Topic: Matrix inverses, and invertible and singular matrices

Question: Are the matrices inverses of one another?

$$A = \begin{bmatrix} 2 & 5 \\ 3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} -\frac{1}{13} & \frac{5}{13} \\ \frac{3}{13} & -\frac{2}{13} \end{bmatrix}$$

Answer choices:

A Yes

B No

C There's not enough information to know

Solution: A

To find the inverse of matrix A, plug it into the formula for the inverse matrix.

$$A^{-1}=\frac{1}{|A|}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$A^{-1}=\frac{1}{\begin{vmatrix}2 & 5\\ 3 & 1\end{vmatrix}}\begin{bmatrix}1 & -5\\ -3 & 2\end{bmatrix}$$

Find the determinant in the denominator of the fraction.

$$A^{-1}=\frac{1}{(2)(1)-(5)(3)}\begin{bmatrix}1 & -5\\ -3 & 2\end{bmatrix}$$

$$A^{-1}=\frac{1}{2-15}\begin{bmatrix}1 & -5\\ -3 & 2\end{bmatrix}$$

$$A^{-1}=-\frac{1}{13}\begin{bmatrix}1 & -5\\ -3 & 2\end{bmatrix}$$

Then distribute the scalar across the matrix.

$$A^{-1}=\begin{bmatrix}-\frac{1}{13}(1) & -\frac{1}{13}(-5)\\ -\frac{1}{13}(-3) & -\frac{1}{13}(2)\end{bmatrix}$$

$$A^{-1}=\begin{bmatrix}-\frac{1}{13} & \frac{5}{13}\\ \frac{3}{13} & -\frac{2}{13}\end{bmatrix}$$

Because the value we found matches matrix B, it means that matrices A and B are inverses of one another.

Topic: Matrix inverses, and invertible and singular matrices

Question: Find the inverse of matrix M.

$$M = \begin{bmatrix} 0 & -2 \\ -4 & 5 \end{bmatrix}$$

Answer choices:

A $\quad M^{-1} = \begin{bmatrix} 0 & -\frac{1}{4} \\ -\frac{1}{2} & -\frac{5}{8} \end{bmatrix}$

B $\quad M^{-1} = \begin{bmatrix} -\frac{5}{8} & -\frac{1}{4} \\ -\frac{1}{2} & 0 \end{bmatrix}$

C $\quad M^{-1} = \begin{bmatrix} 0 & \frac{1}{4} \\ \frac{1}{2} & -\frac{5}{8} \end{bmatrix}$

D $\quad M^{-1} = \begin{bmatrix} -\frac{5}{8} & \frac{1}{4} \\ \frac{1}{2} & 0 \end{bmatrix}$

Solution: B

Plug the values from the matrix into the formula for the inverse matrix.

$$M^{-1}=\frac{1}{|M|}\begin{bmatrix}d & -b\\ -c & a\end{bmatrix}$$

$$M^{-1}=\frac{1}{\begin{vmatrix}0 & -2\\ -4 & 5\end{vmatrix}}\begin{bmatrix}5 & 2\\ 4 & 0\end{bmatrix}$$

Find the determinant in the denominator of the fraction.

$$M^{-1}=\frac{1}{(0)(5)-(-2)(-4)}\begin{bmatrix}5 & 2\\ 4 & 0\end{bmatrix}$$

$$M^{-1}=\frac{1}{0-8}\begin{bmatrix}5 & 2\\ 4 & 0\end{bmatrix}$$

$$M^{-1}=-\frac{1}{8}\begin{bmatrix}5 & 2\\ 4 & 0\end{bmatrix}$$

Then distribute the scalar across the matrix.

$$M^{-1}=\begin{bmatrix}-\frac{1}{8}(5) & -\frac{1}{8}(2)\\ -\frac{1}{8}(4) & -\frac{1}{8}(0)\end{bmatrix}$$

$$M^{-1}=\begin{bmatrix}-\frac{5}{8} & -\frac{1}{4}\\ -\frac{1}{2} & 0\end{bmatrix}$$

Topic: Matrix inverses, and invertible and singular matrices

Question: Classify the matrix.

$$L = \begin{bmatrix} 3 & 7 \\ 0 & -1 \end{bmatrix}$$

Answer choices:

A The matrix is invertible

B The matrix is singular

C The matrix is invertible and singular

D The matrix is neither invertible nor singular

Solution: A

A matrix is either invertible or singular, it can never be both. To determine whether a matrix in the form

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

is invertible or singular, we need to look at the ratio of a to b, compared to the ratio of c to d.

For the given matrix L,

$$\frac{a}{b} = \frac{3}{7}$$

$$\frac{c}{d} = \frac{0}{-1} = 0$$

Because these ratios aren't equivalent, that means matrix L is invertible. If the ratios had been equivalent, the matrix would have been singular.