

Polynomial Regression Coefficients

In polynomial regression with a specified degree, the number of coefficients (including the intercept) is determined by the number of polynomial terms generated from the input features.

Single Feature Polynomial Terms

For a single feature x and polynomial degree $d = 3$, the polynomial terms will be:

- x^0 (intercept)
- x^1
- x^2
- x^3

This results in 4 terms, and thus 4 coefficients (including the intercept).

Multiple Features Polynomial Terms

For multiple features, the number of terms increases due to interaction terms up to the specified polynomial degree.

Example: Two Features and Degree 3

For two original features x_1 and x_2 , and polynomial degree $d = 3$, the polynomial terms include all combinations of these features up to the cubic term:

- $x_1^0 x_2^0$ (the intercept)
- $x_1^1 x_2^0, x_1^0 x_2^1$
- $x_1^2 x_2^0, x_1^1 x_2^1, x_1^0 x_2^2$
- $x_1^3 x_2^0, x_1^2 x_2^1, x_1^1 x_2^2, x_1^0 x_2^3$

To count these terms, use the combinatorial formula for the number of polynomial terms generated from n features and degree d :

$$\text{Number of terms} = (n + d)! / (d! * (n + d - d)!)$$

For $n = 2$ and $d = 3$:

$$\text{Number of terms} = (2 + 3)! / (3! * (2 + 3 - 3)!) = 5! / (3! * 2!) = (5 * 4) / (2 * 1) = 10$$

Thus, with two features and a polynomial degree of 3, there will be 10 terms, resulting in 10 coefficients in the regression model, including the intercept term.