Linear Algebra Workbook

Determinants

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MATH

DETERMINANTS

1. Use the determinant to say whether the matrix A is invertible.

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 3 \end{bmatrix}$$

2. Use the determinant to say whether the matrix A is invertible.

$$A = \begin{bmatrix} -1 & 2 \\ -1 & 2 \end{bmatrix}$$

3. Use the determinant to say whether the matrix A is invertible.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -3 & 0 & 1 \\ 4 & -2 & 0 \end{bmatrix}$$

4. Use the determinant to say whether matrix A is invertible.

$$A = \begin{bmatrix} 1 & -2 & 0 \\ -2 & 1 & 1 \\ 0 & -2 & 1 \end{bmatrix}$$

5. Use the Rule of Sarrus to find the determinant.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ -1 & 0 & 2 \\ 0 & -2 & 3 \end{bmatrix}$$

■ 6. Use the Rule of Sarrus to find the determinant.

$$A = \begin{bmatrix} 0 & 1 & 2 \\ -1 & -2 & -3 \\ 3 & 2 & 1 \end{bmatrix}$$

CRAMER'S RULE FOR SOLVING SYSTEMS

■ 1. Use Cramer's rule to find the expression that would give the value of x. You do not need to solve the system.

$$2x - y = 5$$

$$x + 3y = 15$$

■ 2. Use Cramer's rule to find the expression that would give the value of x. You do not need to solve the system.

$$ax + by = e$$

$$cx + dy = f$$

■ 3. Use Cramer's rule to find the expression that would give the value of y. You do not need to solve the system.

$$3x + 4y = 11$$

$$2x - 3y = -4$$

■ 4. Use Cramer's rule to solve for x.

$$3x + 2y = 1$$

$$6x + 5y = 4$$

5. Use Cramer's rule to solve for y.

$$3x + 2y = 1$$

$$6x + 5y = 4$$

6. Use Cramer's rule to solve for x.

$$3x + 5y = 6$$

$$9x + 10y = 14$$

MODIFYING DETERMINANTS

■ 1. Find the determinant of A if the first row of A gets multiplied by 3.

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$$

■ 2. Find the determinant of A if both rows of A are multiplied by 2.

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

■ 3. Find the determinant of C, using only the determinants of A and B.

$$A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & 4 \\ -1 & 2 \end{bmatrix}$$

$$C = \begin{bmatrix} 5 & 4 \\ 2 & 4 \end{bmatrix}$$

■ 4. Find the determinant of the new matrix if the rows in matrix A are swapped.

$$A = \begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$$

■ 5. Find the determinant of the new matrix after the second and third rows of matrix A are swapped.

$$A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

■ 6. Verify that the row operation $R_2 + 2R_1 \to R_2$ doesn't change the value of $|A|$.

$$A = \begin{bmatrix} 4 & 5 \\ 1 & 2 \end{bmatrix}$$

UPPER AND LOWER TRIANGULAR MATRICES

■ 1. Find the determinant of the upper-triangular matrix.

$$A = \begin{bmatrix} -4 & 1 \\ 0 & -3 \end{bmatrix}$$

■ 2. Find the determinant of the upper-triangular matrix.

$$A = \begin{bmatrix} -4 & 0 & 1 & 3 \\ 0 & -3 & -2 & 1 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

■ 3. Find the determinant of the lower-triangular matrix.

$$A = \begin{bmatrix} 4 & 0 \\ 5 & 3 \end{bmatrix}$$

■ 4. Find the determinant of the lower-triangular matrix.

$$A = \begin{bmatrix} -4 & 0 & 0 \\ 5 & -3 & 0 \\ 3 & -1 & -1 \end{bmatrix}$$

■ 5. Put A into upper or lower triangular form to find the determinant.

$$A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$$

■ 6. Put A into upper or lower triangular form to find the determinant.

$$A = \begin{bmatrix} -1 & 2 & 0 \\ 1 & -1 & 4 \\ 0 & 3 & -4 \end{bmatrix}$$

USING DETERMINANTS TO FIND AREA

■ 1. Find the area of the parallelogram formed by $\vec{v}_1 = (1,4)$ and $\vec{v}_2 = (-2,1)$, if the two vectors form adjacent edges of the parallelogram.

■ 2. Find the area of a parallelogram formed by $\vec{v}_1 = (-3,-3)$ and $\vec{v}_2 = (4,-2)$, if the two vectors form adjacent edges of the parallelogram.

■ 3. Find the area of the parallelogram formed by $\vec{v}_1 = (4,2)$ and $\vec{v}_2 = (1,5)$, if the two vectors form adjacent edges of the parallelogram.

■ 4. The square S is defined by the vertices $(0,3)$, $(0,0)$, $(3,0)$, and $(3,3)$. If the transformation of S by T creates a transformed figure F, find the area of F.

$$T(\vec{x}) = \begin{bmatrix} 2 & -3 \\ 1 & 2 \end{bmatrix} \vec{x}$$

■ 5. A rectangle R is defined by the vertices $(-2,2)$, $(2,2)$, $(-2,-3)$, and $(2,-3)$. If the transformation of S by T creates a transformed figure F, find the area of F.

$$T(\vec{x}) = \begin{bmatrix} -3 & 1 \\ 2 & 0 \end{bmatrix} \vec{x}$$

6. The rectangle R is defined by the vertices $(2,-6)$, $(2,-1)$, $(8,-1)$, and $(8,-6)$. If the transformation of R by T creates a transformed figure L, find the area of L.

$$T(\vec{x})=\begin{bmatrix}2 & -1\\0 & 3\end{bmatrix}\vec{x}$$

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