$\mathcal{H}$: Hilbert space

$\mathfrak{A}$: sub C^*-alg of $\mathfrak{B}(\mathcal{H})$

$\mathfrak{h}_2 = \mathcal{L}_2(\mathcal{H})$: Hilbert - Schmidt space

($\mathcal{L}_2(\mathcal{H}) = \{ A \in \mathfrak{B}(\mathcal{H}) ; \ \mathrm{Tr}\, A^*A < \infty$)

$$\begin{array}{rccc} \ell : & \mathfrak{A} & \longrightarrow & \mathfrak{B}(\mathfrak{h}_2) \\ & \rotatebox{90}{$\in$} & & \rotatebox{90}{$\in$} \\ & A & \longmapsto & \ell(A) \end{array}$$

$$\begin{array}{rccc} \ell(A) : & X & \to & AX \\ & \rotatebox{-90}{$\in$} & & \rotatebox{-90}{$\in$} \\ & \mathfrak{h}_2 & & \mathfrak{h}_2 \end{array}$$: bounded. on $\mathfrak{h}_2$

Note $\quad \ell(A)^* = \ell(A^*)$

ρ : density operator.

$T_\rho = \rho^{1/2}$

$\mathcal{H}_\rho := \overline{\ell(\mathfrak{A})\, T_\rho}$

Prop $\quad \rho$: inj.

$\mathfrak{B}_{fin}(\mathcal{H}) = \{$ finite rank opr on $\mathcal{H} \} \subset \mathfrak{A}$

Then $\quad \mathcal{H}_\rho = \mathcal{L}_2(\mathcal{H})$