

# AI1103 Assignment 2

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And latex-tikz codes from

<https://github.com/MShah134/AI1103/blob/main/Assignment-2/Latex%20codes>

And Python codes from

<https://github.com/MShah134/AI1103/blob/main/Assignment-2/Python%20codes>

As stated before,  $L_1$  segment is shown in black while  $L_2$  segment is shown in black and purple. Therefore, we have:

$$\Pr\left(X \leq \frac{2}{3} \middle| Y = \frac{3}{4}\right) = \frac{\int_{L_1} f_{XY}(x, y) dx}{\int_{L_2} f_{XY}(x, y) dx} \quad (0.0.4)$$

$$= \frac{\int_0^{2/3} 2 dx}{\int_0^{3/4} 2 dx} \quad (0.0.5)$$

$$\Pr\left(X \leq \frac{2}{3} \middle| Y = \frac{3}{4}\right) = \frac{4}{3} \times \frac{2}{3} = \frac{8}{9} \quad (0.0.6)$$

## QUESTION

Let  $X$  and  $Y$  be two random variables having the joint probability density function

$$f(x, y) = \begin{cases} 2 & \text{for } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the conditional probability  $\Pr\left(X \leq \frac{2}{3} \middle| Y = \frac{3}{4}\right)$  is equal to —

- 1) 5/9
- 2) 2/3
- 3) 7/9
- 4) 8/9

Hence the correct option is (d) 8/9.

## SOLUTION

We have

$$\Pr\left(X \leq \frac{2}{3} \middle| Y = \frac{3}{4}\right) = \frac{\Pr\left(X \leq \frac{2}{3}, Y = \frac{3}{4}\right)}{\Pr\left(Y = \frac{3}{4}\right)} \quad (0.0.1)$$

So we have to consider:  $\Pr(X \leq 2/3, Y = 3/4)$  and  $\Pr(Y = 3/4)$ . They are both lines as can be seen in the Python plot. (In black and in both black and purple respectively)

Hence instead of integrating over area in the  $XY$  plane, we have to integrate over these line segments:

$$L_1 : 0 < X \leq 2/3, Y = 3/4 \quad (0.0.2)$$

$$L_2 : 0 < X < 3/4, Y = 3/4 \quad (0.0.3)$$