Topic: Transposes and their determinants

Question: Find the transpose A^T.

$$A = \begin{bmatrix} 4 & -6 \\ 1 & 1 \\ -2 & 9 \end{bmatrix}$$

Answer choices:

A $A^T = \begin{bmatrix} -6 & 4 \\ 1 & 1 \\ 9 & -2 \end{bmatrix}$

B $A^T = \begin{bmatrix} 9 & -2 \\ 1 & 1 \\ -6 & 4 \end{bmatrix}$

C $A^T = \begin{bmatrix} -6 & 1 & 9 \\ 4 & 1 & -2 \end{bmatrix}$

D $A^T = \begin{bmatrix} 4 & 1 & -2 \\ -6 & 1 & 9 \end{bmatrix}$

Solution: D

To take the transpose of a matrix, the first row of the original matrix becomes the first column of the transpose, the second row becomes the second column, the third row becomes the third column, etc.

So for the matrix A,

$$A = \begin{bmatrix} 4 & -6 \\ 1 & 1 \\ -2 & 9 \end{bmatrix}$$

the transpose will be

$$A^T = \begin{bmatrix} 4 & 1 & -2 \\ -6 & 1 & 9 \end{bmatrix}$$

Topic: Transposes and their determinants

Question: Find the transpose B^T.

$$B = \begin{bmatrix} -1 & -5 & 1 \\ -1 & -1 & 1 \\ 2 & 5 & -3 \end{bmatrix}$$

Answer choices:

A $$B^T = \begin{bmatrix} -1 & -1 & 2 \\ -5 & -1 & 5 \\ 1 & 1 & -3 \end{bmatrix}$$

B $$B^T = \begin{bmatrix} 2 & -1 & -1 \\ 5 & -1 & -5 \\ -3 & 1 & 1 \end{bmatrix}$$

C $$B^T = \begin{bmatrix} 2 & 5 & -3 \\ -1 & -1 & 1 \\ -1 & -5 & 1 \end{bmatrix}$$

D $$B^T = \begin{bmatrix} 1 & -5 & -1 \\ 1 & -1 & -1 \\ -3 & 5 & 2 \end{bmatrix}$$

Solution: A

To take the transpose of a matrix, the first row of the original matrix becomes the first column of the transpose, the second row becomes the second column, the third row becomes the third column, etc.

So for the matrix B,

$$B = \begin{bmatrix} -1 & -5 & 1 \\ -1 & -1 & 1 \\ 2 & 5 & -3 \end{bmatrix}$$

the transpose will be

$$B^T = \begin{bmatrix} -1 & -1 & 2 \\ -5 & -1 & 5 \\ 1 & 1 & -3 \end{bmatrix}$$

Topic: Transposes and their determinants

Question: Find the determinant of C^T.

$$C = \begin{bmatrix} 7 & 3 & 4 \\ 1 & 6 & 1 \\ 2 & 2 & 3 \end{bmatrix}$$

Answer choices:

A $|C| = -69$

B $|C| = 69$

C $|C| = -31$

D $|C| = 31$

Solution: B

The determinant of the transpose of a matrix is always equal to the determinant of the original matrix.

So to find the determinant of C^T, instead of finding the transpose and then taking its determinant, we'll just take the determinant of C, starting with breaking down the 3×3 determinant into 2×2 determinants.

$$|C| = \begin{vmatrix} 7 & 3 & 4 \\ 1 & 6 & 1 \\ 2 & 2 & 3 \end{vmatrix}$$

$$|C| = 7 \begin{vmatrix} 6 & 1 \\ 2 & 3 \end{vmatrix} - 3 \begin{vmatrix} 1 & 1 \\ 2 & 3 \end{vmatrix} + 4 \begin{vmatrix} 1 & 6 \\ 2 & 2 \end{vmatrix}$$

Evaluate the 2×2 determinants.

$$|C| = 7[(6)(3) - (1)(2)] - 3[(1)(3) - (1)(2)] + 4[(1)(2) - (6)(2)]$$

$$|C| = 7(18 - 2) - 3(3 - 2) + 4(2 - 12)$$

$$|C| = 7(16) - 3(1) + 4(-10)$$

$$|C| = 112 - 3 - 40$$

$$|C| = 69$$