AI1103 Assignment 2

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And latex-tikz codes from

https://github.com/MShah134/AI1103/blob/main/Assignment−2/main.tex

Question

Let X and Y be two random variables having the joint probability density function

$$f(x,y) = \begin{cases} 2 & \text{for } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the conditional probability $P(X \leq \frac{2}{3} | Y = \frac{3}{4})$ is equal to —-

1) 5/9
2) 2/3
3) 7/9
4) 8/9

Solution

From the definition of joint probability density function (PDF), we have:

$$P((X,Y) \in A) = \iint_A f_{XY}(x.y)\,dx\,dy \tag{0.0.1}$$

Since,

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \tag{0.0.2}$$

Therefore, we have

$$P(X \leq \frac{2}{3} | Y = \frac{3}{4}) = \frac{P(X \leq \frac{2}{3}, Y = \frac{3}{4})}{P(Y = \frac{3}{4})} \tag{0.0.3}$$

So we have to consider: $P(X \leq 2/3, Y = 3/4)$ and $P(Y = 3/4)$. They are both lines.
Hence instead of integrating over area in the XY plane, we have to integrate over line segments:

$$L_1 : 0 < X \leq 2/3, Y = 3/4 \tag{0.0.4}$$
$$L_2 : 0 < X < 3/4, Y = 3/4 \tag{0.0.5}$$

Therefore, we have:

$$P(X \leq \frac{2}{3} | Y = \frac{3}{4}) = \frac{\int_{L_1} f_{XY}(x.y)\,dx}{\int_{L_2} f_{XY}(x.y)\,dx} \tag{0.0.6}$$

$$= \frac{\int_0^{2/3} 2\,dx}{\int_0^{3/4} 2\,dx} \tag{0.0.7}$$

$$= \frac{4}{3} \cdot \frac{2}{3} = \frac{8}{9} \tag{0.0.8}$$

Hence the correct option is (d) 8/9.