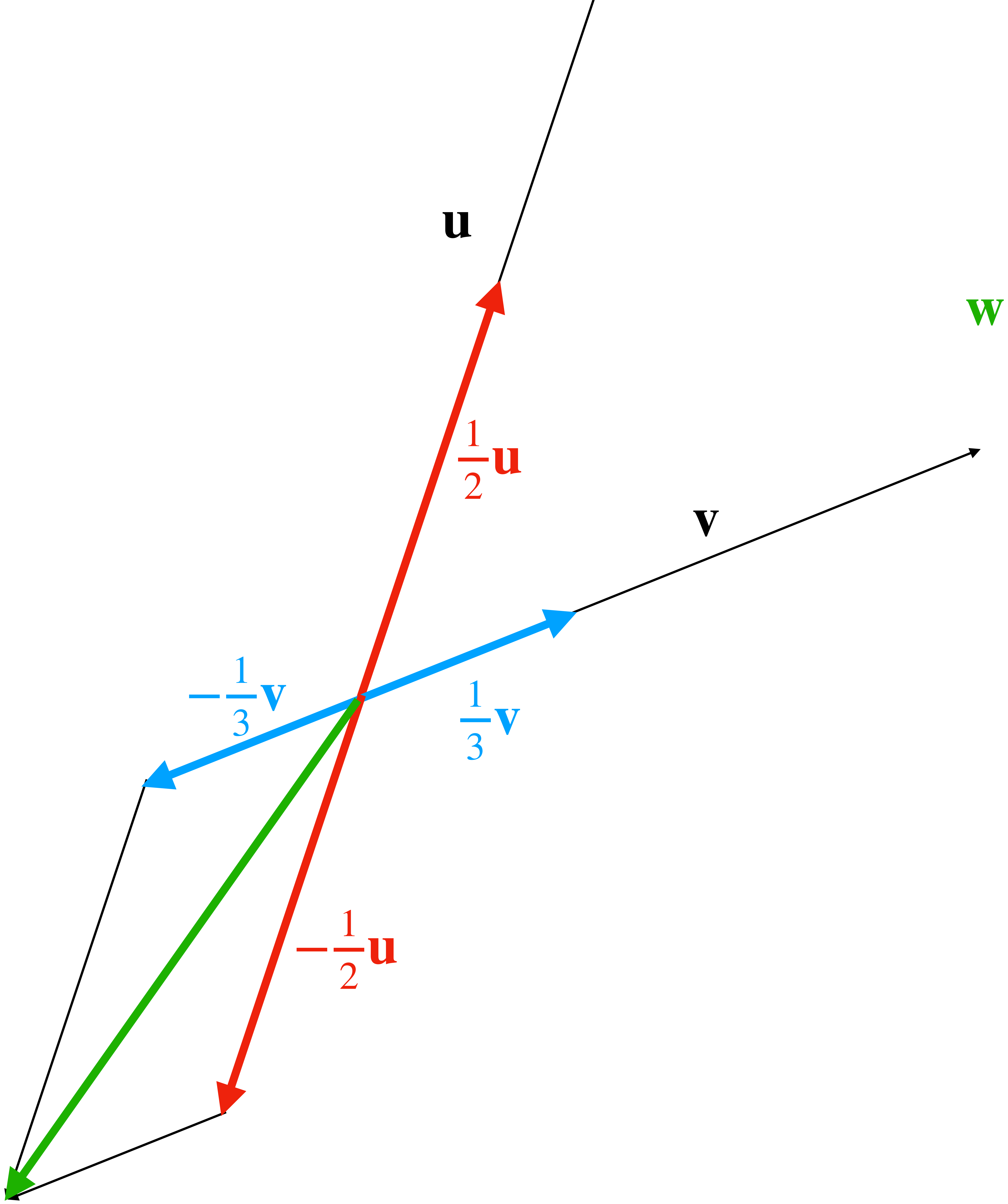

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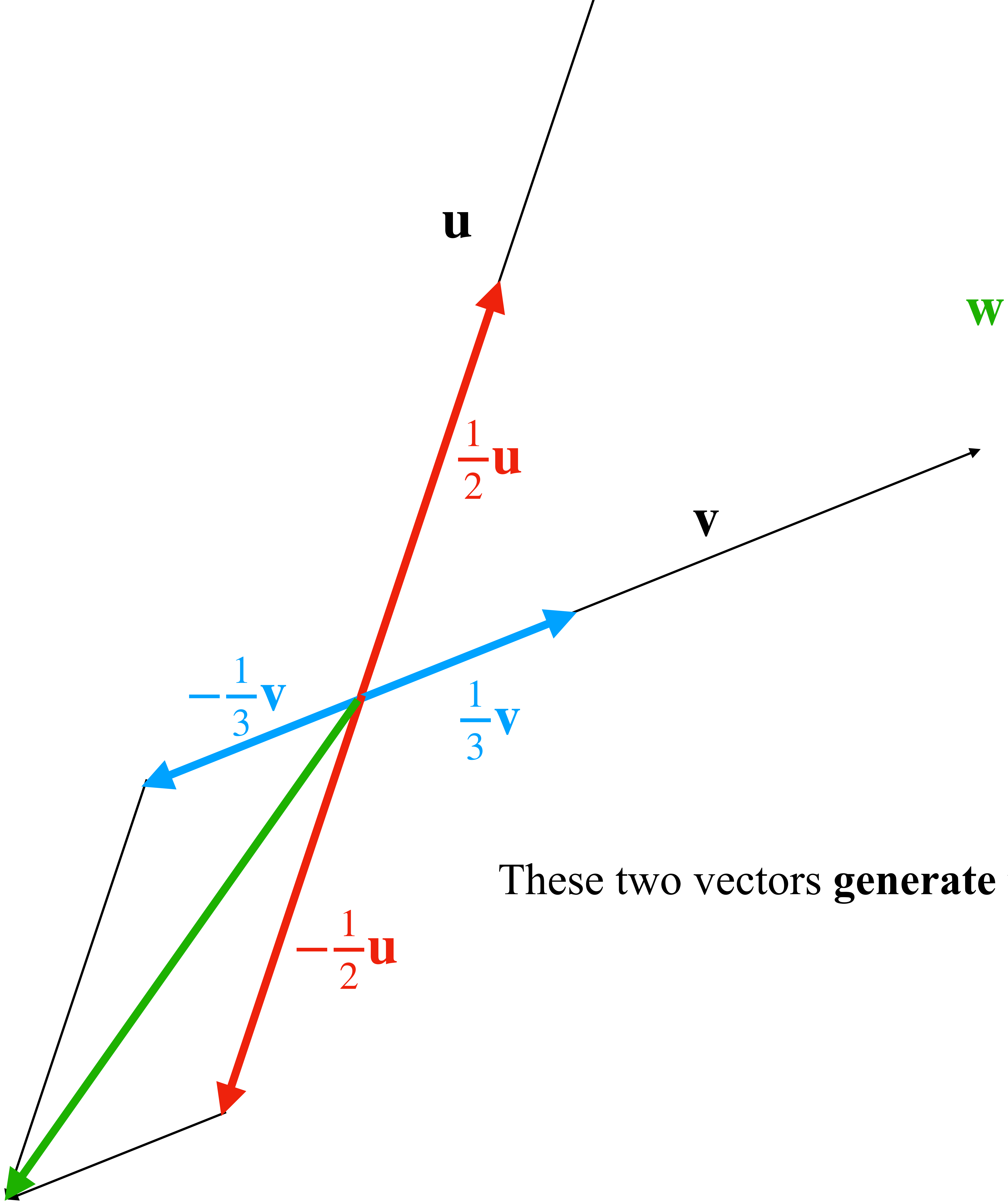


These two vectors **generate** the entire plane they belong to

$$\mathbb{R}^n$$

$$\overrightarrow{v} = (v_1, v_2, \dots, v_n)$$

$$\alpha \overrightarrow{v} = (\alpha v_1, \alpha v_2, \dots, \alpha v_n)$$

$$\mathbf{v} = (v_1, v_2, \dots, v_n)$$

$$\overrightarrow{u} = (u_1, u_2, \dots, u_n)$$

$$\beta \overrightarrow{u} = (\beta u_1, \beta u_2, \dots, \beta u_n)$$

$$\mathbb{R}^n$$

$$\overrightarrow{v} = (v_1, v_2, \ \dots, v_n)$$

$$\alpha \overrightarrow{v} = (\alpha v_1, \alpha v_2, \ \dots \ , \alpha v_n)$$

$$\mathbf{v} = (v_1, v_2, \ \dots, v_n)$$

$$\overrightarrow{u} = (u_1, u_2, \ \dots, u_n)$$

$$\beta \overrightarrow{u} = (\beta u_1, \beta u_2, \ \dots \ , \beta u_n)$$

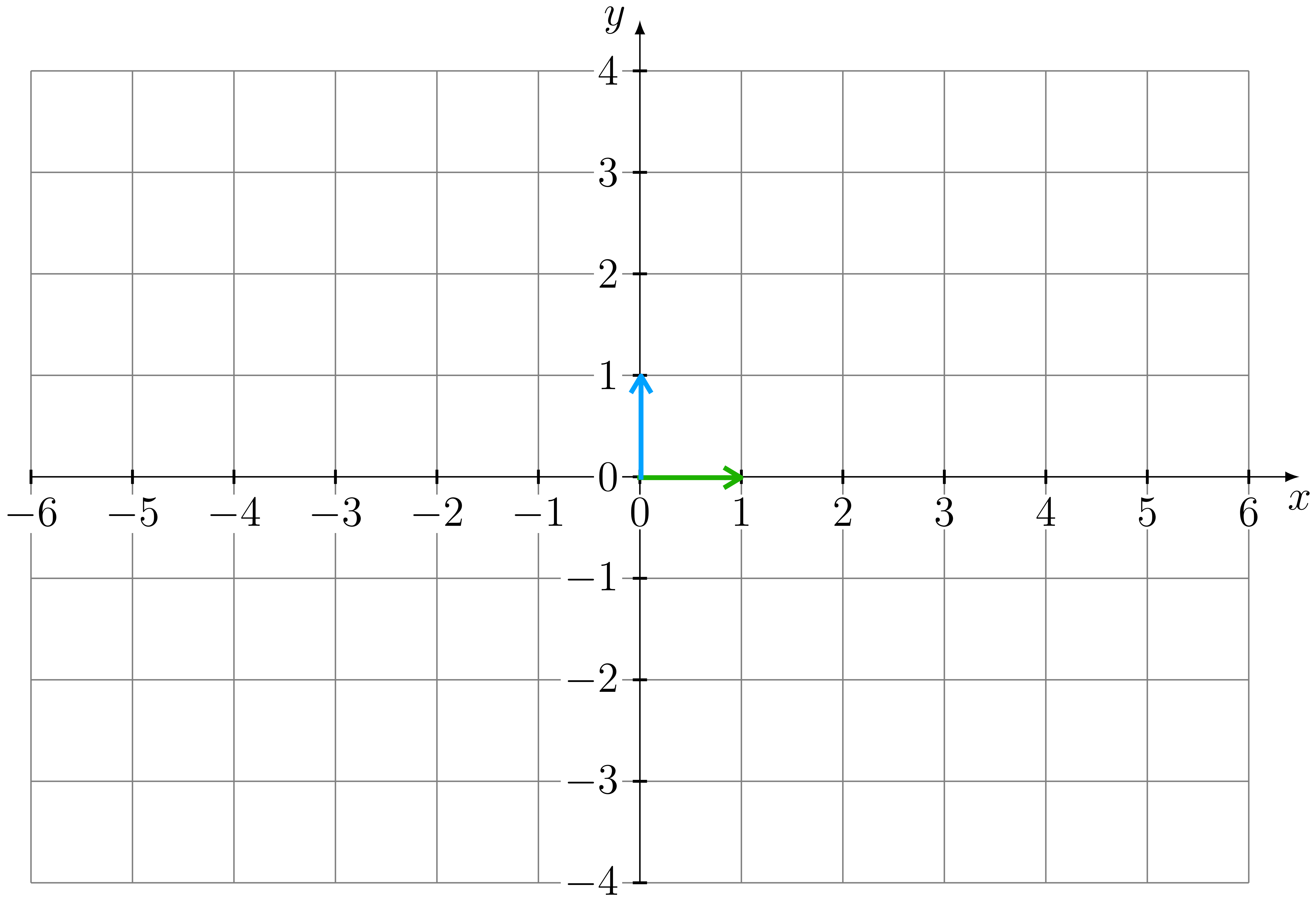
$$\alpha \overrightarrow{v} + \beta \overrightarrow{u} = (\alpha v_1 + \beta u_1, \alpha v_2 + \beta u_2, \ \dots, \alpha v_n + \beta u_n)$$

$$\alpha_1\mathbf{u}_1 + \alpha_2\mathbf{u}_2 + \ldots + \alpha_n\mathbf{u}_n$$

A linear combination of vectors $\mathbf{u}_1, \mathbf{u}_2, \ \ldots \ , \mathbf{u}_n$

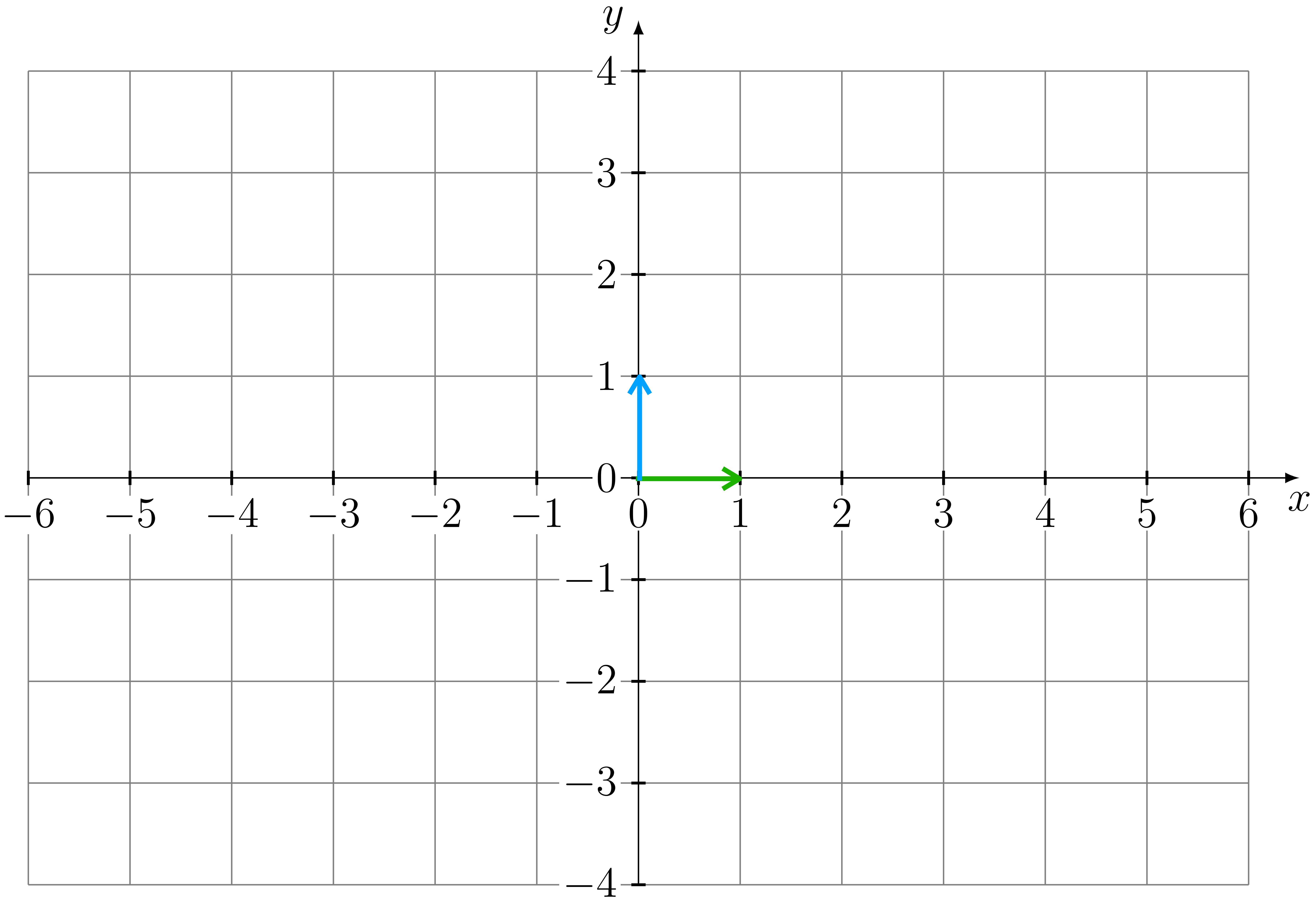
An example on iPad

$\mathbb{R}^2$



$\mathbb{R}^2$

$\vec{i} = \mathbf{i} = \overrightarrow{e}_1 = \mathbf{e}_1 = (1{,}0)$



$\mathbb{R}^2$

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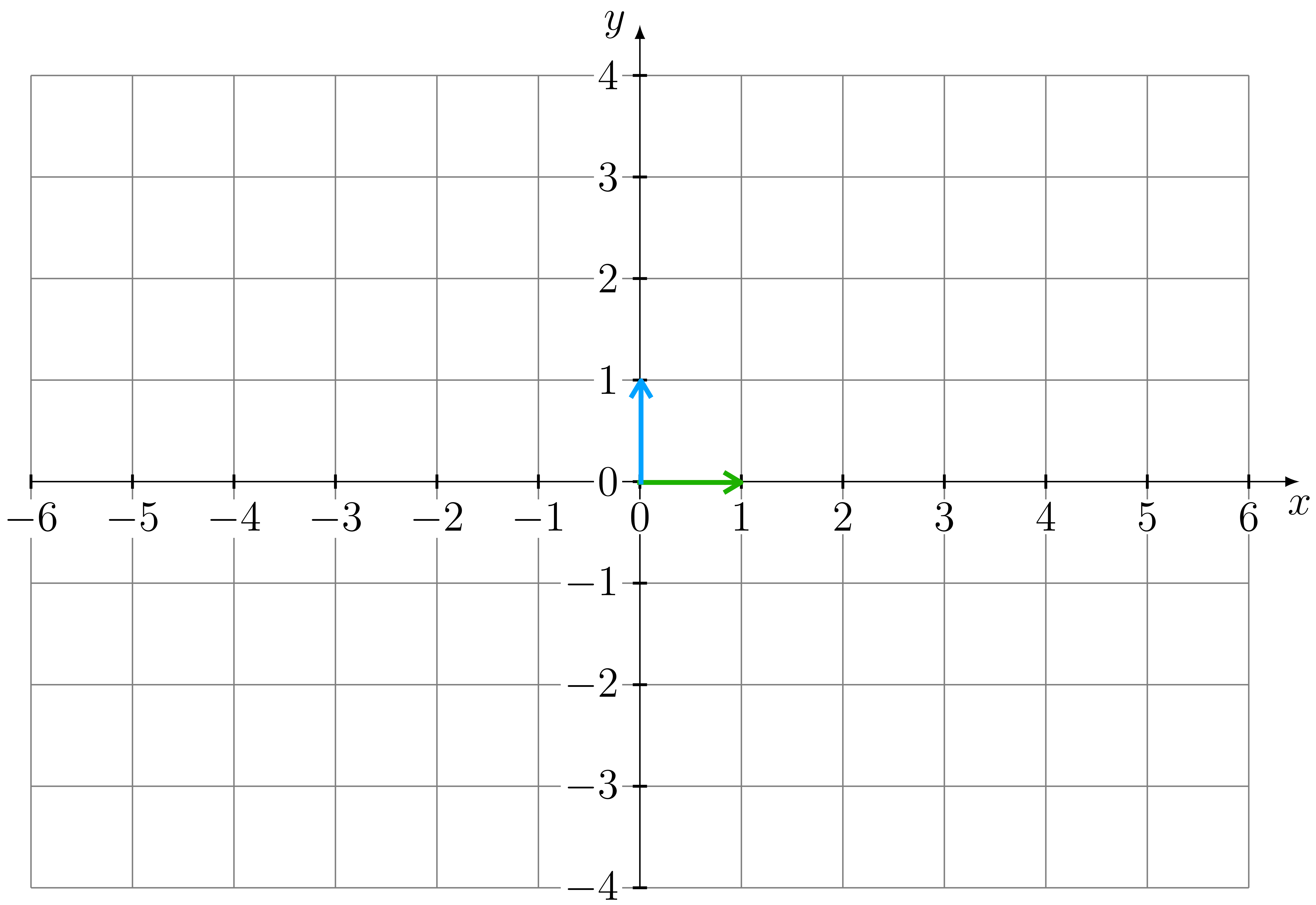

$\mathbb{R}^2$

Each vector in the plane is a linear combination of $\vec{i}$ and $\vec{j}$

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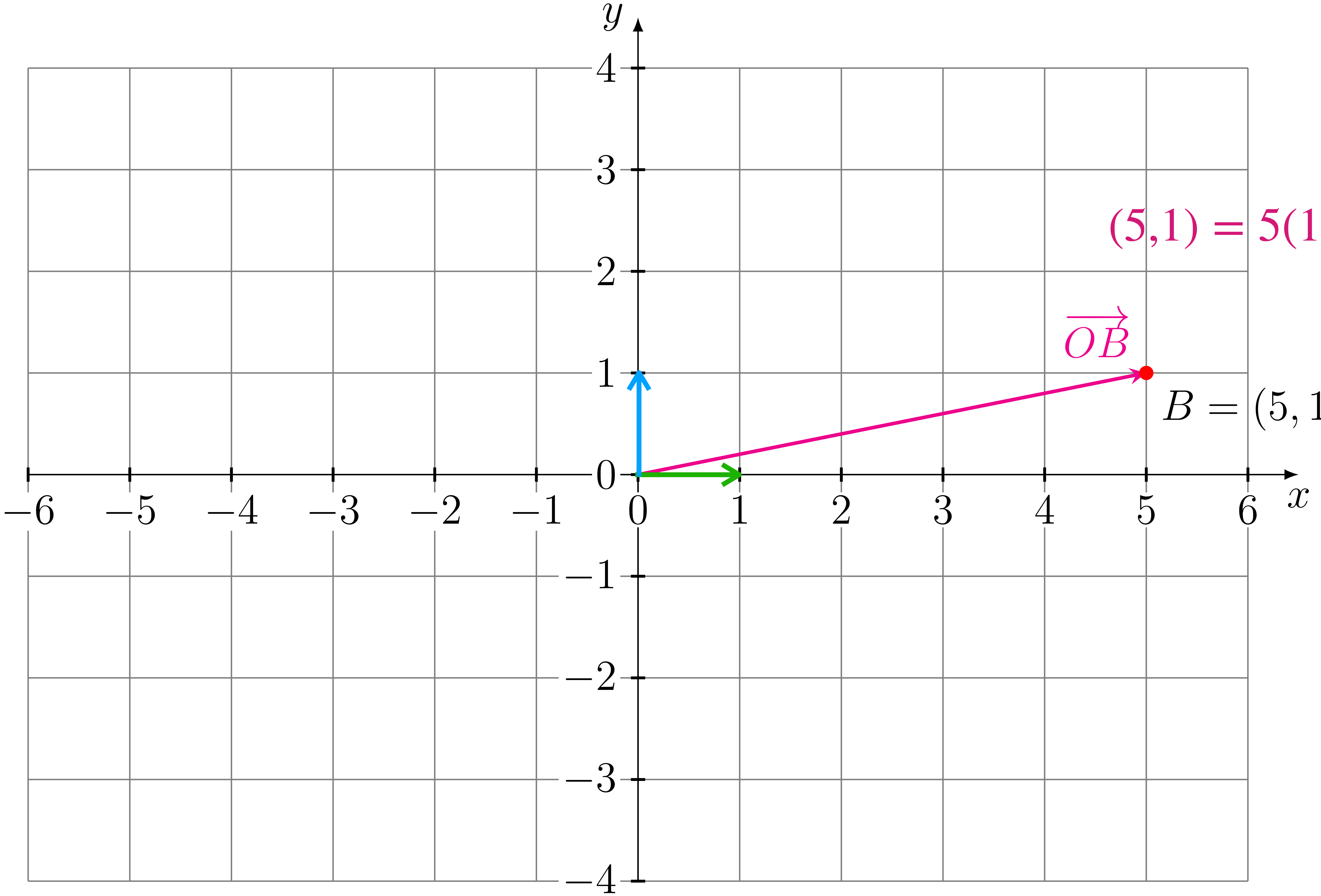
$\vec{j} = \mathbf{j} = \vec{e}_2 = \mathbf{e}_2 = (0,1)$

$(5,1) = 5(1,0) + 1(0,1) = 5\vec{i} + \vec{j}$



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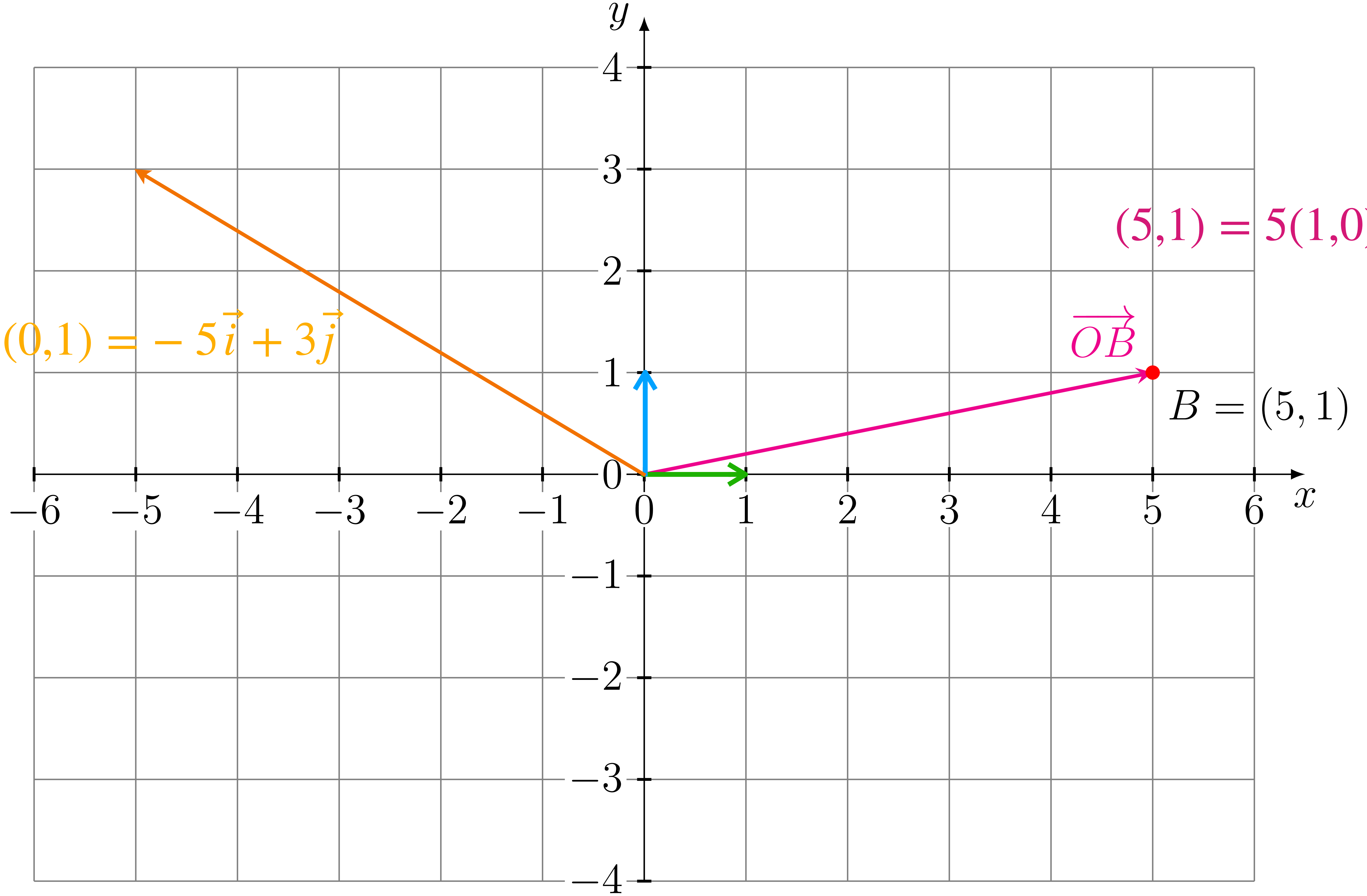
$(-5,3) = -5(1,0) + 3(0,1) = -5\vec{i} + 3\vec{j}$

$(5,1) = 5(1,0) + 1(0,1) = 5\vec{i} + \vec{j}$



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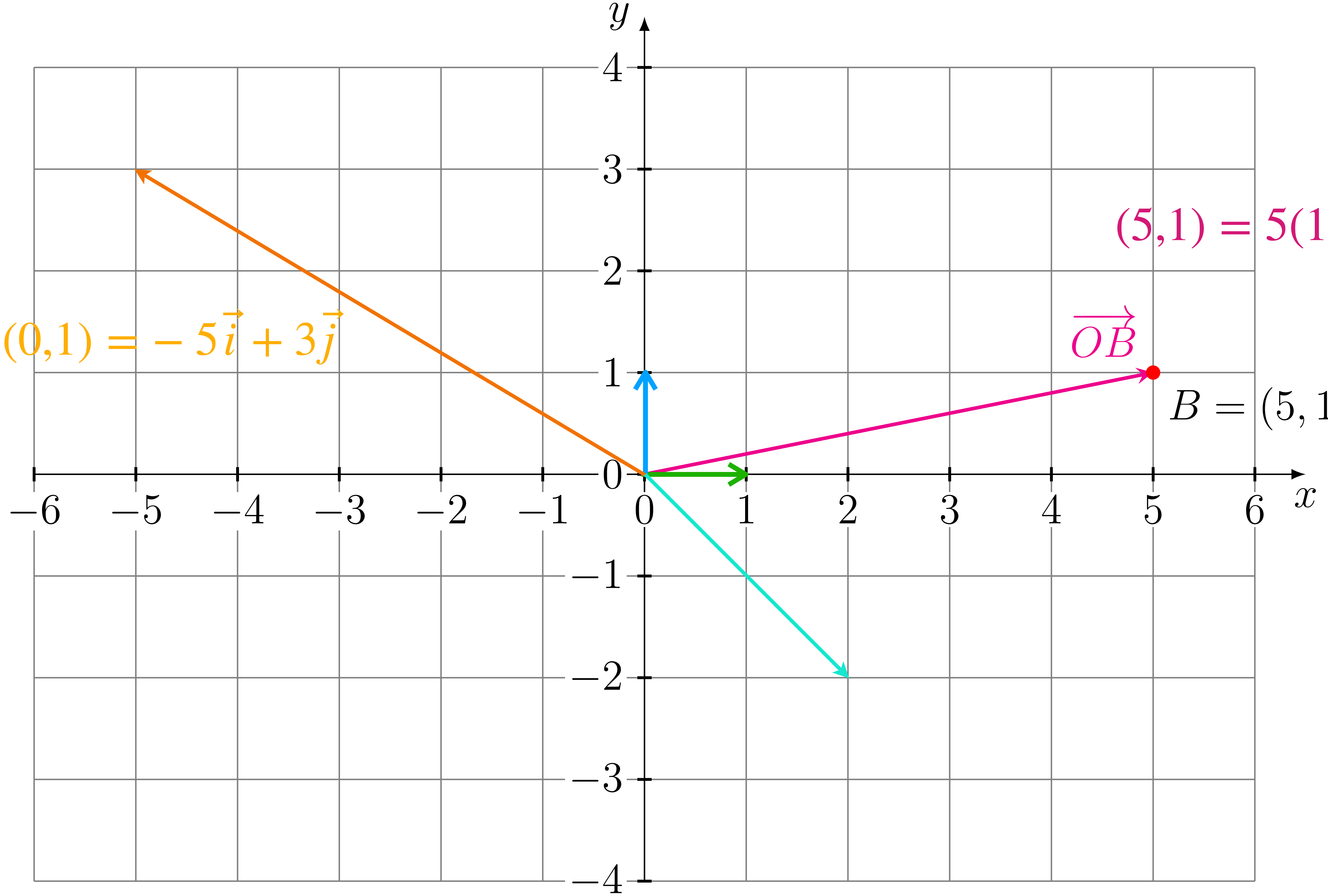

$(5{,}1) = 5(1{,}0) + 1(0{,}1) = 5\vec{i} + \vec{j}$

$(-5{,}3) = -5(1{,}0) + 3(0{,}1) = -5\vec{i} + 3\vec{j}$

$(2, -2) = 2(1{,}0) - 2(0{,}1) = 2\vec{i} - 2\vec{j}$

$\mathbb{R}^3$

User:Jorge Stolfi

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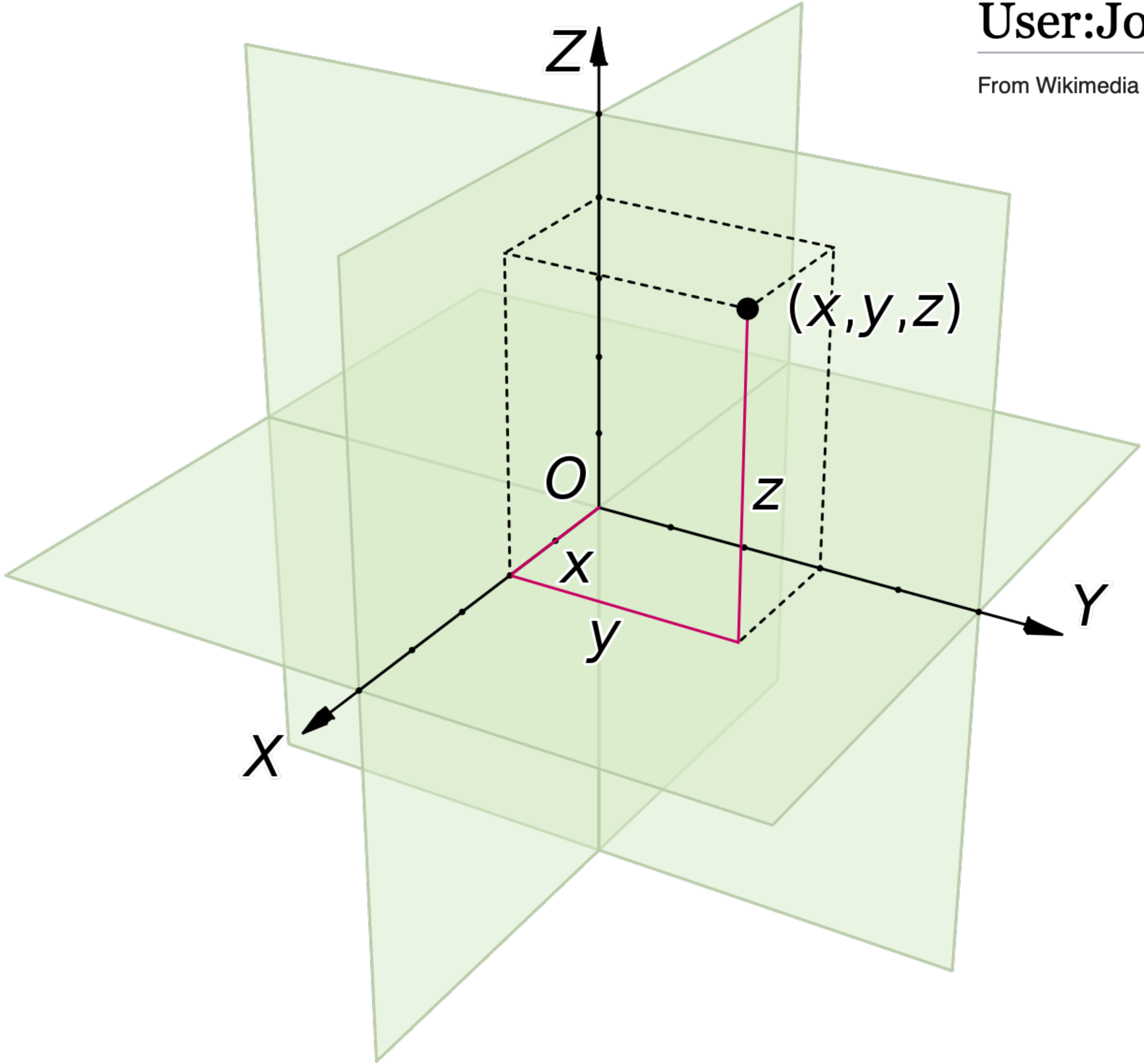
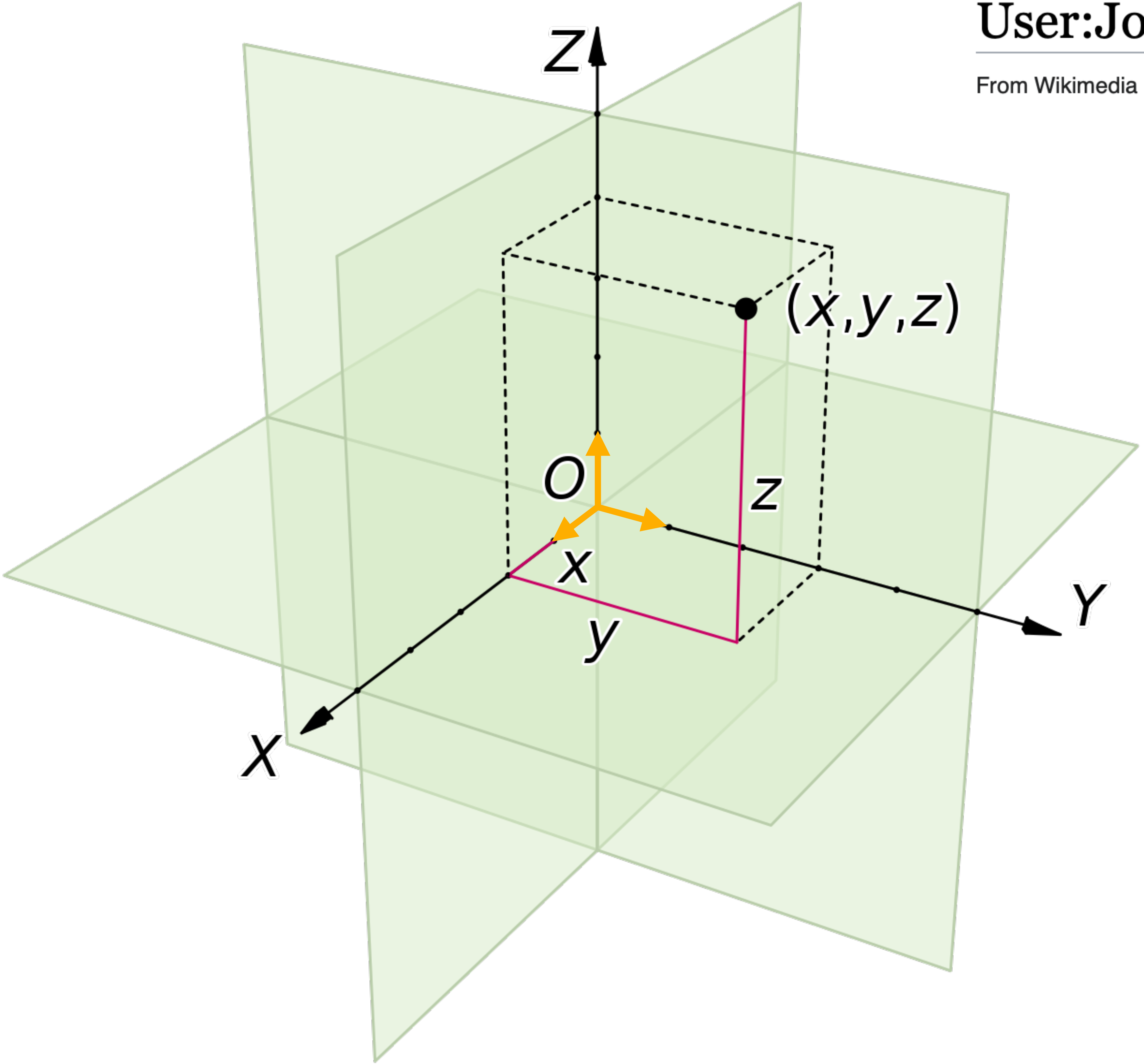


$\mathbb{R}^3$



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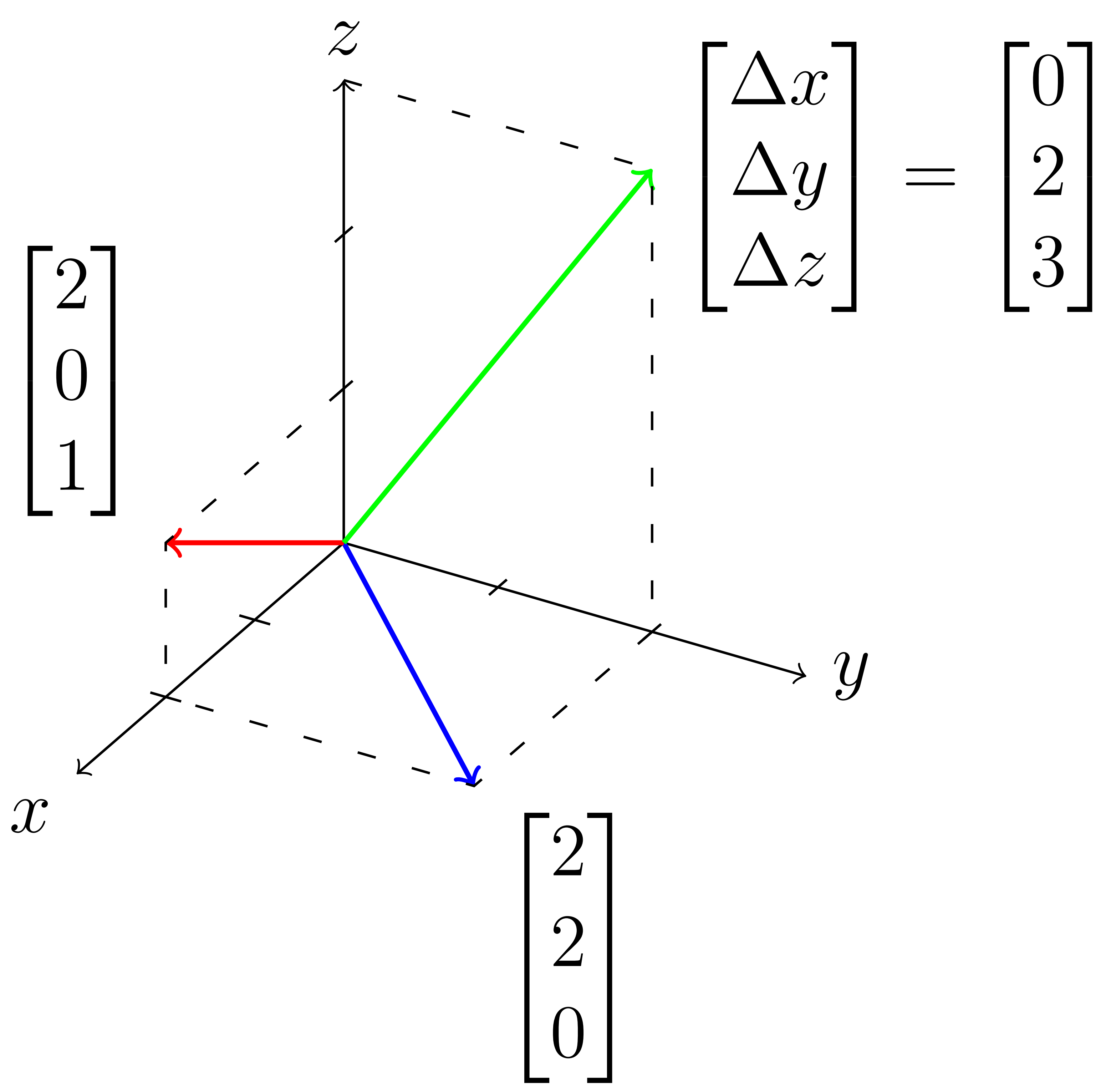
$\mathbb{R}^3$

Each vector in the 3-space is a linear combination of $\vec{i}$, $\vec{j}$ and $\overrightarrow{k}$

$\vec{i} = \mathbf{i} = \overrightarrow{e}_1 = \mathbf{e}_1 = (1,0,0)$

$\vec{j} = \mathbf{j} = \overrightarrow{e}_2 = \mathbf{e}_2 = (0,1,0)$

$\overrightarrow{k} = \mathbf{k} = \overrightarrow{e}_3 = \mathbf{e}_3 = (0,0,1)$



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Each vector in the 3-space is a linear combination of $\vec{i}$, $\vec{j}$ and $\overrightarrow{k}$

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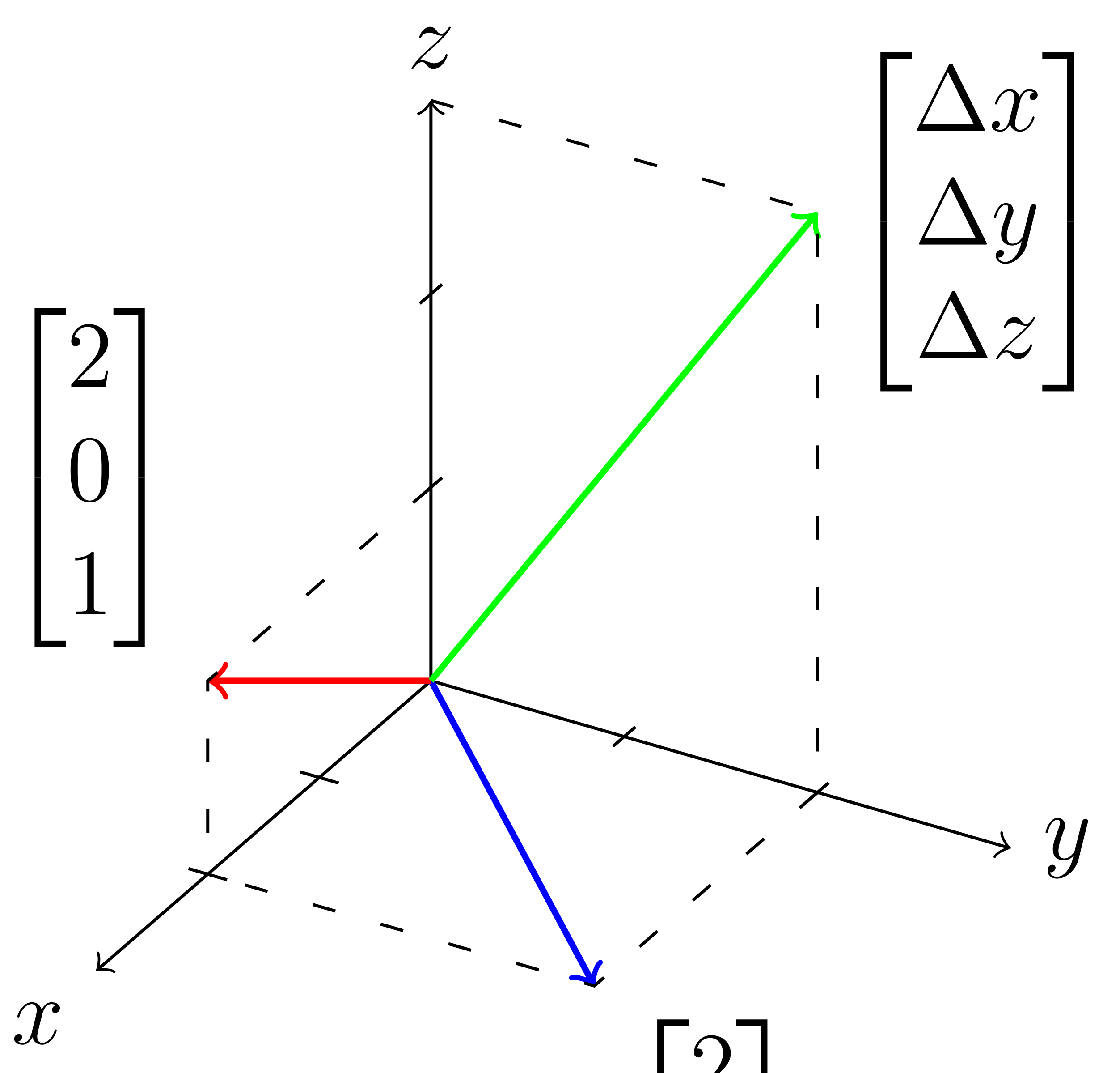

$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$

$\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$

$(2{,}2{,}0) = 2(1{,}0{,}0) + 2(0{,}1{,}0) + 0(0{,}0{,}1) = 2\vec{i} + 2\vec{j} + 0\,\overrightarrow{k}$

$$\mathbb{R}^n$$

$$\overrightarrow{e}_1 = (1, 0, 0, \ \ldots, 0), \ \overrightarrow{e}_2 = (0, 1, 0, \ \ldots, 0), \ \overrightarrow{e}_3 = (0, 0, 1, \ \ldots, 0), \ \ldots \ , \ \overrightarrow{e}_{n-1} = (0, 0, 0, \ \ldots, 1, 0), \ \overrightarrow{e}_n = (0, 0, 0, \ \ldots, 0, 1)$$

$$\mathbb{R}^n$$

$$\vec{e}_1 = (1, 0, 0, \ldots, 0),\ \vec{e}_2 = (0, 1, 0, \ldots, 0),\ \vec{e}_3 = (0, 0, 1, \ldots, 0),\ \ldots\ ,\ \vec{e}_{n-1} = (0, 0, 0, \ldots, 1, 0),\ \vec{e}_n = (0, 0, 0, \ldots, 0, 1)$$

$$\vec{v} = (v_1, v_2, \ldots, v_n) = v_1 \vec{e}_1 + v_2 \vec{e}_2 + \ldots + v_n \vec{e}_n$$

Each vector in $\mathbb{R}^n$ is a linear combination of the unit vectors $\vec{e}_i,\ i = 1, 2, \ldots, n$

Linear combination can be applied to any mathematical objects which can be added and scaled

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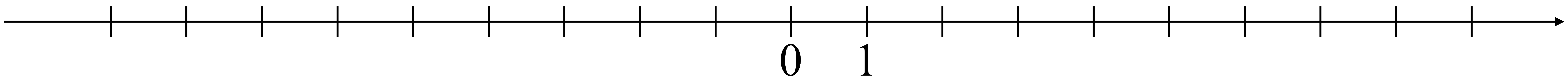
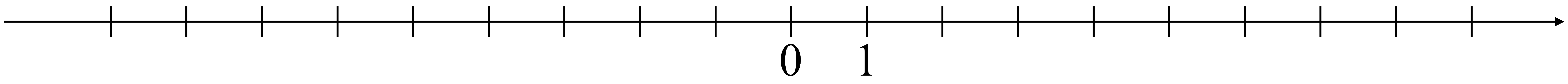


Linear combination can be applied to any mathematical objects which can be added and scaled

$$c_1x_1 + c_2x_2 + \ldots + c_nx_n = b$$

$$6 - 8 = -2$$



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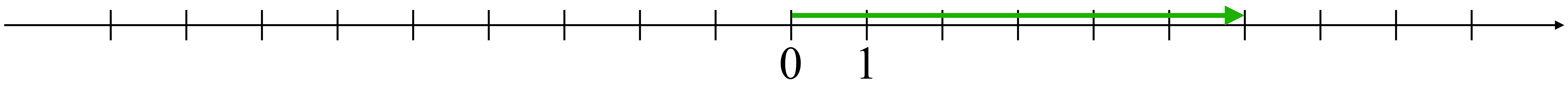

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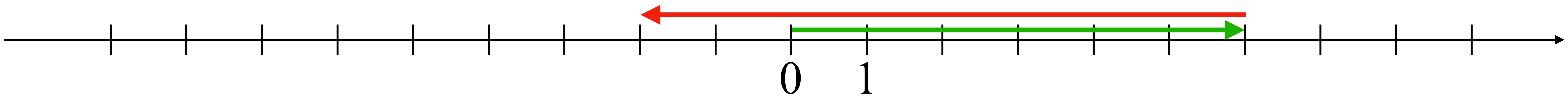

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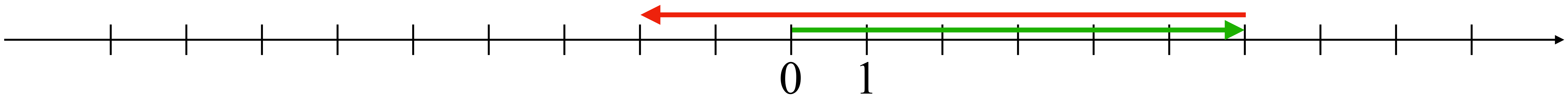
$$c_1x_1 + c_2x_2 + \ldots + c_nx_n = b$$

$$6 - 8 = -2$$



$$3\sin x - 2\cos x$$

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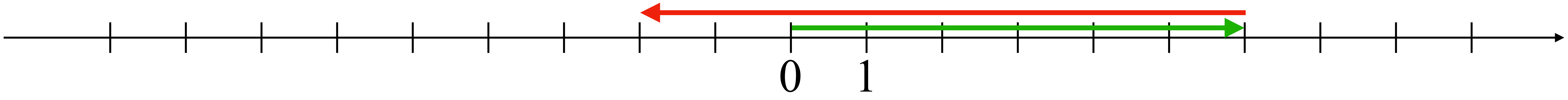

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