POC HW#4

14.16

$N = 5$, $p(x_1) = 1/16$, $p(x_2) = 1/8$, $p(x_3) = 1/4$
$p(x_4) = 1/16$, $p(x_5) = 1/2$

a) $H(X) = \sum_{i=1}^{N} p(x_i) I(x_i) = -\sum_{i=1}^{N} p(x_i) \log_2\left(p(x_i)\right)$

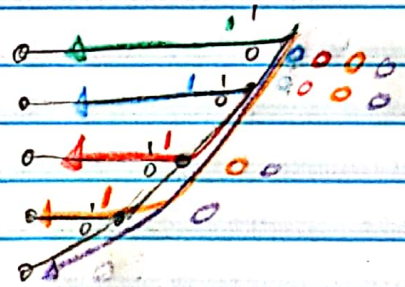
$H(X) = -1/16 \log_2\left(\frac{1}{2^4}\right) - 1/8 \log_2\left(\frac{1}{2^3}\right) - 1/4 \log_2\left(\frac{1}{2^2}\right)$
$- 1/16 \log_2(1/2^4) - 1/2 \cdot \log_2\left(\frac{1}{2^1}\right)$

$= \frac{4}{16} + \frac{3}{8} + \frac{2}{4} + \frac{4}{16} + \frac{1}{2}$

$\boxed{H(X) = 1.875}$

b)

x_i	$p(x_i)$
x_5	0.5
x_3	0.25
x_2	0.125
x_1	0.0625
x_4	0.0625



c)
x_1: 0 0 0 1
x_2: 0 0 1
x_3: 0 1
x_4: 0 0 0 0
x_5: 1

$\frac{4+3+2+4+1}{5} = 14/5 = 2.8$

d) efficiency $= \frac{\text{entropy}}{\text{average \#bits per symbol}} = \frac{1.875}{2.8} = \boxed{0.670}$

e) Naive encoding: 000, 001, 010, 011, 100, 3 bits per symbol
efficiency $= \frac{1.875}{3} = \boxed{0.625}$, worse efficiency than the variable-rate case.