Wedderburn theorem

Every finite dimensional C^*-alg. $\mathfrak{A}$ is $*$-isomorphic to the direct sum of full matrix algebras.

$$\mathfrak{A} \simeq \mathfrak{M}_{n_1} \oplus \cdots\cdots \oplus \mathfrak{M}_{n_k}$$

In particular, every finite dimensional C^*-alg is unital.

If $\phi : \mathfrak{A} \to \mathfrak{B}$ is linear (bij) and satisfies $\phi(A^*) = \phi(A)^*$ and $\phi(AB) = \phi(A)\phi(B)$, ϕ is called $*$-isom.

Assume $\mathfrak{A} \ni I$, (ϕ is $*$-isom and) then $\phi(AI) = \phi(A)\phi(I)$ for $\forall A \in \mathfrak{A}$.

$$\phi(AI) = \phi(A)$$

This means $\Phi(I)$ gives the identity of $\mathfrak{B}$.

○ $\omega : \mathfrak{A} \to \mathbb{C}$ ($I_{\mathfrak{A}} \mapsto$) is state, $\phi : \mathfrak{A} \to \mathfrak{B}$ is $*$-isom.

- ω is linear
- $\omega(A^*A) \geq 0$ (Then we have $\omega(A^*) = \omega(A)^*$)
- $\omega(I_{\mathfrak{A}}) = 1$.

Then $\omega(\phi^{-1}(B))$ gives a state.

$\because$ Since $I_{\mathfrak{B}} = \phi(I_{\mathfrak{A}})$, $\omega(\Phi^{-1}(I_{\mathfrak{B}})) = \omega(I_{\mathfrak{A}}) = 1$

$\omega(\phi^{-1}(B^*B)) = \omega(\phi^{-1}(B)^*\phi^{-1}(B)) \geq 0$