Linear Algebra Workbook Solutions

Determinants

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DETERMINANTS

■ 1. Use the determinant to say whether the matrix A is invertible.

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 3 \end{bmatrix}$$

Solution:

If the determinant of the matrix is nonzero, then the matrix is invertible and an inverse exists.

$$|A| = \begin{vmatrix} 5 & 2 \\ 3 & 3 \end{vmatrix}$$

$$|A| = 5(3) - 2(3)$$

$$|A| = 15 - 6$$

$$|A| = 9$$

Because the determinant is nonzero, the matrix is invertible and an inverse exists.

■ 2. Use the determinant to say whether the matrix A is invertible.

$$A = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}$$

Solution:

If the determinant of the matrix is nonzero, then the matrix is invertible and an inverse exists.

$$|A| = \begin{vmatrix} -1 & 2 \\ -1 & 2 \end{vmatrix}$$

$$|A| = -1(2) - 2(-1)$$

$$|A| = -2 + 2$$

$$|A| = 0$$

Because the determinant is 0, the matrix is not invertible and an inverse does not exists.

■ 3. Use the determinant to say whether the matrix A is invertible.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 1 \\ 4 & -2 & 0 \end{bmatrix}$$

Solution:

If the determinant of the matrix is nonzero, then the matrix is invertible and an inverse exists.

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ -3 & 0 & 1 \\ 4 & -2 & 0 \end{vmatrix}$$

Break the 3×3 determinant into 2×2 determinants.

$$|A| = 1 \begin{vmatrix} 0 & 1 \\ -2 & 0 \end{vmatrix} - 2 \begin{vmatrix} -3 & 1 \\ 4 & 0 \end{vmatrix} + 3 \begin{vmatrix} -3 & 0 \\ 4 & -2 \end{vmatrix}$$

Calculate the 2×2 determinants.

$$|A| = 1((0)(0) - (1)(-2)) - 2((-3)(0) - (1)(4)) + 3((-3)(-2) - (0)(4))$$

$$|A| = 1(0 + 2) - 2(0 - 4) + 3(6 - 0)$$

$$|A| = 1(2) - 2(-4) + 3(6)$$

$$|A| = 2 + 8 + 18$$

$$|A| = 28$$

Because the determinant is nonzero, the matrix is invertible and an inverse exists.

■ 4. Use the determinant to say whether matrix A is invertible.

$$A = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & 1 \\ 0 & -2 & 1 \end{bmatrix}$$

Solution:

If the determinant of the matrix is nonzero, then the matrix is invertible and an inverse exists.

$$|A| = \begin{vmatrix} 1 & -2 & 0 \\ -2 & 1 & 1 \\ 0 & -2 & 1 \end{vmatrix}$$

Break the 3×3 determinant into 2×2 determinants.

$$|A| = 1\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} - (-2)\begin{vmatrix} -2 & 1 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} -2 & 1 \\ 0 & -2 \end{vmatrix}$$

Calculate the 2×2 determinants.

$$|A| = 1((1)(1) - (1)(-2)) + 2((-2)(1) - (1)(0)) + 0((-2)(-2) - (1)(0))$$

$$|A| = 1(1 + 2) + 2(-2 - 0) + 0(4 - 0)$$

$$|A| = 1(3) + 2(-2) + 0(4)$$

$$|A| = 3 - 4 + 0$$

$$|A| = -1$$

Because the determinant is nonzero, the matrix is invertible and an inverse exists.

■ 5. Use the Rule of Sarrus to find the determinant.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 0 & -2 & 3 \end{bmatrix}$$

Solution:

We need to add all but the last column to the right side of the matrix.

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 & 1 \\ -1 & 0 & 2 & -1 & 0 \\ 0 & -2 & 3 & 0 & -2 \end{bmatrix}$$

By the Rule of Sarrus, we add the products of the diagonals from the upper left to the lower right.

$$(1)(0)(3) + (1)(2)(0) + (2)(-1)(-2)$$

Then we subtract the products of the diagonals from the upper right to the lower left.

$$-(2)(0)(0) - (1)(2)(-2) - (1)(-1)(3)$$

The determinant is the sum of these two strings of products.

$$|A| = (1)(0)(3) + (1)(2)(0) + (2)(-1)(-2) - (2)(0)(0) - (1)(2)(-2) - (1)(-1)(3)$$

$$|A| = 0 + 0 + 4 - 0 + 4 + 3$$

$$|A| = 4 + 4 + 3$$

$$|A| = 11$$

6. Use the Rule of Sarrus to find the determinant.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & -2 & -3 \\ 3 & 2 & 1 \end{bmatrix}$$

Solution:

We need to add all but the last column to the right side of the matrix.

$$A = \begin{bmatrix} 0 & 1 & 2 & 0 & 1 \\ -1 & -2 & -3 & -1 & -2 \\ 3 & 2 & 1 & 3 & 2 \end{bmatrix}$$

By the Rule of Sarrus, we add the products of the diagonals from the upper left to the lower right.

$$(0)(-2)(1) + (1)(-3)(3) + (2)(-1)(2)$$

Then we subtract the products of the diagonals from the upper right to the lower left.

$$-(2)(-2)(3) - (0)(-3)(2) - (1)(-1)(1)$$

The determinant is the sum of these two strings of products.

$$|A| = (0)(-2)(1) + (1)(-3)(3) + (2)(-1)(2) - (2)(-2)(3) - (0)(-3)(2) - (1)(-1)(1)$$

$$|A| = 0 - 9 - 4 + 12 + 0 + 1$$

$$|A| = -9 - 4 + 12 + 1$$

$$|A| = 0$$

CRAMER'S RULE FOR SOLVING SYSTEMS

1. Use Cramer's rule to find the expression that would give the value of x. You do not need to solve the system.

$$2x - y = 5$$

$$x + 3y = 15$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}$$

To find D_x, replace the first column of the coefficient matrix with the answer column.

$$D_x = \begin{vmatrix} 5 & -1 \\ 15 & 3 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_x/D.

$$\frac{\begin{vmatrix} 5 & -1 \\ 15 & 3 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix}}$$

■ 2. Use Cramer's rule to find the expression that would give the value of x. You do not need to solve the system.

$$ax + by = e$$

$$cx + dy = f$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$$

To find D_x, replace the first column of the coefficient matrix with the answer column.

$$D_x = \begin{vmatrix} e & b \\ f & d \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_x/D.

$$\frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}}$$

■ 3. Use Cramer's rule to find the expression that would give the value of y. You do not need to solve the system.

$$3x + 4y = 11$$

$$2x - 3y = -4$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 3 & 4 \\ 2 & -3 \end{vmatrix}$$

To find D_y, replace the second column of the coefficient matrix with the answer column.

$$D_y = \begin{vmatrix} 3 & 11 \\ 2 & -4 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_y/D.

$$\frac{\begin{vmatrix} 3 & 11 \\ 2 & -4 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 2 & -3 \end{vmatrix}}$$

■ 4. Use Cramer's rule to solve for x.

$$3x + 2y = 1$$

$$6x + 5y = 4$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}$$

To find D_x, replace the first column of the coefficient matrix with the answer column.

$$D_x = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_x/D.

$$x = \frac{D_x}{D} = \frac{\begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}}$$

Calculate the value of x.

$$x = \frac{1(5) - 2(4)}{3(5) - 2(6)}$$

$$x = \frac{5 - 8}{15 - 12}$$

$$x = \frac{-3}{3}$$

$$x = -1$$

5. Use Cramer's rule to solve for y.

$$3x + 2y = 1$$

$$6x + 5y = 4$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}$$

To find D_y, replace the second column of the coefficient matrix with the answer column.

$$D_y = \begin{vmatrix} 3 & 1 \\ 6 & 4 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_y/D.

$$y = \frac{D_y}{D} = \frac{\begin{vmatrix} 3 & 1 \\ 6 & 4 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 6 & 5 \end{vmatrix}}$$

Calculate the value of y.

$$y = \frac{3(4) - 1(6)}{3(5) - 2(6)}$$

$$y = \frac{12 - 6}{15 - 12}$$

$$y = \frac{6}{3}$$

$$y = 2$$

■ 6. Use Cramer's rule to solve for x.

$$3x + 5y = 6$$

$$9x + 10y = 14$$

Solution:

Find the expression for the determinant of the coefficient matrix D.

$$D = \begin{vmatrix} 3 & 5 \\ 9 & 10 \end{vmatrix}$$

To find D_x, replace the first column of the coefficient matrix with the answer column.

$$D_x = \begin{vmatrix} 6 & 5 \\ 14 & 10 \end{vmatrix}$$

Substitute the determinants into Cramer's rule D_x/D.

$$x=\frac{D_x}{D}=\frac{\begin{vmatrix}6&5\\14&10\end{vmatrix}}{\begin{vmatrix}3&5\\9&10\end{vmatrix}}$$

Calculate the value of x.

$$x=\frac{6(10)-5(14)}{3(10)-5(9)}$$

$$x=\frac{60-70}{30-45}$$

$$x=\frac{-10}{-15}$$

$$x=\frac{2}{3}$$

MODIFYING DETERMINANTS

1. Find the determinant of A if the first row of A gets multiplied by 3.

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

Solution:

The determinant of A is

$$|A| = \begin{vmatrix} 2 & 3 \\ 4 & 1 \end{vmatrix}$$

$$|A| = (2)(1) - (3)(4)$$

When one row of a square matrix is multiplied by a scalar k, the determinant of that matrix gets multiplied by that scalar too, regardless of which row was multiplied by k. So if a row of A was multiplied by $k = 3$, then the determinant will be

$$3|A| = 3((2)(1) - (3)(4))$$

$$3|A| = 3(2 - 12)$$

$$3|A| = 3(-10)$$

$$3|A| = -30$$

2. Find the determinant of A if both rows of A are multiplied by 2.

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

Solution:

The determinant of A is

$$|A| = \begin{vmatrix} 3 & 2 \\ 1 & 4 \end{vmatrix}$$

$$|A| = (3)(4) - (2)(1)$$

When you multiply all rows in a square matrix by a scalar k, the determinant of the resulting matrix will be $k^n|A|$, where n is the number of rows in the matrix. Because there are 2 rows in A, and because $k = 2$, the determinant will be multiplied by $k^n = 2^2$.

$$2^2|A| = 2^2((3)(4) - (2)(1))$$

$$4|A| = 4(12 - 2)$$

$$4|A| = 4(10)$$

$$4|A| = 40$$

3. Find the determinant of C, using only the determinants of A and B.

$$A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 4 \\ -1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 & 4 \\ 2 & 4 \end{bmatrix}$$

Solution:

The three matrices have identical first rows, and the second row of C is the sum of the second rows of A and B. When this occurs, the determinants have the relationship $|C| = |A| + |B|$. So find the determinants of A and B, and then add them together to find the determinant of C.

$$|A| = \begin{vmatrix} 5 & 4 \\ 3 & 2 \end{vmatrix} = (5)(2) - (4)(3) = 10 - 12 = -2$$

$$|B| = \begin{vmatrix} 5 & 4 \\ -1 & 2 \end{vmatrix} = (5)(2) - (4)(-1) = 10 + 4 = 14$$

Then the determinant of C is

$$|C| = -2 + 14$$

$$|C| = 12$$

■ 4. Find the determinant of the new matrix if the rows in matrix A are swapped.

$$A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$$

Solution:

First, create the new matrix by swapping the rows in A, and label it B.

$$B = \begin{bmatrix} 3 & 2 \\ 5 & 4 \end{bmatrix}$$

Based on the "swapped-row" rule, $|B| = -|A|$. So find $-|A|$, and this will be the determinant of the swapped-row matrix B.

$$-|A| = -\begin{vmatrix} 5 & 4 \\ 3 & 2 \end{vmatrix}$$

$$-|A| = -((5)(2) - (4)(3))$$

$$-|A| = -(10 - 12)$$

$$-|A| = -(-2)$$

$$-|A| = 2$$

■ 5. Find the determinant of the new matrix after the second and third rows of matrix A are swapped.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

Solution:

Swapping the second and third rows of A results in the exact same matrix. When any two rows are identical in an $n \times n$ matrix A, the determinant is 0, or $|A| = 0$. We can find the determinant to verify that it's 0.

$$|A| = \begin{vmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{vmatrix}$$

$$|A| = 2\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix} - 0\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 1 & 2 \\ 1 & 2 \end{vmatrix}$$

$$|A| = 2\begin{vmatrix} 2 & 1 \\ 2 & 1 \end{vmatrix}$$

$$|A| = 2((2)(1) - (1)(2))$$

$$|A| = 2(2 - 2)$$

$$|A| = 2(0)$$

$$|A| = 0$$

■ 6. Verify that the row operation $R_2 + 2R_1 \to R_2$ doesn't change the value of $|A|$.

$$A = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$$

Solution:

The determinant of A before the row operation is

$$|A| = \begin{vmatrix} 4 & 5 \\ 1 & 2 \end{vmatrix} = (4)(2) - (5)(1) = 8 - 5 = 3$$

Now apply the row operation $R_2 + 2R_1 \to R_2$,

$$A_R = \begin{bmatrix} 4 & 5 \\ 1 + 2(4) & 2 + 2(5) \end{bmatrix}$$

$$A_R = \begin{bmatrix} 4 & 5 \\ 1 + 8 & 2 + 10 \end{bmatrix}$$

$$A_R = \begin{bmatrix} 4 & 5 \\ 9 & 12 \end{bmatrix}$$

and then find the determinant of the resulting matrix.

$$|A_R| = \begin{vmatrix} 4 & 5 \\ 9 & 12 \end{vmatrix} = (4)(12) - (5)(9) = 48 - 45 = 3$$

Because we get the same determinant before and after the row operation, we can confirm that the row operation didn't affect the value of the determinant.

UPPER AND LOWER TRIANGULAR MATRICES

■ 1. Find the determinant of the upper-triangular matrix.

$$A = \begin{bmatrix} -4 & 1 \\ 0 & -3 \end{bmatrix}$$

Solution:

Because A is an upper-triangular matrix, the determinant can be found just by multiplying the values along the main diagonal. So the determinant is given by

$$|A| = (-4)(-3)$$

$$|A| = 12$$

■ 2. Find the determinant of the upper-triangular matrix.

$$A = \begin{bmatrix} -4 & 0 & 1 & 3 \\ 0 & -3 & -2 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Solution:

Because A is an upper-triangular matrix, the determinant can be found just by multiplying the values along the main diagonal. So the determinant is given by

$$|A| = (-4)(-3)(1)(2)$$

$$|A| = 24$$

■ 3. Find the determinant of the lower-triangular matrix.

$$A = \begin{bmatrix} 4 & 0 \\ 5 & 3 \end{bmatrix}$$

Solution:

Because A is a lower-triangular matrix, the determinant can be found just by multiplying the values along the main diagonal. So the determinant is given by

$$|A| = (4)(3)$$

$$|A| = 12$$

■ 4. Find the determinant of the lower-triangular matrix.

$$A = \begin{bmatrix} -4 & 0 & 0 \\ 5 & -3 & 0 \\ 3 & -1 & -1 \end{bmatrix}$$

Solution:

Because A is a lower-triangular matrix, the determinant can be found just by multiplying the values along the main diagonal. So the determinant is given by

$$|A| = (-4)(-3)(-1)$$

$$|A| = -12$$

5. Put A into upper or lower triangular form to find the determinant.

$$A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

Solution:

We can write A as an upper-triangular matrix by performing $R_1 + R_2 \to R_2$. After the row operation, the matrix is

$$A = \begin{bmatrix} -1 & 2 \\ 0 & 1 \end{bmatrix}$$

Then the determinant of this resulting upper-triangular matrix can be found by multiplying the values along the main diagonal.

$$|A| = (-1)(1)$$

$$|A| = -1$$

6. Put A into upper or lower triangular form to find the determinant.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 4 \\ 0 & 3 & -4 \end{bmatrix}$$

Solution:

In A, we have 0 entries in both the upper right and lower left corners, so we can work in either direction to create an upper- or lower-triangular matrix. Let's create an upper-triangular matrix using row operations.

To make $a_{(2,1)} = 0$, perform $R_1 + R_2 \to R_2$.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 3 & -4 \end{bmatrix}$$

To make $a_{(3,2)} = 0$, perform $-3R_2 + R_3 \to R_3$.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 0 & 1 & 4 \\ 0 & 0 & -16 \end{bmatrix}$$

The matrix is now in upper-triangular form, which means the determinant is given by the product of the values along the main diagonal.

$$|A| = (-1)(1)(-16)$$

$$|A| = 16$$

USING DETERMINANTS TO FIND AREA

■ 1. Find the area of the parallelogram formed by $\vec{v}_1=(1,4)$ and $\vec{v}_2=(-2,1)$, if the two vectors form adjacent edges of the parallelogram.

Solution:

When two vectors form adjacent edges of a parallelogram, we can find the area of the parallelogram by taking the determinant of the matrix of the vectors as column vectors.

In other words, we'll put $\vec{v}_1=(1,4)$ and $\vec{v}_2=(-2,1)$ as column vectors into a matrix

$$A=\begin{bmatrix}1 & -2\\ 4 & 1\end{bmatrix}$$

and then find the determinant of that matrix, which will be the area of the parallelogram.

$$|A|=\begin{vmatrix}1 & -2\\ 4 & 1\end{vmatrix}$$

$$|A|=(1)(1)-(-2)(4)$$

$$|A|=1+8$$

$$|A|=9$$

The area of the parallelogram is 9 square units.

2. Find the area of a parallelogram formed by $\vec{v}_1 = (-3, -3)$ and $\vec{v}_2 = (4, -2)$, if the two vectors form adjacent edges of the parallelogram.

Solution:

When two vectors form adjacent edges of a parallelogram, we can find the area of the parallelogram by taking the determinant of the matrix of the vectors as column vectors.

In other words, we'll put $\vec{v}_1 = (-3, -3)$ and $\vec{v}_2 = (4, -2)$ as column vectors into a matrix

$$A = \begin{bmatrix} -3 & 4 \\ -3 & -2 \end{bmatrix}$$

and then find the determinant of that matrix, which will be the area of the parallelogram.

$$|A| = \begin{vmatrix} -3 & 4 \\ -3 & -2 \end{vmatrix}$$

$$|A| = (-3)(-2) - (4)(-3)$$

$$|A| = 6 + 12$$

$$|A| = 18$$

The area of the parallelogram is 18 square units.

■ 3. Find the area of the parallelogram formed by $\vec{v}_1=(4,2)$ and $\vec{v}_2=(1,5)$, if the two vectors form adjacent edges of the parallelogram.

Solution:

When two vectors form adjacent edges of a parallelogram, we can find the area of the parallelogram by taking the determinant of the matrix of the vectors as column vectors.

In other words, we'll put $\vec{v}_1=(4,2)$ and $\vec{v}_2=(1,5)$ as column vectors into a matrix

$$A=\begin{bmatrix}4&1\\2&5\end{bmatrix}$$

and then find the determinant of that matrix, which will be the area of the parallelogram.

$$|A|=\begin{vmatrix}4&1\\2&5\end{vmatrix}$$

$$|A|=(4)(5)-(1)(2)$$

$$|A|=20-2$$

$$|A|=18$$

The area of the parallelogram is 18 square units.

4. The square S is defined by the vertices $(0,3)$, $(0,0)$, $(3,0)$, and $(3,3)$. If the transformation of S by T creates a transformed figure F, find the area of F.

$$T(\vec{x}) = \begin{bmatrix}2 & -3\\1 & 2\end{bmatrix}\vec{x}$$

Solution:

The area of the transformed figure F can be found using just the area of the square S, and the determinant of the transformation T.

$$\text{Area}_F = |\,\text{Area}_S(\text{Det}(T))\,|$$

The square S is defined between $x=0$ and $x=3$, so its width is 3, and it's defined between $y=0$ and $y=3$, so its height is 3. Therefore, the area of the square is $\text{Area}_S = 3 \cdot 3 = 9$.

The determinant of the transformation matrix is

$$|\,T\,| = \begin{vmatrix}2 & -3\\1 & 2\end{vmatrix}$$

$$|\,T\,| = (2)(2) - (-3)(1)$$

$$|\,T\,| = 4 + 3$$

$$|\,T\,| = 7$$

Then the area of the transformed figure F is

$$\text{Area}_F = |\text{Area}_S(\text{Det}(T))|$$

$$\text{Area}_F = |(9)(7)|$$

$$\text{Area}_F = |63|$$

$$\text{Area}_F = 63$$

5. A rectangle R is defined by the vertices $(-2,2)$, $(2,2)$, $(-2,-3)$, and $(2,-3)$. If the transformation of S by T creates a transformed figure F, find the area of F.

$$T(\vec{x}) = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} \vec{x}$$

Solution:

The area of the transformed figure F can be found using just the area of the rectangle R, and the determinant of the transformation T.

$$\text{Area}_F = |\text{Area}_R(\text{Det}(T))|$$

The rectangle R is defined between $x = -2$ and $x = 2$, so its width is 4, and it's defined between $y = -3$ and $y = 2$, so its height is 5. Therefore, the area of the rectangle is $\text{Area}_R = 4 \cdot 5 = 20$.

The determinant of the transformation matrix is

$$|T| = \begin{vmatrix} -3 & 1 \\ 2 & 0 \end{vmatrix}$$

$$|T| = (-3)(0) - (1)(2)$$

$$|T| = 0 - 2$$

$$|T| = -2$$

Then the area of the transformed figure F is

$$\text{Area}_F = |\text{Area}_R(\text{Det}(T))|$$

$$\text{Area}_F = |(20)(-2)|$$

$$\text{Area}_F = |-40|$$

$$\text{Area}_F = 40$$

6. The rectangle R is defined by the vertices $(2,-6)$, $(2,-1)$, $(8,-1)$, and $(8,-6)$. If the transformation of R by T creates a transformed figure L, find the area of L.

$$T(\vec{x}) = \begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix} \vec{x}$$

Solution:

The area of the transformed figure L can be found using just the area of the rectangle R, and the determinant of the transformation T.

$$\text{Area}_L = |\text{Area}_R(\text{Det}(T))|$$

The rectangle R is defined between $x = 2$ and $x = 8$, so its width is 6, and it's defined between $y = -6$ and $y = -1$, so its height is 5. Therefore, the area of the rectangle is $\text{Area}_R = 6 \cdot 5 = 30$.

The determinant of the transformation matrix is

$$|T| = \begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix}$$

$$|T| = (2)(3) - (-1)(0)$$

$$|T| = 6 - 0$$

$$|T| = 6$$

Then the area of the transformed figure L is

$$\text{Area}_L = |\text{Area}_R(\text{Det}(T))|$$

$$\text{Area}_L = |(30)(6)|$$

$$\text{Area}_L = |180|$$

$$\text{Area}_L = 180$$

FOR THE LOVE OF MATH

W W W . K R I S T A K I N G M A T H . C O M