

Linear Algebra and Geometry 1

Systems of equations, matrices, vectors, and geometry

Matrix—vector multiplication

Hania Uscka-Wehlou, Ph.D. (2009, Uppsala University: Mathematics)

University teacher in mathematics (Associate Professor / Senior Lecturer) at Mälardalen University, Sweden

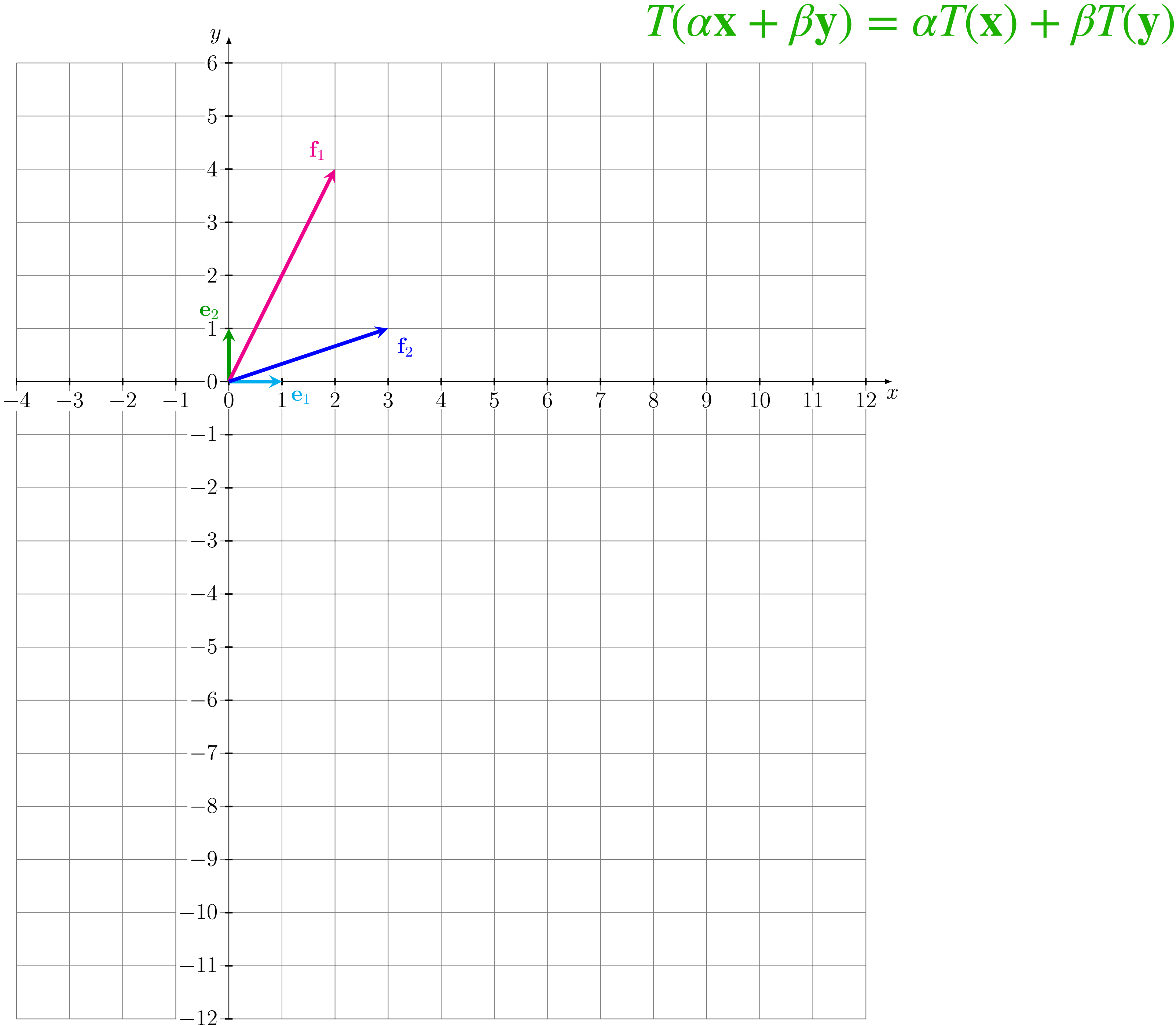


$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

$$T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \mathbf{f}_1, \quad T(\mathbf{e}_2) = \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \mathbf{f}_2$$

$$T \begin{bmatrix} x \\ y \end{bmatrix} = T(x\mathbf{e}_1 + y\mathbf{e}_2) =$$

$$= xT(\mathbf{e}_1) + yT(\mathbf{e}_2) = x \begin{bmatrix} 2 \\ 4 \end{bmatrix} + y \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$



$$T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

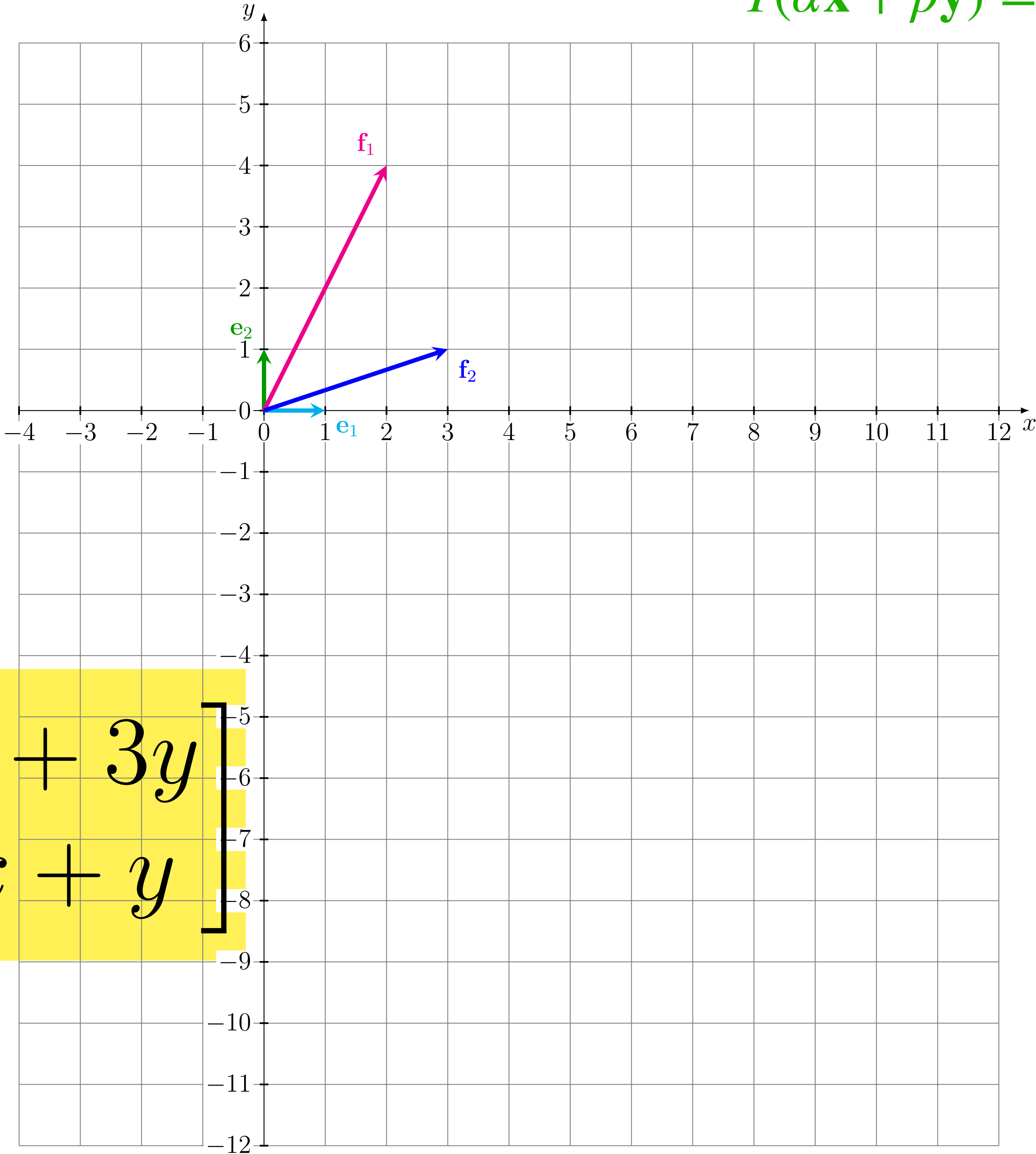
$$T(\mathbf{e}_1) = \begin{bmatrix} 2 \\ 4 \end{bmatrix} = \mathbf{f}_1, \quad T(\mathbf{e}_2) = \begin{bmatrix} 3 \\ 1 \end{bmatrix} = \mathbf{f}_2$$

$$T \begin{bmatrix} x \\ y \end{bmatrix} = T(x\mathbf{e}_1 + y\mathbf{e}_2) =$$

$$= xT(\mathbf{e}_1) + yT(\mathbf{e}_2) = x \begin{bmatrix} 2 \\ 4 \end{bmatrix} + y \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2x + 3y \\ 4x + y \end{bmatrix}$$

$$T(\alpha \mathbf{x} + \beta \mathbf{y}) = \alpha T(\mathbf{x}) + \beta T(\mathbf{y})$$



$$T : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$T(\alpha \mathbf{x} + \beta \mathbf{y}) = \alpha T(\mathbf{x}) + \beta T(\mathbf{y})$$

$$\mathbf{x} = (x_1, x_2, \dots, x_n) = x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \dots + x_n \mathbf{e}_n$$

$$T(\mathbf{e}_1) = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} = \mathbf{f}_1, \quad T(\mathbf{e}_2) = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} = \mathbf{f}_2, \quad \dots, \quad T(\mathbf{e}_n) = \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} = \mathbf{f}_n$$

$$T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = T(x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \dots + x_n \mathbf{e}_n) = x_1 T(\mathbf{e}_1) + x_2 T(\mathbf{e}_2) + \dots + x_n T(\mathbf{e}_n) =$$

$$= x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \dots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix}$$

$$T : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$T(\alpha \mathbf{x} + \beta \mathbf{y}) = \alpha T(\mathbf{x}) + \beta T(\mathbf{y})$$

$$\mathbf{x} = (x_1, x_2, \dots, x_n) = x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \dots + x_n \mathbf{e}_n$$

$$T(\mathbf{e}_1) = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} = \mathbf{f}_1, \quad T(\mathbf{e}_2) = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} = \mathbf{f}_2, \quad \dots, \quad T(\mathbf{e}_n) = \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} = \mathbf{f}_n$$

$$T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = T(x_1 \mathbf{e}_1 + x_2 \mathbf{e}_2 + \dots + x_n \mathbf{e}_n) = x_1 T(\mathbf{e}_1) + x_2 T(\mathbf{e}_2) + \dots + x_n T(\mathbf{e}_n) =$$

$$= x_1 \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{m1} \end{bmatrix} + x_2 \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{m2} \end{bmatrix} + \dots + x_n \begin{bmatrix} a_{1n} \\ a_{2n} \\ \vdots \\ a_{mn} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 a_{11} + x_2 a_{12} + \dots + x_n a_{1n} \\ x_1 a_{21} + x_2 a_{22} + \dots + x_n a_{2n} \\ \vdots \\ x_1 a_{m1} + x_2 a_{m2} + \dots + x_n a_{mn} \end{bmatrix}$$

$$T : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 a_{11} + x_2 a_{12} + \dots + x_n a_{1n} \\ x_1 a_{21} + x_2 a_{22} + \dots + x_n a_{2n} \\ \vdots \\ x_1 a_{m1} + x_2 a_{m2} + \dots + x_n a_{mn} \end{bmatrix}$$

$$T : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$\begin{array}{c}
 T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 a_{11} + x_2 a_{12} + \dots + x_n a_{1n} \\ x_1 a_{21} + x_2 a_{22} + \dots + x_n a_{2n} \\ \vdots \\ x_1 a_{m1} + x_2 a_{m2} + \dots + x_n a_{mn} \end{bmatrix}
 \end{array}$$

$m \times n$

$n \times 1$

$m \times 1$

$$T : \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$\begin{array}{c}
 T \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_1 a_{11} + x_2 a_{12} + \cdots + x_n a_{1n} \\ x_1 a_{21} + x_2 a_{22} + \cdots + x_n a_{2n} \\ \vdots \\ x_1 a_{m1} + x_2 a_{m2} + \cdots + x_n a_{mn} \end{bmatrix} \\
 \begin{array}{ccc} m \times n & n \times 1 & m \times 1 \end{array}
 \end{array}$$

Matrix—vector multiplication is the same as taking the image of the vector in the linear transformation defined by the matrix (the columns are the images of the standard unit vectors \mathbf{e}_i , $i = 1, 2, \dots, n$).