Topic: Cauchy-Schwarz inequality

Question: Use the Cauchy-Schwarz inequality to say which vector set is linearly independent.

Answer choices:

A $\quad \overrightarrow{u} = (2,5)$ and $\overrightarrow{v} = (-6,-15)$

B $\quad \overrightarrow{u} = (-3,1)$ and $\overrightarrow{v} = (9,-3)$

C $\quad \overrightarrow{u} = (1,-4)$ and $\overrightarrow{v} = (-2,8)$

D $\quad \overrightarrow{u} = (6,2)$ and $\overrightarrow{v} = (-5,1)$

Solution: D

Let's plug each answer choice into the Cauchy-Schwarz inequality,

$$|\vec{u} \cdot \vec{v}| = ||\vec{u}|| \ ||\vec{v}||$$

If the two sides are equivalent, the Cauchy-Schwarz inequality tells us that the vectors are linearly dependent. If the two sides are unequal, then we know the vectors are linearly independent.

Answer choice A gives

$$|(2)(-6) + (5)(-15)| = \sqrt{2^2 + 5^2}\sqrt{(-6)^2 + (-15)^2}$$

$$|-12 - 75| = \sqrt{4 + 25}\sqrt{36 + 225}$$

$$|-87| = \sqrt{29}\sqrt{261}$$

$$87 = \sqrt{7{,}569}$$

$$87 = 87$$

Answer choice B gives

$$|(-3)(9) + (1)(-3)| = \sqrt{(-3)^2 + 1^2}\sqrt{9^2 + (-3)^2}$$

$$|-27 - 3| = \sqrt{9 + 1}\sqrt{81 + 9}$$

$$|-30| = \sqrt{10}\sqrt{90}$$

$$30 = \sqrt{900}$$

$$30 = 30$$

Answer choice C gives

$$|(1)(-2)+(-4)(8)|=\sqrt{1^2+(-4)^2}\sqrt{(-2)^2+8^2}$$

$$|-2-32|=\sqrt{1+16}\sqrt{4+64}$$

$$|-34|=\sqrt{17}\sqrt{68}$$

$$34=\sqrt{1{,}156}$$

$$34=34$$

Answer choice D gives

$$|(6)(-5)+(2)(1)|=\sqrt{6^2+2^2}\sqrt{(-5)^2+1^2}$$

$$|-30+2|=\sqrt{36+4}\sqrt{25+1}$$

$$|-28|=\sqrt{40}\sqrt{26}$$

$$28=\sqrt{1{,}040}$$

$$28\approx 32.25$$

Because answer choice D is the only vector set where the two sides of the Cauchy-Schwarz inequality are unequal, answer choice D is the only linearly independent vector set.

Topic: Cauchy-Schwarz inequality

Question: Use the Cauchy-Schwarz inequality to say which vector set is linearly independent.

Answer choices:

A $\quad \vec{u} = (8,6)$ and $\vec{v} = (-4,-3)$

B $\quad \vec{u} = (-6,5)$ and $\vec{v} = (12,-10)$

C $\quad \vec{u} = (7,9)$ and $\vec{v} = (14,18)$

D $\quad \vec{u} = (-5,-5)$ and $\vec{v} = (10,-5)$

Solution: D

Let's plug each answer choice into the Cauchy-Schwarz inequality,

$$|\vec{u} \cdot \vec{v}| = ||\vec{u}|| \, ||\vec{v}||$$

If the two sides are equivalent, the Cauchy-Schwarz inequality tells us that the vectors are linearly dependent. If the two sides are unequal, then we know the vectors are linearly independent.

Answer choice A gives

$$|(8)(-4) + (6)(-3)| = \sqrt{8^2 + 6^2}\sqrt{(-4)^2 + (-3)^2}$$

$$|-32 - 18| = \sqrt{64 + 36}\sqrt{16 + 9}$$

$$|-50| = \sqrt{100}\sqrt{25}$$

$$50 = \sqrt{2{,}500}$$

$$50 = 50$$

Answer choice B gives

$$|(-6)(12) + (5)(-10)| = \sqrt{(-6)^2 + 5^2}\sqrt{12^2 + (-10)^2}$$

$$|-72 - 50| = \sqrt{36 + 25}\sqrt{144 + 100}$$

$$|-122| = \sqrt{61}\sqrt{244}$$

$$122 = \sqrt{14{,}884}$$

$$122 = 122$$

Answer choice C gives

$$|(7)(14)+(9)(18)|=\sqrt{7^2+9^2}\sqrt{14^2+18^2}$$

$$|98+162|=\sqrt{49+81}\sqrt{196+324}$$

$$|260|=\sqrt{130}\sqrt{520}$$

$$260=\sqrt{67{,}600}$$

$$260=260$$

Answer choice D gives

$$|(-5)(10)+(-5)(-5)|=\sqrt{(-5)^2+(-5)^2}\sqrt{10^2+(5)^2}$$

$$|-50+25|=\sqrt{25+25}\sqrt{100+25}$$

$$|-25|=\sqrt{50}\sqrt{125}$$

$$25=\sqrt{6{,}250}$$

$$25\approx 79.06$$

Because answer choice D is the only vector set where the two sides of the Cauchy-Schwarz inequality are unequal, answer choice D is the only linearly independent vector set.

Topic: Cauchy-Schwarz inequality

Question: Use the Cauchy-Schwarz inequality to say which vector set is linearly independent.

Answer choices:

A $\vec{u}=(3,2)$ and $\vec{v}=(-12,-8)$

B $\vec{u}=(6,5)$ and $\vec{v}=(4,0)$

C $\vec{u}=(1,1)$ and $\vec{v}=(2,2)$

D $\vec{u}=(8,-7)$ and $\vec{v}=(-16,14)$

Solution: B

Let's plug each answer choice into the Cauchy-Schwarz inequality,

$$|\vec{u}\cdot\vec{v}| = ||\vec{u}||\ ||\vec{v}||$$

If the two sides are equivalent, the Cauchy-Schwarz inequality tells us that the vectors are linearly dependent. If the two sides are unequal, then we know the vectors are linearly independent.

Answer choice A gives

$$|(3)(-12)+(2)(-8)| = \sqrt{3^2+2^2}\sqrt{-12^2+(-8)^2}$$

$$|-36-16| = \sqrt{9+4}\sqrt{144+64}$$

$$|-52| = \sqrt{13}\sqrt{208}$$

$$52 = \sqrt{2{,}704}$$

$$52 = 52$$

Answer choice B gives

$$|(6)(4)+(5)(0)| = \sqrt{6^2+5^2}\sqrt{4^2+0^2}$$

$$|24+0| = \sqrt{36+25}\sqrt{16+0}$$

$$|24| = \sqrt{61}\sqrt{16}$$

$$24 = \sqrt{976}$$

$$24 \approx 31.24$$

Because answer choice B is a vector set for which the two sides of the Cauchy-Schwarz inequality are unequal, answer choice B is the only linearly independent vector set.