



# Linear Algebra Workbook

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Transposes

## TRANSPPOSES AND THEIR DETERMINANTS

- 1. Find the transpose  $A^T$ .

$$A = [5 \ 6 \ 0 \ 7 \ 5 \ -7]$$

- 2. Find the transpose  $A^T$ .

$$A = \begin{bmatrix} 7 & 9 & -6 \\ 0 & -1 & 9 \end{bmatrix}$$

- 3. Find the transpose  $A^T$ .

$$A = \begin{bmatrix} -4 & -7 \\ 5 & 1 \\ 7 & -2 \\ 4 & -2 \end{bmatrix}$$

- 4. Find the determinant of the transpose of  $A$ .

$$A = \begin{bmatrix} 5 & 3 & 6 & -1 \\ 9 & 0 & 1 & -2 \\ 8 & -2 & -4 & 8 \\ 5 & 4 & 9 & 7 \end{bmatrix}$$



- 5. Find the determinant of the transpose of  $A$ .

$$A = \begin{bmatrix} -9 & -3 & -1 \\ -4 & 7 & 3 \\ -4 & 8 & 7 \end{bmatrix}$$

- 6. Find the determinant of the transpose of  $A$ .

$$A = \begin{bmatrix} -8 & 6 & 8 \\ 3 & -9 & -1 \\ 4 & -9 & 9 \end{bmatrix}$$



## TRANSPOSES OF PRODUCTS, SUMS, AND INVERSES

■ 1. Find  $(AB)^T$ .

$$A = \begin{bmatrix} -1 & 2 \\ 2 & 3 \end{bmatrix}$$

$$B = \begin{bmatrix} -3 & -2 \\ 1 & 2 \end{bmatrix}$$

■ 2. Find  $(AB)^T$ .

$$A = \begin{bmatrix} -1 & 2 & -2 \\ 2 & 3 & 1 \\ 3 & -3 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 2 & -4 & 1 \\ 0 & -3 & -2 \\ -1 & 1 & 2 \end{bmatrix}$$

■ 3. Find  $(X + Y)^T$ .

$$X = \begin{bmatrix} 4 & 1 \\ -2 & 0 \end{bmatrix}$$

$$Y = \begin{bmatrix} -3 & 2 \\ 0 & -1 \end{bmatrix}$$



■ 4. Find  $(X + Y)^T$ .

$$X = \begin{bmatrix} 2 & 0 & 3 \\ 4 & 1 & -1 \\ -2 & 0 & 3 \end{bmatrix}$$

$$Y = \begin{bmatrix} -1 & 2 & -3 \\ 0 & -1 & 2 \\ 4 & -1 & 0 \end{bmatrix}$$

■ 5. Find  $(X^T)^{-1}$ .

$$X = \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix}$$

■ 6. Find  $(A^T)^{-1}$ .

$$A = \begin{bmatrix} 4 & 1 & -3 \\ 1 & 2 & 1 \\ 0 & -1 & 4 \end{bmatrix}$$



## NULL AND COLUMN SPACES OF THE TRANSPOSE

- 1. Find the null and column spaces of the transpose  $M^T$ , identify their spaces  $\mathbb{R}^i$ , and name the dimension of the subspaces.

$$M = \begin{bmatrix} -1 & 0 \\ 2 & 4 \\ -2 & -2 \\ 0 & 4 \end{bmatrix}$$

- 2. Find the row space and left null space of  $A$ , and the dimensions of those spaces.

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \\ -4 & 0 \end{bmatrix}$$

- 3. Find the row space and left null space of  $B$ , and the dimensions of those spaces.

$$B = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 1 & -2 & -1 & 4 \\ 0 & 0 & 2 & -2 \end{bmatrix}$$

- 4. Find the row space and left null space of  $C$ , and the dimensions of those spaces.



$$C = \begin{bmatrix} -1 & 2 & 0 \\ 1 & 4 & 3 \\ 0 & 0 & 3 \end{bmatrix}$$

- 5. Find the row space and left null space of  $A$ , and the dimensions of those spaces.

$$A = \begin{bmatrix} 1 & 3 \\ -3 & 1 \\ 0 & -2 \end{bmatrix}$$

- 6. Find the null and column subspaces of the transpose  $M^T$ , identify their spaces  $\mathbb{R}^i$ , and name the dimension of the subspaces of  $M^T$ .

$$M = \begin{bmatrix} 2 & 4 \\ 1 & 0 \\ -1 & -1 \\ 0 & 3 \end{bmatrix}$$



## THE PRODUCT OF A MATRIX AND ITS TRANSPOSE

■ 1. Is  $A^T A$  invertible?

$$A = \begin{bmatrix} 1 & -2 \\ 0 & 2 \\ 3 & 0 \end{bmatrix}$$

■ 2. Is  $A^T A$  invertible?

$$A = \begin{bmatrix} -12 & 6 \\ 8 & -4 \end{bmatrix}$$

■ 3. Is  $A^T A$  invertible?

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 3 & 2 \\ 1 & 0 & -2 \end{bmatrix}$$

■ 4. Is  $A^T A$  invertible?

$$A = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$$

■ 5. Is  $A^T A$  invertible?





$$A = \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix}$$

■ 6. Is  $A^T A$  invertible?

$$A = \begin{bmatrix} -1 & 0 & 2 \\ 0 & 3 & 3 \end{bmatrix}$$



