Linear Algebra Final Exam

krista king MATH

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This exam is comprehensive over the entire course and includes 12 questions. You have 60 minutes to complete the exam.

The exam is worth 100 points. The 8 multiple choice questions are worth 5 points each (40 points total) and the 4 free response questions are worth 15 points each (60 points total).

Mark your multiple choice answers on this cover page. For the free response questions, show your work and make sure to circle your final answer.

1. (5 pts)	A	B	C	D	E
2. (5 pts)	A	B	C	D	E
3. (5 pts)	A	B	C	D	E
4. (5 pts)	A	B	C	D	E
5. (5 pts)	A	B	C	D	E
6. (5 pts)	A	B	C	D	E
7. (5 pts)	A	B	C	D	E
8. (5 pts)	A	B	C	D	E

1. **(5 pts)** Determine whether the system has one solution, no solutions, or infinitely many solutions.

$$-x + y + 2z + 7w = 4$$

$$3x - y - 4z + 5w = 8$$

$$2x + 4y + 3z - w = 4$$

$$-y - z - 13w = 5$$

A No solutions

B $(x, y, z) = (-1,5, -3,0)$

C Infinitely many solutions

D $(x, y, z) = (-4,0,2,2)$

E $(x, y, z) = (0,4, -2,0)$

2. **(5 pts)** Find the product $B(AC)$.

$$A = \begin{bmatrix} 7 & 3 & -5 \\ -5 & -8 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 3 \\ 4 & -1 \end{bmatrix} \qquad C = \begin{bmatrix} 6 & 1 \\ -7 & -2 \\ 2 & 3 \end{bmatrix}$$

A $B(AC) = \begin{bmatrix} -78 & 47 \\ -40 & 55 \end{bmatrix}$

D $B(AC) = \begin{bmatrix} 78 & 47 \\ -40 & -55 \end{bmatrix}$

B $B(AC) = \begin{bmatrix} -32 & 25 \\ 26 & -55 \end{bmatrix}$

E $B(AC) = \begin{bmatrix} 32 & 25 \\ 26 & -55 \end{bmatrix}$

C The product isn't defined

3. **(5 pts)** Simplify $\vec{w} \cdot (-2\vec{u} + 4\vec{v})$.

$$\vec{u} = (-3,2,1,0)$$

$$\vec{v} = (1, -5, -4,1)$$

$$\vec{w} = (0, -1,1,2)$$

A 14

B $\sqrt{2}$

C $(0,24, -18,8)$

D 2

E $\sqrt{14}$

4. **(5 pts)** Find the equation of the plane passing through A and perpendicular to AB, given $A(2, -1,4)$ and $B(0, -3,2)$.

A $2x - y + 4z = 5$

B $x + y + z = 5$

C $2x - y + 4z = -5$

D $x + y + z = 7$

E $x + y + z = -5$

5. **(5 pts)** Find the general solution to $A\vec{x} = \vec{b}$.

$$A = \begin{bmatrix} 2 & -4 & 6 \\ 3 & -5 & -2 \\ -5 & 7 & 18 \end{bmatrix}$$

$$\vec{b} = (1,1,-1)$$

A $\quad \vec{x} = \begin{bmatrix} \frac{37}{2} \\ \frac{21}{2} \\ 1 \end{bmatrix}$

D $\quad \vec{x} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} 19 \\ 11 \\ 1 \end{bmatrix}$

B $\quad \vec{x} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix} + c_1 \begin{bmatrix} -19 \\ -11 \\ 0 \end{bmatrix}$

E $\quad \vec{x} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ 0 \end{bmatrix}$

C $\quad \vec{x} = c_1 \begin{bmatrix} 19 \\ 11 \\ 1 \end{bmatrix}$

6. **(5 pts)** Transform $\vec{x} = (-2,5)$ with $S \circ T$, if $S: \mathbb{R}^2 \to \mathbb{R}^2$ and $T: \mathbb{R}^2 \to \mathbb{R}^2$.

$$S(\vec{x}) = \begin{bmatrix} x_1 + x_2 \\ 2x_2 - x_1 \end{bmatrix}$$

$$T(\vec{x}) = \begin{bmatrix} x_1 + 3x_2 \\ 2x_2 \end{bmatrix}$$

A $(S \circ T)(\vec{x}) = (7,23)$

B $(S \circ T)(\vec{x}) = (39,24)$

C $(S \circ T)(\vec{x}) = (23,7)$

D $(S \circ T)(\vec{x}) = (24,39)$

E $(S \circ T)(\vec{x}) = (-23,7)$

7. **(5 pts)** Find the orthogonal complement of V.

$$V = \text{Span}\left(\begin{bmatrix}-1\\0\\-2\\3\end{bmatrix}, \begin{bmatrix}-1\\-2\\0\\-5\end{bmatrix}\right)$$

A $\quad V^{\perp} = \text{Span}\left(\begin{bmatrix}-2\\3\\1\\-4\end{bmatrix}\right)$

B $\quad V^{\perp} = \text{Span}\left(\begin{bmatrix}2\\-1\\1\\0\end{bmatrix}, \begin{bmatrix}-3\\4\\0\\1\end{bmatrix}\right)$

C $\quad V^{\perp} = \text{Span}\left(\begin{bmatrix}-2\\1\\1\\0\end{bmatrix}\right)$

D $\quad V^{\perp} = \text{Span}\left(\begin{bmatrix}1\\0\\2\\-3\end{bmatrix}, \begin{bmatrix}0\\1\\-1\\4\end{bmatrix}\right)$

E $\quad V^{\perp} = \text{Span}\left(\begin{bmatrix}-2\\1\\1\\0\end{bmatrix}, \begin{bmatrix}3\\-4\\0\\1\end{bmatrix}\right)$

8. **(5 pts)** Use the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ to transform $[\vec{x}]_B = (-1,4)$ in the basis B in the domain to a vector in the basis B in the codomain.

$$T(\vec{x}) = \begin{bmatrix} -3 & 10 \\ 0 & 4 \end{bmatrix} \vec{x}$$

$$B = \text{Span}\left(\begin{bmatrix} -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -4 \\ 0 \end{bmatrix} \right)$$

A $[T(\vec{x})]_B = \begin{bmatrix} -60 \\ -48 \end{bmatrix}$

B $[T(\vec{x})]_B = \begin{bmatrix} -4 \\ -1 \end{bmatrix}$

C $[T(\vec{x})]_B = \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

D $[T(\vec{x})]_B = \begin{bmatrix} 23 \\ -\frac{49}{2} \end{bmatrix}$

E $[T(\vec{x})]_B = \begin{bmatrix} \frac{4}{3} \\ -\frac{5}{12} \end{bmatrix}$

9. **(15 pts)** Find the four fundamental subspaces of M, including their spaces and dimensions.

$$M = \begin{bmatrix} 3 & 0 & 6 \\ -3 & 1 & 0 \\ 6 & -2 & 0 \\ 0 & -1 & -6 \end{bmatrix}$$

10. **(15 pts)** Find the least squares solution to the system.

$$x + 3y = -6$$

$$y - x = 4$$

$$y = 1$$

11. **(15 pts)** The subspace V is a space in $\mathbb{R}^3$. Use a Gram-Schmidt process to change the basis of V into an orthonormal basis.

$$V = \text{Span}\left(\begin{bmatrix} 2 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ -2 \\ 4 \end{bmatrix}, \begin{bmatrix} -2 \\ -2 \\ 3 \end{bmatrix}\right)$$

12. **(15 pts)** Find the Eigenvalues and Eigenvectors of the matrix, then describe what's happening in the Eigenbases.

$$A = \begin{bmatrix} -1 & 4 & -2 \\ 0 & 2 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$