AI1103 Assignment 3

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Download all latex-tikz codes from

https://github.com/MShah134/AI1103/blob/main/Assignment−4/main.tex

Question

X_1 and X_2 are independent Poisson variables such that $\Pr(X_1 = 2) = \Pr(X_1 = 1)$ and $\Pr(X_2 = 2) = \Pr(X_2 = 3)$. What is the variance of $(X_1 - 2X_2)$?

(a) 14
(b) 4
(c) 3
(d) 2

Solution

For a Poisson variable X,

$$\Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{0.0.1}$$

Since $\Pr(X_1 = 2) = \Pr(X_1 = 1)$,

$$\frac{{\lambda_1}^2 e^{-\lambda_1}}{2!} = \frac{{\lambda_1}^1 e^{-\lambda_1}}{1!} \tag{0.0.2}$$

$$\lambda_1 = 2!/1! = 2 \tag{0.0.3}$$

Similarly, as $\Pr(X_2 = 2) = \Pr(X_2 = 3)$,

$$\frac{{\lambda_2}^2 e^{-\lambda_2}}{2!} = \frac{{\lambda_2}^3 e^{-\lambda_2}}{3!} \tag{0.0.4}$$

$$\lambda_2 = 3!/2! = 3 \tag{0.0.5}$$

Also we know for a Poisson variable X, the following holds true:

$$E[X] = \lambda \tag{0.0.6}$$

$$\mathrm{Var}[X] = \lambda \tag{0.0.7}$$

$$\mathrm{Var}[X] = E[X^2] - (E[X])^2 \tag{0.0.8}$$

Now, for the variance of $(X_1 - 2X_2)$

$$\begin{aligned}
\mathrm{Var}[X_1 - 2X_2] &= E[(X_1 - 2X_2)^2] - (E[X_1 - 2X_2])^2 \\
&= E[X_1^2 + 4X_2^2 - 4X_1X_2] \\
&- (E[X_1] - 2E[X_2])^2 \\
&= E[X_1^2] - (E[X_1])^2 + 4(E[X_2^2] - (E[X_2])^2) \\
&- 4E[X_1X_2] + 4E[X_1]E[X_2]
\end{aligned} \tag{0.0.9}$$

Since the variables are independent:

$$E[X_1X_2] = E[X_1]E[X_2] \tag{0.0.10}$$

Substituting equations (0.0.7) and (0.0.8), we get:

$$\begin{aligned}
\mathrm{Var}[X_1 - 2X_2] &= \mathrm{Var}[X_1] + 4(\mathrm{Var}[X_2]) \\
&- 4E[X_1][X_2] + 4E[X_1][X_2] \\
&= \lambda_1 + 4\lambda_2 = 2 + 4(3) = 14
\end{aligned} \tag{0.0.11}$$

Hence option (a) 14 is correct.