Active Learning and Covering Problems with Precedence

author names withheld

Abstract

In the Bayesian Active Learning a hidden hypothesis is required to be uncovered. To do so, the learner is allowed to perform tests, each of which reveals partial information about the hidden hypothesis. Upon receiving this information, the learner adaptively selects the next test to be performed. The goal is to uncover the hidden hypothesis while performing as few tests as possible in the worst or average case.

In the covering problems, we are given a set of items and a collection of subsets that cover these items. The objective is to select a sequence of tests that covers all items which again, minimized the worst or average covering cost.

For both types of problems, a natural constraint may arise that some tests can only be performed only after certain other tests (or some subsets can only be selected after selecting certain other subsets). We model such constraints using directed acyclic graphs (DAGs) that impose precedence on the tests or subsets. This paper explores the connection of active learning and covering problems under such constraints.

We show that given any $(O(1), \alpha)$-approximation ratio for the precedence constrained set cover problem, we can obtain an $O(\alpha \cdot \log n)$-approximation ratio for the worst case active learning problem with precedence constraints, where n is the number of hypothesis. Similarly for the average case version, we prove that given any $O(\beta)$-approximation ratio for the precedence constrained min-sum set cover, we can obtain an $O(\beta \cdot \log n)$-approximation ratio for the average case active learning problem with precedence constraints. Finally, we provide several approximation algorithms for the set cover and min-sum set cover problems with precedence constraints.

Keywords: Bayesian active learning, Set cover, Precedence constraints, Approximation Algorithms, Decision Trees

1. Introduction

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Appendix A. My Proof of Theorem 1

This is a boring technical proof.

Appendix B. My Proof of Theorem 2

This is a complete version of a proof sketched in the main text.