Topic: Adding and scaling linear transformations

Question: Find the product of a scalar $c = -3$ and the transformation $T(\vec{x})$.

$$T(\vec{x}) = \begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Answer choices:

A $$cT(\vec{x}) = \begin{bmatrix} 6 & -3 \\ 0 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

B $$cT(\vec{x}) = \begin{bmatrix} -6 & 3 \\ 0 & -15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

C $$cT(\vec{x}) = \begin{bmatrix} 6 & 3 \\ 0 & 15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

D $$cT(\vec{x}) = \begin{bmatrix} -6 & -3 \\ 0 & -15 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Solution: B

The transformation T is given as a matrix-vector product. If we call the matrix that's in the transformation T the matrix B, then multiplying the transformation by the scalar $c = -3$ gives

$$cT(\vec{x}) = -3\begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

First find cB.

$$cB = -3\begin{bmatrix} 2 & -1 \\ 0 & 5 \end{bmatrix}$$

$$cB = \begin{bmatrix} -3(2) & -3(-1) \\ -3(0) & -3(5) \end{bmatrix}$$

$$cB = \begin{bmatrix} -6 & 3 \\ 0 & -15 \end{bmatrix}$$

So the scaled transformation would be

$$cT(\vec{x}) = \begin{bmatrix} -6 & 3 \\ 0 & -15 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Topic: Adding and scaling linear transformations

Question: Find the sum of the transformations $S(\vec{x})$ and $T(\vec{x})$.

$$S(\vec{x}) = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$T(\vec{x}) = \begin{bmatrix} -1 & 4 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Answer choices:

A $$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} 2 & 10 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

B $$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} -2 & 10 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

C $$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} 2 & -10 \\ -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

D $$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} -2 & -10 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Solution: A

The sum of the transformations is the sum of the matrices A and B given in their matrix vector products,

$$S(\vec{x}) = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$T(\vec{x}) = \begin{bmatrix} -1 & 4 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

The sum of the matrices is

$$A + B = \begin{bmatrix} 3 & 6 \\ 0 & -1 \end{bmatrix} + \begin{bmatrix} -1 & 4 \\ 2 & 4 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3-1 & 6+4 \\ 0+2 & -1+4 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 2 & 10 \\ 2 & 3 \end{bmatrix}$$

So the sum of the transformations would be

$$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} 2 & 10 \\ 2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Topic: Adding and scaling linear transformations

Question: Find the sum of the transformations $S(\vec{x})$ and $T(\vec{x})$.

$$S(\vec{x}) = \begin{bmatrix} 4 & -1 & 1 \\ 0 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$T(\vec{x}) = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & -1 \\ 3 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Answer choices:

A $$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} 5 & 3 & 1 \\ 0 & 2 & 0 \\ 0 & 2 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

B $$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 2 \\ 6 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

C $$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} 5 & -3 & 1 \\ 0 & 2 & 0 \\ 0 & -2 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

D $$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & -2 \\ 6 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Solution: D

The sum of the transformations is the sum of the matrices A and B given in their matrix vector products,

$$S(\vec{x}) = \begin{bmatrix} 4 & -1 & 1 \\ 0 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$T(\vec{x}) = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & -1 \\ 3 & 3 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

The sum of the matrices is

$$A + B = \begin{bmatrix} 4 & -1 & 1 \\ 0 & 2 & -1 \\ 3 & 1 & 4 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 0 \\ 0 & 0 & -1 \\ 3 & 3 & -2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 4-1 & -1+2 & 1+0 \\ 0+0 & 2+0 & -1-1 \\ 3+3 & 1+3 & 4-2 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & -2 \\ 6 & 4 & 2 \end{bmatrix}$$

So the sum of the transformations would be

$$S(\vec{x}) + T(\vec{x}) = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & -2 \\ 6 & 4 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$