

AI1103 Assignment 1

Megh Shah - CS20BTECH11032

Download all latex-tikz codes from:

<https://github.com/MShah134/AI1103/blob/main/Assignment-1/main.tex>

PROBLEM

A man is known to speak truth 3 out of 4 times. He throws a die and reports that it is a six. Find the probability that it is actually a six.

SOLUTION

- 1) Let $\Pr(X = i)$ be the probability that number i is obtained on the die.
- 2) Let $\Pr(Y = i)$ be the probability that number i is reported on the die.
- 3) Let $\Pr(Z = 0)$ be the probability the man is lying.
- 4) Let $\Pr(Z = 1)$ be the probability that the man is telling the truth.

We have to find: $\Pr(X = 6|Y = 6)$

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \quad (0.0.1)$$

Now, $X = 6, Y = 6 \implies X = 6, Z = 1$
(man is telling the truth and the die rolls a six)

$$\Pr(X = 6, Y = 6) = \Pr(X = 6, Z = 1)$$

Since these are independent events:

$$\Pr(X = 6, Z = 1) = \Pr(X = 6) \Pr(Z = 1) \quad (0.0.2)$$

$$\Pr(X = 6, Z = 1) = (1/6) \times (3/4) = 1/8 \quad (0.0.3)$$

$$\implies \Pr(X = 6, Y = 6) = 1/8$$

By symmetry we get,

$$\Pr(Y = i) = \Pr(Y = j) \quad (0.0.4)$$

$$\forall i, j \in \{1, 2, 3, 4, 5, 6\}$$

Also, since these are all disjoint cases whose union is 1, we get:

$$\sum_{i=1}^6 \Pr(Y = i) = 1 \quad (0.0.5)$$

From eq 0.0.4 and 0.0.5, we have

$$\Pr(Y = 6) = 1/6$$

Putting the obtained results back in eq 0.0.1,

$$\Pr(X = 6|Y = 6) = \frac{\Pr(X = 6, Y = 6)}{\Pr(Y = 6)}$$

$$\Pr(X = 6|Y = 6) = \frac{1/8}{1/6} = 3/4$$

Hence, the required probability is $3/4$.

Consider table 4:

Case(s)	Notes	Probability
$\Pr(X = i)$	$i=1,2,\dots,6$	$1/6$
$\Pr(Y = i)$	$i=1,2,\dots,6$	$1/6$
$\Pr(Z = 0)$	-	$1/4$
$\Pr(Z = 1)$	-	$3/4$

TABLE 4: Probabilities of random variables