Fairness Assumptions in the Modal μ-Calculus

Myrthe Spronck

Supervisor: Tim Willemse

September 7, 2023

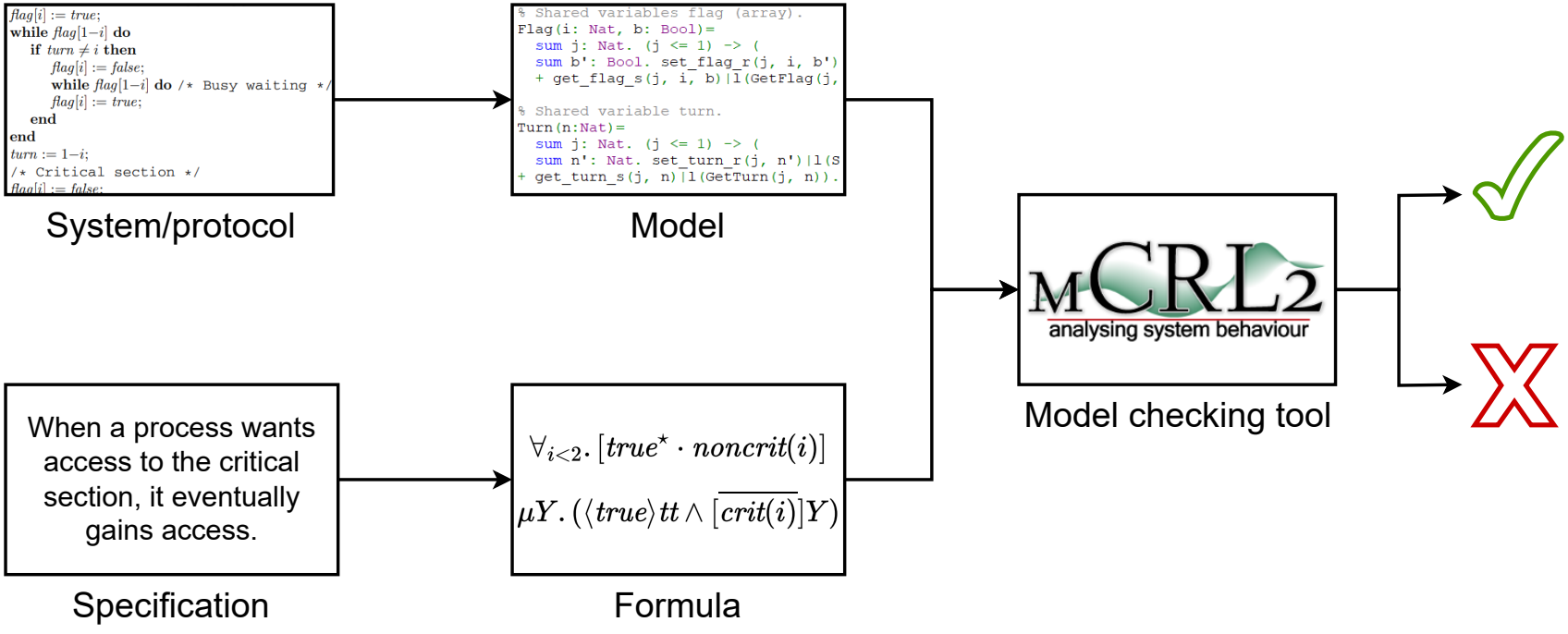
Motivation

Motivation

- Model checking

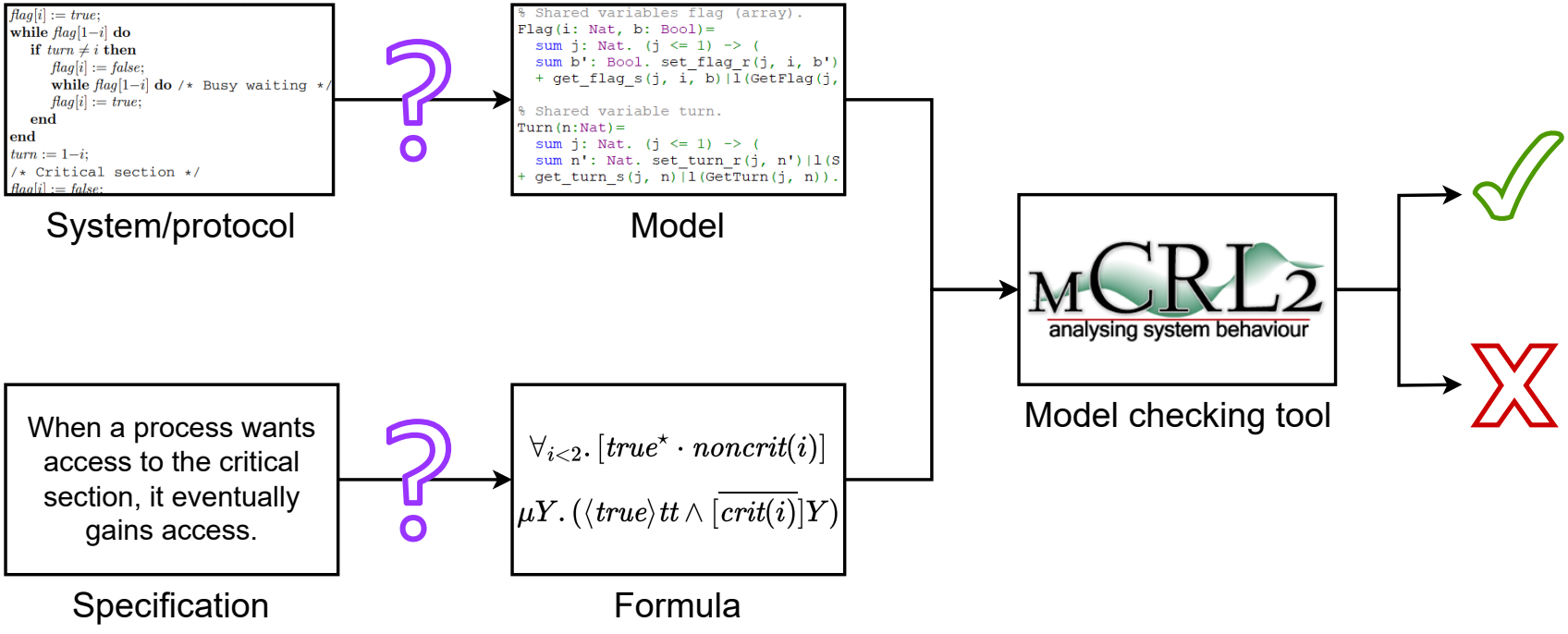
Motivation

- Model checking



Motivation

- Model checking



Motivation

Motivation

- Question: did I design my formula correctly?

Motivation

- Question: did I design my formula correctly?
- The modal μ-calculus:

Motivation

- Question: did I design my formula correctly?
- The modal μ-calculus:
  - Used in mCRL2

Motivation

- Question: did I design my formula correctly?
- The modal μ-calculus:
  - Used in mCRL2
  - Highly expressive

Motivation

- Question: did I design my formula correctly?
- The modal μ-calculus:
  - Used in mCRL2
  - Highly expressive
  - Difficult to understand formulae

Motivation

- Question: did I design my formula correctly?

Motivation

- Question: did I design my formula correctly?
- Consider specific types of formulae

Motivation

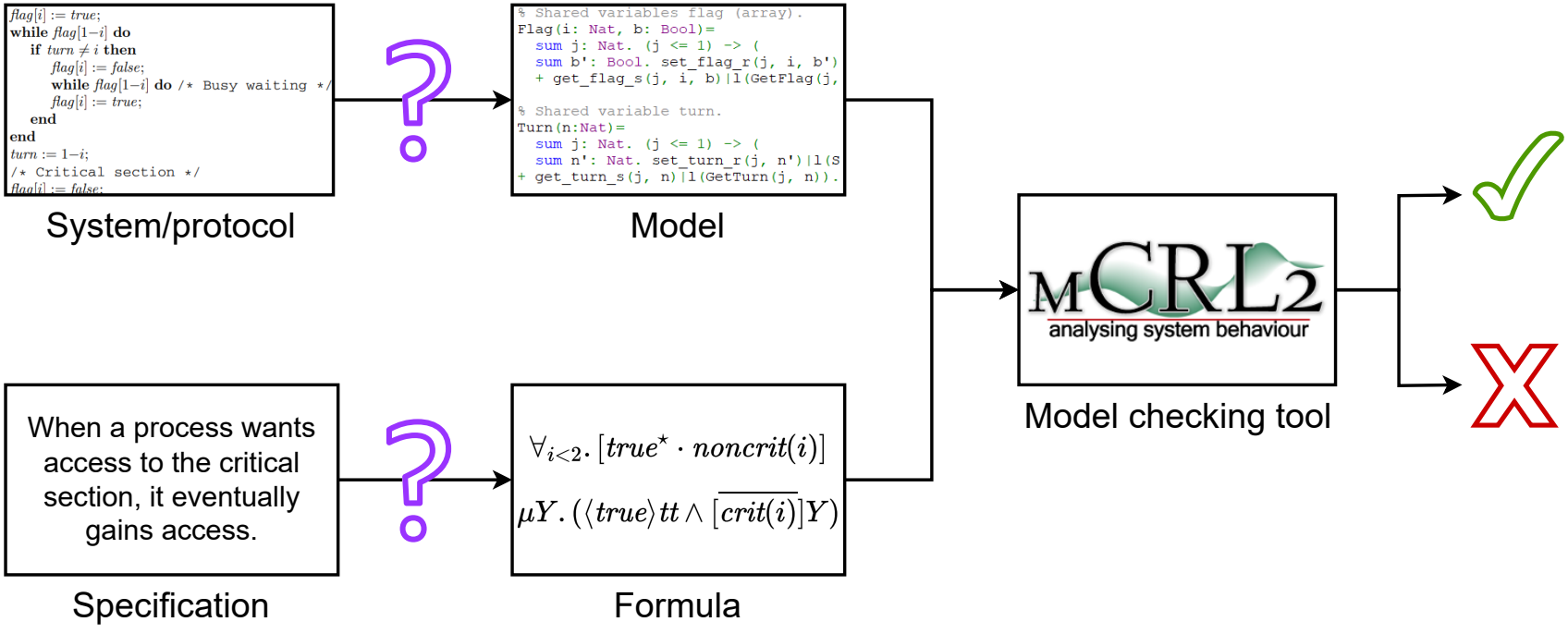
- Question: did I design my formula correctly?
- Consider specific types of formulae
  - Commonly required

Motivation

- Question: did I design my formula correctly?
- Consider specific types of formulae
  - Commonly required
  - Non-trivial

Motivation: Fairness

Motivation: Fairness



Motivation: Fairness

Motivation: Fairness

- Model abstracts reality

Motivation: Fairness

- Model abstracts reality
  - Unrealistic executions included in model

Motivation: Fairness

- Model abstracts reality
  - Unrealistic executions included in model
  - Property may be violated in model but not reality

Motivation: Fairness

- Model abstracts reality
  - Unrealistic executions included in model
  - Property may be violated in model but not reality
- Fairness assumptions address this

Motivation: Fairness

- Model abstracts reality
  - Unrealistic executions included in model
  - Property may be violated in model but not reality
- Fairness assumptions address this
  - Fair and unfair executions

Motivation: Fairness

- Model abstracts reality
  - Unrealistic executions included in model
  - Property may be violated in model but not reality
- Fairness assumptions address this
  - Fair and unfair executions
  - Properties are satisfied if all violations are unfair

Motivation: Which Formulae

Motivation: Which Formulae

- Properties under fairness assumptions

Motivation: Which Formulae

- Properties under fairness assumptions
  - Which properties?

Motivation: Which Formulae

- Properties under fairness assumptions
  - Which properties?
  - Which fairness assumptions?

Interlude: LTS

Interlude: LTS

- For models: labelled transition systems

Interlude: LTS

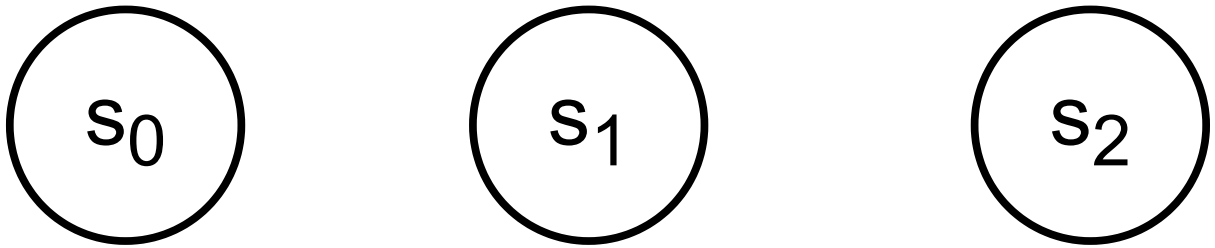
- For models: labelled transition systems
- Consists of:

Interlude: LTS

- For models: labelled transition systems
- Consists of:
  -
  -
  -
  -

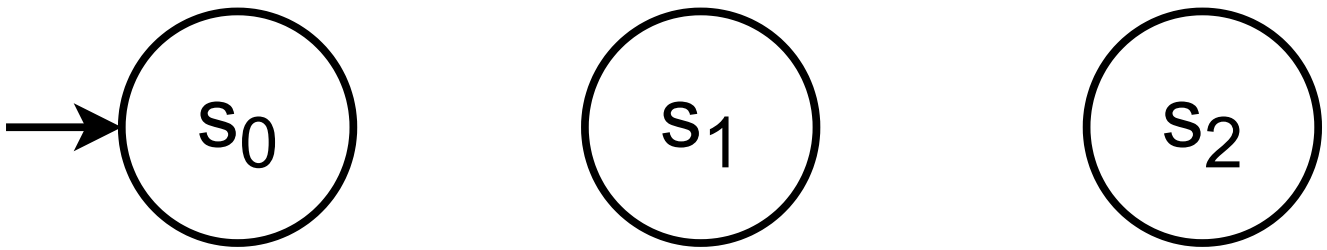
Interlude: LTS

- For models: labelled transition systems
- Consists of:
  - States
  -
  -
  -



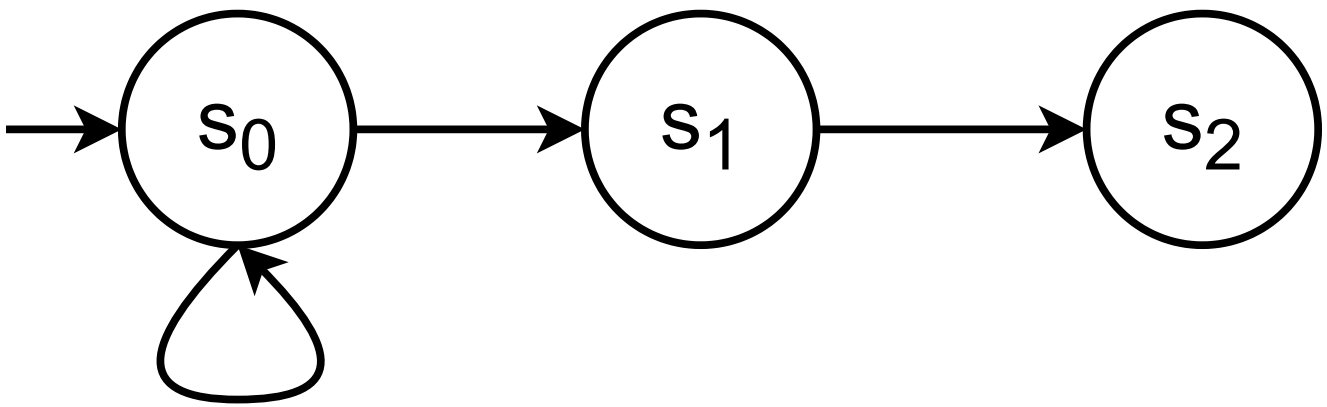
Interlude: LTS

- For models: labelled transition systems
- Consists of:
  - States
  - Initial state
  -
  -



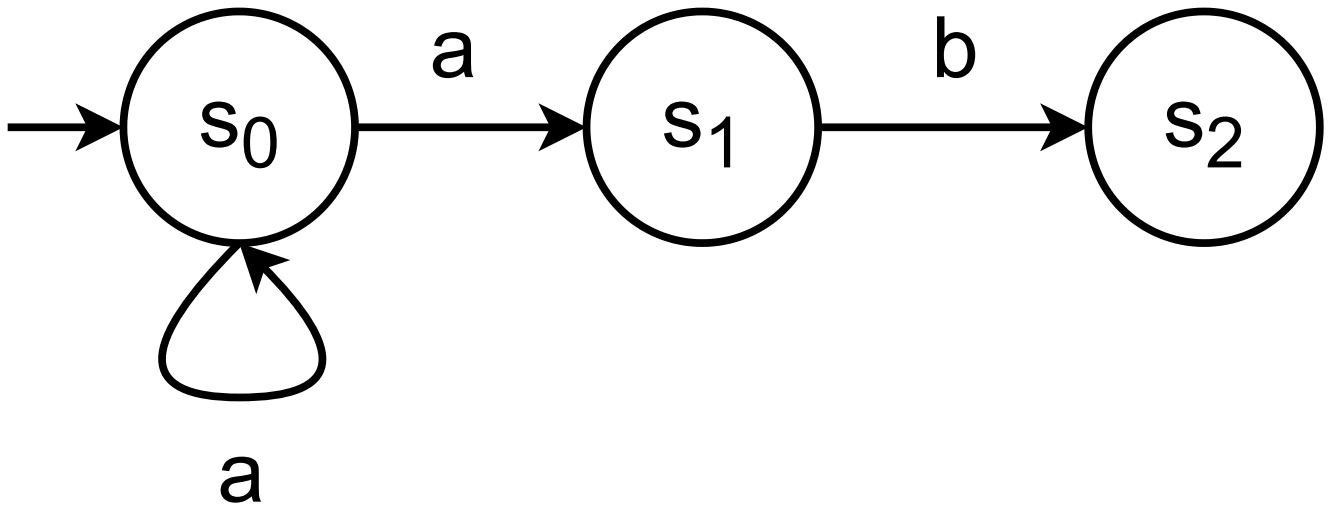
Interlude: LTS

- For models: labelled transition systems
- Consists of:
  - States
  - Initial state
  - Transitions
  -



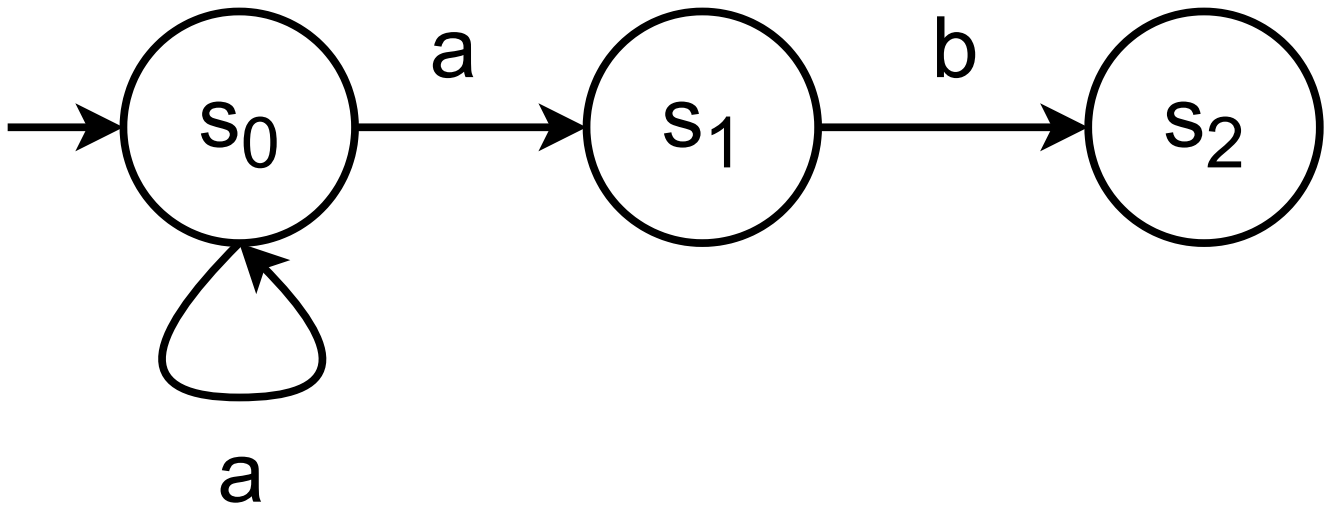
Interlude: LTS

- For models: labelled transition systems
- Consists of:
  - States
  - Initial state
  - Transitions
  - Actions



Interlude: LTS

- For models: labelled transition systems
- Consists of:
  - States
  - Initial state
  - Transitions
  - Actions



- Finite systems

Which Properties

Which Properties

- *Property specification patterns* (PSP) by Dwyer, Avrunin and Corbett (1999)

Which Properties

- *Property specification patterns* (PSP) by Dwyer, Avrunin and Corbett (1999)
  - Most practical properties covered

Which Properties

- *Property specification patterns* (PSP) by Dwyer, Avrunin and Corbett (1999)
  - Most practical properties covered
- Combine *behaviour* with *scope*

Which Properties

- *Property specification patterns* (PSP) by Dwyer, Avrunin and Corbett (1999)
  - Most practical properties covered
- Combine *behaviour* with *scope*
  - Behaviour: “*r* action never occurs” (absence), “every *q* is eventually followed by an *r*” (response)

Which Properties

- *Property specification patterns* (PSP) by Dwyer, Avrunin and Corbett (1999)
  - Most practical properties covered
- Combine *behaviour* with *scope*
  - Behaviour: “*r* action never occurs” (absence), “every *q* is eventually followed by an *r*” (response)
  - Scope: “throughout entire path” (global), “after first *a* action” (after-first)

Which Assumptions

Which Assumptions

- General shared shape

Which Assumptions

- General shared shape
  - “if something is possible often enough, it occurs often enough”

Which Assumptions

- General shared shape
  - “if something is possible often enough, it occurs often enough”
- Many different fairness assumptions in the literature

Which Assumptions

- General shared shape
  - “if something is possible often enough, it occurs often enough”
- Many different fairness assumptions in the literature
- Focus on three:

Which Assumptions

- General shared shape
  - “if something is possible often enough, it occurs often enough”
- Many different fairness assumptions in the literature
- Focus on three:
  - Weak fairness of actions (WFA)

Which Assumptions

- General shared shape
  - “if something is possible often enough, it occurs often enough”
- Many different fairness assumptions in the literature
- Focus on three:
  - Weak fairness of actions (WFA)
  - Strong fairness of actions (SFA)

Which Assumptions

- General shared shape
  - “if something is possible often enough, it occurs often enough”
- Many different fairness assumptions in the literature
- Focus on three:
  - Weak fairness of actions (WFA)
  - Strong fairness of actions (SFA)
  - Fair reachability of actions (FRA)

Project

Project

- Research questions (paraphrased)

Project

- Research questions (paraphrased)
  1. Which fairness assumptions

Project

- Research questions (paraphrased)
  1. Which fairness assumptions
  2. Single pattern under fairness

Project

- Research questions (paraphrased)
  1. Which fairness assumptions
  2. Single pattern under fairness
  3. Generalise to other properties

Project

- Research questions (paraphrased)
  1. Which fairness assumptions
  2. Single pattern under fairness
  3. Generalise to other properties
  4. Practical use in mCRL2

Global Response Formulae

Global Response Formulae

- RQ2: First design formulae for global response

Interlude: μ-calculus

Interlude: μ-calculus

$$\phi, \psi ::= tt \mid ff \mid X \mid \neg\phi \mid \phi \wedge \psi \mid \phi \vee \psi \mid \phi \Rightarrow \psi \mid$$
$$[a]\phi \mid \langle a \rangle \phi \mid \mu X.\phi \mid \nu X.\phi$$

Interlude: μ-calculus

$$\phi, \psi ::= \mathit{tt} \mid \mathit{ff} \mid X \mid \neg\phi \mid \phi \wedge \psi \mid \phi \vee \psi \mid \phi \Rightarrow \psi \mid$$
$$[a]\phi \mid \langle a \rangle \phi \mid \mu X.\phi \mid \nu X.\phi$$

- *tt* is all states, *ff* none
- X is a set of states, determined by environment
- $\neg$, $\wedge$, $\vee$ and $\Rightarrow$ as expected
- $[a]\phi$ if all a-transitions go to a state satisfying ϕ
- $\langle a \rangle \phi$ if there exists an a-transition to a state satisfying ϕ.
- $\mu X.\phi$ is the least fixpoint of ϕ, $\nu X.\phi$ the greatest
  - Rough intuition: finite and infinite recursion

Global Response Formulae

Global Response Formulae

- Pattern: “along whole path, any occurrence of q is eventually followed by an occurrence of r”

Global Response Formulae

- Pattern: “along whole path, any occurrence of q is eventually followed by an occurrence of r”
- Without fairness: $[\mathit{true}^\star \cdot q]\mu Y.(\langle \mathit{true} \rangle \mathit{tt} \wedge [\overline{r}]\, Y)$

Global Response Formulae

- Pattern: “along whole path, any occurrence of q is eventually followed by an occurrence of r”
- Without fairness: $[\mathit{true}^\star \cdot q]\mu Y.(\langle \mathit{true} \rangle \mathit{tt} \wedge [\overline{r}]\, Y)$
- Adding fairness

Global Response Formulae: WFA

Global Response Formulae: WFA

- Weak fairness of actions (WFA): an action that is *perpetually enabled* from some point onwards must occur infinitely often

Global Response Formulae: WFA

Global Response Formulae: WFA

- “there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often”

Global Response Formulae: WFA

- “there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often”

$$\neg(\langle true^{\star} \cdot q\rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle\lambda\rangle tt \Rightarrow (\mu Y.(([\lambda]ff \wedge X) \vee \langle \lambda \setminus r\rangle X \vee \langle \overline{\lambda \cup r}\rangle Y)))))$$

Global Response Formulae: WFA

- "there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often"

$$\neg(\langle true^{\star} \cdot q \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(([\lambda] ff \wedge X) \vee \langle \lambda \setminus r \rangle X \vee \langle \overline{\lambda \cup r} \rangle Y)))))$$

Global Response Formulae: WFA

- "there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often"

$$\neg(\langle true^{\star} \cdot q \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(([\lambda] ff \wedge X) \vee \langle \lambda \setminus r \rangle X \vee \langle \overline{\lambda \cup r} \rangle Y)))))$$

Global Response Formulae: WFA

- "there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often"

$$\neg(\langle true^{\star} \cdot q \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(([\lambda] ff \wedge X) \vee \langle \lambda \setminus r \rangle X \vee \langle \overline{\lambda \cup r} \rangle Y)))))$$

Global Response Formulae: WFA

- "there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often"

$$\neg(\langle true^{\star} \cdot q \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(([\lambda] ff \wedge X) \vee \langle \lambda \setminus r \rangle X \vee \langle \overline{\lambda \cup r} \rangle Y)))))$$

Global Response Formulae: WFA

- "there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often"

$$\neg(\langle true^{\star} \cdot q \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(([\lambda] ff \wedge X) \vee \langle \lambda \setminus r \rangle X \vee \langle \overline{\lambda \cup r} \rangle Y)))))$$

- If an action is perpetually enabled, it occurs infinitely often

Global Response Formulae: WFA

- "there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often"

$$\neg(\langle true^{\star} \cdot q \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(([\lambda] ff \wedge X) \vee \langle \lambda \setminus r \rangle X \vee \langle \overline{\lambda \cup r} \rangle Y)))))$$

- If an action is perpetually enabled, it occurs infinitely often
- If an action is not perpetually enabled, it is disabled infinitely often

Global Response Formulae: WFA

- "there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often"

$$\neg(\langle true^{\star} \cdot q\rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle\lambda\rangle tt \Rightarrow (\mu Y.(([\lambda]ff \wedge X) \vee \langle \lambda \setminus r\rangle X \vee \langle \overline{\lambda \cup r}\rangle Y)))))$$

- Proof in thesis

Global Response Formulae: WFA

- "there does not exist a path, on which after a q there is never an r, and on this path every action that is eventually perpetually enabled occurs infinitely often"

$$\neg(\langle \mathit{true}^\star \cdot q \rangle \nu X.(\bigwedge_{\lambda \in \mathit{Act}} (\langle \lambda \rangle \mathit{tt} \Rightarrow (\mu Y.(([\lambda]\mathit{ff} \wedge X) \vee \langle \lambda \setminus r \rangle X \vee \langle \overline{\lambda \cup r} \rangle Y)))))$$

- Proof in thesis
- Formula design with Bas Luttik, inspired by justness formula from Bouwman, Luttik and Willemse (2020)

Global Response Formulae: FRA

Global Response Formulae: FRA

- FRA formula extremely similar to WFA

Global Response Formulae: FRA

- FRA formula extremely similar to WFA
  - Perpetually *enabled* actions vs. perpetually *reachable* actions

Global Response Formulae: FRA

- FRA formula extremely similar to WFA
  - Perpetually *enabled* actions vs. perpetually *reachable* actions

$$\neg(\langle true^{\star} \cdot q \rangle \nu X.(\bigwedge_{\lambda \in Act} \langle \lambda \rangle tt \Rightarrow (\mu Y.(([\lambda] ff \wedge X) \vee \langle \lambda \setminus r \rangle X \vee \langle \overline{\lambda \cup r} \rangle Y))))$$

Global Response Formulae: FRA

- FRA formula extremely similar to WFA
  - Perpetually *enabled* actions vs. perpetually *reachable* actions

$$\neg(\langle true^{\star} \cdot q\rangle \nu X.(\bigwedge_{\lambda \in Act} \langle true^{\star} \cdot \lambda\rangle tt \Rightarrow (\mu Y.(([true^{\star} \cdot \lambda]ff \wedge X) \vee \langle \lambda \setminus r\rangle X \vee \langle \overline{\lambda \cup r}\rangle Y))))$$

Global Response Formulae: SFA

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design
  - Not currently enabled $\Rightarrow$ not perpetually enabled (yet)

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design
  - Not currently enabled $\Rightarrow$ not perpetually enabled (yet)
  - Does not work for relentlessly enabled

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design
  - Not currently enabled $\Rightarrow$ not perpetually enabled (yet)
  - Does not work for relentlessly enabled
  - Need to know in advance which actions are relentlessly enabled

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design
  - Not currently enabled $\Rightarrow$ not perpetually enabled (yet)
  - Does not work for relentlessly enabled
  - Need to know in advance which actions are relentlessly enabled
- Consider all possible subsets

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design
  - Not currently enabled $\Rightarrow$ not perpetually enabled (yet)
  - Does not work for relentlessly enabled
  - Need to know in advance which actions are relentlessly enabled
- Consider all possible subsets
- Observation: on a strongly fair path, all actions either

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design
  - Not currently enabled $\Rightarrow$ not perpetually enabled (yet)
  - Does not work for relentlessly enabled
  - Need to know in advance which actions are relentlessly enabled
- Consider all possible subsets
- Observation: on a strongly fair path, all actions either
  - Occur infinitely often

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design
  - Not currently enabled $\Rightarrow$ not perpetually enabled (yet)
  - Does not work for relentlessly enabled
  - Need to know in advance which actions are relentlessly enabled
- Consider all possible subsets
- Observation: on a strongly fair path, all actions either
  - Occur infinitely often
  - Are perpetually disabled in a suffix

Global Response Formulae: SFA

- Strong fairness of actions (SFA): an action that is *relentlessly enabled* must occur infinitely often
- Struggled with design
  - Not currently enabled $\Rightarrow$ not perpetually enabled (yet)
  - Does not work for relentlessly enabled
  - Need to know in advance which actions are relentlessly enabled
- Consider all possible subsets
- Observation: on a strongly fair path, all actions either
  - Occur infinitely often
  - Are perpetually disabled in a suffix
- Observation also given in *Fairness* by Francez (1986)

Global Response Formulae: SFA

Global Response Formulae: SFA

$$\neg(\langle true^{\star} \cdot q \rangle(\bigvee_{F \subseteq Act} (\mu Y.(\langle \overline{r} \rangle Y \vee \nu X.inf_{SFA}(F)))))$$

Where

$$\begin{aligned} inf_{SFA}(F) &= exec_{SFA}(F, n) \\ exec_{SFA}(F, 0) &= [\overline{F}]ff \wedge X \\ exec_{SFA}(F, k+1) &= \mu W_{k+1}.([\overline{F}]ff \wedge \\ &\qquad (\langle \overline{r} \rangle W_{k+1} \vee \langle \alpha_{k+1} \setminus r \rangle exec_{SFA}(F, k))) \end{aligned}$$

With an arbitrary order on the actions in F such that α_1 is the first action, α_2 the second, etc. until α_n

Global Response Formulae: SFA

$$\neg(\langle true^{\star} \cdot q \rangle(\bigvee_{F \subseteq Act} (\mu Y.(\langle \overline{r} \rangle Y \vee \nu X.inf_{SFA}(F)))))$$

Where

$$\begin{aligned} inf_{SFA}(F) &= exec_{SFA}(F, n) \\ exec_{SFA}(F, 0) &= [\overline{F}]ff \wedge X \\ exec_{SFA}(F, k+1) &= \mu W_{k+1}.([\overline{F}]ff \wedge \\ &\qquad (\langle \overline{r} \rangle W_{k+1} \vee \langle \alpha_{k+1} \setminus r \rangle exec_{SFA}(F, k))) \end{aligned}$$

With an arbitrary order on the actions in F such that α_1 is the first action, α_2 the second, etc. until α_n

Global Response Formulae: SFA

$$\neg(\langle true^{\star} \cdot q \rangle(\bigvee_{F \subseteq Act} (\mu Y.(\langle \overline{r} \rangle Y \vee \nu X.inf_{SFA}(F)))))$$

Where

$$\begin{aligned} inf_{SFA}(F) &= exec_{SFA}(F, n) \\ exec_{SFA}(F, 0) &= [\overline{F}]ff \wedge X \\ exec_{SFA}(F, k+1) &= \mu W_{k+1}.([\overline{F}]ff \wedge \\ &\qquad (\langle \overline{r} \rangle W_{k+1} \vee \langle \alpha_{k+1} \setminus r \rangle exec_{SFA}(F, k))) \end{aligned}$$

With an arbitrary order on the actions in F such that α_1 is the first action, α_2 the second, etc. until α_n

Global Response Formulae: SFA

$$\neg(\langle true^{\star} \cdot q \rangle(\bigvee_{F \subseteq Act} (\mu Y.(\langle \overline{r} \rangle Y \vee \nu X.inf_{SFA}(F)))))$$

Where

$$\begin{aligned} inf_{SFA}(F) &= exec_{SFA}(F, n) \\ exec_{SFA}(F, 0) &= [\overline{F}]ff \wedge X \\ exec_{SFA}(F, k+1) &= \mu W_{k+1}.([\overline{F}]ff \wedge \\ &\qquad (\langle \overline{r} \rangle W_{k+1} \vee \langle \alpha_{k+1} \setminus r \rangle exec_{SFA}(F, k))) \end{aligned}$$

With an arbitrary order on the actions in F such that α_1 is the first action, α_2 the second, etc. until α_n

Global Response Formulae: SFA

$$\neg(\langle true^{\star} \cdot q \rangle(\bigvee_{F \subseteq Act} (\mu Y.(\langle \overline{r} \rangle Y \vee \nu X.inf_{SFA}(F)))))$$

Where

$$\begin{aligned} inf_{SFA}(F) &= exec_{SFA}(F, n) \\ exec_{SFA}(F, 0) &= [\overline{F}]ff \wedge X \\ exec_{SFA}(F, k+1) &= \mu W_{k+1}.([\overline{F}]ff \wedge \\ &\qquad (\langle \overline{r} \rangle W_{k+1} \vee \langle \alpha_{k+1} \setminus r \rangle exec_{SFA}(F, k))) \end{aligned}$$

With an arbitrary order on the actions in F such that α_1 is the first action, α_2 the second, etc. until α_n

Global Response Formulae: SFA

Global Response Formulae: SFA

- Correctness proven in thesis

Global Response Formulae: SFA

- Correctness proven in thesis
- Due to subsets, exponential in number of actions

Other Patterns

Other Patterns

- RQ3: Generalise to other patterns

Other Patterns

- RQ3: Generalise to other patterns
- These fairness assumption are not relevant for all

Other Patterns

Behaviour	Relevant	Covered	Scope	Relevant	Covered
Absence	×	×	Global	✓	✓
Existence	✓	✓	Before	×	×
Existence (Most)	×	×	After-First	✓	✓
Existence (Least)	✓	×	Between-First	×	×
Existence (Exactly)	✓	×	Until-First	✓	✓
Universality	×	×	Before-Variant	✓	✓
Precedence	×	×	After-Last	✓	✓
Response	✓	✓	Between-Last	×	×
Chain-Precedence	×	×	Until-Last	✓	✓
Chain-Response	✓	×	After-Any	✓	✓
Always-Enabled	×	×	Between-Any	×	×
Precedence-Variant	✓	×	Until-Any	✓	✓
Response-Variant	✓	×			
Constrained-Chain	✓	×			

Other Patterns

Other Patterns

- First, designed separate formulae

Other Patterns

- First, designed separate formulae
- Mostly identical, same fairness assumption

Other Patterns

- First, designed separate formulae
- Mostly identical, same fairness assumption
  - Identify impact of behaviour and scope

Other Patterns

- First, designed separate formulae
- Mostly identical, same fairness assumption
  - Identify impact of behaviour and scope
  - Represent with 5 variables: δ_1, δ_2, δ_3, δ_4, δ_5

Other Patterns

Other Patterns

- Create “base” formulae with placeholder variables

Other Patterns

- Create “base” formulae with placeholder variables

$$\neg(\langle \delta_1 \cdot \delta_2 \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(\langle \delta_3 \rangle tt \vee ([\lambda] ff \wedge X) \vee \langle \lambda \setminus (\delta_4 \cup \delta_5) \rangle X \vee \langle \overline{\lambda \cup \delta_4 \cup \delta_5} \rangle Y)))))$$

Other Patterns

- Create “base” formulae with placeholder variables

$$\neg(\langle \delta_1 \cdot \delta_2 \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(\langle \delta_3 \rangle tt \vee ([\lambda] ff \wedge X) \vee \langle \lambda \setminus (\delta_4 \cup \delta_5) \rangle X \vee \langle \overline{\lambda \cup \delta_4 \cup \delta_5} \rangle Y)))))$$

- Similar for FRA and SFA

Other Patterns

Other Patterns

- Only need to know how to fill in

Other Patterns

- Only need to know how to fill in

Behaviour	Scope		δ_1	δ_2	δ_3	δ_4	δ_5
Response	Global		ε	$true^\star \cdot q$	*false*	*r*	*false*
	Before-Var		ε	$\overline{b}^\star \cdot q$	*b*	*r*	*false*
	After	Any	$true^\star \cdot a$	$true^\star \cdot q$	*false*	*r*	*false*
		First	$\overline{a}^\star \cdot a$	$true^\star \cdot q$	*false*	*r*	*false*
		Last	$true^\star \cdot a$	$\overline{a}^\star \cdot q$	*false*	*r*	*a*
	Until	Any	$true^\star \cdot a$	$\overline{b}^\star \cdot q$	*b*	*r*	*false*
		First	$(\overline{a}^\star \cdot a) + (true^\star \cdot b \cdot \overline{a}^\star \cdot a)$	$\overline{b}^\star \cdot q$	*b*	*r*	*false*
		Last	$true^\star \cdot a$	$\overline{a \cup b}^\star \cdot q$	*b*	*r*	*a*
Existence	Global		ε	ε	*false*	*r*	*false*
	Before-Var		ε	ε	*b*	*r*	*false*
	After	Any	$true^\star \cdot a$	ε	*false*	*r*	*false*
		First	$\overline{a}^\star \cdot a$	ε	*false*	*r*	*false*
		Last	$true^\star \cdot a$	ε	*false*	*r*	*a*
	Until	Any	$true^\star \cdot a$	ε	*b*	*r*	*false*
		First	$(\overline{a}^\star \cdot a) + (true^\star \cdot b \cdot \overline{a}^\star \cdot a)$	ε	*b*	*r*	*false*
		Last	$true^\star \cdot a$	ε	*b*	*r*	*a*

Additional Work

Additional Work

- Generalise fairness assumptions

Additional Work

- Generalise fairness assumptions
  1. Treat only some actions fairly

Additional Work

- Generalise fairness assumptions
  1. Treat only some actions fairly
  2. Fairness of task

Additional Work

- Generalise fairness assumptions
  1. Treat only some actions fairly
  2. Fairness of task
- Guide on use in mCRL2

Additional Work

- Generalise fairness assumptions
  1. Treat only some actions fairly
  2. Fairness of task
- Guide on use in mCRL2
  - How to modify the model

Additional Work

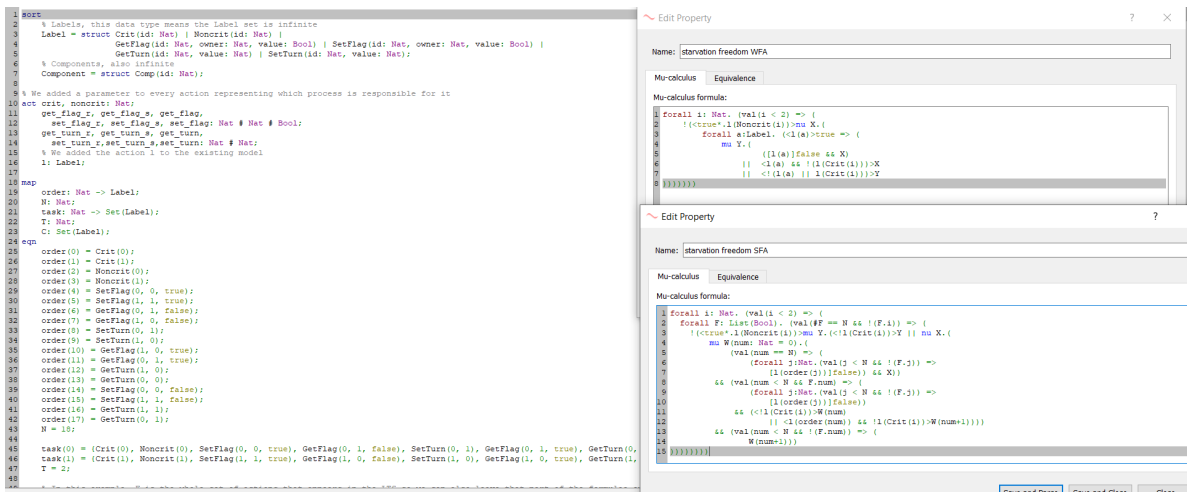
- Generalise fairness assumptions
  1. Treat only some actions fairly
  2. Fairness of task
- Guide on use in mCRL2
  - How to modify the model
  - How to represent the formulae

Additional Work

- Generalise fairness assumptions
  1. Treat only some actions fairly
  2. Fairness of task
- Guide on use in mCRL2
  - How to modify the model
  - How to represent the formulae
  - Example: Dekker's algorithm

Additional Work

- Generalise fairness assumptions
  1. Treat only some actions fairly
  2. Fairness of task
- Guide on use in mCRL2
  - How to modify the model
  - How to represent the formulae
  - Example: Dekker's algorithm



Conclusion

Conclusion

- Primary contributions:

Conclusion

- Primary contributions:
  - Designed and proved formulae for global response under WFA, SFA and FRA

Conclusion

- Primary contributions:
  - Designed and proved formulae for global response under WFA, SFA and FRA
  - Generalised to cover more patterns (and proven)

Conclusion

- Primary contributions:
  - Designed and proved formulae for global response under WFA, SFA and FRA
  - Generalised to cover more patterns (and proven)
  - Generalised to cover variants of fairness assumptions

Conclusion

- Primary contributions:
  - Designed and proved formulae for global response under WFA, SFA and FRA
  - Generalised to cover more patterns (and proven)
  - Generalised to cover variants of fairness assumptions
  - Explained how to use these in practical model checking

Conclusion

- Primary contributions:
  - Designed and proved formulae for global response under WFA, SFA and FRA
  - Generalised to cover more patterns (and proven)
  - Generalised to cover variants of fairness assumptions
  - Explained how to use these in practical model checking
- Additionally in thesis:

Conclusion

- Primary contributions:
  - Designed and proved formulae for global response under WFA, SFA and FRA
  - Generalised to cover more patterns (and proven)
  - Generalised to cover variants of fairness assumptions
  - Explained how to use these in practical model checking
- Additionally in thesis:
  - Explanation of placeholder table

Conclusion

- Primary contributions:
  - Designed and proved formulae for global response under WFA, SFA and FRA
  - Generalised to cover more patterns (and proven)
  - Generalised to cover variants of fairness assumptions
  - Explained how to use these in practical model checking
- Additionally in thesis:
  - Explanation of placeholder table
  - Comparisons to other formulae in the literature

Conclusion

- Primary contributions:
  - Designed and proved formulae for global response under WFA, SFA and FRA
  - Generalised to cover more patterns (and proven)
  - Generalised to cover variants of fairness assumptions
  - Explained how to use these in practical model checking
- Additionally in thesis:
  - Explanation of placeholder table
  - Comparisons to other formulae in the literature
  - Discussions of unconditional fairness of actions

Future Work

Future Work

- More fairness, more properties

Future Work

- More fairness, more properties
- Remainder of PSP

Future Work

- More fairness, more properties
- Remainder of PSP

Behaviour	Relevant	Covered	Scope	Relevant	Covered
Absence	×	×	Global	✓	✓
Existence	✓	✓	Before	×	×
Existence (Most)	×	×	After-First	✓	✓
Existence (Least)	✓	×	Between-First	×	×
Existence (Exactly)	✓	×	Until-First	✓	✓
Universality	×	×	Before-Variant	✓	✓
Precedence	×	×	After-Last	✓	✓
Response	✓	✓	Between-Last	×	×
Chain-Precedence	×	×	Until-Last	✓	✓
Chain-Response	✓	×	After-Any	✓	✓
Always-Enabled	×	×	Between-Any	×	×
Precedence-Variant	✓	×	Until-Any	✓	✓
Response-Variant	✓	×			
Constrained-Chain	✓	×			

Future Work

- More fairness, more properties
- Remainder of PSP

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already

$$\neg(\langle true^{\star} \cdot q_0 \cdot \overline{q_1}^{\star} \cdot q_1 \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow ($$
$$\mu Y.(([\lambda] ff \wedge X) \vee$$
$$\langle \lambda \setminus r_0 \rangle X \vee \langle \overline{\lambda \cup r_0} \rangle Y)))))$$
$$\wedge \neg(\langle true^{\star} \cdot q_0 \cdot \overline{q_1}^{\star} \cdot q_1 \cdot \overline{r_0}^{\star} \cdot r_0 \rangle \nu X.(\bigwedge_{\lambda \in Act} (\langle \lambda \rangle tt \Rightarrow ($$
$$\mu Y.(([\lambda] ff \wedge X) \vee$$
$$\langle \lambda \setminus r_1 \rangle X \vee \langle \overline{\lambda \cup r_1} \rangle Y)))))$$

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already
- Consider blocking actions

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already
- Consider blocking actions
  - Also started

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already
- Consider blocking actions
  - Also started

$$\neg(\langle \delta_1 \cdot \delta_2 \rangle \nu X.(\bigwedge_{\lambda \in (Act \setminus B)} (\langle \lambda \rangle tt \Rightarrow (\mu Y.(\langle \delta_3 \rangle tt \vee ([\lambda] ff \wedge X) \vee \langle \lambda \setminus (\delta_4 \cup \delta_5) \rangle X \vee \langle \overline{\lambda \cup \delta_4 \cup \delta_5} \rangle Y)))))$$

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already
- Consider blocking actions
  - Also started

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already
- Consider blocking actions
  - Also started
- Confirm proofs with formal proof assistant

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already
- Consider blocking actions
  - Also started
- Confirm proofs with formal proof assistant
  - Make formulae more trustworthy

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already
- Consider blocking actions
  - Also started
- Confirm proofs with formal proof assistant
  - Make formulae more trustworthy
  - Was planned as part of this project

Future Work

- More fairness, more properties
- Remainder of PSP
  - Started this already
- Consider blocking actions
  - Also started
- Confirm proofs with formal proof assistant
  - Make formulae more trustworthy
  - Was planned as part of this project
  - Isabelle/HOL

Final Thoughts

Final Thoughts

- Still a lot I want to do

Final Thoughts

- Still a lot I want to do
- Work done so far is already very useful