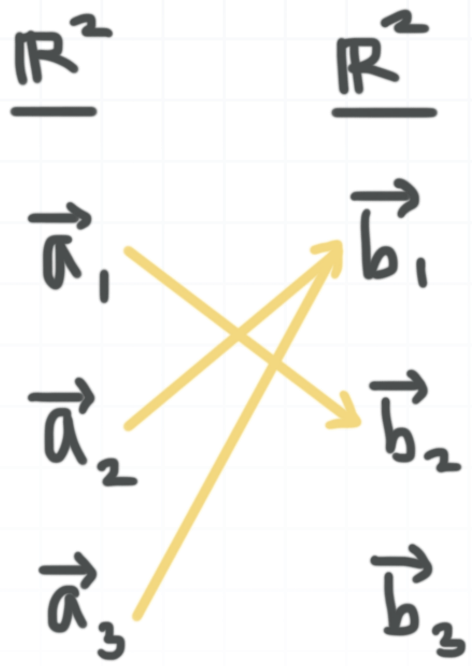
Topic: Inverse of a transformation

Question: If the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ is mapping vectors in A to vectors in B, then...



Answer choices:

A T is surjective

B T is injective

C T is both surjective and injective

D T is neither surjective nor injective

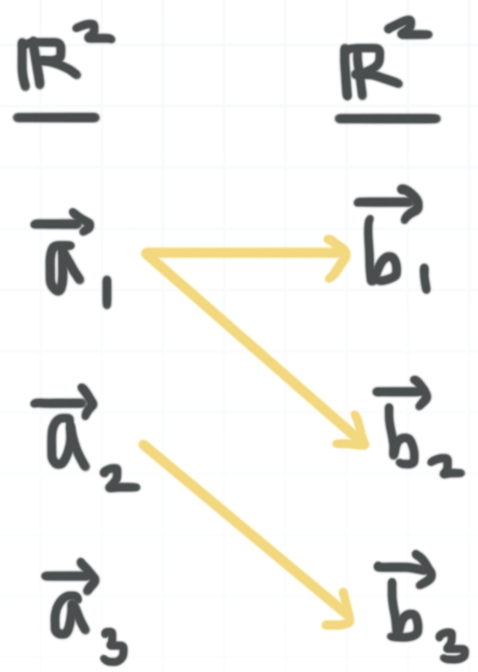
Solution: D

Not every vector $\vec{b}$ is being mapped to, since no $\vec{a}$ is mapping to $\vec{b}_3$, so T is not surjective. Every vector $\vec{a}$ is being mapped from, but not all the vectors $\vec{a}$ are mapping to a unique $\vec{b}$, since both $\vec{a}_2$ and $\vec{a}_3$ are mapping to $\vec{b}_1$, so T is not injective.

Because T has to be both surjective and injective in order to be invertible, and since T is neither surjective nor injective, the transformation is not invertible, and a unique T^{-1} is not defined.

Topic: Inverse of a transformation

Question: If the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ is mapping vectors in A to vectors in B, then...



Answer choices:

A $\quad T$ is surjective

B $\quad T$ is injective

C $\quad T$ is both surjective and injective

D $\quad T$ is neither surjective nor injective

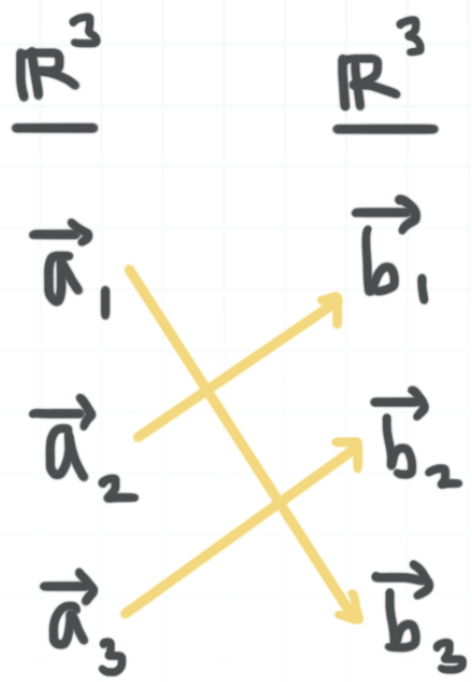
Solution: A

Every vector $\vec{b}$ is being mapped to, so T is surjective. But every vector $\vec{a}$ is not being mapped from, since $\vec{a}_3$ is not being mapped to any $\vec{b}$, so T is not injective.

Because T has to be both surjective and injective in order to be invertible, and since T is not injective, the transformation is not invertible, and a unique T^{-1} is not defined.

Topic: Inverse of a transformation

Question: If the transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ is mapping vectors in A to vectors in B, then...



Answer choices:

A T is onto

B T is one-to-one

C T is both onto and one-to-one

D T is neither onto nor one-to-one

Solution: C

Every vector $\vec{b}$ is being mapped to, so T is onto. And every vector $\vec{a}$ is being mapped from, to a unique $\vec{b}$, so T is one-to-one.

Because T is both onto and one-to-one, the transformation is invertible, and a unique T^{-1} is defined.