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$\Omega = \{\omega_1, \omega_2, \cdots, \omega_r\}$ 事象系

P : Ω 上の確率

$P(\omega_1), \cdots, P(\omega_r)$

$\Omega \supset A = \{\omega_{i_1}, \cdots, \omega_{i_s}\}$

$P(A) = P(\omega_{i_1}) + \cdots + P(\omega_{i_s})$

$P(\Omega) = P(\omega_1) + \cdots + P(\omega_r) = 1$

例 サイコロ ω_i :「iの目がでる」

$P(\omega_i)$: iの目がでる確率

$A = \{\omega_2, \omega_4, \omega_6\}$

$P(A) = P(\omega_2) + P(\omega_4) + P(\omega_6)$: 偶数の目がでる確率

$B = \{\omega_1, \omega_3, \omega_5\}$

$P(B)$: 奇数の目がでる確率

例. サイコロ

$1 = X(\omega_1) \longleftarrow \omega_1 \longrightarrow \frac{1}{12} = P(\omega_1)$

$2 = X(\omega_2) \longleftarrow \omega_2 \longrightarrow \frac{1}{6} = P(\omega_2)$

$1 = X(\omega_3) \longleftarrow \omega_3 \longrightarrow \frac{1}{12} = P(\omega_3)$

$2 = X(\omega_4) \longleftarrow \omega_4 \longrightarrow \frac{1}{3} = P(\omega_4)$

$1 = X(\omega_5) \longleftarrow \omega_5 \longrightarrow \frac{1}{6} = P(\omega_5)$

$2 = X(\omega_6) \longleftarrow \omega_6 \longrightarrow \frac{1}{6} = P(\omega_6)$

$P_X(1) =$ Xの値が1となる確率 $P(\omega_1) + P(\omega_3) + P(\omega_5) = P(B)$

$P_X(2) =$ Xの値が2となる確率 $P(\omega_2) + P(\omega_4) + P(\omega_6) = P(A)$

$P_X(1) + P_X(2) = 1.$

Xの期待値 $1 \cdot P_X(1) + 2 \cdot P_X(2)$

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$1\cdot p(\omega_1)+2p(\omega_2)+1\cdot p(\omega_3)+2p(\omega_4)+1\cdot p(\omega_5)+2p(\omega_6)$

$= 1\cdot(\underbrace{p(\omega_1)+p(\omega_3)+p(\omega_5)}_{=P_X(1)}) + 2(\underbrace{p(\omega_2)+p(\omega_4)+p(\omega_6)}_{=P_X(2)})$

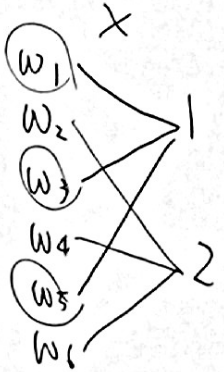
$= 1\cdot P_X(1)+2\cdot P_X(2)$

$B = \{X$の値が1となる事象$\} = \{\omega_1, \omega_3, \omega_5\}$

$= \{\omega \in \Omega \,;\, X(\omega)=1\}$

$= X^{-1}(1)$

$A = X^{-1}(2)$



$P_X(1)=P(X^{-1}(1)),\ P_X(2)=P(X^{-1}(2))$

$\boxed{\Omega=\{\omega_1,\cdots,\omega_r\},\quad P:\Omega\text{上の確率}}$

$X:\Omega$ 上の確率変数

$P_X(a) = P(X^{-1}(a))$

Xの確率分布

$1 = Y(\omega_1) \xleftarrow{Y} \omega_1 \xrightarrow{P} \frac{1}{12}$

$1 = Y(\omega_2) \leftarrow \omega_2 \rightarrow \frac{1}{6}$

$1 = Y(\omega_3) \leftarrow \omega_3 \rightarrow \frac{1}{12}$

$2 = Y(\omega_4) \leftarrow \omega_4 \rightarrow \frac{1}{3}$

$2 = Y(\omega_5) \leftarrow \omega_5 \rightarrow \frac{1}{6}$

$2 = Y(\omega_6) \leftarrow \omega_6 \rightarrow \frac{1}{6}$

$P_Y(1)=P(Y^{-1}(1))$

$=P(\{\omega_1,\omega_2,\omega_3\})$

$=\frac{1}{12}+\frac{1}{6}+\frac{1}{12}=\frac{4}{12}=\frac{1}{3}$

$P_Y(2)=P(Y^{-1}(2))$

$=P(\{\omega_4,\omega_5,\omega_6\})$

$=\frac{1}{3}+\frac{1}{6}+\frac{1}{6}=\frac{2}{3}$

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$\Omega = \{\omega_1, \omega_2\}$

表 裏

$P(\omega_1)$, $P(\omega_2)$: 確率

積事象 Ω^2	確率
$\omega'_1 = (\omega_1, \omega_1)$	$P(\omega_1)P(\omega_1) = P(\omega'_1)$
$\omega'_2 = (\omega_1, \omega_2)$	$P(\omega_1)P(\omega_2) = P(\omega'_2)$
$\omega'_3 = (\omega_2, \omega_1)$	$P(\omega_2)P(\omega_1) = P(\omega'_3)$
$\omega'_4 = (\omega_2, \omega_2)$	$P(\omega_2)P(\omega_2) = P(\omega'_4)$

積事象 $\Omega \times \Omega = \Omega^2$

Ω^N

$N = 2$, $\Omega = \{\omega_1, \omega_2\}$

$\hat{\theta}$: 確率変数

推定量

	Ω^2: 入力	出力
$\theta^2 = P(\omega'_1) \leftarrow$	$\omega'_1 = (\omega_1, \omega_1)$	$1 = \hat{\theta}(\omega'_1)$
$\theta(1-\theta) = P(\omega'_2) \leftarrow$	$\omega'_2 = (\omega_1, \omega_2)$	$1/2 = \hat{\theta}(\omega'_2)$
$(1-\theta)\theta = P(\omega'_3) \leftarrow$	$\omega'_3 = (\omega_2, \omega_1)$	$1/2 = \hat{\theta}(\omega'_3)$
$(1-\theta)^2 = P(\omega'_4) \leftarrow$	$\omega'_4 = (\omega_2, \omega_2)$	$0 = \hat{\theta}(\omega'_4)$

確率変数 $\hat{\theta}$ の 確率分布

$$P_{\hat{\theta}}(x) = P(\hat{\theta}^{-1}(x))$$

$$P_{\hat{\theta}}(1) = P(\hat{\theta}^{-1}(1)) = P(\omega'_1) = \theta^2$$

$$P_{\hat{\theta}}(1/2) = P(\hat{\theta}^{-1}(1/2)) = P(\{\omega'_2, \omega'_3\}) = \theta(1-\theta) + (1-\theta)\theta = 2\theta(1-\theta)$$

$$P_{\hat{\theta}}(0) = (1-\theta)^2$$

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推定量 $\hat{\Theta}$ は確率変数である

よって $\hat{\Theta}$ に対する期待値が計算できて

$$E[\hat{\Theta}] = 1 \cdot P_{\hat{\Theta}}(1) + \frac{1}{2} P_{\hat{\Theta}}\left(\frac{1}{2}\right) + 0 \cdot P_{\hat{\Theta}}(0)$$

$$= 1 \cdot \theta^2 + \frac{1}{2} \cdot 2\theta(1-\theta)$$

$$= \theta^2 + \theta - \theta^2 = \theta$$

$$E_\theta[\hat{\Theta}] = \sum_{(x_1, \cdots, x_N) \in \Omega^N} P_\theta^N(x_1, \cdots, x_N) \hat{\Theta}(x_1, \cdots, x_N)$$

$$= \sum_a a \, P_{\hat{\Theta}}(a)$$

ここで $P_{\hat{\Theta}}(a) = P(\hat{\Theta}^{-1}(a))$