Topic: Transformation matrices and the image of the subset

Question: If $\vec{a} = (-4,2)$ becomes $\vec{b}$ after undergoing a transformation by matrix Q, find $\vec{b}$.

$$Q = \begin{bmatrix} 11 & 1 \\ 0 & -6 \end{bmatrix}$$

Answer choices:

A $\vec{b} = \begin{bmatrix} 42 \\ -12 \end{bmatrix}$

B $\vec{b} = \begin{bmatrix} -42 \\ 12 \end{bmatrix}$

C $\vec{b} = \begin{bmatrix} -42 \\ -12 \end{bmatrix}$

D $\vec{b} = \begin{bmatrix} 42 \\ 12 \end{bmatrix}$

Solution: C

To apply a transformation matrix to $\vec{a}$, we'll multiply the matrix by the vector.

$$\vec{b} = M\vec{a} = \begin{bmatrix} 11 & 1 \\ 0 & -6 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

$$\vec{b} = M\vec{a} = \begin{bmatrix} 11(-4) + 1(2) \\ 0(-4) - 6(2) \end{bmatrix}$$

$$\vec{b} = M\vec{a} = \begin{bmatrix} -44 + 2 \\ 0 - 12 \end{bmatrix}$$

$$\vec{b} = M\vec{a} = \begin{bmatrix} -42 \\ -12 \end{bmatrix}$$

Topic: Transformation matrices and the image of the subset

Question: What are the vertices of the transformation of the polygon given by $(-2,1)$, $(1,3)$, $(2,-2)$, and $(-3,-1)$ after it's transformed by matrix P.

$$P = \begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$$

Answer choices:

A $(2,2)$, $(-3,7)$, $(-3,2)$, and $(4,-3)$

B $(2,2)$, $(-3,7)$, $(-4,-4)$, and $(6,-6)$

C $(4,1)$, $(-2,10)$, $(-3,2)$, and $(4,-3)$

D $(4,1)$, $(-2,10)$, $(-4,-4)$, and $(6,-6)$

Solution: D

Put the vertices of the polygon into a matrix.

$$\begin{bmatrix} -2 & 1 & 2 & -3 \\ 1 & 3 & -2 & -1 \end{bmatrix}$$

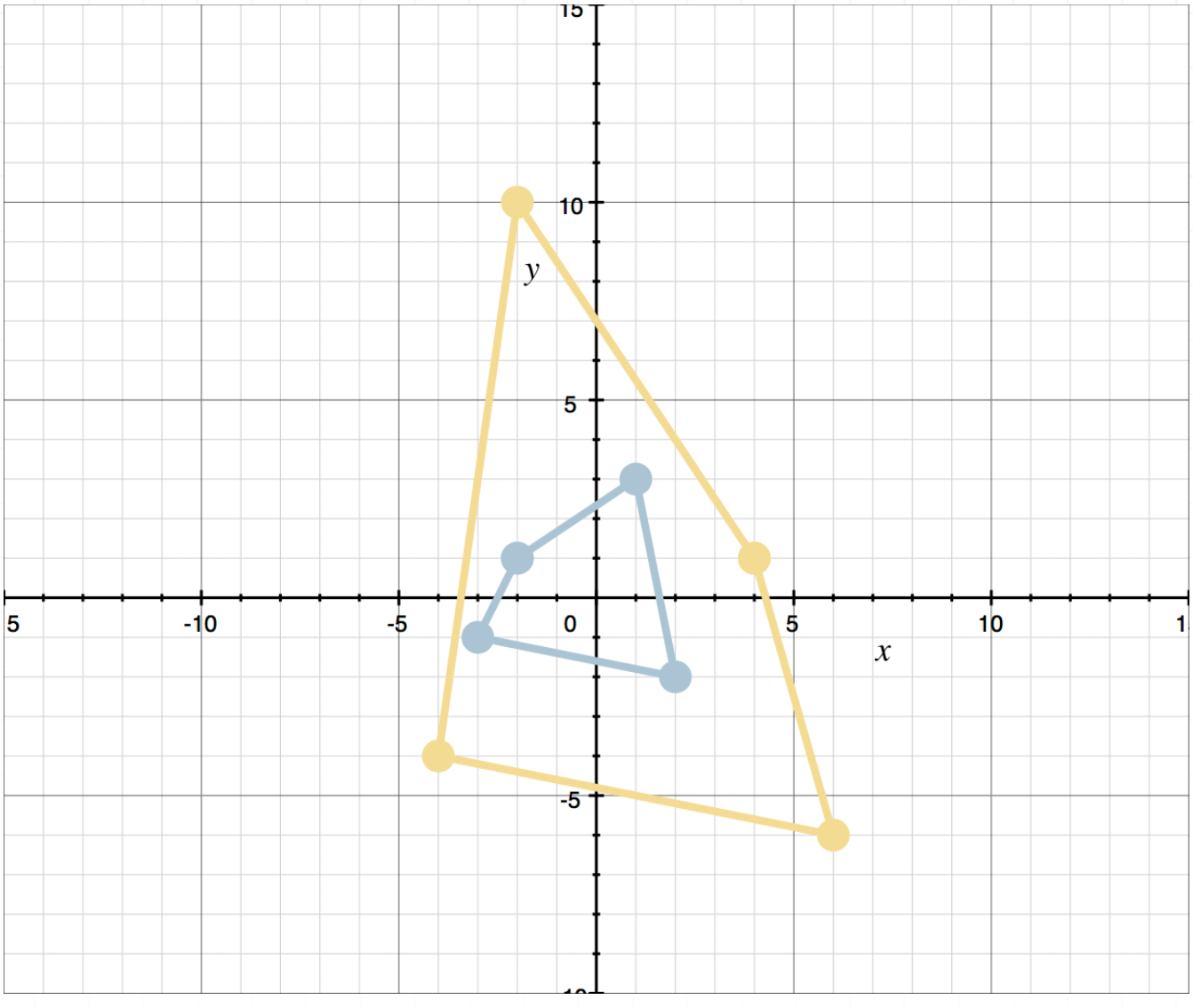
Apply the transformation of P to the vertex matrix.

$$\begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} -2 & 1 & 2 & -3 \\ 1 & 3 & -2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} -2(-2)+0(1) & -2(1)+0(3) & -2(2)+0(-2) & -2(-3)+0(-1) \\ 1(-2)+3(1) & 1(1)+3(3) & 1(2)+3(-2) & 1(-3)+3(-1) \end{bmatrix}$$

$$\begin{bmatrix} 4 & -2 & -4 & 6 \\ 1 & 10 & -4 & -6 \end{bmatrix}$$

The original polygon is sketched in light blue, and its transformation after P is in yellow.



Topic: Transformation matrices and the image of the subset

Question: What are the vertices of the transformation of the triangle with vertices $(-3,0)$, $(1,2)$, and $(1,-2)$ after it's transformed by matrix S.

$$S = \begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix}$$

Answer choices:

A $(0,-6)$, $(-2,4)$, and $(2,0)$

B $(0,-4)$, $(-1,3)$, and $(2,2)$

C $(1,-3)$, $(-1,6)$, and $(3,1)$

D $(2,-1)$, $(0,2)$, and $(1,4)$

Solution: A

Put the vertices of the triangle into a matrix.

$$\begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -2 \end{bmatrix}$$

Apply the transformation of S to the vertex matrix.

$$\begin{bmatrix} 0 & -1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} -3 & 1 & 1 \\ 0 & 2 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 0(-3)-1(0) & 0(1)-1(2) & 0(1)-1(-2) \\ 2(-3)+1(0) & 2(1)+1(2) & 2(1)+1(-2) \end{bmatrix}$$

$$\begin{bmatrix} 0 & -2 & 2 \\ -6 & 4 & 0 \end{bmatrix}$$

The original triangle is sketched in light blue, and its transformation after S is in yellow.