Topic: Coordinates in a new basis

Question: The vectors $\overrightarrow{v}=(2,2,3)$, $\vec{s}=(-6,0,2)$, and $\overrightarrow{w}=(2,-2,-5)$ form an alternate basis for $\mathbb{R}^3$. Use them to transform $\overrightarrow{x}=-2i+k$ into the alternate basis.

Answer choices:

A $\quad [\overrightarrow{x}]_B=\begin{bmatrix}-\frac{1}{2}\\ 0\\ -\frac{1}{2}\end{bmatrix}$

B $\quad [\overrightarrow{x}]_B=\begin{bmatrix}-2\\ 0\\ 1\end{bmatrix}$

C $\quad [\overrightarrow{x}]_B=\begin{bmatrix}0\\ \frac{1}{3}\\ 0\end{bmatrix}$

D $\quad [\overrightarrow{x}]_B=\begin{bmatrix}\frac{1}{2}\\ 0\\ \frac{1}{2}\end{bmatrix}$

Solution: A

The vector $\overrightarrow{x} = (-2,0,1)$ is given in terms of the standard basis, and we need to transform it into an alternate basis that's defined by $\overrightarrow{v} = (2,2,3)$, $\vec{s} = (-6,0,2)$, and $\overrightarrow{w} = (2,-2,-5)$.

So let's plug the values we've been given into the matrix equation.

$$A[\overrightarrow{x}]_B = \overrightarrow{x}$$

$$\begin{bmatrix} 2 & -6 & 2 \\ 2 & 0 & -2 \\ 3 & 2 & -5 \end{bmatrix} [\overrightarrow{x}]_B = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$$

To find the representation of $\overrightarrow{x}$ in the alternate basis, $[\overrightarrow{x}]_B$, we'll put the augmented matrix into reduced row-echelon form.

$$\begin{bmatrix} 2 & -6 & 2 & | & -2 \\ 2 & 0 & -2 & | & 0 \\ 3 & 2 & -5 & | & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 1 & | & -1 \\ 2 & 0 & -2 & | & 0 \\ 3 & 2 & -5 & | & 1 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 1 & | & -1 \\ 0 & 6 & -4 & | & 2 \\ 3 & 2 & -5 & | & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 1 & | & -1 \\ 0 & 6 & -4 & | & 2 \\ 0 & 11 & -8 & | & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 1 & | & -1 \\ 0 & 1 & -\frac{2}{3} & | & \frac{1}{3} \\ 0 & 11 & -8 & | & 4 \end{bmatrix} \to \begin{bmatrix} 1 & -3 & 1 & | & -1 \\ 0 & 1 & -\frac{2}{3} & | & \frac{1}{3} \\ 0 & 0 & -\frac{2}{3} & | & \frac{1}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -\frac{2}{3} & | & \frac{1}{3} \\ 0 & 0 & -\frac{2}{3} & | & \frac{1}{3} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & -\frac{2}{3} & | & \frac{1}{3} \\ 0 & 0 & 1 & | & -\frac{1}{2} \end{bmatrix} \to \begin{bmatrix} 1 & 0 & -1 & | & 0 \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -\frac{1}{2} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & | & -\frac{1}{2} \\ 0 & 1 & 0 & | & 0 \\ 0 & 0 & 1 & | & -\frac{1}{2} \end{bmatrix}$$

So $\vec{x} = (-2,0,1)$, expressed in the alternate basis, is

$$[\vec{x}]_B = \begin{bmatrix} -\frac{1}{2} \\ 0 \\ -\frac{1}{2} \end{bmatrix}$$

Topic: Coordinates in a new basis

Question: The vectors $\vec{v}=(1,-4)$ and $\vec{w}=(-3,2)$ form an alternate basis for $\mathbb{R}^2$. Use them to transform $\vec{x}=-i-6j$ into the alternate basis.

Answer choices:

A $\quad [\vec{x}]_B=\begin{bmatrix}-2\\-1\end{bmatrix}$

B $\quad [\vec{x}]_B=\begin{bmatrix}2\\1\end{bmatrix}$

C $\quad [\vec{x}]_B=\begin{bmatrix}-1\\1\end{bmatrix}$

D $\quad [\vec{x}]_B=\begin{bmatrix}1\\-1\end{bmatrix}$

Solution: B

The vector $\vec{x}=(-1,-6)$ is given in terms of the standard basis, and we need to transform it into an alternate basis that's defined by $\vec{v}=(1,-4)$ and $\vec{w}=(-3,2)$.

So let's plug the values we've been given into the matrix equation.

$$A[\vec{x}]_B=\vec{x}$$

$$\begin{bmatrix}1 & -3\\ -4 & 2\end{bmatrix}[\vec{x}]_B=\begin{bmatrix}-1\\ -6\end{bmatrix}$$

To find the representation of $\vec{x}$ in the alternate basis, $[\vec{x}]_B$, we'll put the augmented matrix into reduced row-echelon form.

$$\begin{bmatrix}1 & -3 & | & -1\\ -4 & 2 & | & -6\end{bmatrix}\to\begin{bmatrix}1 & -3 & | & -1\\ 0 & -10 & | & -10\end{bmatrix}$$

$$\begin{bmatrix}1 & -3 & | & -1\\ 0 & 1 & | & 1\end{bmatrix}\to\begin{bmatrix}1 & 0 & | & 2\\ 0 & 1 & | & 1\end{bmatrix}$$

So $\vec{x}=(-1,-6)$, expressed in the alternate basis, is

$$[\vec{x}]_B=\begin{bmatrix}2\\ 1\end{bmatrix}$$

Topic: Coordinates in a new basis

Question: The vectors $\overrightarrow{v}=(1,-5)$ and $\overrightarrow{w}=(-2,0)$ form an alternate basis for $\mathbb{R}^2$. Use them, and an inverse matrix, to transform $\overrightarrow{x}=3i-5j$ into the alternate basis.

Answer choices:

A $\quad [\overrightarrow{x}]_B=\begin{bmatrix}-1\\2\end{bmatrix}$

B $\quad [\overrightarrow{x}]_B=\begin{bmatrix}-1\\1\end{bmatrix}$

C $\quad [\overrightarrow{x}]_B=\begin{bmatrix}1\\-2\end{bmatrix}$

D $\quad [\overrightarrow{x}]_B=\begin{bmatrix}1\\-1\end{bmatrix}$

Solution: D

The vector $\vec{x}=(3,-5)$ is given in terms of the standard basis, and we need to transform it into an alternate basis that's defined by $\vec{v}=(1,-5)$ and $\vec{w}=(-2,0)$.

So let's plug the values we've been given into the matrix equation.

$$A[\vec{x}]_B=\vec{x}$$

$$\begin{bmatrix}1 & -2\\ -5 & 0\end{bmatrix}[\vec{x}]_B=\begin{bmatrix}3\\ -5\end{bmatrix}$$

Find A^{-1} from A.

$$[A\ |\ I]=\begin{bmatrix}1 & -2 & | & 1 & 0\\ -5 & 0 & | & 0 & 1\end{bmatrix}$$

$$[A\ |\ I]=\begin{bmatrix}1 & -2 & | & 1 & 0\\ 0 & -10 & | & 5 & 1\end{bmatrix}$$

$$[A\ |\ I]=\begin{bmatrix}1 & -2 & | & 1 & 0\\ 0 & 1 & | & -\frac{1}{2} & -\frac{1}{10}\end{bmatrix}$$

$$[A\ |\ I]=\begin{bmatrix}1 & 0 & | & 0 & -\frac{1}{5}\\ 0 & 1 & | & -\frac{1}{2} & -\frac{1}{10}\end{bmatrix}$$

So the inverse matrix is

$$A^{-1} = \begin{bmatrix} 0 & -\frac{1}{5} \\ -\frac{1}{2} & -\frac{1}{10} \end{bmatrix}$$

Now to find the representation of $\vec{x} = (3, -5)$ in the alternate basis, we simply multiply the inverse matrix by the vector.

$$[\vec{x}]_B = A^{-1}\vec{x}$$

$$[\vec{x}]_B = \begin{bmatrix} 0 & -\frac{1}{5} \\ -\frac{1}{2} & -\frac{1}{10} \end{bmatrix} \begin{bmatrix} 3 \\ -5 \end{bmatrix}$$

$$[\vec{x}]_B = \begin{bmatrix} 0(3) - \frac{1}{5}(-5) \\ -\frac{1}{2}(3) - \frac{1}{10}(-5) \end{bmatrix}$$

$$[\vec{x}]_B = \begin{bmatrix} 0 + 1 \\ -\frac{3}{2} + \frac{1}{2} \end{bmatrix}$$

$$[\vec{x}]_B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$