Progress, Justness and Fairness in Modal μ-Calculus Formulae

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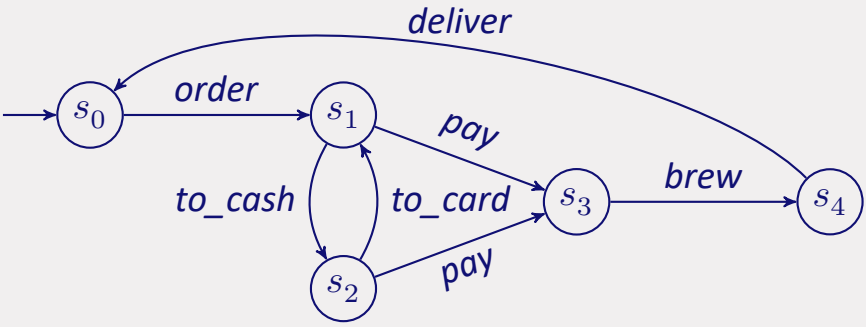
Example



- *Inevitable delivery*: when *order* occurs, inevitably *deliver* will occur

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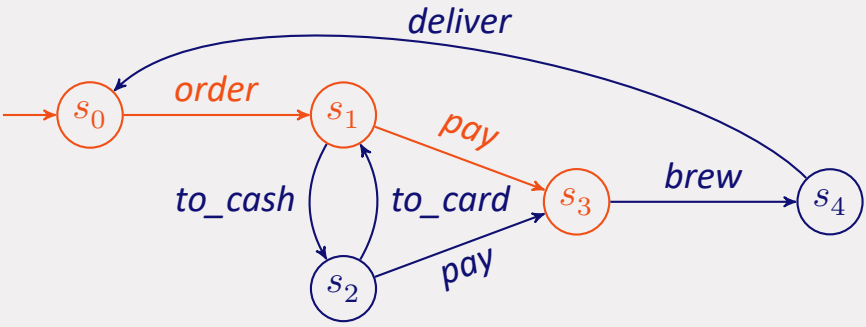

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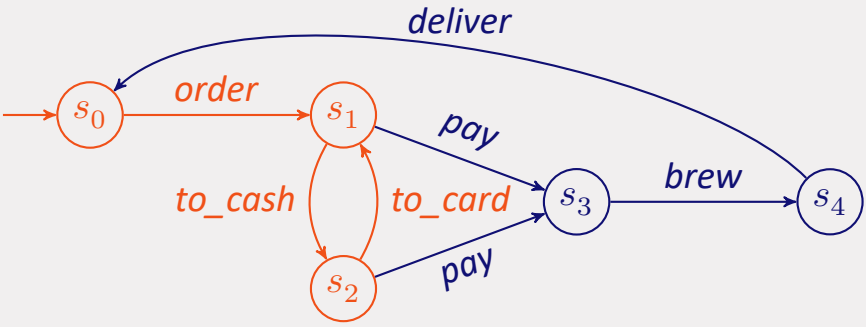

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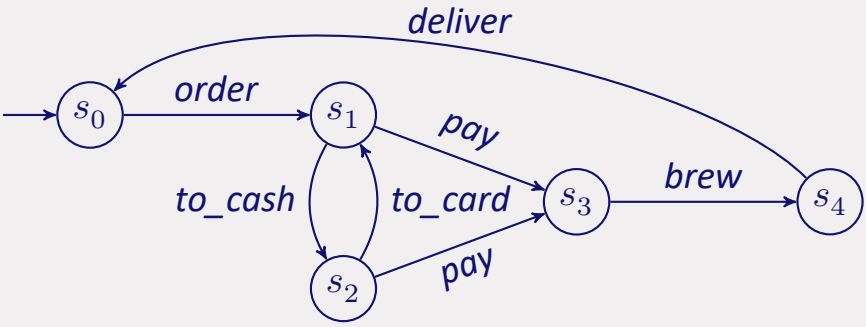

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- How to restrict verification to complete paths?
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- Syntax:

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- State-based logic
  - Interweave completeness criterion and property

Designing Formulae

1. Consider multiple properties
    - Single template for many properties
2. Consider multiple completeness criteria
    - Different template formulae for different criteria

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1. Consider multiple properties
    - Single template for many properties
2. Consider multiple completeness criteria
    - Different template formulae for different criteria
3. Proven correctness

Many Properties

- Dwyer, Avrunin, and Corbett (1999):
  Patterns in property specifications for finite-state verification

Many Properties

- Dwyer, Avrunin, and Corbett (1999):
  Patterns in property specifications for finite-state verification
- Cover subset of patterns
  - 4 behaviours $\times$ 4 scope $=$ 16 patterns
  - Only relevant (liveness) patterns

Template Property

- “A path that violates the property”

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- *Inevitable delivery*: when *order* occurs, inevitably *deliver* will occur
  - $\rho = \mathit{Act}^\star \cdot \mathit{order}$
  - $\alpha_f = \{\mathit{deliver}\}$
  - $\alpha_e = \emptyset$

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Completeness Criteria

- In presentation:

- In paper:

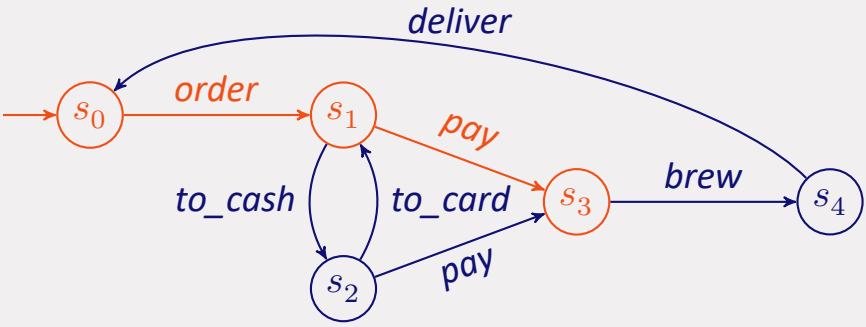
Completeness Criteria

- In presentation:
  - Progress
  - Weak fairness of actions (WFA)
- In paper:

Completeness Criteria

- In presentation:
  - Progress
  - Weak fairness of actions (WFA)
- In paper:
  - Strong fairness of actions (SFA)
  - Justness of actions (JA)
  - Weak hyperfairness of actions (WHFA)
  - Strong hyperfairness of actions (SHFA)

Progress



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Progress Formula

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$$\langle \rho \rangle$$

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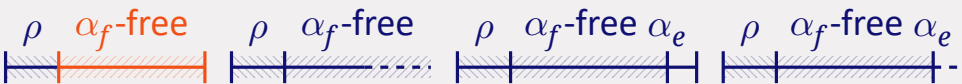

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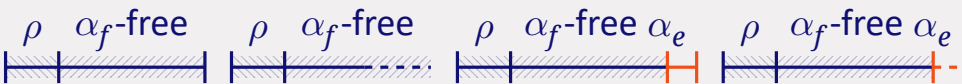

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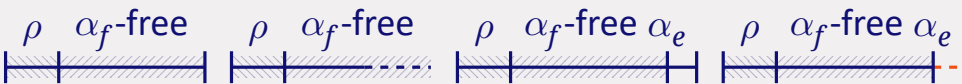

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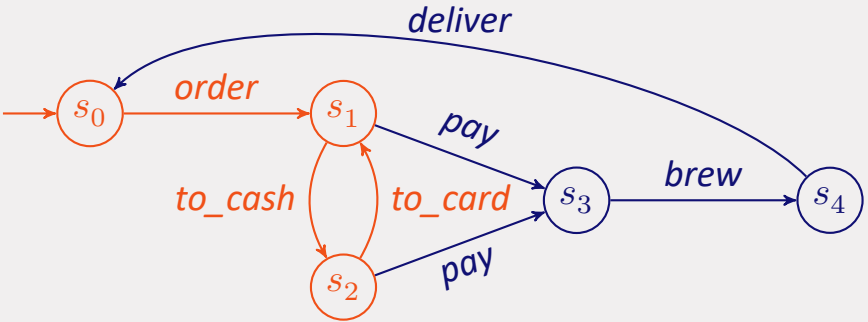
Weak Fairness of Actions



- WFA: on every suffix of a complete path, every perpetually enabled action occurs
  - Perpetually enabled: enabled in every state

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Weak Fairness Formula

- "No path that satisfies WFA, progress, and is $(\rho, \alpha_f, \alpha_e)$-violating"

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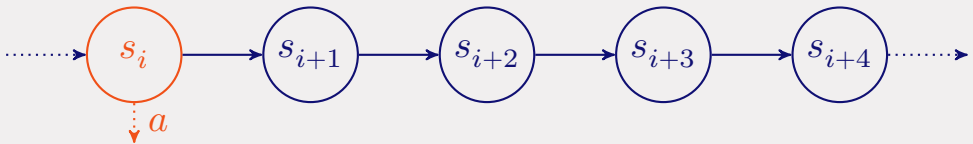

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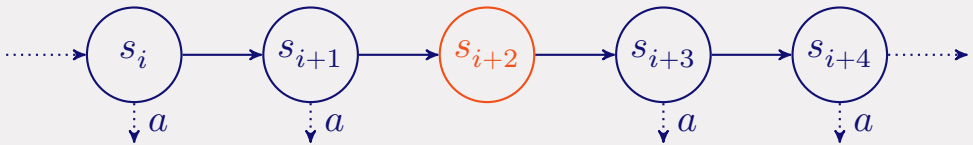

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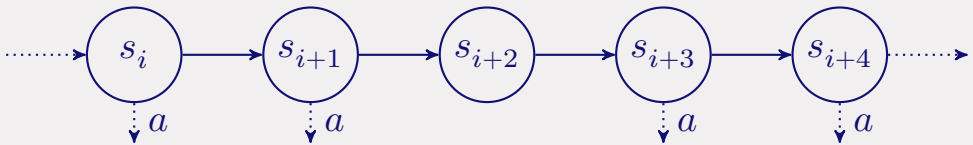

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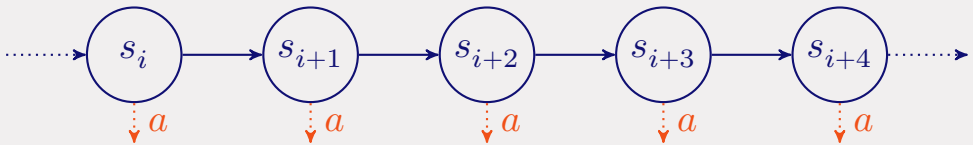

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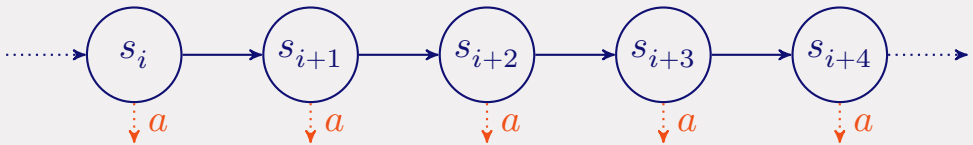

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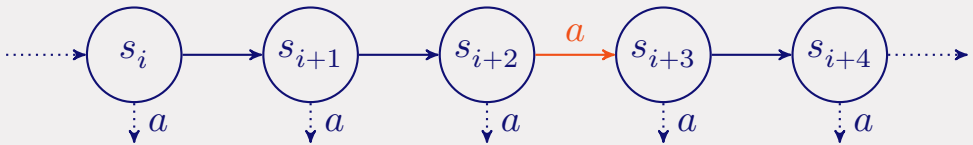

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Strong Fairness, Briefly

- “If infinitely often enabled, occur infinitely often”
- Formula:

$$\neg\langle \rho \cdot \overline{\alpha_f}^{\star} \rangle (\langle \alpha_e \rangle tt \vee [Act]ff \vee \bigvee_{\emptyset \neq F \subseteq Act} \nu X.(\bigwedge_{a \in F} \mu W.([\overline{F}]ff \wedge (\langle a \setminus \alpha_f \rangle X \vee \langle \overline{\alpha_f} \rangle W))))$$

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- Dividing actions into two sets
  - Exponential

Contributions

- Definition of $(\rho, \alpha_f, \alpha_e)$-violating paths
- Template formulae for multiple completeness criteria
  - Progress
  - Weak fairness of actions
  - Strong fairness of actions
  - Justness of actions
  - Weak hyperfairness of actions
  - Strong hyperfairness of actions
- With blocking actions
- Characterisation of finitely realisable path predicates
- Correctness proofs

Future Work

- Formalisation of proofs
- More completeness criteria and more properties
  - E.g. fairness over sets of actions, state properties
- Verifications with formulae