Reflection and Transmission of Light from Multilayer Films: An easy approach, using MATLAB

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When optical beam hits a multilayered system of different refractive indices, it gets reflected, refracted, and absorbed in a way that can be derived from the Fresnel equations. But, with increasing number of layers, math becomes complicated. We have designed a MATLAB algorithm underlying the transfer–matrix method for the calculation of the optical properties of multilayered system and have verified it with experimental observations.

OCIS codes: .

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1. INTRODUCTION

The optical beam passing through the interface of different refractive indices, changes its direction towards/away normal to the interface, and the changed direction can be calculated mathematically using refractive indices making that interface. To understand this, we can consider an example illustrated in Figure 1. Assuming n_2 to be sand. If a car enters into a region with higher refractive index at oblique angle, its right front wheel enters into an area of n_2 earlier than the left front wheel, hence starts facing lossy force earlier, causing a change in direction towards the normal. Same intuition can help to predict diverted optical path of optical beam.

But if a linearly polarized light faces an interface of higher refractive index it gets refracted and reflected. The direction of beam propagation ($\vec{k}$) is shown in figure 2 and sinusoidal waves shows that the oscillation of electric field is perpendicular to the direction of wave propagation. Optical beam is characterized as p–polarized, if electric field oscillations are perpendicular to the plane formed by incident, reflected and transmitted beam, and p if oscillations are in the same plane. The direction of reflected and transmitted beams can be calculated by using Snell's Law and intensities can be computed via Fresnel coefficients as:

$$r_p = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2}, \tag{1}$$

$$r_s = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2}, \tag{2}$$

$$t_p = \frac{2n_1 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2}, \tag{3}$$

$$r_s = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2}. \tag{4}$$

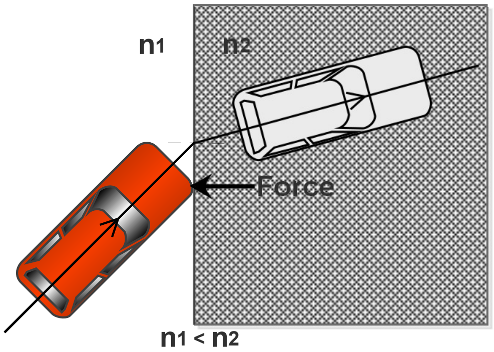


Fig. 1. Car entering into sand (intuition)

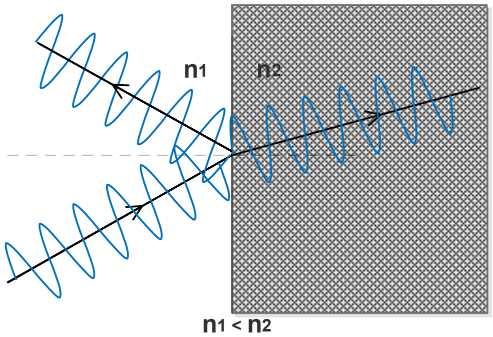


Fig. 2. Refraction of optical beam at the interface between two media of different refractive indices

These coefficients can easily be used on single interface, but for multilayered system, matrix transformation method is more useful.

A. Matrix Method

Suppose we have a multilayered system[1] having N refractive indices stacked together making $N-1$ interfaces with refractive index n_j, impedance Z_j, thickness d_j for layer j. Also the layer 0 is semi–infinite with $Z = -\infty$ and layer N is being treated semi–infinite with $Z = \infty$ and phase change is ϕ_j.

$$\begin{pmatrix} E_{j-1} \\ H_{j-1} \end{pmatrix} = \begin{pmatrix} \cos\phi_j & i\sin\phi_j/Z_j \\ Z_j i\sin\phi_j & \sin\phi_j \end{pmatrix} \begin{pmatrix} E_j \\ H_j \end{pmatrix} \tag{5}$$

Eq. (5) relates amplitudes in one layer to the next adjacent layer and therefore repeated application of transfer matrix allows us to propagate waves from one side of the multilayer system to the other using

$$\begin{pmatrix} E_1 \\ H_1 \end{pmatrix} = \prod_{j=2}^{N-1} \begin{pmatrix} \cos\phi_j & i\sin\phi_j/Z_j \\ Z_j i\sin\phi_j & \sin\phi_j \end{pmatrix} \begin{pmatrix} E_N \\ H_N \end{pmatrix}. \tag{6}$$

And the characteristic matrix[2] for the entire system will be

$$M = \begin{pmatrix} m_{11} & m_{12} \\ m_{21} & m_{22} \end{pmatrix} = \prod_{j=1}^{N-1} \begin{pmatrix} \cos\phi_j & i\sin\phi_j/Z_j \\ Z_j i\sin\phi_j & \sin\phi_j \end{pmatrix}. \tag{7}$$

Here, $Z_j = \sqrt{\frac{\epsilon_0}{\mu_0}} n_j \cos\theta_j$ and by applying the boundary conditions for figure (3), we have

$$Z_0 = \sqrt{\frac{\epsilon_0}{\mu_0}} n_0 \cos\theta_i \quad \text{and} \quad Z_N = \sqrt{\frac{\epsilon_0}{\mu_0}} n_N \cos\theta_t. \tag{8}$$

Consequently,

$$r = \frac{Z_1 m_{11} + Z_1 Z_N m_{12} - m_{21} - Z_N m_{22}}{Z_1 m_{11} + Z_1 Z_N m_{12} + m_{21} + Z_N m_{22}} \tag{9}$$

$$t = \frac{2Z_1}{Z_1 m_{11} + Z_1 Z_N m_{12} + m_{21} + Z_N m_{22}} \tag{10}$$

To find r or t for any configuration of multilayered system, we only need to compute the characteristic matrices for each film, multiply them and substitute resulting matrix elements into the Eqs. (9) and (10).

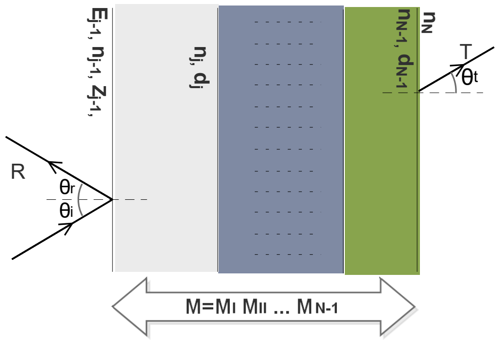


Fig. 3. Propagation of optical beam through a multilayer structure consisting of the materials with different indices of refraction.

To find reflection and transmission coefficients, we have

$$R = rr' \quad \text{and} \quad T = tt', \tag{11}$$

where r' and t' are the complex conjugates of r and t.

2. ALGORITHM

We implement the matrix transformation method via MATLAB. Syntax of such function is

$$[\theta_{\text{incident}}, R_s, R_p, T_s, T_p] = \mathit{MultiLayerFilm}(n_{1\to N}, d_{2\to K}, \theta_{\text{incident}}, \lambda.]$$

Here *MultiLayerFilm* is the MATLAB function whose algorithm is shown in algorithm (1), $n_{1\to N}, d_{2\to K}, \theta_{\text{incident}}, \lambda$ are input arguments and function gives output values.

Algorithm 1. $\mathit{MultiLayerFilm}(n_{1\to N}, d_{2\to K}, \theta_{Incident}, Lambda)$

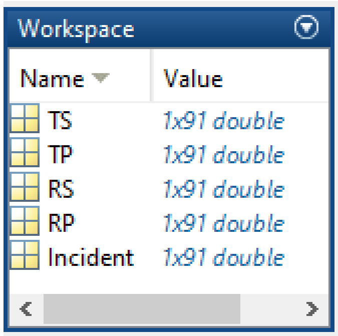
```
for $\theta_i := \theta^*_{Incident}$ do
    $\theta_{i_{1\to N}} = SneilsLaw(n_{1\to N}, \theta_i)$
    $\phi_{2\to K} = n_{2\to K} d_{2\to K} \frac{2\pi}{\lambda}$        ▷ here $K = N - 1$
    $Z_s = \sqrt{\frac{\epsilon_0}{\mu_0}} n_{1\to N} \cos\theta_{1\to N}$
    $m_s = Matrix(\phi_{2\to K}, Z_s)$
    $[R_s, T_s] = RT(m_s, Z_{s1}, Z_{sN})$
    $Z_p = \sqrt{\frac{\epsilon_0}{\mu_0}} n_{1\to N} / \cos\theta_{1\to N}$
    $m_p = Matrix(\phi_{2\to K}, Z_{)}$
    $[R_p, T_p] = RT(m_p, Z_{p1}, Z_{pN})$
End.
rtplot($\theta^*_{Incident}, R_s, R_p, T_s, T_p$)
```

$^*\theta_{Incident}$ is an array.
User defined functions are in boldface.

Let we have two layers of thickness 1 mm and 0.2 mm separated with distance of 0.3 mm having refractive indices 1.4, 1.5, wavelength 1547 nm. Now we want to find R_s, R_p, T_s, T_p for incident angles from 0° to 90° with the following function.

```
% n (Glass) = 1.47
% n (Borosilicate Glass) = 1.5007
% d (Glass) = 1mm
% d (Air) = 0.302 mm
% d (Borosilicate Glass) = 2.228 mm
[Incident,RS,RP,TS,TP] = ...
    MultiLayerFilm([1 1.47 1.5007 1],[1e−3 0.302e−3
        2.228e−3],0:90,1547e−9);
```

This generate arrays `Incident`, `RS`, `RP`, `TS`, `TP` corresponding to $\theta_{\text{incident}}, R_s, R_p, T_s, T_p$, as shown in figure 4



Name	Value
TS	1x91 double
TP	1x91 double
RS	1x91 double
RP	1x91 double
Incident	1x91 double

Fig. 4. Workspace of MATLAB showing output values of function *MultiLayerFilm*.

3. EXPERIMENTAL SETUP

Experimental setup is shown in Figure 7. We used two transparent materials of thickness 1 mm, 2.288 mm having refractive indices 1.47, 1.5007 (at $\lambda = 1547$ nm) with separation of 0.302 mm and placed on a rotating table. Optical power sensor that can be rotated along the table to measure transmitted/reflected beam. Actual setup is shown in Figure 6 and measured reflection and transmission coefficients for θ_i from 1° to 90° and measured corresponding transmission and reflection power amplitudes, for both s and p polarized optical beams.

4. RESULTS AND DISCUSSION

Figure 5 show results of our experimentally measured transmission/reflection intensities, measured for different incident angles $\theta_i(0° \to 90°)$. Here lines represent theoretical plots generated by algorithm (1) and stars represent experimental measurements which exactly matches to the observations based on MATLAB algorithm.

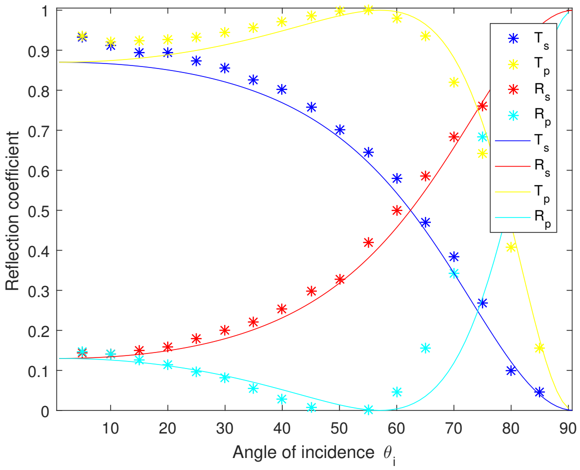


Fig. 5. Power coefficients, lines show theoretical and stars show experimental results.

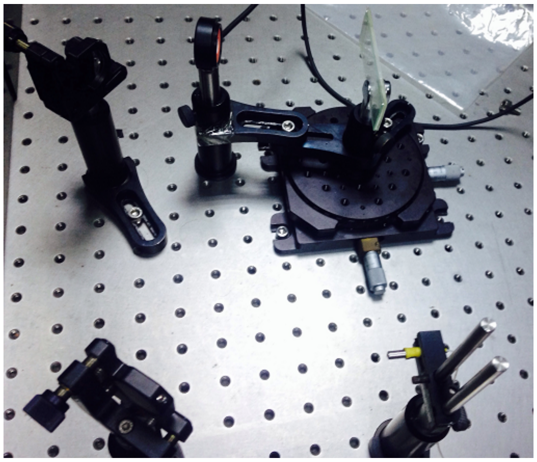


Fig. 6. Screen shot of experimental setup

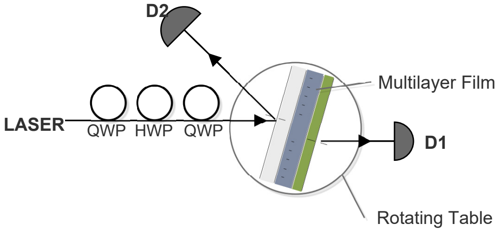


Fig. 7. Schematic diagram of experimental setup.

5. APPENDIX

A. MATLAB Functions

A.1. MultiLayerFilm

```
function [Incident,RS,RP,TS,TP] = MultiLayerFilm(n,d,Incident,Lambda)
BOUNDRY = length(n)-1;
INC=Incident;
for INCIDENCE=Incident
%% Calculation of angles on boundries starting from first
Theta=SnellsLaw(n,INCIDENCE);
%%
Phi(2:BOUNDRY) = n(2:BOUNDRY).*d(1:BOUNDRY-1).*(2.*pi./(Lambda));
Z_s = (2.6544e-3).*n(1:BOUNDRY+1).*cosd(Theta(1:BOUNDRY+1));
Z_p = (2.6544e-3).*n(1:BOUNDRY+1)./cosd(Theta(1:BOUNDRY+1));
m1=Matrix(Phi,Z_s);
[R_s(INCIDENCE+1),T_s(INCIDENCE+1)]=R_T(m1,Z_s(1),Z_s(BOUNDRY+1));
m2=Matrix(Phi,Z_p);
[R_p(INCIDENCE+1),T_p(INCIDENCE+1)]=R_T(m2,Z_p(1),Z_p(BOUNDRY+1));
end
RS=R_s;,RP=R_p;,TS=T_s;,TP=T_p;
rtplot(Incident,R_s,R_p,T_s,T_p);
end
```

A.2. Matrix

```
function m = Matrix(Phi,Z)
Limit=length(Phi);
M1=[1 0;0 1];
for j=2:Limit
    M{j} = [cosd(Phi(j)) (1i*sind(Phi(j)))/Z(j) ; 1i*Z(j)*sind(Phi(j)) cosd(Phi(j))];
    M1=M1*M{j};
end
m=M1;
end
```

A.3. R_T

```
function [R,T] = R_T(m,Z_o,Z_s)
r=(Z_o*m(1,1)+Z_o*Z_s*m(1,2)-m(2,1)-Z_s*m(2,2))/...
           (Z_o*m(1,1)+Z_o*Z_s*m(1,2)+m(2,1)+Z_s*m(2,2));
t=(2*Z_o)/...
```

```
            (Z_o*m(1,1)+Z_o*Z_s*m(1,2)+m(2,1)+Z_s*m
                (2,2));
T=abs(t.*t');
R=abs(r.*r');
end
```

A.4. norm2unity

```
function norm = norm2unity(input)
norm=(input-min(input))/(max(input)-min(input))
end
```

A.5. rtplot

```
function Output = rtplot(Incident,R_s,R_p,T_s,T_p)
Incident=Incident+1;
plot(Incident,T_s,'r');
hold on
plot(Incident,R_s,'r');
hold on
plot(Incident,T_p,'b');
plot(Incident,abs(R_p),'b');
hold off
%[K,I] = min(R_p);
%X_b=I.*(ones(1,length(Incident)));
%plot(X_b,R_p,'--');
end
```

A.6. SnellsLaw

```
function angles = SnellsLaw(n, incident)
Theta=zeros(1,length(n));
Theta(1)=incident;
for j=2:length(n)
    Theta(j) = asind((n(j-1)/n(j))*sind(Theta(j-1)));
end
angles=Theta;
end
```

REFERENCES

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2. E. Hecht and E. Hecht, *9. Interference* (Pearson Education, 2017), p. 441–446.