AI1103 Assignment 1

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Problem 2.18:

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution 2.18:

Let $\Pr(x = i)$ denote the probability that the number i is obtained on the die.
Let $\Pr(y = i)$ denote the probability that the i is reported as the number on the die.
Let $\Pr(z = 0)$ denote the probability the man is lying and $\Pr(z = 1)$ denotes the probability that the man is telling the truth.

The notation used here is:
$\Pr(A = i \cdot B = j) \equiv \Pr(A = i \wedge B = j)$

Now, we have to find out:

$$\Pr(x = 6|y = 6)$$

Recalling Bayes' Theorem:

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)}...[1]$$

Now, $\Pr(x = 6 \cdot y = 6)$ is only possible when the man is telling the truth (z=1) and the die rolls a 6 (x=6)
$\Pr(x = 6 \cdot y = 6) = \Pr(x = 6 \cdot z = 1)$

Both of these are independent events, hence by definition:

$$\Pr(x = 6 \cdot z = 1) = \Pr(x = 6) \Pr(z = 1)$$

$$\Pr(x = 6 \cdot z = 1) = (1/6) * (3/4) = 1/8$$

Hence, we have:
$\Pr(x = 6 \cdot y = 6) = 1/8$

Now for $\Pr(y = 6)$:
We know by symmetry that

$$\Pr(y = i) = \Pr(y = j)...[2]$$

$\forall\ i, j \in \{1, 2, 3, 4, 5, 6\}$

Also, since these are all disjoint cases whose union covers all cases, we also have:

$$\Pr(y = 1)+\Pr(y = 2)+... \Pr(y = 6) = 1$$

From [2], we have

$$\Pr(y = 6) + \Pr(y = 6)... \Pr(y = 6) = 1$$

$$6 \Pr(y = 6) = 1$$

$$\Pr(y = 6) = 1/6$$

Putting the obtained results back in [1],

$$\Pr(x = 6|y = 6) = \frac{\Pr(x = 6 \cdot y = 6)}{\Pr(y = 6)}$$

$$\Pr(x = 6|y = 6) = \frac{1/8}{1/6} = 3/4$$

Hence, the required probability is 0.75