

AI1103 Assignment 1

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Download all latex-tikz codes from:

<https://github.com/MShah134/AI1103/blob/main/Assignment-1/main.tex>

$$\forall i, j \in \{1, 2, 3, 4, 5, 6\}$$

Also, since these are all disjoint cases whose union is 1, we also have:

$$\sum_{i=1}^6 \Pr(Y = i) = 1 \quad (0.0.3)$$

PROBLEM

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

SOLUTION

- 1) Let $\Pr(X = i)$ denote the probability that the number i is obtained on the die.
- 2) Let $\Pr(Y = i)$ denote the probability that the i is reported as the number on the die.
- 3) Let $\Pr(Z = 0)$ denote the probability the man is lying.
- 4) Let $\Pr(Z = 1)$ denote the probability that the man is telling the truth.

Now, we have to find out: $\Pr(X = 6|Y = 6)$

Recalling Bayes' Theorem:

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \quad (0.0.1)$$

Now, $\Pr(X = 6, Y = 6)$ is only possible when the man is telling the truth ($Z = 1$) and the die rolls a six. ($X = 6$)

$$\Pr(X = 6, Y = 6) = \Pr(X = 6, Z = 1)$$

These are independent events, hence by definition:

$$\Pr(X = 6, Z = 1) = \Pr(X = 6) \Pr(Z = 1)$$

$$\Pr(X = 6, Z = 1) = (1/6) \times (3/4) = 1/8$$

Hence, we have:

$$\Pr(X = 6, Y = 6) = 1/8$$

Now by symmetry we have,

$$\Pr(Y = i) = \Pr(Y = j) \quad (0.0.2)$$

From (0.0.2) and (0.0.3), we have

$$\Pr(Y = 6) = 1/6$$

Putting the obtained results back in (0.0.1),

$$\Pr(X = 6|Y = 6) = \frac{\Pr(X = 6, Y = 6)}{\Pr(Y = 6)}$$

$$\Pr(X = 6|Y = 6) = \frac{1/8}{1/6} = 3/4$$

Hence, the required probability is $3/4$.

The table containing the random variables and their probabilities is given by:

Case(s)	Notes	Probability
$\Pr(X = i)$	$i=1,2,\dots,6$	$1/6$
$\Pr(Y = i)$	$i=1,2,\dots,6$	$1/6$
$\Pr(Z = 0)$	-	$1/4$
$\Pr(Z = 1)$	-	$3/4$