Topic: Vector triangle inequality

Question: Use the vector triangle inequality to say which vector set is linearly independent.

Answer choices:

A $\vec{u} = (-1, -6)$, $\vec{v} = (-4, -24)$

B $\vec{u} = (2, -7)$, $\vec{v} = (4, -14)$

C $\vec{u} = (9,4)$, $\vec{v} = (72,32)$

D $\vec{u} = (-9, -8)$, $\vec{v} = (-5,6)$

Solution: D

Let's plug each answer choice into the vector triangle inequality,

$$||\overrightarrow{u}+\overrightarrow{v}|| \le ||\overrightarrow{u}|| + ||\overrightarrow{v}||$$

If the left side is less than the right side, the vector set is linearly independent. But if the sides are equivalent (or if the left side is 0), then the vector set is linearly dependent.

Let's test answer choice A in the vector triangle inequality.

$$||\overrightarrow{u}+\overrightarrow{v}|| \le ||\overrightarrow{u}|| + ||\overrightarrow{v}||$$

$$\sqrt{(u_1+v_1)^2+(u_2+v_2)^2} \le \sqrt{u_1^2+u_2^2} + \sqrt{v_1^2+v_2^2}$$

$$\sqrt{(-1-4)^2+(-6-24)^2} \le \sqrt{(-1)^2+(-6)^2} + \sqrt{(-4)^2+(-24)^2}$$

$$\sqrt{(-5)^2+(-30)^2} \le \sqrt{1+36} + \sqrt{16+576}$$

$$\sqrt{25+900} \le \sqrt{37} + \sqrt{592}$$

$$\sqrt{925} \le \sqrt{37} + \sqrt{592}$$

$$30.41 \le 6.08 + 24.33$$

$$30.41 = 30.41$$

Because the sides are equal, the vector set in answer choice A is linearly dependent, so let's test answer choice B.

$$||\overrightarrow{u}+\overrightarrow{v}|| \le ||\overrightarrow{u}|| + ||\overrightarrow{v}||$$

$$\sqrt{(u_1 + v_1)^2 + (u_2 + v_2)^2} \le \sqrt{u_1^2 + u_2^2} + \sqrt{v_1^2 + v_2^2}$$

$$\sqrt{(2 + 4)^2 + (-7 - 14)^2} \le \sqrt{2^2 + (-7)^2} + \sqrt{4^2 + (-14)^2}$$

$$\sqrt{6^2 + (-21)^2} \le \sqrt{4 + 49} + \sqrt{16 + 196}$$

$$\sqrt{36 + 441} \le \sqrt{53} + \sqrt{212}$$

$$\sqrt{477} \le \sqrt{53} + \sqrt{212}$$

$$21.84 \le 7.28 + 14.56$$

$$21.84 = 21.84$$

Because the sides are equal, the vector set in answer choice B is linearly dependent, so let's test answer choice C.

$$||\vec{u} + \vec{v}|| \le ||\vec{u}|| + ||\vec{v}||$$

$$\sqrt{(u_1 + v_1)^2 + (u_2 + v_2)^2} \le \sqrt{u_1^2 + u_2^2} + \sqrt{v_1^2 + v_2^2}$$

$$\sqrt{(9 + 72)^2 + (4 + 32)^2} \le \sqrt{9^2 + 4^2} + \sqrt{72^2 + 32^2}$$

$$\sqrt{6{,}561 + 1{,}296} \le \sqrt{81 + 16} + \sqrt{5{,}184 + 1{,}024}$$

$$\sqrt{7{,}857} \le \sqrt{97} + \sqrt{6{,}208}$$

$$88.64 \le 9.85 + 78.79$$

$$88.64 = 88.64$$

Because the sides are equal, the vector set in answer choice C is linearly dependent, so let's test answer choice D.

$$||\vec{u}+\vec{v}|| \le ||\vec{u}|| + ||\vec{v}||$$

$$\sqrt{(u_1+v_1)^2+(u_2+v_2)^2} \le \sqrt{u_1^2+u_2^2} + \sqrt{v_1^2+v_2^2}$$

$$\sqrt{(-9-5)^2+(-8+6)^2} \le \sqrt{(-9)^2+(-8)^2} + \sqrt{(-5)^2+6^2}$$

$$\sqrt{(-14)^2+(-2)^2} \le \sqrt{81+64} + \sqrt{25+36}$$

$$\sqrt{196+4} \le \sqrt{81+64} + \sqrt{25+36}$$

$$\sqrt{200} \le \sqrt{145} + \sqrt{61}$$

$$14.14 \le 12.04 + 7.81$$

$$14.14 < 19.85$$

Because the left side is less than the right side, the vector set in answer choice D is linearly independent.

Topic: Vector triangle inequality

Question: Use the vector triangle inequality to say which vector set is linearly independent.

Answer choices:

A $\vec{u} = (-1,4)$, $\vec{v} = (-7,28)$

B $\vec{u} = (-8, -3)$, $\vec{v} = (2,8)$

C $\vec{u} = (-2,7)$, $\vec{v} = (-4,14)$

D $\vec{u} = (0, -9)$, $\vec{v} = (0,72)$

Solution: B

Let's plug each answer choice into the vector triangle inequality,

$$||\overrightarrow{u}+\overrightarrow{v}|| \le ||\overrightarrow{u}|| + ||\overrightarrow{v}||$$

If the left side is less than the right side, the vector set is linearly independent. But if the sides are equivalent (or if the left side is 0), then the vector set is linearly dependent.

Let's test answer choice A in the vector triangle inequality.

$$||\overrightarrow{u}+\overrightarrow{v}|| \le ||\overrightarrow{u}|| + ||\overrightarrow{v}||$$

$$\sqrt{(u_1+v_1)^2+(u_2+v_2)^2} \le \sqrt{u_1^2+u_2^2} + \sqrt{v_1^2+v_2^2}$$

$$\sqrt{(-1-7)^2+(4+28)^2} \le \sqrt{(-1)^2+4^2} + \sqrt{(-7)^2+28^2}$$

$$\sqrt{(-8)^2+32^2} \le \sqrt{1+16} + \sqrt{49+784}$$

$$\sqrt{64+1{,}024} \le \sqrt{17} + \sqrt{833}$$

$$\sqrt{1{,}088} \le \sqrt{17} + \sqrt{833}$$

$$32.98 \le 4.12 + 28.86$$

$$32.98 = 32.98$$

Because the sides are equal, the vector set in answer choice A is linearly dependent, so let's test answer choice B.

$$||\overrightarrow{u}+\overrightarrow{v}|| \le ||\overrightarrow{u}|| + ||\overrightarrow{v}||$$

$$\sqrt{(u_1 + v_1)^2 + (u_2 + v_2)^2} \le \sqrt{u_1^2 + u_2^2} + \sqrt{v_1^2 + v_2^2}$$

$$\sqrt{(-8 + 2)^2 + (-3 + 8)^2} \le \sqrt{(-8)^2 + (-3)^2} + \sqrt{2^2 + 8^2}$$

$$\sqrt{(-6)^2 + 5^2} \le \sqrt{64 + 9} + \sqrt{4 + 64}$$

$$\sqrt{36 + 25} \le \sqrt{73} + \sqrt{68}$$

$$\sqrt{61} \le \sqrt{73} + \sqrt{68}$$

$$7.81 \le 8.54 + 8.25$$

$$7.81 \le 16.79$$

Because the left side is less than the right side, the vector set in answer choice B is linearly independent.

Topic: Vector triangle inequality

Question: Use the vector triangle inequality to say which vector set is linearly independent.

Answer choices:

A $\quad \vec{u} = (9, -2),\ \vec{v} = (54, -12)$

B $\quad \vec{u} = (3, -1),\ \vec{v} = (9, -3)$

C $\quad \vec{u} = (5, -3),\ \vec{v} = (0,6)$

D $\quad \vec{u} = (-9,3),\ \vec{v} = (-72,24)$

Solution: C

Let's plug each answer choice into the vector triangle inequality,

$$||\vec{u}+\vec{v}|| \leq ||\vec{u}|| + ||\vec{v}||$$

If the left side is less than the right side, the vector set is linearly independent. But if the sides are equivalent (or if the left side is 0), then the vector set is linearly dependent.

Let's test answer choice A in the vector triangle inequality.

$$||\vec{u}+\vec{v}|| \leq ||\vec{u}|| + ||\vec{v}||$$

$$\sqrt{(u_1+v_1)^2+(u_2+v_2)^2} \leq \sqrt{u_1^2+u_2^2} + \sqrt{v_1^2+v_2^2}$$

$$\sqrt{(9+54)^2+(-2-12)^2} \leq \sqrt{9^2+(-2)^2} + \sqrt{54^2+(-12)^2}$$

$$\sqrt{63^2+(-14)^2} \leq \sqrt{81+4} + \sqrt{2{,}916+144}$$

$$\sqrt{3{,}969+196} \leq \sqrt{81+4} + \sqrt{2{,}916+144}$$

$$\sqrt{4{,}165} \leq \sqrt{85} + \sqrt{3{,}060}$$

$$64.54 = 64.54$$

Because the sides are equal, the vector set in answer choice A is linearly dependent, so let's test answer choice B.

$$||\vec{u}+\vec{v}|| \leq ||\vec{u}|| + ||\vec{v}||$$

$$\sqrt{(u_1+v_1)^2+(u_2+v_2)^2} \le \sqrt{u_1^2+u_2^2}+\sqrt{v_1^2+v_2^2}$$

$$\sqrt{(3+9)^2+(-1-3)^2} \le \sqrt{3^2+(-1)^2}+\sqrt{9^2+(-3)^2}$$

$$\sqrt{12^2+(-4)^2} \le \sqrt{9+1}+\sqrt{81+9}$$

$$\sqrt{144+16} \le \sqrt{10}+\sqrt{90}$$

$$\sqrt{160} \le \sqrt{10}+\sqrt{90}$$

$$12.65 \le 3.16+9.49$$

$$12.65 = 12.65$$

Because the sides are equal, the vector set in answer choice B is linearly dependent, so let's test answer choice C.

$$||\vec{u}+\vec{v}|| \le ||\vec{u}||+||\vec{v}||$$

$$\sqrt{(u_1+v_1)^2+(u_2+v_2)^2} \le \sqrt{u_1^2+u_2^2}+\sqrt{v_1^2+v_2^2}$$

$$\sqrt{(5+0)^2+(-3+6)^2} \le \sqrt{5^2+(-3)^2}+\sqrt{0^2+6^2}$$

$$\sqrt{5^2+3^2} \le \sqrt{25+9}+\sqrt{0+36}$$

$$\sqrt{25+9} \le \sqrt{34}+\sqrt{36}$$

$$\sqrt{34} \le \sqrt{34}+\sqrt{36}$$

$$5.83 \le 5.83+6$$

$$5.83 < 11.83$$

Because the left side is less than the right side, the vector set in answer choice C is linearly independent.

We can also verify that answer choice D is linearly dependent.

$$||\vec{u}+\vec{v}|| \le ||\vec{u}|| + ||\vec{v}||$$

$$\sqrt{(u_1+v_1)^2+(u_2+v_2)^2} \le \sqrt{u_1^2+u_2^2} + \sqrt{v_1^2+v_2^2}$$

$$\sqrt{(-9+(-72))^2+(3+24)^2} \le \sqrt{(-9)^2+3^2} + \sqrt{(-72)^2+24^2}$$

$$\sqrt{(-81)^2+27^2} \le \sqrt{81+9} + \sqrt{5{,}184+576}$$

$$\sqrt{7{,}290} \le \sqrt{90} + \sqrt{5{,}760}$$

$$85.38 = 85.38$$