AI1103 Assignment 1

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Download all latex-tikz codes from:

https://github.com/MShah134/AI1103/blob/main/Assignment−1/main.tex

Problem

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

Solution

1) Let $\Pr(X = i)$ denote the probability that the number i is obtained on the die.

2) Let $\Pr(Y = i)$ denote the probability that the i is reported as the number on the die.

3) Let $\Pr(Z = 0)$ denote the probability the man is lying.

4) Let $\Pr(Z = 1)$ denote the probability that the man is telling the truth.

[The notation used here is:
$\Pr(A = i \cdot B = j) \equiv \Pr(A = i \wedge B = j)$]

Now, we have to find out: $\Pr(X = 6|Y = 6)$
Recalling Bayes' Theorem:

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{0.0.1}$$

Now, $\Pr(X = 6 \cdot Y = 6)$ is only possible when the man is telling the truth (Z = 1) and the die rolls a six. (X = 6)

$\Pr(X = 6 \cdot Y = 6) = \Pr(X = 6 \cdot Z = 1)$
These are independent events, hence by definition:

$$\Pr(X = 6 \cdot Z = 1) = \Pr(X = 6)\Pr(Z = 1)$$

$$\Pr(X = 6 \cdot Z = 1) = (1/6) * (3/4) = 1/8$$

Hence, we have:
$\Pr(X = 6 \cdot Y = 6) = 1/8$

Now for $\Pr(Y = 6)$,
We know by symmetry that

$$\Pr(Y = i) = \Pr(Y = j) \tag{0.0.2}$$

$\forall\ i, j \in \{1, 2, 3, 4, 5, 6\}$

Also, since these are all disjoint cases whose union is 1, (covers all cases) we also have:

$$\sum_{i=1}^{6} \Pr(Y = i) = 1 \tag{0.0.3}$$

From (0.0.2) and (0.0.3), we have

$$6\Pr(Y = 6) = 1$$

$$\Pr(Y = 6) = 1/6$$

Putting the obtained results back in (0.0.1),

$$\Pr(X = 6|Y = 6) = \frac{\Pr(X = 6 \cdot Y = 6)}{\Pr(Y = 6)}$$

$$\Pr(X = 6|Y = 6) = \frac{1/8}{1/6} = 3/4$$

Hence, the required probability is 3/4.