Topic: Solving systems with inverse matrices

Question: Use an inverse matrix to find the solution to the system.

$$3x + 12y = 51$$

$$-2x + 6y = -6$$

Answer choices:

A $x = -9$ and $y = -2$

B $x = -9$ and $y = 2$

C $x = 9$ and $y = -2$

D $x = 9$ and $y = 2$

Solution: D

Start by transferring the system into a matrix equation.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$$

$$\begin{bmatrix} 3 & 12 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 51 \\ -6 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M = \begin{bmatrix} 3 & 12 \\ -2 & 6 \end{bmatrix}$$

$$M^{-1} = \frac{1}{(3)(6) - (12)(-2)} \begin{bmatrix} 6 & -12 \\ 2 & 3 \end{bmatrix}$$

$$M^{-1} = \frac{1}{42} \begin{bmatrix} 6 & -12 \\ 2 & 3 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} \frac{1}{7} & -\frac{2}{7} \\ \frac{1}{21} & \frac{1}{14} \end{bmatrix}$$

Then we can say that the solution to the system is

$$\overrightarrow{a} = M^{-1}\overrightarrow{b}$$

$$\overrightarrow{a} = \begin{bmatrix} \frac{1}{7} & -\frac{2}{7} \\ \frac{1}{21} & \frac{1}{14} \end{bmatrix} \begin{bmatrix} 51 \\ -6 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{1}{7}(51) - \frac{2}{7}(-6) \\ \frac{1}{21}(51) + \frac{1}{14}(-6) \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{51}{7} + \frac{12}{7} \\ \frac{51}{21} - \frac{6}{14} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{63}{7} \\ \frac{17}{7} - \frac{3}{7} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{63}{7} \\ \frac{14}{7} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 9 \\ 2 \end{bmatrix}$$

Using this process with the inverse matrix, we conclude that $x = 9$ and $y = 2$.

Topic: Solving systems with inverse matrices

Question: Use an inverse matrix to find the solution to the system.

$$y - 5x = -15$$

$$3x + 8y = 95$$

Answer choices:

A $x = 5$ and $y = 10$

B $x = -5$ and $y = 10$

C $x = 5$ and $y = -10$

D $x = -5$ and $y = -10$

Solution: A

Start by transferring the system into a matrix equation.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} f \\ g \end{bmatrix}$$

$$\begin{bmatrix} -5 & 1 \\ 3 & 8 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -15 \\ 95 \end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M = \begin{bmatrix} -5 & 1 \\ 3 & 8 \end{bmatrix}$$

$$M^{-1} = \frac{1}{(-5)(8) - (1)(3)} \begin{bmatrix} 8 & -1 \\ -3 & -5 \end{bmatrix}$$

$$M^{-1} = -\frac{1}{43} \begin{bmatrix} 8 & -1 \\ -3 & -5 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} -\frac{8}{43} & \frac{1}{43} \\ \frac{3}{43} & \frac{5}{43} \end{bmatrix}$$

Then we can say that the solution to the system is

$$\overrightarrow{a} = M^{-1}\overrightarrow{b}$$

$$\overrightarrow{a} = \begin{bmatrix} -\frac{8}{43} & \frac{1}{43} \\ \frac{3}{43} & \frac{5}{43} \end{bmatrix} \begin{bmatrix} -15 \\ 95 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\frac{8}{43}(-15) + \frac{1}{43}(95) \\ \frac{3}{43}(-15) + \frac{5}{43}(95) \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{120}{43} + \frac{95}{43} \\ -\frac{45}{43} + \frac{475}{43} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{215}{43} \\ \frac{430}{43} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$$

Using this process with the inverse matrix, we conclude that $x = 5$ and $y = 10$.

Topic: Solving systems with inverse matrices

Question: Use an inverse matrix to find the solution to the system.

$$4x + 8y = -20$$

$$-12x - 3y = -66$$

Answer choices:

A $x = 7$ and $y = -6$

B $x = -7$ and $y = 6$

C $x = 7$ and $y = 6$

D $x = -7$ and $y = -6$

Solution: A

We could divide through both equations in the system to reduce them. The first equation $4x+8y=-20$ becomes

$$\frac{4}{4}x+\frac{8}{4}y=-\frac{20}{4}$$

$$x+2y=-5$$

And the equation $-12x-3y=-66$ becomes

$$\frac{-12}{-3}x+\frac{-3}{-3}y=\frac{-66}{-3}$$

$$4x+y=22$$

Then transfer the system into a matrix equation.

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}f\\ g\end{bmatrix}$$

$$\begin{bmatrix}1 & 2\\ 4 & 1\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}-5\\ 22\end{bmatrix}$$

Find the inverse of the coefficient matrix.

$$M=\begin{bmatrix}1 & 2\\ 4 & 1\end{bmatrix}$$

$$M^{-1}=\frac{1}{(1)(1)-(2)(4)}\begin{bmatrix}1 & -2\\ -4 & 1\end{bmatrix}$$

$$M^{-1} = -\frac{1}{7}\begin{bmatrix} 1 & -2 \\ -4 & 1 \end{bmatrix}$$

$$M^{-1} = \begin{bmatrix} -\frac{1}{7} & \frac{2}{7} \\ \frac{4}{7} & -\frac{1}{7} \end{bmatrix}$$

Then we can say that the solution to the system is

$$\vec{a} = M^{-1}\vec{b}$$

$$\vec{a} = \begin{bmatrix} -\frac{1}{7} & \frac{2}{7} \\ \frac{4}{7} & -\frac{1}{7} \end{bmatrix}\begin{bmatrix} -5 \\ 22 \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} -\frac{1}{7}(-5) + \frac{2}{7}(22) \\ \frac{4}{7}(-5) - \frac{1}{7}(22) \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{5}{7} + \frac{44}{7} \\ -\frac{20}{7} - \frac{22}{7} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} \frac{49}{7} \\ -\frac{42}{7} \end{bmatrix}$$

$$\vec{a} = \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ -6 \end{bmatrix}$$

Using this process with the inverse matrix, we conclude that $x = 7$ and $y = -6$.