$C : \mathfrak{h} \to \mathfrak{h}$: anti linear operator

satisfying $\|C\psi\| = \|\psi\|$, $\psi \in \mathfrak{h}$, $C^2 = I$.

we have $\langle C\psi, C\phi \rangle = \langle \phi, \psi \rangle$.

$\boxed{\text{Theorem}}$

There is a unique unitary operator

$$I_C : \mathcal{L}_2(\mathfrak{h}) \to \mathfrak{h} \otimes \mathfrak{h}$$

s.t.

$$I_C(|\phi\rangle\langle\psi|) = \phi \otimes C\psi, \quad \psi, \phi \in \mathfrak{h}$$

☺ Let $\{e_n\}_{n=1}^{\infty}$ be CONS of $\mathfrak{h}$.

Then $\{Ce_n\}_{n=1}^{\infty}$ is also CONS of $\mathfrak{h}$;

and hence $\{e_m \otimes Ce_n\}$ is CONS of $\mathfrak{h} \otimes \mathfrak{h}$.

So there exists $\exists$ a unique unitary operator

$I_C : \mathcal{L}_2(\mathfrak{h}) \to \mathfrak{h} \otimes \mathfrak{h}$ s.t. $I_C(|e_m\rangle\langle e_n|) = e_m \otimes Ce_n$.

$\forall \psi, \phi \in \mathfrak{h}$. $\quad |\phi\rangle = \sum \langle e_m|\phi\rangle \cancel{e_m} |e_m\rangle$, $|\psi\rangle = \sum \langle e_n|\psi\rangle |e_n\rangle$

$$|\phi\rangle\langle\psi| = \sum_{n,m} \langle e_m|\phi\rangle \langle\psi|e_n\rangle |e_n\rangle\langle e_n|$$

$$I_C(|\phi\rangle\langle\psi|) = \sum_{n,m} \langle e_m|\phi\rangle \langle\psi|e_n\rangle \, e_m \otimes Ce_n$$

$$= \sum_m \langle e_m|\phi\rangle e_m \otimes \sum_n C(\langle e_n|\psi\rangle e_n$$

$$= \phi \otimes C\psi$$