

AI1103 Assignment 4

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Download all latex-tikz codes from

<https://github.com/MShah134/AI1103/blob/main/Assignment-4/main.tex>

QUESTION

X_1 and X_2 are independent Poisson variables such that $\Pr(X_1 = 2) = \Pr(X_1 = 1)$ and $\Pr(X_2 = 2) = \Pr(X_2 = 3)$. What is the variance of $(X_1 - 2X_2)$?

- 1) 14
- 2) 4
- 3) 3
- 4) 2

SOLUTION

For a Poisson variable X ,

$$\Pr(X = k) = \frac{\lambda^k e^{-\lambda}}{k!} \quad (0.0.1)$$

Since $\Pr(X_1 = 2) = \Pr(X_1 = 1)$,

$$\frac{\lambda_1^2 e^{-\lambda_1}}{2!} = \frac{\lambda_1^1 e^{-\lambda_1}}{1!} \quad (0.0.2)$$

$$\lambda_1 = 2!/1! = 2 \quad (0.0.3)$$

Similarly, as $\Pr(X_2 = 2) = \Pr(X_2 = 3)$,

$$\frac{\lambda_2^2 e^{-\lambda_2}}{2!} = \frac{\lambda_2^3 e^{-\lambda_2}}{3!} \quad (0.0.4)$$

$$\lambda_2 = 3!/2! = 3 \quad (0.0.5)$$

Also we know for a Poisson variable X , the following holds true:

$$E[X] = \lambda \quad (0.0.6)$$

$$\text{Var}[X] = \lambda \quad (0.0.7)$$

$$\text{Var}[X] = E[X^2] - (E[X])^2 \quad (0.0.8)$$

Now, for the variance of $(X_1 - 2X_2)$

$$\begin{aligned} \text{Var}[X_1 - 2X_2] &= E[(X_1 - 2X_2)^2] - (E[X_1 - 2X_2])^2 \\ &= E[X_1^2 + 4X_2^2 - 4X_1X_2] \\ &\quad - (E[X_1] - 2E[X_2])^2 \\ &= E[X_1^2] - (E[X_1])^2 + 4E[X_2^2] \\ &\quad - 4(E[X_2])^2 + 4E[X_1X_2] \\ &\quad + 4E[X_1]E[X_2] \end{aligned} \quad (0.0.9)$$

Since the variables are independent:

$$E[X_1X_2] = E[X_1]E[X_2] \quad (0.0.10)$$

Substituting equations (0.0.7) and (0.0.8), we get:

$$\begin{aligned} \text{Var}[X_1 - 2X_2] &= \text{Var}[X_1] + 4(\text{Var}[X_2]) \\ &\quad - 4E[X_1][X_2] + 4E[X_1][X_2] \\ &= \lambda_1 + 4\lambda_2 = 2 + 4(3) = 14 \end{aligned} \quad (0.0.11)$$

Hence option (a) 14 is correct.