Topic: Projection onto the subspace

Question: If $\vec{x}$ is a vector in $\mathbb{R}^3$, find an expression for the projection of any $\vec{x}$ onto the subspace V.

$$V = \text{Span}\left(\begin{bmatrix}-2\\0\\-2\end{bmatrix}, \begin{bmatrix}0\\4\\2\end{bmatrix}\right)$$

Answer choices:

A $\quad \text{Proj}_V\vec{x} = \frac{1}{9}\begin{bmatrix}5 & -2 & 4\\-2 & 8 & 2\\4 & 2 & 5\end{bmatrix}\vec{x}$

B $\quad \text{Proj}_V\vec{x} = \begin{bmatrix}-4 & -2 & -5\\-2 & 8 & 2\\-5 & 2 & -4\end{bmatrix}\vec{x}$

C $\quad \text{Proj}_V\vec{x} = \begin{bmatrix}5 & -2 & 4\\-2 & 8 & 2\\4 & 2 & 5\end{bmatrix}\vec{x}$

D $\quad \text{Proj}_V\vec{x} = \frac{1}{9}\begin{bmatrix}-4 & -2 & -5\\-2 & 8 & 2\\-5 & 2 & -4\end{bmatrix}\vec{x}$

Solution: A

Because the vectors that span V are linearly independent, the matrix A of the basis vectors that define V is

$$A = \begin{bmatrix} -2 & 0 \\ 0 & 4 \\ -2 & 2 \end{bmatrix}$$

The transpose A^T is

$$A^T = \begin{bmatrix} -2 & 0 & -2 \\ 0 & 4 & 2 \end{bmatrix}$$

Find A^TA.

$$A^TA = \begin{bmatrix} -2 & 0 & -2 \\ 0 & 4 & 2 \end{bmatrix} \begin{bmatrix} -2 & 0 \\ 0 & 4 \\ -2 & 2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} -2(-2) + 0(0) - 2(-2) & -2(0) + 0(4) - 2(2) \\ 0(-2) + 4(0) + 2(-2) & 0(0) + 4(4) + 2(2) \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 4 + 0 + 4 & 0 + 0 - 4 \\ 0 + 0 - 4 & 0 + 16 + 4 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 8 & -4 \\ -4 & 20 \end{bmatrix}$$

Find the inverse of A^TA.

$$[A^TA \mid I_2] = \begin{bmatrix} 8 & -4 & | & 1 & 0 \\ -4 & 20 & | & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_2] = \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{1}{8} & 0 \\ -4 & 20 & | & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_2] = \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{1}{8} & 0 \\ 0 & 18 & | & \frac{1}{2} & 1 \end{bmatrix}$$

$$[A^TA \mid I_2] = \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{1}{8} & 0 \\ 0 & 1 & | & \frac{1}{36} & \frac{1}{18} \end{bmatrix}$$

$$[A^TA \mid I_2] = \begin{bmatrix} 1 & 0 & | & \frac{5}{36} & \frac{1}{36} \\ 0 & 1 & | & \frac{1}{36} & \frac{1}{18} \end{bmatrix}$$

So $(A^TA)^{-1}$ is

$$(A^TA)^{-1} = \begin{bmatrix} \frac{5}{36} & \frac{1}{36} \\ \frac{1}{36} & \frac{1}{18} \end{bmatrix}$$

$$(A^TA)^{-1} = \frac{1}{36}\begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix}$$

Now the projection of $\vec{x}$ onto the subspace V will be

$$\text{Proj}_V\vec{x} = A(A^TA)^{-1}A^T\vec{x}$$

$$\text{Proj}_V\vec{x} = \begin{bmatrix} -2 & 0 \\ 0 & 4 \\ -2 & 2 \end{bmatrix} \frac{1}{36}\begin{bmatrix} 5 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 0 & -2 \\ 0 & 4 & 2 \end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x}=\frac{1}{36}\begin{bmatrix}-2&0\\0&4\\-2&2\end{bmatrix}\begin{bmatrix}5&1\\1&2\end{bmatrix}\begin{bmatrix}-2&0&-2\\0&4&2\end{bmatrix}\vec{x}$$

First, simplify $(A^TA)^{-1}A^T$.

$$\text{Proj}_V\vec{x}=\frac{1}{36}\begin{bmatrix}-2&0\\0&4\\-2&2\end{bmatrix}\begin{bmatrix}5(-2)+1(0)&5(0)+1(4)&5(-2)+1(2)\\1(-2)+2(0)&1(0)+2(4)&1(-2)+2(2)\end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x}=\frac{1}{36}\begin{bmatrix}-2&0\\0&4\\-2&2\end{bmatrix}\begin{bmatrix}-10&4&-8\\-2&8&2\end{bmatrix}\vec{x}$$

Next, simplify $A(A^TA)^{-1}A^T$.

$$\text{Proj}_V\vec{x}=\frac{1}{36}\begin{bmatrix}-2(-10)+0(-2)&-2(4)+0(8)&-2(-8)+0(2)\\0(-10)+4(-2)&0(4)+4(8)&0(-8)+4(2)\\-2(-10)+2(-2)&-2(4)+2(8)&-2(-8)+2(2)\end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x}=\frac{1}{36}\begin{bmatrix}20&-8&16\\-8&32&8\\16&8&20\end{bmatrix}\vec{x}$$

To simplify the matrix, factor out a 4.

$$\text{Proj}_V\vec{x}=\frac{4}{36}\begin{bmatrix}5&-2&4\\-2&8&2\\4&2&5\end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x}=\frac{1}{9}\begin{bmatrix}5&-2&4\\-2&8&2\\4&2&5\end{bmatrix}\vec{x}$$

Topic: Projection onto the subspace

Question: If $\vec{x}$ is a vector in $\mathbb{R}^4$, find an expression for the projection of any $\vec{x}$ onto the subspace S, if S is spanned by $\vec{x}_1$ and $\vec{x}_2$.

$$\vec{x}_1 = \frac{1}{3}\begin{bmatrix} 1 \\ 0 \\ -1 \\ 2 \end{bmatrix} \text{ and } \vec{x}_2 = \frac{1}{3}\begin{bmatrix} 0 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

Answer choices:

A $\quad \text{Proj}_S\vec{x} = \frac{1}{27}\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & -1 & 2 \end{bmatrix}\vec{x}$

B $\quad \text{Proj}_S\vec{x} = \frac{1}{9}\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & -1 & 2 \end{bmatrix}\vec{x}$

C $\quad \text{Proj}_S\vec{x} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & -1 & 2 \end{bmatrix}\vec{x}$

D $\quad \text{Proj}_S\vec{x} = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & -1 & 2 \end{bmatrix}\vec{x}$

Solution: C

Because the vectors that span S are linearly independent, the matrix A of the basis vectors that define S is

$$A = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 2 & -1 \end{bmatrix}$$

The transpose A^T is

$$A^T = \frac{1}{3}\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}$$

Find A^TA.

$$A^TA = \frac{1}{3}\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 2 & -1 \end{bmatrix}$$

$$A^TA = \frac{1}{9}\begin{bmatrix} 1(1)+0(0)-1(-1)+2(2) & 1(0)+0(1)-1(1)+2(-1) \\ 0(1)+1(0)+1(-1)-1(2) & 0(0)+1(1)+1(1)-1(-1) \end{bmatrix}$$

$$A^TA = \frac{1}{9}\begin{bmatrix} 1+0+1+4 & 0+0-1-2 \\ 0+0-1-2 & 0+1+1+1 \end{bmatrix}$$

$$A^TA = \frac{1}{9}\begin{bmatrix} 6 & -3 \\ -3 & 3 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} \frac{6}{9} & -\frac{3}{9} \\ -\frac{3}{9} & \frac{3}{9} \end{bmatrix}$$

$$A^TA = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

Find the inverse of A^TA.

$$[A^TA \mid I_2] = \begin{bmatrix} \frac{2}{3} & -\frac{1}{3} & | & 1 & 0 \\ -\frac{1}{3} & \frac{1}{3} & | & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_2] = \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{3}{2} & 0 \\ -\frac{1}{3} & \frac{1}{3} & | & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_2] = \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{3}{2} & 0 \\ 0 & \frac{1}{6} & | & \frac{1}{2} & 1 \end{bmatrix}$$

$$[A^TA \mid I_2] = \begin{bmatrix} 1 & -\frac{1}{2} & | & \frac{3}{2} & 0 \\ 0 & 1 & | & 3 & 6 \end{bmatrix}$$

$$[A^TA \mid I_2] = \begin{bmatrix} 1 & 0 & | & 3 & 3 \\ 0 & 1 & | & 3 & 6 \end{bmatrix}$$

So $(A^TA)^{-1}$ is

$$(A^TA)^{-1} = \begin{bmatrix} 3 & 3 \\ 3 & 6 \end{bmatrix}$$

The projection of $\vec{x}$ onto the subspace S will be

$$\text{Proj}_S\vec{x} = A(A^TA)^{-1}A^T\vec{x}$$

$$\text{Proj}_S\vec{x} = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 3 & 6 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}\vec{x}$$

$$\text{Proj}_S\vec{x} = \frac{1}{9}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 3 \\ 3 & 6 \end{bmatrix}\begin{bmatrix} 1 & 0 & -1 & 2 \\ 0 & 1 & 1 & -1 \end{bmatrix}\vec{x}$$

First, simplify $(A^TA)^{-1}A^T$.

$$\text{Proj}_S\vec{x} = \frac{1}{9}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3(1)+3(0) & 3(0)+3(1) & 3(-1)+3(1) & 3(2)+3(-1) \\ 3(1)+6(0) & 3(0)+6(1) & 3(-1)+6(1) & 3(2)+6(-1) \end{bmatrix}\vec{x}$$

$$\text{Proj}_S\vec{x} = \frac{1}{9}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 3 & 3 & 0 & 3 \\ 3 & 6 & 3 & 0 \end{bmatrix}\vec{x}$$

Now, simplify $A(A^TA)^{-1}A^T$.

$$\text{Proj}_S\vec{x} = \frac{1}{9}\begin{bmatrix} 1(3)+0(3) & 1(3)+0(6) & 1(0)+0(3) & 1(3)+0(0) \\ 0(3)+1(3) & 0(3)+1(6) & 0(0)+1(3) & 0(3)+1(0) \\ -1(3)+1(3) & -1(3)+1(6) & -1(0)+1(3) & -1(3)+1(0) \\ 2(3)-1(3) & 2(3)-1(6) & 2(0)-1(3) & 2(3)-1(0) \end{bmatrix}\vec{x}$$

$$\text{Proj}_S\vec{x} = \frac{1}{9}\begin{bmatrix} 3 & 3 & 0 & 3 \\ 3 & 6 & 3 & 0 \\ 0 & 3 & 3 & -3 \\ 3 & 0 & -3 & 6 \end{bmatrix}\vec{x}$$

$$\text{Proj}_S\vec{x} = \frac{3}{9}\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & -1 & 2 \end{bmatrix}\vec{x}$$

$$\text{Proj}_S\vec{x} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & -1 \\ 1 & 0 & -1 & 2 \end{bmatrix}\vec{x}$$

Topic: Projection onto the subspace

Question: If $\vec{x}$ is a vector in $\mathbb{R}^3$, find an expression for the projection of any $\vec{x}$ onto the subspace V.

$$V = \text{Span}\left(\begin{bmatrix}1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\2\\0\end{bmatrix}, \begin{bmatrix}1\\0\\-2\end{bmatrix}\right)$$

Answer choices:

A $\quad \text{Proj}_V\vec{x} = \begin{bmatrix}9 & -7 & -12\\-7 & 26 & 2\\-9 & -4 & 18\end{bmatrix}\vec{x}$

B $\quad \text{Proj}_V\vec{x} = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\vec{x}$

C $\quad \text{Proj}_V\vec{x} = \begin{bmatrix}24 & 0 & -16\\-8 & 16 & 16\\0 & 0 & 16\end{bmatrix}\vec{x}$

D $\quad \text{Proj}_V\vec{x} = \frac{1}{8}\begin{bmatrix}3 & 0 & -2\\-1 & 2 & 2\\0 & 0 & 2\end{bmatrix}\vec{x}$

Solution: B

Because the vectors that span V are linearly independent, the matrix A of the basis vectors that define V is

$$A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

The transpose A^T is

$$A^T = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & -2 \end{bmatrix}$$

Find A^TA.

$$A^TA = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 2 & 0 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 0 \\ 0 & 0 & -2 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1(1) - 1(-1) + 0(0) & 1(0) - 1(2) + 0(0) & 1(1) - 1(0) + 0(-2) \\ 0(1) + 2(-1) + 0(0) & 0(0) + 2(2) + 0(0) & 0(1) + 2(0) + 0(-2) \\ 1(1) + 0(-1) - 2(0) & 1(0) + 0(2) - 2(0) & 1(1) + 0(0) - 2(-2) \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 1 + 1 + 0 & 0 - 2 + 0 & 1 + 0 + 0 \\ 0 - 2 + 0 & 0 + 4 + 0 & 0 + 0 + 0 \\ 1 + 0 + 0 & 0 + 0 + 0 & 1 + 0 + 4 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 2 & -2 & 1 \\ -2 & 4 & 0 \\ 1 & 0 & 5 \end{bmatrix}$$

Find the inverse of A^TA.

$$[A^TA \mid I_3] = \begin{bmatrix} 2 & -2 & 1 & | & 1 & 0 & 0 \\ -2 & 4 & 0 & | & 0 & 1 & 0 \\ 1 & 0 & 5 & | & 0 & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_3] = \begin{bmatrix} 1 & -1 & \frac{1}{2} & | & \frac{1}{2} & 0 & 0 \\ -2 & 4 & 0 & | & 0 & 1 & 0 \\ 1 & 0 & 5 & | & 0 & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_3] = \begin{bmatrix} 1 & -1 & \frac{1}{2} & | & \frac{1}{2} & 0 & 0 \\ 0 & 2 & 1 & | & 1 & 1 & 0 \\ 1 & 0 & 5 & | & 0 & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_3] = \begin{bmatrix} 1 & -1 & \frac{1}{2} & | & \frac{1}{2} & 0 & 0 \\ 0 & 2 & 1 & | & 1 & 1 & 0 \\ 0 & 1 & \frac{9}{2} & | & -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_3] = \begin{bmatrix} 1 & -1 & \frac{1}{2} & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{2} & | & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 1 & \frac{9}{2} & | & -\frac{1}{2} & 0 & 1 \end{bmatrix}$$

$$[A^TA \mid I_3] = \begin{bmatrix} 1 & -1 & \frac{1}{2} & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & \frac{1}{2} & | & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 4 & | & -1 & -\frac{1}{2} & 1 \end{bmatrix}$$

$$[A^TA\ |\ I_3] = \begin{bmatrix} 1 & 0 & 1 & | & 1 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & | & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 4 & | & -1 & -\frac{1}{2} & 1 \end{bmatrix}$$

$$[A^TA\ |\ I_3] = \begin{bmatrix} 1 & 0 & 1 & | & 1 & \frac{1}{2} & 0 \\ 0 & 1 & \frac{1}{2} & | & \frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & | & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{4} \end{bmatrix}$$

$$[A^TA\ |\ I_3] = \begin{bmatrix} 1 & 0 & 1 & | & 1 & \frac{1}{2} & 0 \\ 0 & 1 & 0 & | & \frac{5}{8} & \frac{9}{16} & -\frac{1}{8} \\ 0 & 0 & 1 & | & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{4} \end{bmatrix}$$

$$[A^TA\ |\ I_3] = \begin{bmatrix} 1 & 0 & 0 & | & \frac{5}{4} & \frac{5}{8} & -\frac{1}{4} \\ 0 & 1 & 0 & | & \frac{5}{8} & \frac{9}{16} & -\frac{1}{8} \\ 0 & 0 & 1 & | & -\frac{1}{4} & -\frac{1}{8} & \frac{1}{4} \end{bmatrix}$$

So $(A^TA)^{-1}$ is

$$(A^TA)^{-1} = \begin{bmatrix} \frac{5}{4} & \frac{5}{8} & -\frac{1}{4} \\ \frac{5}{8} & \frac{9}{16} & -\frac{1}{8} \\ -\frac{1}{4} & -\frac{1}{8} & \frac{1}{4} \end{bmatrix}$$

$$(A^TA)^{-1}=\frac{1}{16}\begin{bmatrix}20 & 10 & -4\\ 10 & 9 & -2\\ -4 & -2 & 4\end{bmatrix}$$

Then the projection of $\vec{x}$ onto the subspace V will be

$$\text{Proj}_V\vec{x}=A(A^TA)^{-1}A^T\vec{x}$$

$$\text{Proj}_V\vec{x}=\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 0\\ 0 & 0 & -2\end{bmatrix}\frac{1}{16}\begin{bmatrix}20 & 10 & -4\\ 10 & 9 & -2\\ -4 & -2 & 4\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 0 & 2 & 0\\ 1 & 0 & -2\end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x}=\frac{1}{16}\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 0\\ 0 & 0 & -2\end{bmatrix}\begin{bmatrix}20 & 10 & -4\\ 10 & 9 & -2\\ -4 & -2 & 4\end{bmatrix}\begin{bmatrix}1 & -1 & 0\\ 0 & 2 & 0\\ 1 & 0 & -2\end{bmatrix}\vec{x}$$

First, simplify $(A^TA)^{-1}A^T$.

$$\text{Proj}_V\vec{x}=\frac{1}{16}\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 0\\ 0 & 0 & -2\end{bmatrix}\begin{bmatrix}20(1)+10(0)-4(1) & 20(-1)+10(2)-4(0) & 20(0)+10(0)-4(-2)\\ 10(1)+9(0)-2(1) & 10(-1)+9(2)-2(0) & 10(0)+9(0)-2(-2)\\ -4(1)-2(0)+4(1) & -4(-1)-2(2)+4(0) & -4(0)-2(0)+4(-2)\end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x}=\frac{1}{16}\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 0\\ 0 & 0 & -2\end{bmatrix}\begin{bmatrix}20+0-4 & -20+20-0 & 0+0+8\\ 10+0-2 & -10+18-0 & 0+0+4\\ -4-0+4 & 4-4+0 & 0-0-8\end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x}=\frac{1}{16}\begin{bmatrix}1 & 0 & 1\\ -1 & 2 & 0\\ 0 & 0 & -2\end{bmatrix}\begin{bmatrix}16 & 0 & 8\\ 8 & 8 & 4\\ 0 & 0 & -8\end{bmatrix}\vec{x}$$

Next, simplify $A(A^TA)^{-1}A^T$.

$$\text{Proj}_V\vec{x}=\frac{1}{16}\begin{bmatrix}1(16)+0(8)+1(0) & 1(0)+0(8)+1(0) & 1(8)+0(4)+1(-8)\\ -1(16)+2(8)+0(0) & -1(0)+2(8)+0(0) & -1(8)+2(4)+0(-8)\\ 0(16)+0(8)-2(0) & 0(0)+0(8)-2(0) & 0(8)+0(4)-2(-8)\end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x} = \frac{1}{16}\begin{bmatrix} 16+0+0 & 0+0+0 & 8+0-8 \\ -16+16+0 & 0+16+0 & -8+8+0 \\ 0+0-0 & 0+0-0 & 0+0+16 \end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x} = \frac{1}{16}\begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x} = \frac{16}{16}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x} = 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\vec{x}$$

$$\text{Proj}_V\vec{x} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\vec{x}$$