Topic: Preimage, image, and the kernel

Question: Find the preimage A_1 of the subset B_1 under the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$.

$$B_1 = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$$

$$T(\overrightarrow{x}) = \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Answer choices:

A $\quad A_1 = \left\{ \begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}, \begin{bmatrix} \frac{14}{3} \\ \frac{5}{3} \end{bmatrix} \right\}$

B $\quad A_1 = \left\{ \begin{bmatrix} -\frac{1}{3} \\ \frac{2}{3} \end{bmatrix}, \begin{bmatrix} \frac{14}{3} \\ \frac{5}{3} \end{bmatrix} \right\}$

C $\quad A_1 = \left\{ \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}, \begin{bmatrix} \frac{14}{3} \\ -\frac{5}{3} \end{bmatrix} \right\}$

D $\quad A_1 = \left\{ \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}, \begin{bmatrix} -\frac{14}{3} \\ \frac{5}{3} \end{bmatrix} \right\}$

Solution: A

We're trying to find the preimage of B_1 under T, which we'll call $T^{-1}(B_1)$.

$$T^{-1}(B_1) = \left\{ \vec{x} \in \mathbb{R}^2 \ \middle| \ T(\vec{x}) \in B_1 \right\}$$

$$T^{-1}(B_1) = \left\{ \vec{x} \in \mathbb{R}^2 \ \middle| \ \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix} \right\}$$

In other words, we're just trying to find all the vectors $\vec{x}$ in $\mathbb{R}^2$ that satisfy either of these matrix equations. So let's rewrite both augmented matrices in reduced row-echelon form. We get

$$\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & | & 1 \\ 0 & 3 & | & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & | & 1 \\ 0 & 1 & | & -\frac{2}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & \frac{1}{3} \\ 0 & 1 & | & -\frac{2}{3} \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & -1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & | & 3 \\ 0 & 3 & | & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & | & 3 \\ 0 & 1 & | & \frac{5}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & \frac{14}{3} \\ 0 & 1 & | & \frac{5}{3} \end{bmatrix}$$

From the first augmented matrix, we get $x_1 = 1/3$ and $x_2 = -2/3$. And from the second augmented matrix we get $x_1 = 14/3$ and $x_2 = 5/3$. Therefore,

$\begin{bmatrix} \frac{1}{3} \\ -\frac{2}{3} \end{bmatrix}$ in the pre-image A_1 would map to $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ in the subset B_1 under T

$\begin{bmatrix} \frac{14}{3} \\ \frac{5}{3} \end{bmatrix}$ in the pre-image A_1 would map to $\begin{bmatrix} 3 \\ 5 \end{bmatrix}$ in the subset B_1 under T

Topic: Preimage, image, and the kernel

Question: Find the preimage A_1 of the subset B_1 under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$.

$$B_1 = \left\{ \begin{bmatrix} -3 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right\}$$

$$T(\overrightarrow{x}) = \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Answer choices:

A $\quad A_1 = \left\{ \begin{bmatrix} -\frac{3}{2} \\ \frac{3}{8} \end{bmatrix}, \begin{bmatrix} -1 \\ \frac{3}{4} \end{bmatrix} \right\}$

B $\quad A_1 = \left\{ \begin{bmatrix} -\frac{3}{2} \\ \frac{3}{8} \end{bmatrix}, \begin{bmatrix} 1 \\ -\frac{3}{4} \end{bmatrix} \right\}$

C $\quad A_1 = \left\{ \begin{bmatrix} \frac{3}{2} \\ -\frac{3}{8} \end{bmatrix}, \begin{bmatrix} 1 \\ -\frac{3}{4} \end{bmatrix} \right\}$

D $\quad A_1 = \left\{ \begin{bmatrix} \frac{3}{2} \\ -\frac{3}{8} \end{bmatrix}, \begin{bmatrix} -1 \\ \frac{3}{4} \end{bmatrix} \right\}$

Solution: D

We're trying to find the preimage of B_1 under T, which we'll call $T^{-1}(B_1)$.

$$T^{-1}(B_1) = \{\vec{x} \in \mathbb{R}^2 \ | \ T(\vec{x}) \in B_1\}$$

$$T^{-1}(B_1) = \left\{\vec{x} \in \mathbb{R}^2 \ \middle| \ \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix} \text{ or } \begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}\right\}$$

In other words, we're just trying to find all the vectors $\vec{x}$ in $\mathbb{R}^2$ that satisfy either of these matrix equations. So let's rewrite both augmented matrices in reduced row-echelon form. We get

$$\begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & | & -3 \\ 1 & 4 & | & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & | & 0 \\ -2 & 0 & | & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & | & 0 \\ 0 & 8 & | & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & | & 0 \\ 0 & 1 & | & -\frac{3}{8} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & \frac{3}{2} \\ 0 & 1 & | & -\frac{3}{8} \end{bmatrix}$$

and

$$\begin{bmatrix} -2 & 0 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -2 & 0 & | & 2 \\ 1 & 4 & | & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & | & 2 \\ -2 & 0 & | & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & | & 2 \\ 0 & 8 & | & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & | & 2 \\ 0 & 1 & | & \frac{3}{4} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & -1 \\ 0 & 1 & | & \frac{3}{4} \end{bmatrix}$$

From the first augmented matrix, we get $x_1 = 3/2$ and $x_2 = -3/8$. And from the second augmented matrix we get $x_1 = -1$ and $x_2 = 3/4$. Therefore,

$\begin{bmatrix} \frac{3}{2} \\ -\frac{3}{8} \end{bmatrix}$ in the pre-image A_1 would map to $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$ in the subset B_1 under T

$\begin{bmatrix} -1 \\ \frac{3}{4} \end{bmatrix}$ in the pre-image A_1 would map to $\begin{bmatrix} 2 \\ 2 \end{bmatrix}$ in the subset B_1 under T

Topic: Preimage, image, and the kernel

Question: Find the preimage A_1 of the subset B_1 under the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$.

$$B_1 = \left\{ \begin{bmatrix} 5 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right\}$$

$$T(\overrightarrow{x}) = \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

Answer choices:

A $A_1 = \left\{ \begin{bmatrix} -\frac{8}{3} \\ \frac{7}{6} \end{bmatrix}, \begin{bmatrix} -\frac{5}{3} \\ \frac{2}{3} \end{bmatrix} \right\}$

B $A_1 = \left\{ \begin{bmatrix} \frac{8}{3} \\ -\frac{7}{6} \end{bmatrix}, \begin{bmatrix} -\frac{5}{3} \\ \frac{2}{3} \end{bmatrix} \right\}$

C $A_1 = \left\{ \begin{bmatrix} -\frac{8}{3} \\ \frac{7}{6} \end{bmatrix}, \begin{bmatrix} \frac{5}{3} \\ -\frac{2}{3} \end{bmatrix} \right\}$

D $A_1 = \left\{ \begin{bmatrix} \frac{8}{3} \\ -\frac{7}{6} \end{bmatrix}, \begin{bmatrix} \frac{5}{3} \\ -\frac{2}{3} \end{bmatrix} \right\}$

Solution: B

We're trying to find the preimage of B_1 under T, which we'll call $T^{-1}(B_1)$.

$$T^{-1}(B_1) = \left\{ \vec{x} \in \mathbb{R}^2 \ \middle| \ T(\vec{x}) \in B_1 \right\}$$

$$T^{-1}(B_1) = \left\{ \vec{x} \in \mathbb{R}^2 \ \middle| \ \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix} \text{ or } \begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix} \right\}$$

In other words, we're just trying to find all the vectors $\vec{x}$ in $\mathbb{R}^2$ that satisfy either of these matrix equations. So let's rewrite both augmented matrices in reduced row-echelon form. We get

$$\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & | & 5 \\ 1 & 4 & | & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & | & 5 \\ 0 & 6 & | & -7 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & | & 5 \\ 0 & 1 & | & -\frac{7}{6} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & \frac{8}{3} \\ 0 & 1 & | & -\frac{7}{6} \end{bmatrix}$$

and

$$\begin{bmatrix} 1 & -2 \\ 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & | & -3 \\ 1 & 4 & | & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & | & -3 \\ 0 & 6 & | & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 & | & -3 \\ 0 & 1 & | & \frac{2}{3} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & | & -\frac{5}{3} \\ 0 & 1 & | & \frac{2}{3} \end{bmatrix}$$

From the first augmented matrix, we get $x_1 = 8/3$ and $x_2 = -7/6$. And from the second augmented matrix we get $x_1 = -5/3$ and $x_2 = 2/3$. Therefore,

$\begin{bmatrix} \frac{8}{3} \\ -\frac{7}{6} \end{bmatrix}$ in the pre-image A_1 would map to $\begin{bmatrix} 5 \\ -2 \end{bmatrix}$ in the subset B_1 under T

$\begin{bmatrix} -\frac{5}{3} \\ \frac{2}{3} \end{bmatrix}$ in the pre-image A_1 would map to $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$ in the subset B_1 under T