Topic: Inverse transformations are linear

Question: Find B^{-1}.

$$B = \begin{bmatrix} -4 & 1 \\ 2 & -3 \end{bmatrix}$$

Answer choices:

A $\quad B^{-1} = \begin{bmatrix} -\frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

B $\quad B^{-1} = \begin{bmatrix} \frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{5} & \frac{2}{5} \end{bmatrix}$

C $\quad B^{-1} = \begin{bmatrix} -\frac{3}{10} & \frac{1}{10} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$

D $\quad B^{-1} = \begin{bmatrix} \frac{3}{10} & \frac{1}{10} \\ \frac{1}{5} & \frac{2}{5} \end{bmatrix}$

Solution: A

To find the inverse of the 2×2 matrix B, we'll augment it with I_2,

$$[B \mid I] = \begin{bmatrix} -4 & 1 & | & 1 & 0 \\ 2 & -3 & | & 0 & 1 \end{bmatrix}$$

and then work on putting the left side of the augmented matrix into reduced row-echelon form. We'll use Gauss-Jordan elimination, and start by finding the pivot in the first column, then zeroing out the rest of the column.

$$[B \mid I] = \begin{bmatrix} 1 & -\frac{1}{4} & | & -\frac{1}{4} & 0 \\ 2 & -3 & | & 0 & 1 \end{bmatrix}$$

$$[B \mid I] = \begin{bmatrix} 1 & -\frac{1}{4} & | & -\frac{1}{4} & 0 \\ 0 & -\frac{5}{2} & | & \frac{1}{2} & 1 \end{bmatrix}$$

Find the pivot entry in the second column, then finish zeroing out the rest of the second column.

$$[B \mid I] = \begin{bmatrix} 1 & -\frac{1}{4} & | & -\frac{1}{4} & 0 \\ 0 & 1 & | & -\frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

$$[B \mid I] = \begin{bmatrix} 1 & 0 & | & -\frac{3}{10} & -\frac{1}{10} \\ 0 & 1 & | & -\frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

Now that the left side of the augmented matrix has been changed into the identity matrix, the right side of the augmented matrix must be the inverse, B^{-1}.

$$B^{-1} = \begin{bmatrix} -\frac{3}{10} & -\frac{1}{10} \\ -\frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

Topic: Inverse transformations are linear

Question: Find M^{-1}.

$$M = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 1 \\ 4 & -2 & 0 \end{bmatrix}$$

Answer choices:

A $\quad M^{-1} = \begin{bmatrix} \frac{1}{14} & \frac{3}{14} & \frac{1}{14} \\ -\frac{1}{7} & -\frac{3}{7} & -\frac{5}{14} \\ \frac{3}{14} & \frac{5}{14} & -\frac{3}{14} \end{bmatrix}$

B $\quad M^{-1} = \begin{bmatrix} -\frac{1}{14} & -\frac{3}{14} & -\frac{1}{14} \\ \frac{1}{7} & \frac{3}{7} & \frac{5}{14} \\ -\frac{3}{14} & -\frac{5}{14} & \frac{3}{14} \end{bmatrix}$

C $\quad M^{-1} = \begin{bmatrix} \frac{1}{14} & -\frac{3}{14} & \frac{1}{14} \\ \frac{1}{7} & -\frac{3}{7} & -\frac{5}{14} \\ \frac{3}{14} & \frac{5}{14} & \frac{3}{14} \end{bmatrix}$

D $\quad M^{-1} = \begin{bmatrix} -\frac{1}{14} & \frac{3}{14} & -\frac{1}{14} \\ -\frac{1}{7} & \frac{3}{7} & \frac{5}{14} \\ -\frac{3}{14} & -\frac{5}{14} & -\frac{3}{14} \end{bmatrix}$

Solution: C

To find the inverse of the 3×3 matrix M, we'll augment it with I_3,

$$[M \mid I] = \begin{bmatrix} 1 & 2 & 3 & \mid & 1 & 0 & 0 \\ -3 & 0 & 1 & \mid & 0 & 1 & 0 \\ 4 & -2 & 0 & \mid & 0 & 0 & 1 \end{bmatrix}$$

and then work on putting the left side of the augmented matrix into reduced row-echelon form. We'll use Gauss-Jordan elimination, and start by zeroing out the first column.

$$[M \mid I] = \begin{bmatrix} 1 & 2 & 3 & \mid & 1 & 0 & 0 \\ 0 & 6 & 10 & \mid & 3 & 1 & 0 \\ 4 & -2 & 0 & \mid & 0 & 0 & 1 \end{bmatrix}$$

$$[M \mid I] = \begin{bmatrix} 1 & 2 & 3 & \mid & 1 & 0 & 0 \\ 0 & 6 & 10 & \mid & 3 & 1 & 0 \\ 0 & -10 & -12 & \mid & -4 & 0 & 1 \end{bmatrix}$$

Find the pivot entry in the second column, then finish zeroing out the rest of the second column.

$$[M \mid I] = \begin{bmatrix} 1 & 2 & 3 & \mid & 1 & 0 & 0 \\ 0 & 1 & \frac{5}{3} & \mid & \frac{1}{2} & \frac{1}{6} & 0 \\ 0 & -10 & -12 & \mid & -4 & 0 & 1 \end{bmatrix}$$

$$[M \mid I] = \begin{bmatrix} 1 & 0 & -\frac{1}{3} & | & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{5}{3} & | & \frac{1}{2} & \frac{1}{6} & 0 \\ 0 & -10 & -12 & | & -4 & 0 & 1 \end{bmatrix}$$

$$[M \mid I] = \begin{bmatrix} 1 & 0 & -\frac{1}{3} & | & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{5}{3} & | & \frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & \frac{14}{3} & | & 1 & \frac{5}{3} & 1 \end{bmatrix}$$

Find the pivot entry in the third column, then zero out the rest of the third column.

$$[M \mid I] = \begin{bmatrix} 1 & 0 & -\frac{1}{3} & | & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & \frac{5}{3} & | & \frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & | & \frac{3}{14} & \frac{5}{14} & \frac{3}{14} \end{bmatrix}$$

$$[M \mid I] = \begin{bmatrix} 1 & 0 & 0 & | & \frac{1}{14} & -\frac{3}{14} & \frac{1}{14} \\ 0 & 1 & \frac{5}{3} & | & \frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 1 & | & \frac{3}{14} & \frac{5}{14} & \frac{3}{14} \end{bmatrix}$$

$$[M \mid I] = \begin{bmatrix} 1 & 0 & 0 & | & \frac{1}{14} & -\frac{3}{14} & \frac{1}{14} \\ 0 & 1 & 0 & | & \frac{1}{7} & -\frac{3}{7} & -\frac{5}{14} \\ 0 & 0 & 1 & | & \frac{3}{14} & \frac{5}{14} & \frac{3}{14} \end{bmatrix}$$

Now that the left side of the augmented matrix has been changed into the identity matrix, the right side of the augmented matrix must be the inverse, M^{-1}.

$$M^{-1} = \begin{bmatrix} \frac{1}{14} & -\frac{3}{14} & \frac{1}{14} \\ \frac{1}{7} & -\frac{3}{7} & -\frac{5}{14} \\ \frac{3}{14} & \frac{5}{14} & \frac{3}{14} \end{bmatrix}$$

Topic: Inverse transformations are linear

Question: Find K^{-1}.

$$K = \begin{bmatrix} -2 & 1 & 0 & 4 \\ 1 & -3 & 0 & 1 \\ 0 & 4 & -1 & 2 \\ 2 & 0 & 1 & -3 \end{bmatrix}$$

Answer choices:

A $$K^{-1} = \begin{bmatrix} -\frac{1}{11} & -\frac{17}{33} & \frac{5}{33} & \frac{5}{33} \\ \frac{1}{11} & -\frac{2}{11} & -\frac{6}{33} & \frac{6}{33} \\ -\frac{8}{11} & -\frac{4}{33} & -\frac{7}{33} & \frac{26}{33} \\ \frac{2}{11} & \frac{10}{33} & \frac{1}{33} & -\frac{1}{33} \end{bmatrix}$$

B $$K^{-1} = \begin{bmatrix} \frac{1}{11} & \frac{17}{33} & -\frac{5}{33} & -\frac{5}{33} \\ -\frac{1}{11} & \frac{2}{11} & \frac{6}{33} & -\frac{6}{33} \\ \frac{8}{11} & \frac{4}{33} & \frac{7}{33} & -\frac{26}{33} \\ -\frac{2}{11} & -\frac{10}{33} & -\frac{1}{33} & \frac{1}{33} \end{bmatrix}$$

C $$K^{-1} = \begin{bmatrix} -\frac{1}{45} & \frac{17}{45} & \frac{13}{45} & \frac{13}{45} \\ \frac{1}{15} & -\frac{2}{15} & \frac{2}{15} & \frac{6}{45} \\ \frac{32}{45} & -\frac{4}{45} & -\frac{11}{45} & \frac{34}{45} \\ \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

D $$K^{-1} = \begin{bmatrix} \frac{1}{45} & -\frac{17}{45} & -\frac{13}{45} & -\frac{13}{45} \\ -\frac{1}{15} & \frac{2}{15} & -\frac{2}{15} & -\frac{6}{45} \\ -\frac{32}{45} & \frac{4}{45} & \frac{11}{45} & -\frac{34}{45} \\ -\frac{2}{9} & -\frac{2}{9} & -\frac{1}{9} & -\frac{1}{9} \end{bmatrix}$$

Solution: C

To find the inverse of the 4×4 matrix K, we'll augment it with I_4,

$$[K \mid I] = \begin{bmatrix} -2 & 1 & 0 & 4 & | & 1 & 0 & 0 & 0 \\ 1 & -3 & 0 & 1 & | & 0 & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & | & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & -3 & | & 0 & 0 & 0 & 1 \end{bmatrix}$$

and then work on putting the left side of the augmented matrix into reduced row-echelon form. We'll use Gauss-Jordan elimination, and start by finding the pivot in the first column, then zeroing out the rest of the column.

$$[K \mid I] = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -2 & | & -\frac{1}{2} & 0 & 0 & 0 \\ 1 & -3 & 0 & 1 & | & 0 & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & | & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & -3 & | & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[K \mid I] = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -2 & | & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{5}{2} & 0 & 3 & | & \frac{1}{2} & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & | & 0 & 0 & 1 & 0 \\ 2 & 0 & 1 & -3 & | & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[K \mid I] = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -2 & | & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & -\frac{5}{2} & 0 & 3 & | & \frac{1}{2} & 1 & 0 & 0 \\ 0 & 4 & -1 & 2 & | & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & | & 1 & 0 & 0 & 1 \end{bmatrix}$$

Find the pivot entry in the second column, then finish zeroing out the rest of the second column.

$$[K \mid I] = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & -2 & | & -\frac{1}{2} & 0 & 0 & 0 \\ 0 & 1 & 0 & -\frac{6}{5} & | & -\frac{1}{5} & -\frac{2}{5} & 0 & 0 \\ 0 & 4 & -1 & 2 & | & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & | & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$[K \mid I] = \begin{bmatrix} 1 & 0 & 0 & -\frac{13}{5} & | & -\frac{3}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & -\frac{6}{5} & | & -\frac{1}{5} & -\frac{2}{5} & 0 & 0 \\ 0 & 4 & -1 & 2 & | & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & | & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$[K \mid I] = \begin{bmatrix} 1 & 0 & 0 & -\frac{13}{5} & | & -\frac{3}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & -\frac{6}{5} & | & -\frac{1}{5} & -\frac{2}{5} & 0 & 0 \\ 0 & 0 & -1 & \frac{34}{5} & | & \frac{4}{5} & \frac{8}{5} & 1 & 0 \\ 0 & 1 & 1 & 1 & | & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$[K \mid I] = \begin{bmatrix} 1 & 0 & 0 & -\frac{13}{5} & | & -\frac{3}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & -\frac{6}{5} & | & -\frac{1}{5} & -\frac{2}{5} & 0 & 0 \\ 0 & 0 & -1 & \frac{34}{5} & | & \frac{4}{5} & \frac{8}{5} & 1 & 0 \\ 0 & 0 & 1 & \frac{11}{5} & | & \frac{6}{5} & \frac{2}{5} & 0 & 1 \end{bmatrix}$$

Find the pivot entry in the third column, then zero out the rest of the third column.

$$[K \mid I] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -\frac{13}{5} & -\frac{3}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & -\frac{6}{5} & -\frac{1}{5} & -\frac{2}{5} & 0 & 0 \\ 0 & 0 & 1 & -\frac{34}{5} & -\frac{4}{5} & -\frac{8}{5} & -1 & 0 \\ 0 & 0 & 1 & \frac{11}{5} & \frac{6}{5} & \frac{2}{5} & 0 & 1 \end{array}\right]$$

$$[K \mid I] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -\frac{13}{5} & -\frac{3}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & -\frac{6}{5} & -\frac{1}{5} & -\frac{2}{5} & 0 & 0 \\ 0 & 0 & 1 & -\frac{34}{5} & -\frac{4}{5} & -\frac{8}{5} & -1 & 0 \\ 0 & 0 & 0 & 9 & 2 & 2 & 1 & 1 \end{array}\right]$$

Find the pivot entry in the fourth column, then zero out the rest of the fourth column.

$$[K \mid I] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & -\frac{13}{5} & -\frac{3}{5} & -\frac{1}{5} & 0 & 0 \\ 0 & 1 & 0 & -\frac{6}{5} & -\frac{1}{5} & -\frac{2}{5} & 0 & 0 \\ 0 & 0 & 1 & -\frac{34}{5} & -\frac{4}{5} & -\frac{8}{5} & -1 & 0 \\ 0 & 0 & 0 & 1 & \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \end{array}\right]$$

$$[K \mid I] = \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{1}{45} & \frac{17}{45} & \frac{13}{45} & \frac{13}{45} \\ 0 & 1 & 0 & -\frac{6}{5} & -\frac{1}{5} & -\frac{2}{5} & 0 & 0 \\ 0 & 0 & 1 & -\frac{34}{5} & -\frac{4}{5} & -\frac{8}{5} & -1 & 0 \\ 0 & 0 & 0 & 1 & \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \end{array}\right]$$

$$[K \mid I] = \begin{bmatrix} 1 & 0 & 0 & 0 & | & -\frac{1}{45} & \frac{17}{45} & \frac{13}{45} & \frac{13}{45} \\ 0 & 1 & 0 & 0 & | & \frac{1}{15} & -\frac{2}{15} & \frac{2}{15} & \frac{6}{45} \\ 0 & 0 & 1 & -\frac{34}{5} & | & -\frac{4}{5} & -\frac{8}{5} & -1 & 0 \\ 0 & 0 & 0 & 1 & | & \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

$$[K \mid I] = \begin{bmatrix} 1 & 0 & 0 & 0 & | & -\frac{1}{45} & \frac{17}{45} & \frac{13}{45} & \frac{13}{45} \\ 0 & 1 & 0 & 0 & | & \frac{1}{15} & -\frac{2}{15} & \frac{2}{15} & \frac{6}{45} \\ 0 & 0 & 1 & 0 & | & \frac{32}{45} & -\frac{4}{45} & -\frac{11}{45} & \frac{34}{45} \\ 0 & 0 & 0 & 1 & | & \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$

Now that the left side of the augmented matrix has been changed into the identity matrix, the right side of the augmented matrix must be the inverse, K^{-1}.

$$K^{-1} = \begin{bmatrix} -\frac{1}{45} & \frac{17}{45} & \frac{13}{45} & \frac{13}{45} \\ \frac{1}{15} & -\frac{2}{15} & \frac{2}{15} & \frac{6}{45} \\ \frac{32}{45} & -\frac{4}{45} & -\frac{11}{45} & \frac{34}{45} \\ \frac{2}{9} & \frac{2}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix}$$