Topic: The null space and Ax=O

Question: Is $\vec{x}=(-5,1,3)$ in the null space of A?

$$A=\begin{bmatrix}1 & -4 & 3\\ 2 & 4 & 2\\ -1 & -5 & 0\end{bmatrix}$$

Answer choices:

A Yes, $\vec{x}$ is in the null space of A.

B No, $\vec{x}$ is not in the null space of A.

C It’s impossible to say whether or not $\vec{x}$ is in the null space of A.

Solution: A

If $\vec{x} = (-5,1,3)$ is in the null space of A, then the product of A and $\vec{x}$ should satisfy the homogeneous equation.

$$A\vec{x} = \vec{O}$$

$$\begin{bmatrix} 1 & -4 & 3 \\ 2 & 4 & 2 \\ -1 & -5 & 0 \end{bmatrix} \begin{bmatrix} -5 \\ 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If we perform the matrix multiplication on the left side of the equation, we should get the zero vector.

$$\begin{bmatrix} 1(-5) - 4(1) + 3(3) \\ 2(-5) + 4(1) + 2(3) \\ -1(-5) - 5(1) + 0(3) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 - 4 + 9 \\ -10 + 4 + 6 \\ 5 - 5 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Because we get a true equation, we know that $\vec{x} = (-5,1,3)$ is in the null space of A.

Topic: The null space and Ax=O

Question: Which of the vectors is in the null space of A?

$$A = \begin{bmatrix} -3 & 1 & 9 \\ 1 & 1 & 1 \end{bmatrix}$$

Answer choices:

A $\begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$

B $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

C $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$

D $\begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}$

Solution: D

If a vector $\vec{x}$ is in the null space of A, then the product of A and $\vec{x}$ should satisfy the homogeneous equation.

$$A\vec{x} = \vec{O}$$

$$\begin{bmatrix} -3 & 1 & 9 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

If we perform the matrix multiplication on the left side of the equation, we should get the zero vector. So consider $\vec{x} = (2,3, -1)$ first.

$$\begin{bmatrix} -3 & 1 & 9 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -3(2) + 1(3) + 9(-1) \\ 1(2) + 1(3) + 1(-1) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -6 + 3 - 9 \\ 2 + 3 - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -12 \\ 4 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Because we get a false equation, we know that $\vec{x} = (2,3, -1)$ is not in the null space of A. So consider $\vec{x} = (1, -1,0)$.

$$\begin{bmatrix} -3 & 1 & 9 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix}-3(1)+1(-1)+9(0)\\1(1)+1(-1)+1(0)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}-3-1+0\\1-1+0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}-4\\0\end{bmatrix}\neq\begin{bmatrix}0\\0\end{bmatrix}$$

Because we get a false equation, we know that $\vec{x}=(1,-1,0)$ is not in the null space of A. So consider $\vec{x}=(0,1,0)$.

$$\begin{bmatrix}-3&1&9\\1&1&1\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}-3(0)+1(1)+9(0)\\1(0)+1(1)+1(0)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}0+1+0\\0+1+0\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}1\\1\end{bmatrix}\neq\begin{bmatrix}0\\0\end{bmatrix}$$

Because we get a false equation, we know that $\vec{x}=(0,1,0)$ is not in the null space of A. So consider the last vector, $\vec{x}=(2,-3,1)$.

$$\begin{bmatrix}-3&1&9\\1&1&1\end{bmatrix}\begin{bmatrix}2\\-3\\1\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix}-3(2)+1(-3)+9(1)\\1(2)+1(-3)+1(1)\end{bmatrix}=\begin{bmatrix}0\\0\end{bmatrix}$$

$$\begin{bmatrix} -6 - 3 + 9 \\ 2 - 3 + 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Because we get a true equation, we know that $\vec{x} = (2, -3, 1)$ is in the null space of A.

Topic: The null space and Ax=O

Question: Which of the vectors is in the null space of A?

$$A = \begin{bmatrix} 5 & 3 & 1 & 5 \\ -10 & -2 & 1 & -3 \\ -5 & 1 & 2 & 4 \\ 7 & 1 & -1 & -2 \end{bmatrix}$$

Answer choices:

A $\quad \vec{x} = (1,0,1,1)$

B $\quad \vec{x} = (-1,3,-4,0)$

C $\quad \vec{x} = (0,-1,0,0)$

D $\quad \vec{x} = (1,2,0,-4)$

Solution: B

If vector is in the null space of A, then the product of A and $\overrightarrow{x}$ should satisfy the homogeneous equation.

$$A\overrightarrow{x}=\overrightarrow{O}$$

$$\begin{bmatrix}5&3&1&5\\-10&-2&1&-3\\-5&1&2&4\\7&1&-1&-2\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

If we perform the matrix multiplication on the left side of the equation, we should get the zero vector. So consider $\overrightarrow{x}=(1,0,1,1)$ first.

$$\begin{bmatrix}5&3&1&5\\-10&-2&1&-3\\-5&1&2&4\\7&1&-1&-2\end{bmatrix}\begin{bmatrix}1\\0\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}5(1)+3(0)+1(1)+5(1)\\-10(1)-2(0)+1(1)-3(1)\\-5(1)+1(0)+2(1)+4(1)\\7(1)+1(0)-1(1)-2(1)\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix}5+0+1+5\\-10+0+1-3\\-5+0+2+4\\7+0-1-2\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\end{bmatrix}$$

$$\begin{bmatrix} 11 \\ -12 \\ 1 \\ 4 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Because we get a false equation, we know that $\vec{x} = (1,0,1,1)$ is not in the null space of A. So consider $\vec{x} = (-1,3,-4,0)$.

$$\begin{bmatrix} 5 & 3 & 1 & 5 \\ -10 & -2 & 1 & -3 \\ -5 & 1 & 2 & 4 \\ 7 & 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} -1 \\ 3 \\ -4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 5(-1) + 3(3) + 1(-4) + 5(0) \\ -10(-1) - 2(3) + 1(-4) - 3(0) \\ -5(-1) + 1(3) + 2(-4) + 4(0) \\ 7(-1) + 1(3) - 1(-4) - 2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 + 9 - 4 + 0 \\ 10 - 6 - 4 - 0 \\ 5 + 3 - 8 + 0 \\ -7 + 3 + 4 - 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Because we get a true equation, we know that $\vec{x} = (-1,3,-4,0)$ is in the null space of A.