Topic: Equation of a plane, and normal vectors

Question: What is the normal vector to the plane?

$$3x + 5y + 9z = -26$$

Answer choices:

A $\vec{n} = (3,5,9)$

B $\vec{n} = (-3, -5, 9)$

C $\vec{n} = (3, -5, -9)$

D $\vec{n} = (-3, -5, -9)$

Solution: A

Given a plane $Ax + By + Cz = D$, the normal vector to that plane is

$$\vec{n} = (A, B, C)$$

So from the plane $3x + 5y + 9z = -26$, we can simply pull out the coefficients on x, y, and z to get the components of the normal vector.

$$\vec{n} = (3, 5, 9)$$

Topic: Equation of a plane, and normal vectors

Question: Find the equation of the plane, given a point in the plane and the normal vector to the plane.

$$(x, y, z) = (5, -8, -9)$$

$$\overrightarrow{n} = (8, 2, -1)$$

Answer choices:

A $8x - 2y - z = 17$

B $8x + 2y - z = 33$

C $8x + 2y + z = 33$

D $-8x - 2y - z = 17$

Solution: B

Plugging the normal vector and the point on the plane into the plane equation gives

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$8(x - 5) + 2(y - (-8)) - 1(z - (-9)) = 0$$

Now we'll simplify and get the equation of the plane into standard form.

$$8(x - 5) + 2(y + 8) - (z + 9) = 0$$

$$8x - 40 + 2y + 16 - z - 9 = 0$$

$$8x + 2y - z - 33 = 0$$

$$8x + 2y - z = 33$$

Topic: Equation of a plane, and normal vectors

Question: Find the equation of the plane, given a point in the plane and the normal vector to the plane.

$$(x, y, z) = (-5, 3, -3)$$

$$\vec{n} = (-4, -3, 9)$$

Answer choices:

A $\quad 4x + 3y + 9z = -16$

B $\quad -4x - 3y - 9z = -16$

C $\quad 4x + 3y - 9z = -16$

D $\quad -4x - 3y + 9z = -16$

Solution: D

Plugging the normal vector and the point on the plane into the plane equation gives

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$-4(x - (-5)) - 3(y - 3) + 9(z - (-3)) = 0$$

Now we'll simplify and get the equation of the plane into standard form.

$$-4(x + 5) - 3(y - 3) + 9(z + 3) = 0$$

$$-4x - 20 - 3y + 9 + 9z + 27 = 0$$

$$-4x - 3y + 9z + 16 = 0$$

$$-4x - 3y + 9z = -16$$