

AI1103 Assignment 2

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And latex-tikz codes from

<https://github.com/MShah134/AI1103/blob/main/Assignment-2/main.tex>

Therefore, we have:

$$\Pr\left(X \leq \frac{2}{3} | Y = \frac{3}{4}\right) = \frac{\int_{L_1} f_{XY}(x,y) dx}{\int_{L_2} f_{XY}(x,y) dx} \quad (0.0.6)$$

QUESTION

Let X and Y be two random variables having the joint probability density function

$$f(x,y) = \begin{cases} 2 & \text{for } 0 < x < y < 1 \\ 0 & \text{otherwise} \end{cases}$$

Then the conditional probability $\Pr\left(X \leq \frac{2}{3} | Y = \frac{3}{4}\right)$ is equal to —

- 1) 5/9
- 2) 2/3
- 3) 7/9
- 4) 8/9

$$= \frac{\int_0^{2/3} 2 dx}{\int_0^{3/4} 2 dx} \quad (0.0.7)$$

$$= \frac{4}{3} \cdot \frac{2}{3} = \frac{8}{9} \quad (0.0.8)$$

Hence the correct option is (d) 8/9.

SOLUTION

From the definition of joint probability density function (PDF), we have:

$$\Pr((X,Y) \in A) = \iint_A f_{XY}(x,y) dx dy \quad (0.0.1)$$

Since,

$$\Pr(A|B) = \frac{\Pr(AB)}{\Pr(B)} \quad (0.0.2)$$

Therefore, we have

$$\Pr\left(X \leq \frac{2}{3} | Y = \frac{3}{4}\right) = \frac{\Pr\left(X \leq \frac{2}{3}, Y = \frac{3}{4}\right)}{\Pr\left(Y = \frac{3}{4}\right)} \quad (0.0.3)$$

So we have to consider: $\Pr(X \leq 2/3, Y = 3/4)$ and $\Pr(Y = 3/4)$. They are both lines.

Hence instead of integrating over area in the XY plane, we have to integrate over line segments:

$$L_1 : 0 < X \leq 2/3, Y = 3/4 \quad (0.0.4)$$

$$L_2 : 0 < X < 3/4, Y = 3/4 \quad (0.0.5)$$