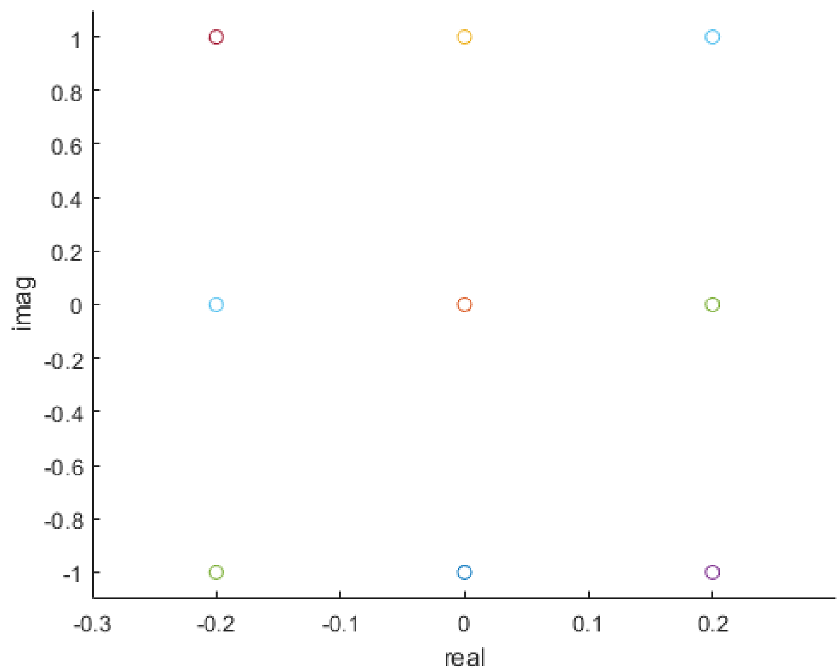
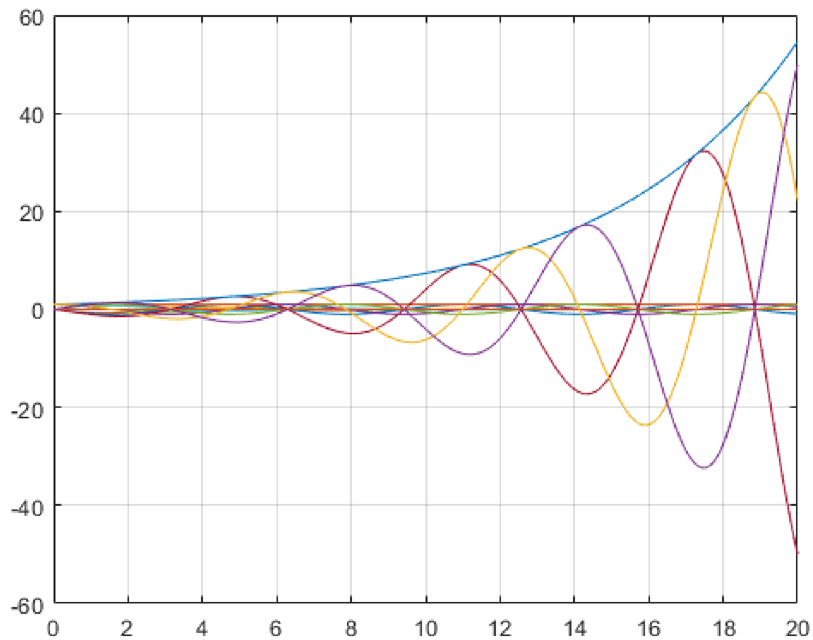
1. Complex Exponentials

1.2 Plotting complex exponentials



MatLab code:

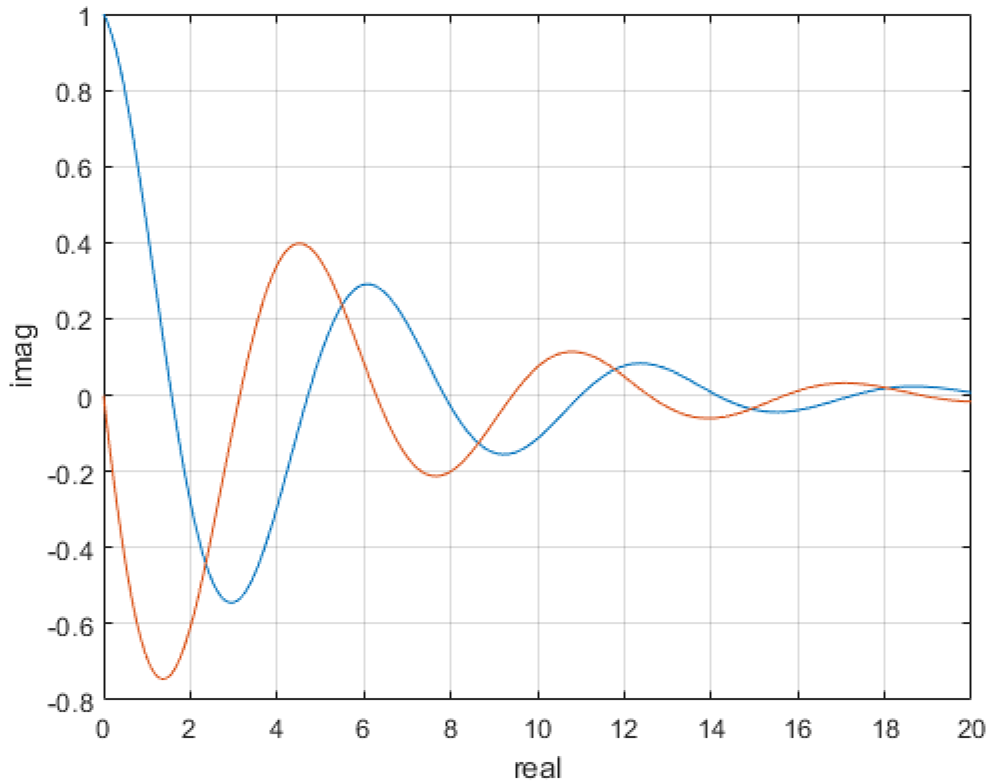
```
for s = -0.2:0.2:0.2
    for r = -1:1
        scatter(s, r);
        xlabel("real");
        ylabel("imag");
        hold on;
    end
end
```



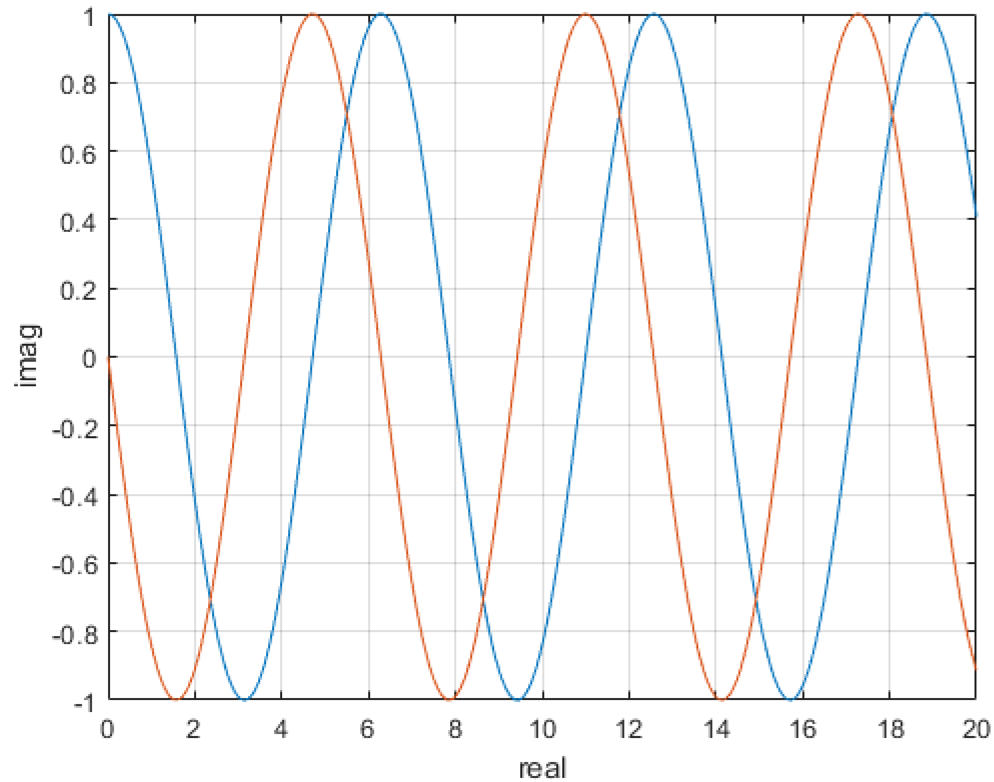
MatLab code:

```
for re = -0.2:0.2:0.2
    for im = -1:1
        c = re + im*j;
        t = 0:0.01:20;
        x = exp(c*t);
        plot(t, real(x), t, imag(x)), grid;
        hold on;
    end
end
```

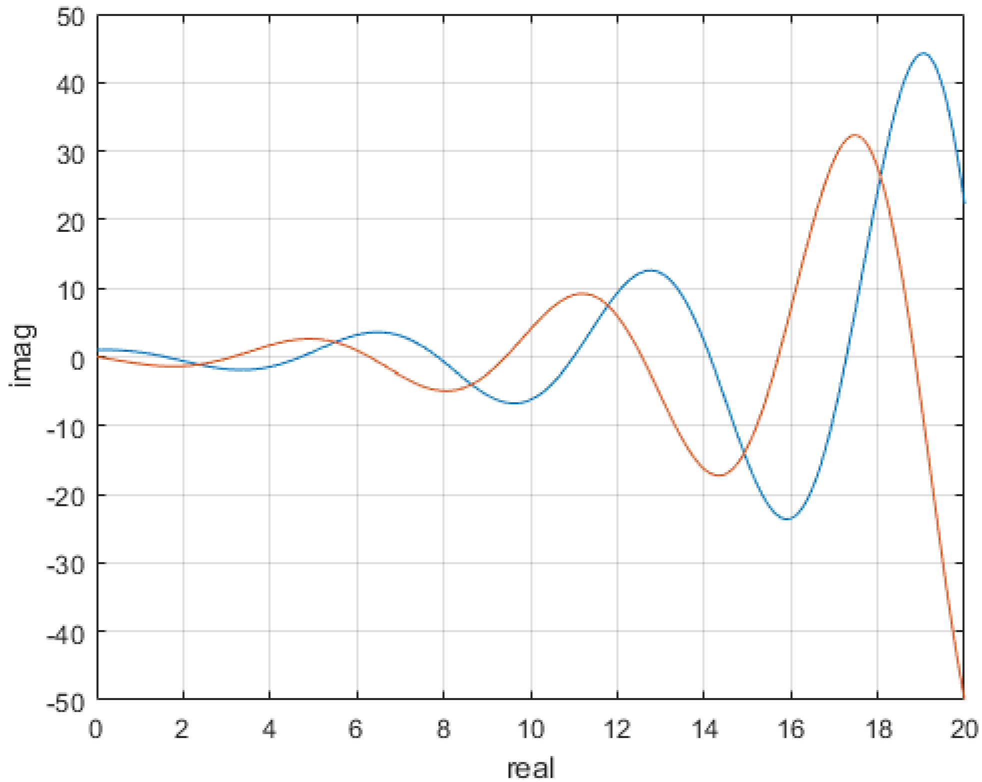
This graph represents all graphs below together



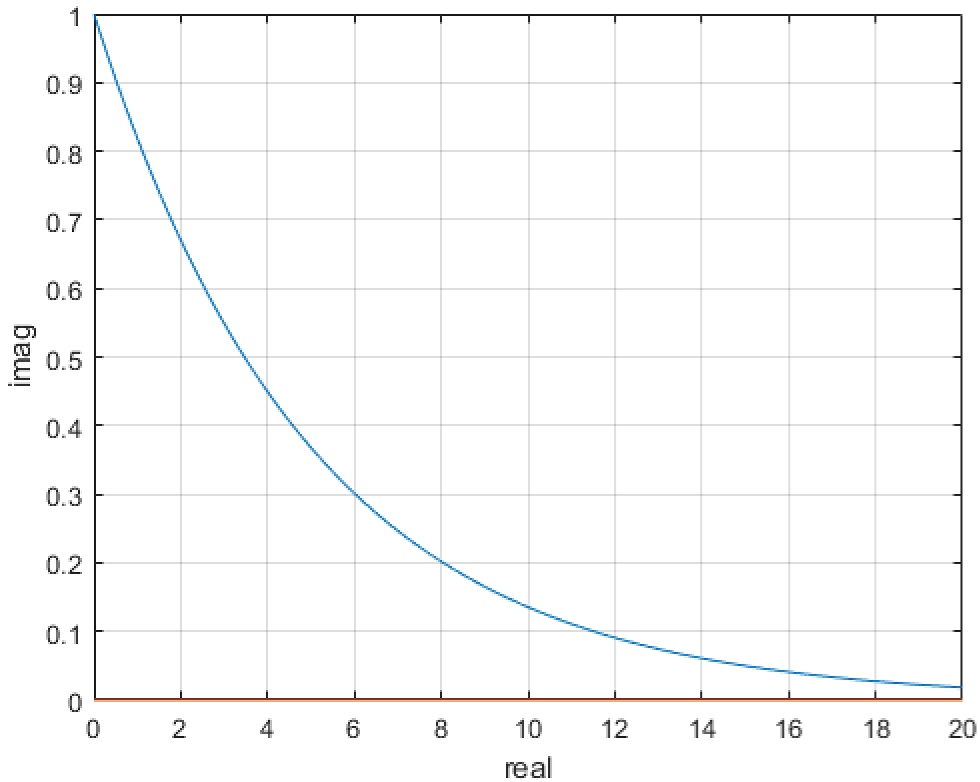
s = -0.2-j1



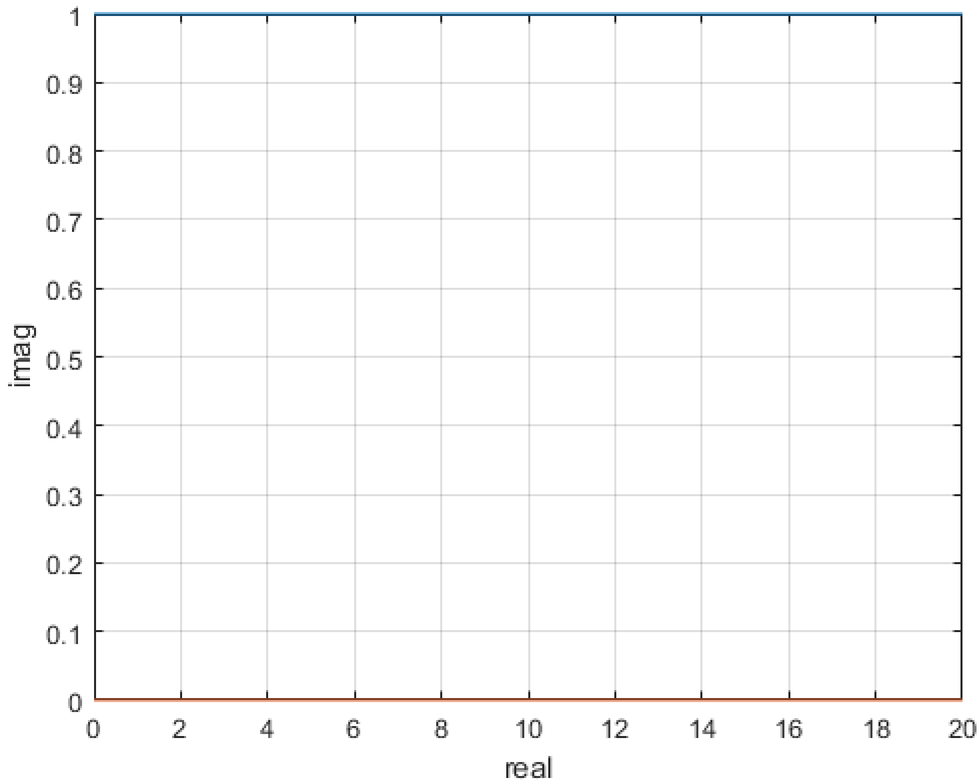
s = 0-j1



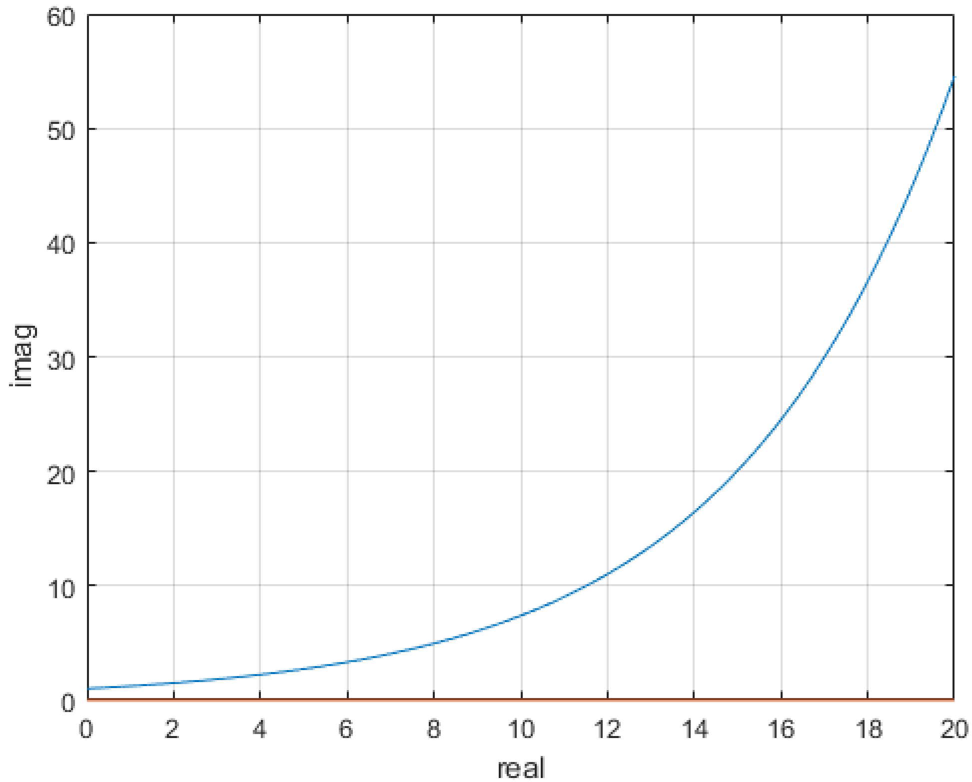
s = 0.2-j1



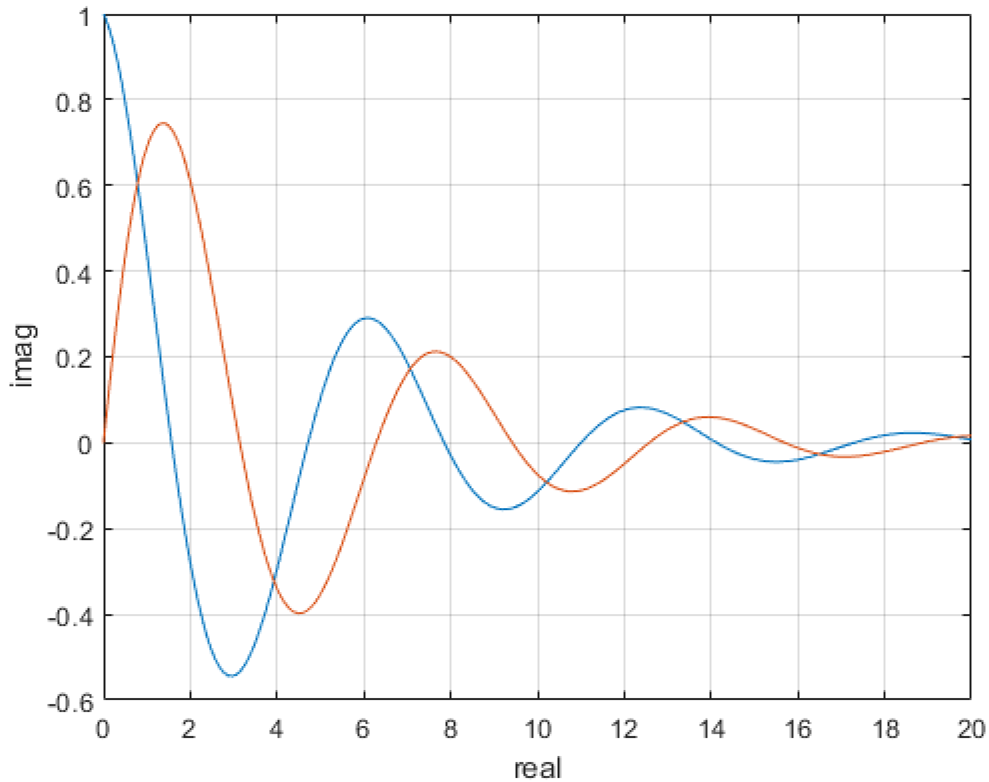
s = -0.2+j0



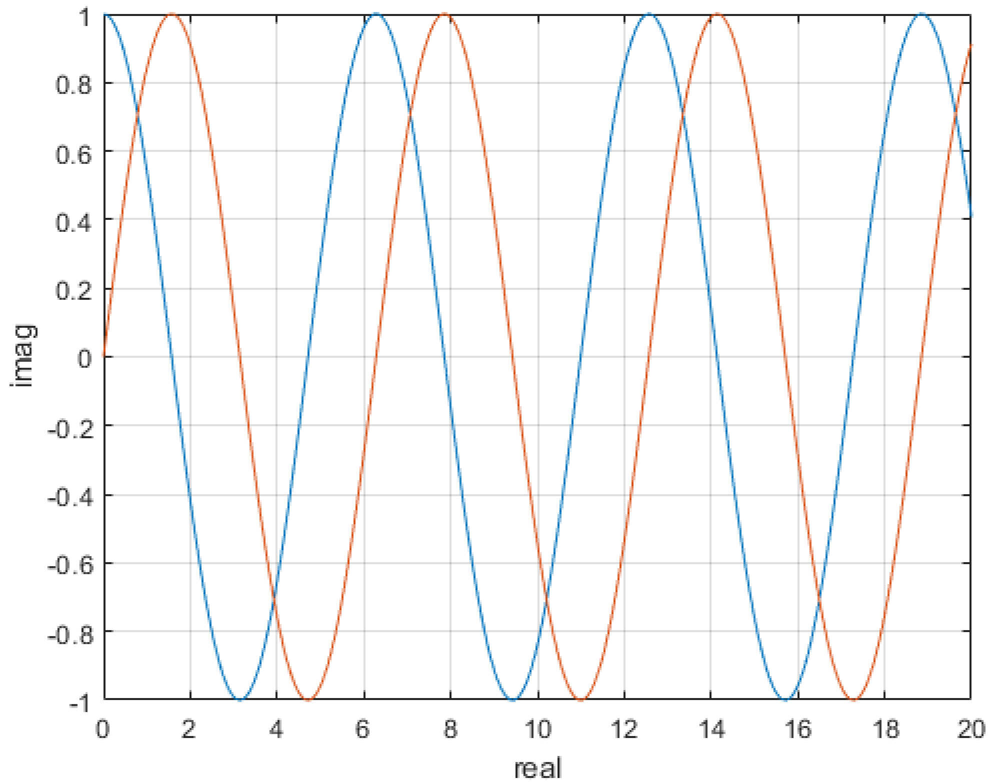
s = 0+j0



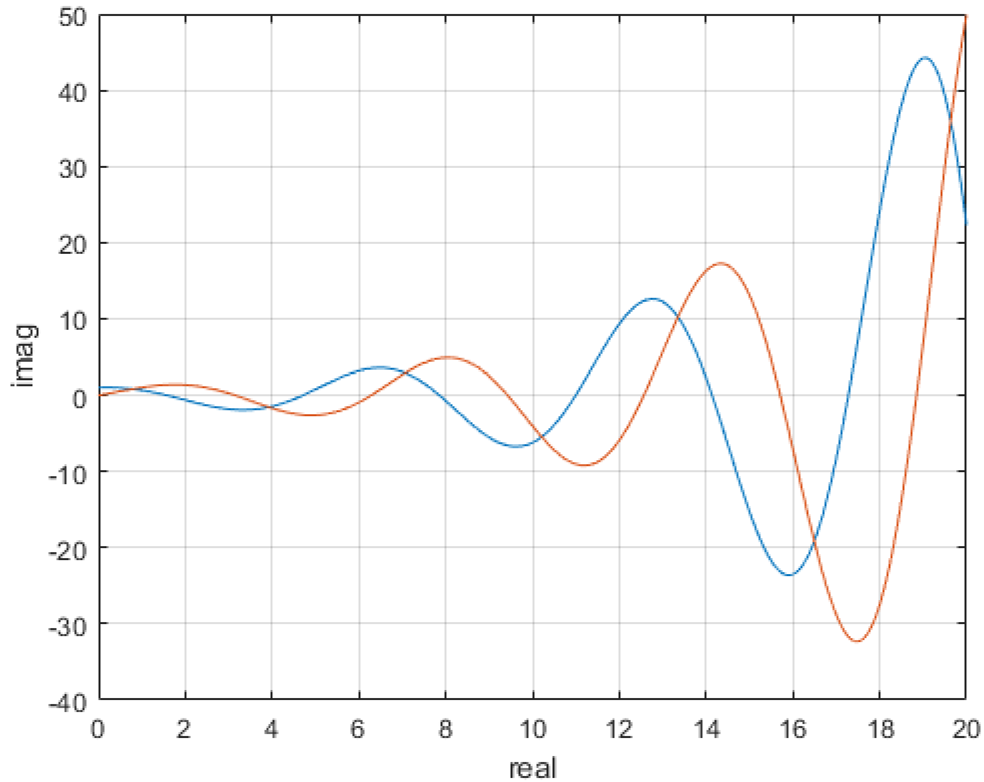
s = 0.2+j0



s = -0.2+j1



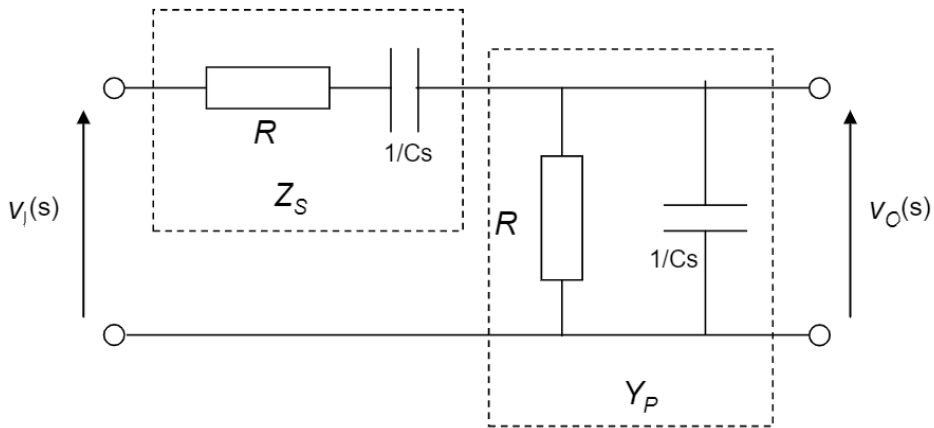
s = 0+j1



s = 0+j1

2. The Wien Bridge Circuit

2.1 Introduction



$Z_s(s) = R + 1/Cs$

$Y_p(s) = 1/R + 1/(1/Cs) = 1/R + Cs$

$$H_{(s)} = \frac{1}{1 + (R + \frac{1}{Cs})(\frac{1}{R} + Cs)}$$

$$H_{(s)} = \frac{1}{1 + (1 + RCs + \frac{1}{RCs} + 1)}$$

$$H_{(s)} = \frac{1}{3 + RCs + \frac{1}{RCs}}$$

RC = 1

=>

$$H_{(s)} = \frac{1}{3 + s + \frac{1}{s}}$$

$$H_{(s)} = \frac{s}{s^2 + 3s + 1}$$

2.2 Impulse response

D = $b^2 - 4ac = 9 - 4 = 5$

$$S_{1,2} = \frac{-3 \pm \sqrt{5}}{2}$$

$$H_{(s)} = \frac{s}{\left(\mathrm{s} + \frac{3+\sqrt{5}}{2}\right)\left(\mathrm{s} + \frac{3-\sqrt{5}}{2}\right)}$$

$$H_{(s)} = \frac{s}{\left(\mathrm{s} - \frac{-3-\sqrt{5}}{2}\right)\left(\mathrm{s} - \frac{-3+\sqrt{5}}{2}\right)}$$

Poles:

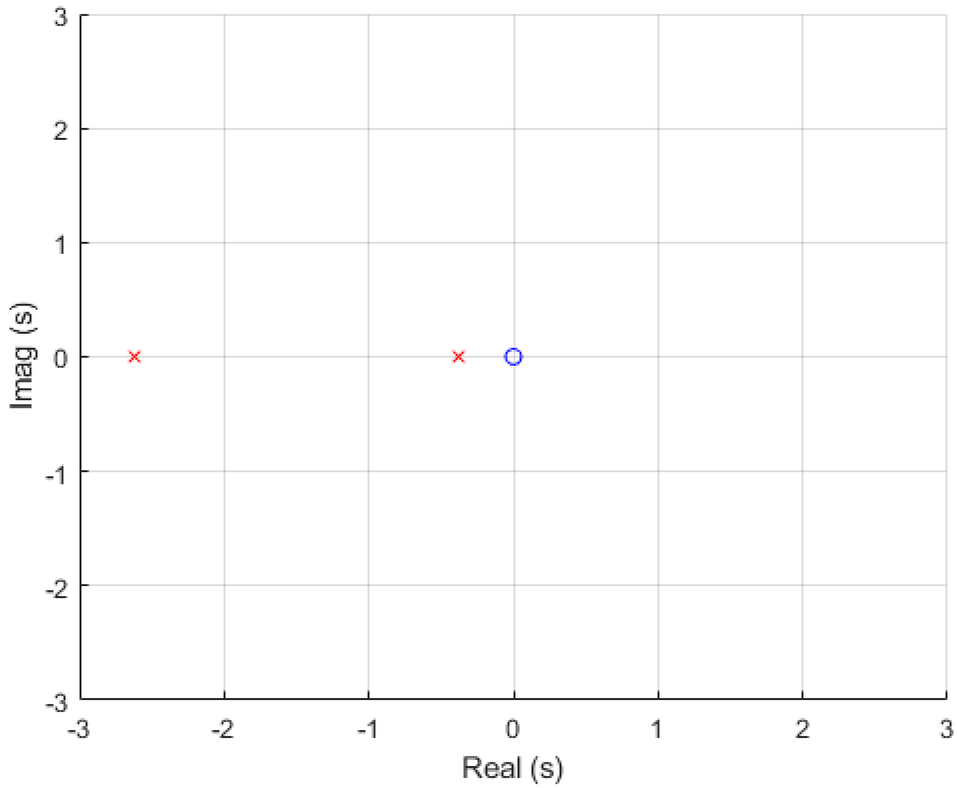
$$\frac{-3-\sqrt{5}}{2} = -2.62$$

$$\frac{-3+\sqrt{5}}{2} = -0.38$$

One zero:

Z = 0

Plot:



Code:

```
z = 0;
p1 = (-3-sqrt(5))/2;
p2 = (-3+sqrt(5))/2;
scatter(0, z, "b", "o");
hold on;
scatter(p1, 0, "r", "x");
hold on;
scatter(p2, 0, "r", "x");
hold on;
```

Partial fractions:

$$H_{(s)} = \frac{\frac{3\sqrt{5}+5}{10}}{s - \frac{-3-\sqrt{5}}{2}} + \frac{\frac{5-3\sqrt{5}}{10}}{s - \frac{-3+\sqrt{5}}{2}}$$

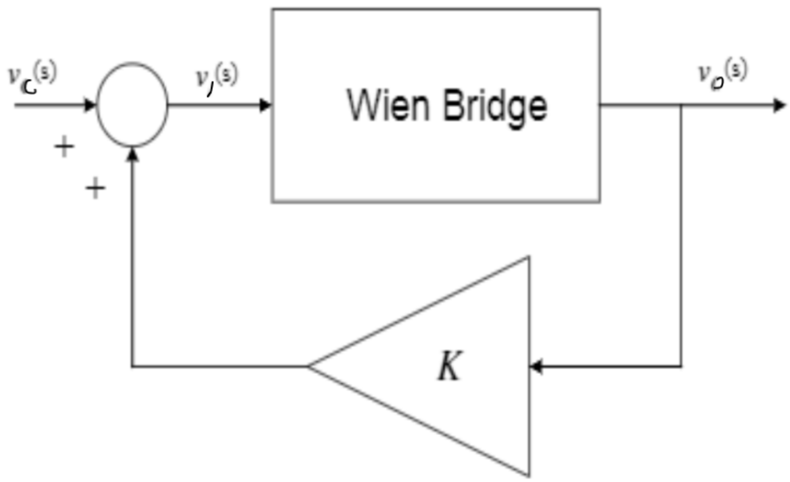
$$H_{(s)} = \frac{1.17}{s-(-2.62)} + \frac{4.33}{s-(-0.38)}$$

Inverse Laplace transform of $H_{(s)}$:

$$h_{(t)} = \frac{3+\sqrt{5}}{2\sqrt{5}} e^{-\frac{(3+\sqrt{5})}{2}t} + \frac{\sqrt{5}-3}{2\sqrt{5}} e^{\frac{\sqrt{5}-3}{2}t}$$

3. The Wien Bridge with Feedback

3.1 The closed-loop transfer function



Transfer function of The Wien Bridge with Feedback:

$H_{(s)}/(1-G_{(s)}H_{(s)})$

$$G_{(s)} = \frac{H_{(s)}}{K/s + \; H_{(s)}}$$

$$G_{(s)} = \frac{1}{\frac{K/s}{H_{(s)}} + 1}$$

$$G_{(s)} = \frac{1}{\frac{\frac{K}{s}}{\frac{s}{s^2+3s+1}} + 1}$$

$$G_{(s)} = \frac{1}{\mathrm{K}(s^2+3s+1)+1}$$

i)K = 1

$$G_{(s)} = \frac{1}{s^2 + 3s + 2}$$

$$G_{(s)} = \frac{1}{(s - (-2))(s - (1))}$$

Poles:

-1

-2

ii)K = 2

$$G_{(s)} = \frac{1}{2s^2 + 6s + 3}$$

$$G_{(s)} = \frac{1}{(s - \frac{3 + \sqrt{3}}{2})(s - \frac{3 - \sqrt{3}}{2})}$$

Poles:

$$\frac{3 + \sqrt{3}}{2} = 2.37$$

$$\frac{3 - \sqrt{3}}{2} = 0.634$$

iii)K=3

$$G_{(s)} = \frac{1}{3s^2 + 9s + 4}$$

$$G_{(s)} = \frac{1}{(s - \frac{9 + \sqrt{33}}{6})(s - \frac{9 - \sqrt{33}}{6})}$$

Poles:

$$\frac{9 + \sqrt{33}}{6} = 2.46$$

$$\frac{9 - \sqrt{33}}{6} = 0.543$$

iv)K=4

$$G_{(s)} = \frac{1}{4s^2 + 12s + 5}$$

$$G_{(s)} = \frac{1}{(s - \frac{5}{2})(s - \frac{1}{2})}$$

Poles:

$$\frac{5}{2} = 2.5$$

$$\frac{1}{2} = 0.5$$

3.2 Impulse response of the closed-loop system

Couldn’t relate this exercise with what we have learned in the class. Couldn’t solve it neither.