Linear Algebra Workbook

Inverses

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INVERSE OF A TRANSFORMATION

■ 1. Given a vector $\vec{v}$ in $\mathbb{R}^3$, what would the identity transformation be?

■ 2. If a transformation T is invertible, what are the three conclusions that we can make about it?

■ 3. If you can prove that a transformation T is both injective and surjective, and if you know that its inverse is unique, then what can you say about the transformation?

■ 4. Is the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ invertible?

$$T(x) = x^2$$

■ 5. Prove that $(T^{-1})^{-1} = T$.

■ 6. Prove that the inverse of a transformation is unique.

INVERTIBILITY FROM THE MATRIX-VECTOR PRODUCT

■ 1. Is the matrix invertible?

$$\begin{bmatrix} 1 & 2 & 0 \\ -3 & 5 & -1 \end{bmatrix}$$

■ 2. Is the matrix invertible?

$$\begin{bmatrix} \pi & -\pi \\ -\pi & \pi \end{bmatrix}$$

■ 3. Is the matrix invertible?

$$\begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix}$$

■ 4. Find the dimensions of the transformation matrix for each transformation, if each transformation were written as a matrix-vector product, $T(\vec{x}) = M\vec{x}$.

$$T : \mathbb{R}^3 \to \mathbb{R}^6$$

$$T : \mathbb{R}^4 \to \mathbb{R}^2$$

$$T : \mathbb{R}^u \to \mathbb{R}^w$$

■ 5. Using the transformations from the previous question, state the dimensions of $\vec{x}$, and then state the dimensions of $T(\vec{x})$.

■ 6. What can we say about the invertibility of the transformation $T: \mathbb{R}^u \to \mathbb{R}^w$ from the last two questions?

INVERSE TRANSFORMATIONS ARE LINEAR

■ 1. Given two $n \times n$ matrices, A and B, if we know that $AB = I$ and $BA = I$, where I is the $n \times n$ identity matrix, then what else do we know about A and B?

■ 2. Find the inverse of the matrix.

$$\begin{bmatrix} \pi & -\pi \\ \pi & \pi \end{bmatrix}$$

■ 3. Prove that the matrix found in the previous question is actually the inverse of the original matrix.

■ 4. Find the inverse of the matrix.

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 5 \\ 5 & 6 & 0 \end{bmatrix}$$

■ 5. Prove that the matrix we found in the previous question is actually the inverse of the original matrix.

■ 6. Prove that the inverse of an invertible linear transformation T is also a linear transformation.

MATRIX INVERSES, AND INVERTIBLE AND SINGULAR MATRICES

1. Find the inverse of matrix G.

$$G = \begin{bmatrix} -3 & 8 \\ 0 & -2 \end{bmatrix}$$

2. Find the inverse of matrix N.

$$N = \begin{bmatrix} 11 & -4 \\ 5 & -3 \end{bmatrix}$$

3. What is the inverse of matrix K?

$$K = \begin{bmatrix} 3 & 3 \\ -6 & 0 \end{bmatrix}$$

4. Is the matrix invertible or singular?

$$Z = \begin{bmatrix} 4 & 2 \\ -2 & -1 \end{bmatrix}$$

5. Is the matrix invertible or singular?

$$Y = \begin{bmatrix} 0 & 6 \\ 2 & -1 \end{bmatrix}$$

6. Is B invertible?

$$B = \begin{bmatrix} -4 & 1 \\ -5 & 0 \end{bmatrix}$$

SOLVING SYSTEMS WITH INVERSE MATRICES

■ 1. Use an inverse matrix to solve the system.

$$-4x + 3y = -14$$

$$7x - 4y = 32$$

■ 2. Use an inverse matrix to solve the system.

$$6x - 11y = 2$$

$$-10x + 7y = -26$$

■ 3. Use an inverse matrix to solve the system.

$$13y - 6x = -81$$

$$7x + 17 = -22y$$

■ 4. Sketch a graph of vectors to visually find the solution to the system.

$$3x = 3$$

$$x - y = -2$$

■ 5. Sketch a graph of vectors to visually find the solution to the system.

$$-y = -4$$

$$2x - y = -2$$

■ 6. Sketch a graph of vectors to visually find the solution to the system.

$$x - y = 0$$

$$x + y = 2$$