

AI1103 Assignment 2

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And latex-tikz codes from

<https://github.com/MShah134/AI1103/blob/main/Assignment-2/Latex%20codes>

And Python codes from

<https://github.com/MShah134/AI1103/blob/main/Assignment-2/Python%20codes>

As stated before, L_1 segment is shown in black while L_2 segment is shown in black and purple. Therefore, we have:

$$\Pr\left(X \leq \frac{2}{3} \middle| Y = \frac{3}{4}\right) = \frac{\int_{L_1} f_{XY}(x, y) dx}{\int_{L_2} f_{XY}(x, y) dx} \quad (0.0.4)$$

$$= \frac{\int_0^{2/3} 2 dx}{\int_0^{3/4} 2 dx} \quad (0.0.5)$$

$$\Pr\left(X \leq \frac{2}{3} \middle| Y = \frac{3}{4}\right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9} \quad (0.0.6)$$

QUESTION

GATE EC Q.60: Let X and Y be two random variables having the joint probability density function

$$f(x, y) = \begin{cases} 2 & \text{for } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the conditional probability $\Pr\left(X \leq \frac{2}{3} \middle| Y = \frac{3}{4}\right)$ is equal to —

- 1) 5/9
- 2) 2/3
- 3) 7/9
- 4) 8/9

Hence the correct option is (d) 8/9.

SOLUTION

We have

$$\Pr\left(X \leq \frac{2}{3} \middle| Y = \frac{3}{4}\right) = \frac{\Pr\left(X \leq \frac{2}{3}, Y = \frac{3}{4}\right)}{\Pr\left(Y = \frac{3}{4}\right)} \quad (0.0.1)$$

So we have to consider: $\Pr(X \leq 2/3, Y = 3/4)$ and $\Pr(Y = 3/4)$. They are both lines as can be seen in the Python plot. (In black and in both black and purple respectively)

Hence instead of integrating over area in the XY plane, we have to integrate over these line segments:

$$L_1 : 0 < X \leq 2/3, Y = 3/4 \quad (0.0.2)$$

$$L_2 : 0 < X < 3/4, Y = 3/4 \quad (0.0.3)$$