FRE 6711 Final Project Report

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1. Introduction

In this project, we first collected the historical data of 12 indexes, including:
1.CurrencyShares Euro Trust (FXE)
2.iShares MSCI Japan Index (EWJ)
3.SPDR GOLD Trust (GLD)
4. Powershares NASDAQ-100 Trust (QQQ)
5. SPDR S&P 500 (SPY)
6. iShares Lehman Short Treasury Bond (SHV)
7. PowerShares DB Agriculture Fund (DBA)
8. United States Oil Fund LP (USO)
9. SPDR Biotech ETF(XBI)
10. iShares S&P Latin America 40 Index (ILF)
11. iShares MSCI Pacific ex-Japan Index Fund (EPP)
12. SPDR DJ Euro Stoxx 50 (FEZ).

Next, we built factor-based model allocation namely a Long/Short Global Macro Strategy with a Target Beta, where Beta is the usual single factor market risk measure. We derive the factor data from Ken French's website. The portfolio is re-optimized weekly for a period of analysis from March 2007 to the end of March 2020. Since we assume that the portfolio we created can represent the world global economy, we use the French Fama 3-factor model.

After constructing the factor model, we did the back-tests based on 3 cases: a long look-back period ($\geq$ 120 days), a short look-back period ($\leq$ 40 days), and a medium look-back period. We also use trend following estimators for the expected returns, which are also called sample mean and sample covariance. Besides, the estimation of the coefficients of the model as they are computed by regressions on the factors is also related to the length of the look-back period. In summary, we assess the impact of the length of those regression-based estimators on the realized performance and risk indicators of the optimized portfolio. The behavior of the optimal portfolio built from a specific combination of estimators for covariance and expected return may change with the market environment and the target Beta.

2. Notations

The ETF data we used is downloaded from Yahoo Finance. The French Fama 3-factor model data is downloaded from Ken French's website. After simplification and combinations of the above two datasets, we define three time periods for analysis:

Before the crisis: 2007/3/1 - 2008/3/1
During the crisis: 2008/3/2 - 2010/7/1
After the crisis: 2010/7/2 - 2020/6/30
The whole period: 2007/3/1 - 2020/6/30

Besides, we also define three time periods for backtesting and estimation, and for each time-period, we use the same number of days to estimate expected returns and variances:

Short term: 40 days
Middle term: 80 days
Long term: 120 days

We separate backtests for each sub-period and choose the angle to compare the impact of different target Beta under a given Term-Structure. As our classification, there are three combinations: $P_{40}^{40}, P_{80}^{80}, P_{120}^{120}$

where the upper right number represents the number of days to estimate expected return and lower right number represents the number of days to estimate variance matrix.

The target Beta is denoted as β_T^m. As demonstrated in the project memo, we consider 6 different target Betas:

$\beta_T^m = -1$

$\beta_T^m = -0.5$

$\beta_T^m = 0$

$\beta_T^m = 0.5$

$\beta_T^m = 1$

$\beta_T^m = 1.5$

3 Result & Graphs

3.1 Before the crisis

Before the Crisis

	E[r]_40,Cov_40						E[r]_80,Cov_80						E[r]_120,Cov_120						SPY
	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	
cum_return	0.34932	0.34996	0.34820	0.34399	0.33742	0.32863	0.85968	0.84353	0.82912	0.81034	0.78700	0.76008	1.84848	1.79804	1.73793	1.66925	1.59317	1.51097	0.96226
ari_mean_return	-0.00315	-0.00312	-0.00309	-0.00306	-0.00303	-0.00300	0.00012	0.00006	0.00002	-0.00001	-0.00004	-0.00007	0.00273	0.00256	0.00240	0.00224	0.00207	0.00191	-0.00006
geo_mean_return	-1.03338	-1.03156	-1.03654	-1.04857	-1.06761	-1.09368	-0.14206	-0.16085	-0.17794	-0.20065	-0.22961	-0.26411	0.61815	0.59064	0.55683	0.51675	0.47038	0.41773	-0.03025
daily_min_return	-50.74158	-52.45820	-54.17246	-55.88500	-57.59568	-59.30436	-36.24578	-35.87810	-35.38372	-34.88655	-34.38956	-34.59271	-15.15433	-14.49277	-13.83007	-14.84438	-16.29099	-17.73898	-7.40842
drawdown	-0.41112	-0.41154	-0.41466	-0.42161	-0.42868	-0.43584	-0.25400	-0.26423	-0.27423	-0.28414	-0.29399	-0.30374	-0.22419	-0.19573	-0.16700	-0.14272	-0.13893	-0.16601	-0.07481
vol	10.97702	11.09525	11.37027	11.79176	12.34454	13.01187	9.28782	9.34938	9.57698	9.96053	10.48409	11.12595	5.65430	5.03836	4.67689	4.63007	4.90710	5.45905	2.70226
sharpe_ratio	-0.00029	-0.00028	-0.00027	-0.00026	-0.00025	-0.00023	0.00001	0.00001	0.00000	0.00000	0.00000	-0.00001	0.00048	0.00051	0.00051	0.00048	0.00042	0.00035	-0.00002
skewness	-0.42027	-0.44943	-0.43731	-0.39271	-0.32854	-0.25708	0.06382	0.10599	0.17321	0.25178	0.33018	0.39975	0.19561	0.33038	0.42200	0.40177	0.28931	0.17168	-0.29924
kurtosis	2.25933	2.21941	2.11831	1.97884	1.82575	1.67781	1.92430	2.00270	2.12174	2.29403	2.49856	2.70578	1.72936	1.95240	2.22766	2.34927	2.32254	2.33357	0.45167
modified_VaR	-124.07172	-125.91235	-128.77603	-132.63607	-137.46290	-143.22386	-93.07534	-92.98861	-93.98562	-96.13672	-99.43191	-103.80884	-51.24722	-44.04582	-39.88006	-39.80747	-43.71471	-50.44264	-29.47950
C_VaR	-159.72906	-162.31745	-166.71532	-171.76109	-177.44837	-185.09565	-132.10653	-130.90784	-132.27844	-136.20596	-142.46272	-152.29393	-77.03392	-64.88818	-60.11641	-61.09449	-67.05814	-77.56697	-39.76631

As shown above, we find that:

1. Under the same period of comparison, the larger the target Beta tends to be, the lower the cumulative return is. However, when beta decreases below 0, the cumulative return also decreases for P_{40}^{40}. Both arithmetic and the geometric mean of return increase as target Beta increases.

	E[r]_40,Cov_40					
	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5
cum_return	0.34932	0.34996	0.34820	0.34399	0.33742	0.32863
ari_mean_return	-0.00315	-0.00312	-0.00309	-0.00306	-0.00303	-0.00300
geo_mean_return	-1.03338	-1.03156	-1.03654	-1.04857	-1.06761	-1.09368

2. Under the same target Beta, the portfolio with a longer period of estimation tends to have a higher return and smaller variance. Take target Beta = -1 as example:

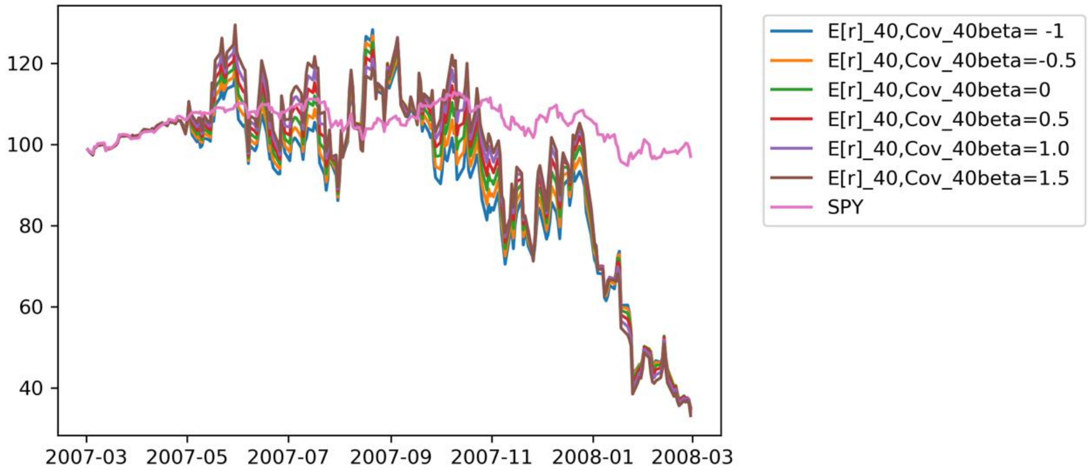
	E[r]_40,Cov_40	E[r]_80,Cov_80	E[r]_120,Cov_120
	beta= -1	beta= -1	beta= -1
cum_return	0.349318073	0.859675281	1.848478488
ari_mean_return	-0.003149683	0.000122839	0.002727213
geo_mean_return	-1.033378413	-0.142055232	0.61815487
modified_VaR	-124.0717228	-93.07534368	-51.24721588
C_VaR	-159.7290591	-132.1065302	-77.03391996

3. Compared with the benchmark SPY, only the long-term back-test provides a better return. However, its variance is higher than the SPY's.

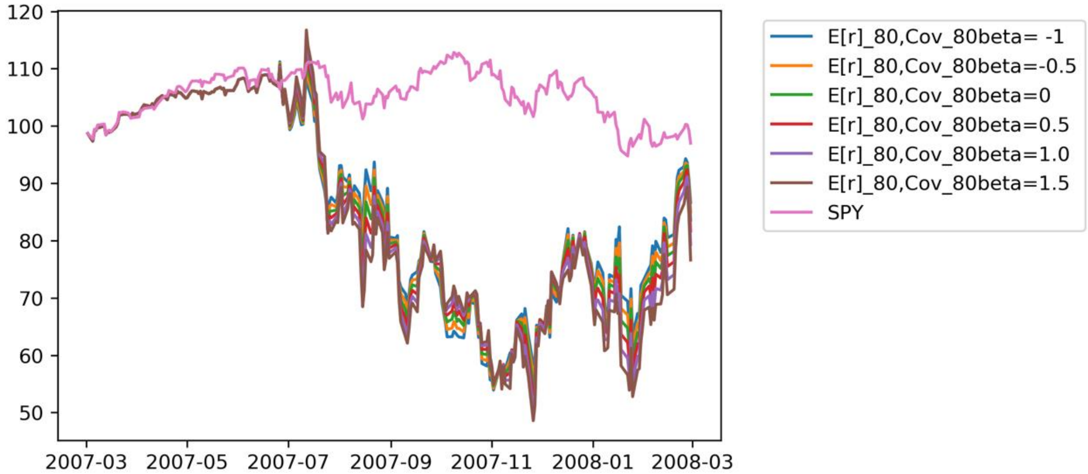
	E[r]_40,Cov_40	E[r]_80,Cov_80	E[r]_120,Cov_120	SPY
	beta= -1	beta= -1	beta= -1	
cum_return	0.349318073	0.859675281	1.848478488	0.9622645
ari_mean_return	-0.003149683	0.000122839	0.002727213	-6.25E-05
geo_mean_return	-1.033378413	-0.142055232	0.61815487	-0.030254
modified_VaR	-124.0717228	-93.07534368	-51.24721588	-29.4795
C_VaR	-159.7290591	-132.1065302	-77.03391996	-39.76631

In conclusion, our portfolio with a long-term period performs better than the benchmark but with a larger risk. However, in terms of the Sharpe Ratio, we can find that it has a better chance to make a profit than the benchmark.

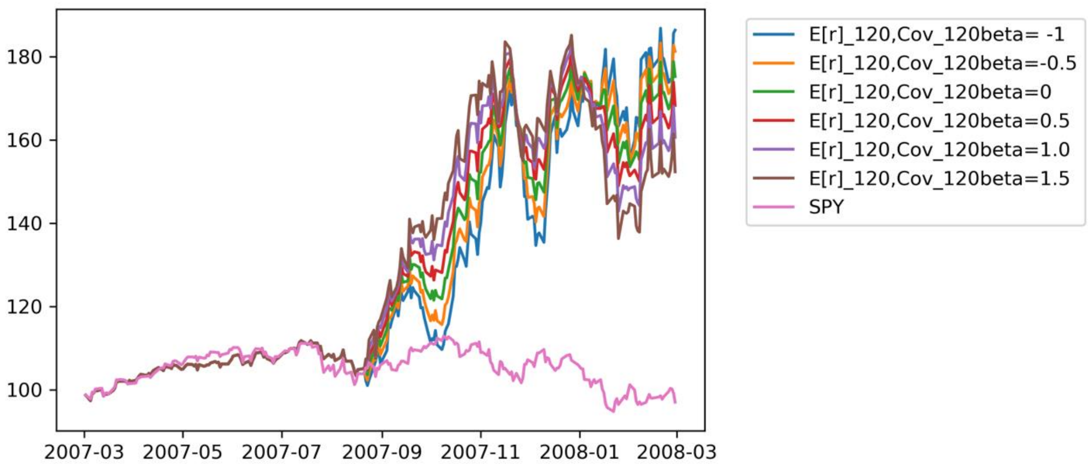
There are also graphs of the changes of Net Value and the distributions of return before the crisis:



E[r]_40,Cov_40beta= -1
E[r]_40,Cov_40beta=-0.5
E[r]_40,Cov_40beta=0
E[r]_40,Cov_40beta=0.5
E[r]_40,Cov_40beta=1.0
E[r]_40,Cov_40beta=1.5
SPY
120
100
80
60
40
2007-03
2007-05
2007-07
2007-09
2007-11
2008-01
2008-03

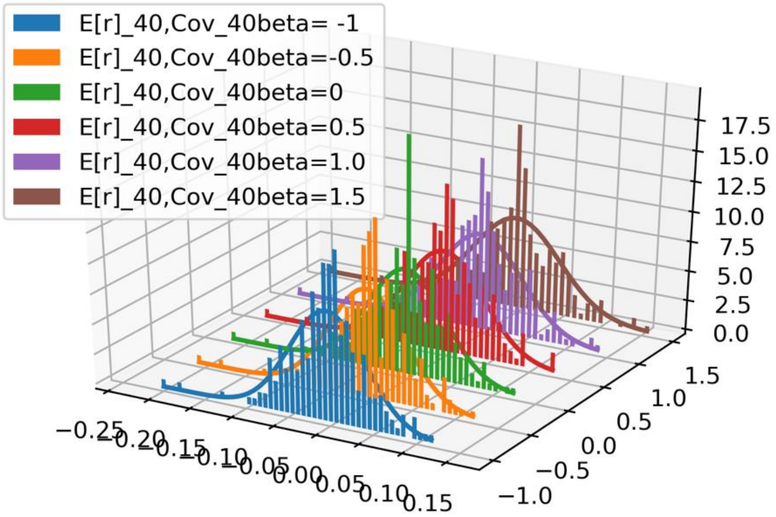


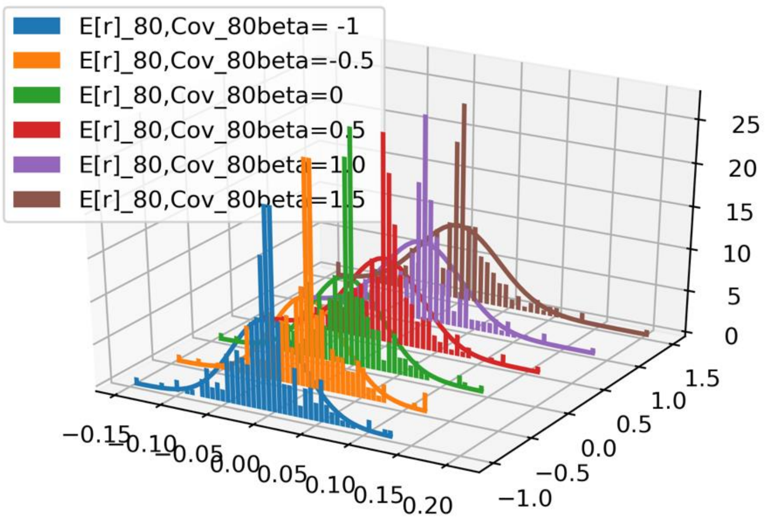
E[r]_80,Cov_80beta= -1
E[r]_80,Cov_80beta=-0.5
E[r]_80,Cov_80beta=0
E[r]_80,Cov_80beta=0.5
E[r]_80,Cov_80beta=1.0
E[r]_80,Cov_80beta=1.5
SPY
120
110
100
90
80
70
60
50
2007-03
2007-05
2007-07
2007-09
2007-11
2008-01
2008-03

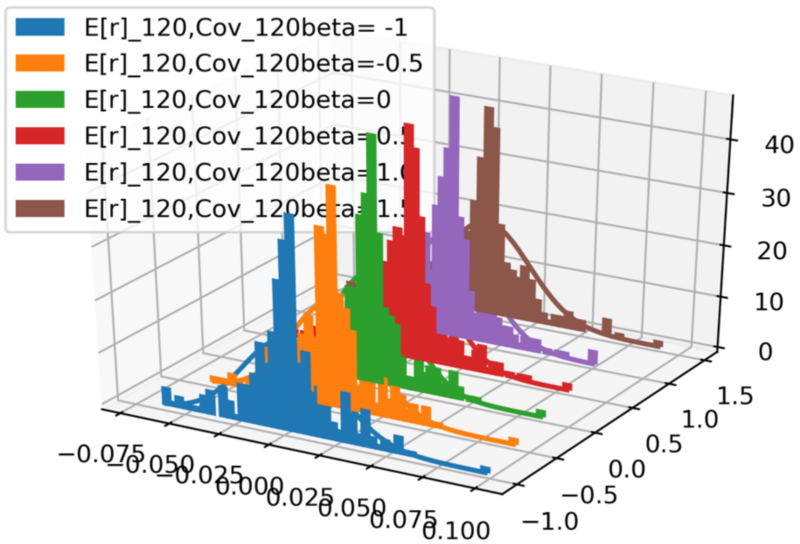


E[r]_120,Cov_120beta= -1
E[r]_120,Cov_120beta=-0.5
E[r]_120,Cov_120beta=0
E[r]_120,Cov_120beta=0.5
E[r]_120,Cov_120beta=1.0
E[r]_120,Cov_120beta=1.5
SPY
180
160
140
120
100
2007-03
2007-05
2007-07
2007-09
2007-11
2008-01
2008-03

Daily return distributions:







3.2 During the crisis

During the Crisis																			
	E[r]_40,Cov_40						E[r]_80,Cov_80						E[r]_120,Cov_120						SPY
	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	
cum_return	0.06585	0.07789	0.08741	0.09814	0.10413	0.10448	0.00742	0.00642	0.00521	0.00371	0.00240	0.00143	0.01297	0.01172	0.01011	0.00746	0.00492	0.00301	0.34667
ari_mean_return	0.00284	0.00261	0.00242	0.00235	0.00228	0.00220	-0.00432	-0.00417	-0.00406	-0.00408	-0.00411	-0.00413	-0.00298	-0.00281	-0.00264	-0.00254	-0.00246	-0.00238	-0.00013
geo_mean_return	-0.79172	-0.72054	-0.67168	-0.62255	-0.59743	-0.59600	-1.71542	-1.77669	-1.86463	-2.00809	-2.19174	-2.40911	-1.47942	-1.52207	-1.58459	-1.71319	-1.88879	-2.09584	-0.08676
daily_min_return	-99.38768	-91.51476	-83.71352	-76.23971	-75.91386	-79.43537	-62.48706	-71.88673	-85.29690	-97.37635	-109.70994	-122.04353	-66.67618	-76.13181	-85.56702	-96.74327	-108.63479	-120.54437	-24.61193
drawdown	-0.67724	-0.66543	-0.65248	-0.63810	-0.65123	-0.66424	-0.61779	-0.62718	-0.63560	-0.66378	-0.69790	-0.72936	-0.59502	-0.60220	-0.60951	-0.64768	-0.70819	-0.76076	-0.24948
vol	27.21897	26.00954	25.05854	24.36882	23.90340	23.67612	17.69692	18.99614	20.39690	21.97576	23.82220	25.83490	19.06914	20.10476	21.32293	22.98223	24.89948	26.93564	5.21266
sharpe_ratio	0.00010	0.00010	0.00010	0.00010	0.00010	0.00009	-0.00024	-0.00022	-0.00020	-0.00019	-0.00017	-0.00016	-0.00016	-0.00014	-0.00012	-0.00011	-0.00010	-0.00009	-0.00003
skewness	0.12168	0.06681	0.00820	-0.04876	-0.09906	-0.13412	0.02644	0.04469	0.07644	0.11385	0.15849	0.20540	0.17516	0.16507	0.16498	0.17106	0.18438	0.20229	0.40174
kurtosis	1.63991	1.41049	1.22633	1.10081	1.02357	1.00869	2.47110	2.95268	3.51716	4.02226	4.49350	4.91311	3.29669	3.60444	3.97641	4.35410	4.72601	5.07892	7.58995
modified_VaR	-266.97504	-258.63583	-252.56871	-248.50064	-246.17275	-245.42812	-184.50224	-195.51780	-206.63924	-219.25057	-233.79398	-249.42629	-188.98497	-198.31219	-208.77915	-223.17342	-239.54514	-256.59192	-45.51890
C_VaR	-379.01707	-368.28593	-359.90217	-352.50174	-347.93227	-347.18851	-273.18551	-298.43923	-324.04559	-351.58183	-381.96967	-412.35752	-292.78261	-312.48049	-334.70950	-364.04840	-395.33742	-426.71370	-79.48971

As shown above, we find that:

1. Under the same period of comparison, the larger the target Beta tends to be, the lower the cumulative return is. Both arithmetic and the geometric mean of return increase as target Beta increases. Meanwhile, the variances are also increasing as the beta gets larger. Take a look-back period of 80 days as an instance:

	E[r]_80,Cov_80					
	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5
cum_return	0.00742	0.00642	0.00521	0.00371	0.00240	0.00143
ari_mean_return	-0.00432	-0.00417	-0.00406	-0.00408	-0.00411	-0.00413
geo_mean_return	-1.71542	-1.77669	-1.86463	-2.00809	-2.19174	-2.40911
modified_VaR	-266.97504	-258.63583	-252.56871	-248.50064	-246.17275	-245.42812
C_VaR	-379.01707	-368.28593	-359.90217	-352.50174	-347.93227	-347.18851

2. Under the same target Beta, the portfolio with the shortest period of estimation tends to have a higher return, and the portfolios with the medium and long period of estimation have close returns. The portfolio of the three periods has similar variances. Take target Beta = 0.5 as example:

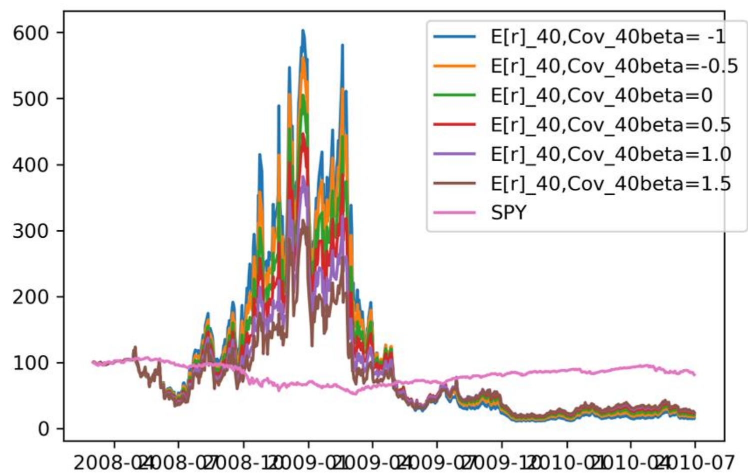
	E[r]_40,Cov_40	E[r]_80,Cov_80	E[r]_120,Cov_120
	beta=0.5	beta=0.5	beta=0.5
cum_return	0.09814	0.00371	0.00371
ari_mean_return	0.00235	-0.00408	-0.00408
geo_mean_return	-0.62255	-2.00809	-2.00809
modified_VaR	-248.50064	-219.25057	-219.25057
C_VaR	-352.50174	-351.58183	-351.58183

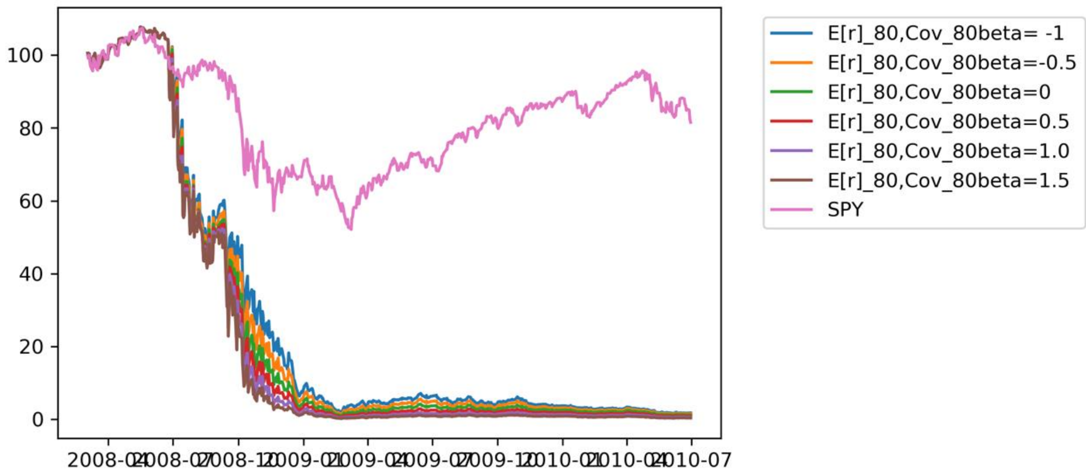
3. Compared with the benchmark SPY, none of the portfolios provides a better result than the benchmark, although the portfolio of the short period of estimation shows higher arithmetic mean of return than the benchmark's. Take target Beta = 0.5 as example:

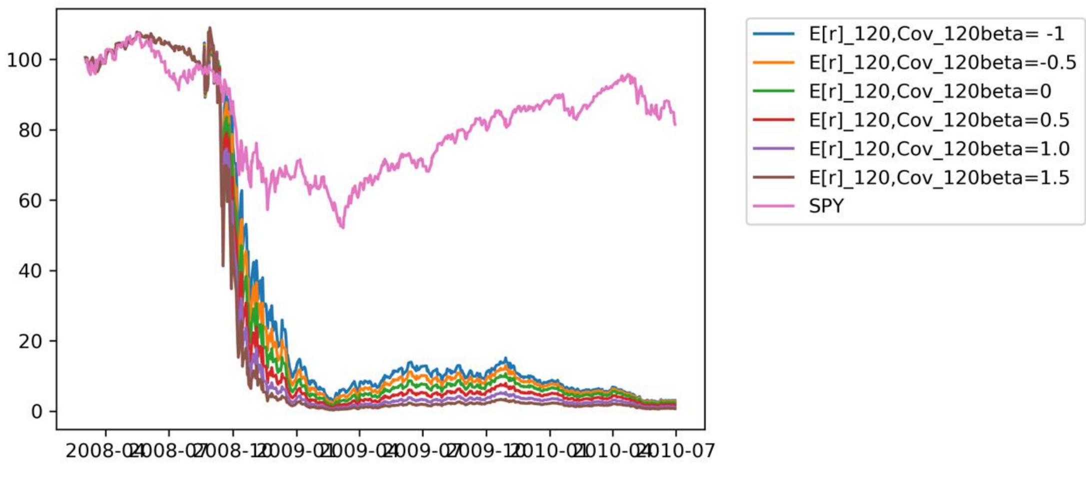
	E[r]_40,Cov_40	E[r]_80,Cov_80	E[r]_120,Cov_120	SPY
	beta=0.5	beta=0.5	beta=0.5	
cum_return	0.09814	0.00371	0.00371	0.3466734
ari_mean_return	0.00235	-0.00408	-0.00408	-0.00013
geo_mean_return	-0.62255	-2.00809	-2.00809	-0.086765
modified_VaR	-248.50064	-219.25057	-219.25057	-45.5189
C_VaR	-352.50174	-351.58183	-351.58183	-79.48971

In conclusion, none of our portfolios perform better than the benchmark during the crisis.

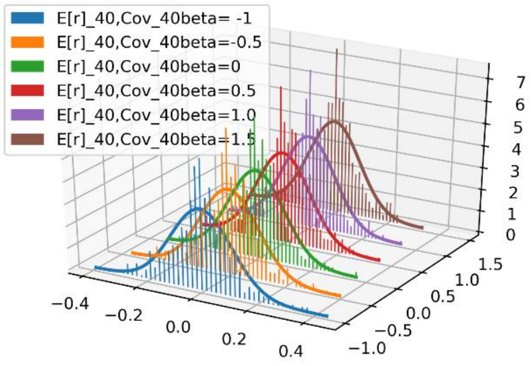
There are also graphs of the changes of Net Value and the distributions of return during the crisis:

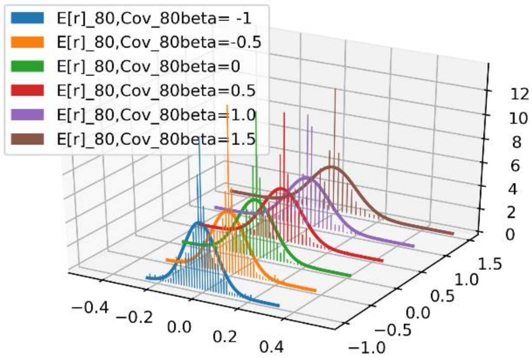


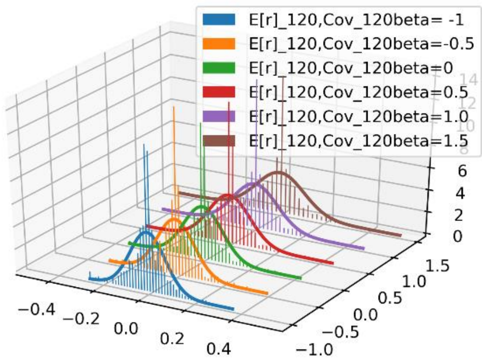




Daily return distributions:


E[r]_40,Cov_40beta= -1
E[r]_40,Cov_40beta=-0.5
E[r]_40,Cov_40beta=0
E[r]_40,Cov_40beta=0.5
E[r]_40,Cov_40beta=1.0
E[r]_40,Cov_40beta=1.5


E[r]_80,Cov_80beta= -1
E[r]_80,Cov_80beta=-0.5
E[r]_80,Cov_80beta=0
E[r]_80,Cov_80beta=0.5
E[r]_80,Cov_80beta=1.0
E[r]_80,Cov_80beta=1.5


E[r]_120,Cov_120beta= -1
E[r]_120,Cov_120beta=-0.5
E[r]_120,Cov_120beta=0
E[r]_120,Cov_120beta=0.5
E[r]_120,Cov_120beta=1.0
E[r]_120,Cov_120beta=1.5

3.3 After the Crisis

After the Crisis																			
	E[r]_40,Cov_40						E[r]_80,Cov_80						E[r]_120,Cov_120						SPY
	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	
cum_return	0.01120	0.01781	0.02620	0.03370	0.02674	0.03707	0.35620	0.33996	0.36199	0.29792	0.16125	0.11611	0.00007	0.00012	0.00013	0.00014	0.00013	0.00011	0.35847
ari_mean_return	0.00068	0.00087	0.00105	0.00117	0.00122	0.00145	0.00235	0.00240	0.00252	0.00235	0.00195	0.00191	-0.00180	-0.00156	-0.00153	-0.00154	-0.00155	-0.00157	0.00057
geo_mean_returr	-0.21685	-0.17074	-0.13242	-0.10740	-0.13040	-0.09796	0.12689	0.12225	0.12849	0.10913	0.04811	0.01548	-0.72288	-0.66843	-0.65539	-0.65354	-0.65942	-0.67323	0.12752
daily_min_return	-100.29420	-102.16869	-104.04330	-103.48877	-105.01878	-106.89350	-90.76423	-95.96224	-100.37172	-103.32556	-119.91701	-136.37118	-59.36603	-57.41337	-57.51382	-57.92839	-58.32852	-61.24061	-27.35590
drawdown	-0.61742	-0.62839	-0.63939	-0.63839	-0.65088	-0.66112	-0.57808	-0.59371	-0.60377	-0.57215	-0.63443	-0.72611	-0.46295	-0.45529	-0.45472	-0.45409	-0.48987	-0.55560	-0.23336
vol	13.88712	13.88108	14.01066	14.10137	14.69010	15.09141	15.18286	15.43174	15.79333	15.36547	14.69090	14.99393	11.68424	11.77529	11.63594	11.52691	11.57762	11.78759	2.69762
sharpe_ratio	0.00005	0.00006	0.00007	0.00008	0.00008	0.00010	0.00015	0.00016	0.00016	0.00015	0.00013	0.00013	-0.00015	-0.00013	-0.00013	-0.00013	-0.00013	-0.00013	0.00021
skewness	0.18205	0.14240	0.10295	0.05197	0.03831	-0.00717	0.19369	0.12181	0.08402	-0.02788	-0.20587	-0.31235	0.08762	0.03480	-0.01941	-0.07206	-0.12627	-0.18676	-0.63290
kurtosis	3.98033	4.01061	4.07993	4.02815	3.77283	4.04632	2.94098	2.95619	3.33899	3.42452	3.86030	5.05194	1.22729	1.29225	1.17503	1.18417	1.43849	1.97080	14.96252
modified_VaR	-131.73783	-132.34413	-134.18563	-136.26842	-142.80352	-147.10119	-143.17550	-147.51193	-151.18166	-150.19023	-147.86786	-151.53777	-120.70881	-122.28279	-122.13052	-122.09765	-123.38056	-126.04765	-24.95362
C_VaR	-187.03928	-188.64795	-192.12778	-195.44321	-203.88403	-210.82350	-205.77258	-211.57259	-217.02153	-212.90668	-207.99343	-216.12500	-161.13809	-164.57479	-164.27111	-164.58082	-166.81571	-171.38428	-42.13842

As shown above, we find that:

1. Under the same period of comparison, there is no significant difference of return and variance under different target Betas. Take a look-back period of 120 days as an instance:

	E[r]_120,Cov_120					
	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5
cum_return	0.00007	0.00012	0.00013	0.00014	0.00013	0.00011
ari_mean_return	-0.00180	-0.00156	-0.00153	-0.00154	-0.00155	-0.00157
geo_mean_return	-0.72288	-0.66843	-0.65539	-0.65354	-0.65942	-0.67323
modified_VaR	-120.70881	-122.28279	-122.13052	-122.09765	-123.38056	-126.04765
C_VaR	-161.13809	-164.57479	-164.27111	-164.58082	-166.81571	-171.38428

2. Under the same target Beta, the portfolio with the medium period of estimation has the highest return. The variances of the three cases are similar. Take target Beta = 0 as example:

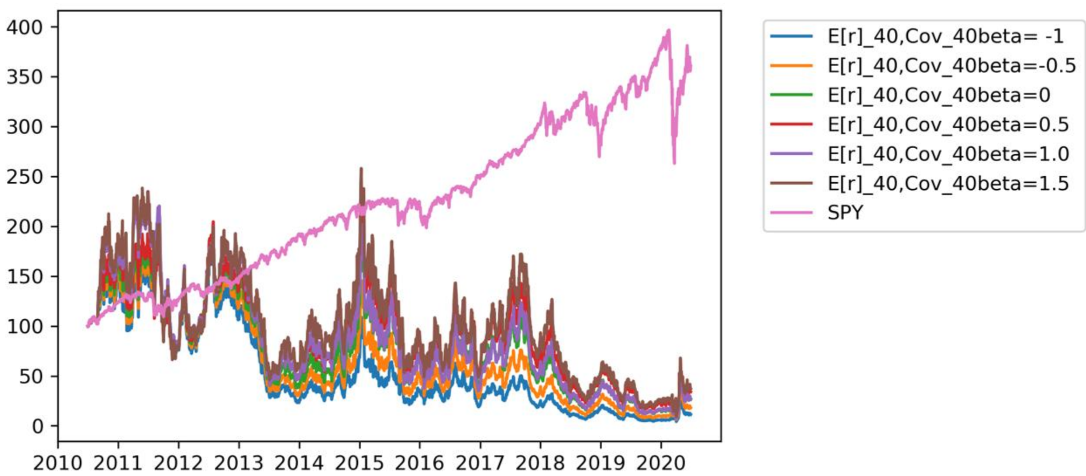
	E[r]_40,Cov_40	E[r]_80,Cov_80	E[r]_120,Cov_120
	beta=0	beta=0	beta=0
cum_return	0.02620	0.36199	0.00013
ari_mean_return	0.00105	0.00252	-0.00153
geo_mean_return	-0.13242	0.12849	-0.65539
modified_VaR	-134.18563	-151.18166	-122.13052
C_VaR	-192.12778	-217.02153	-164.27111

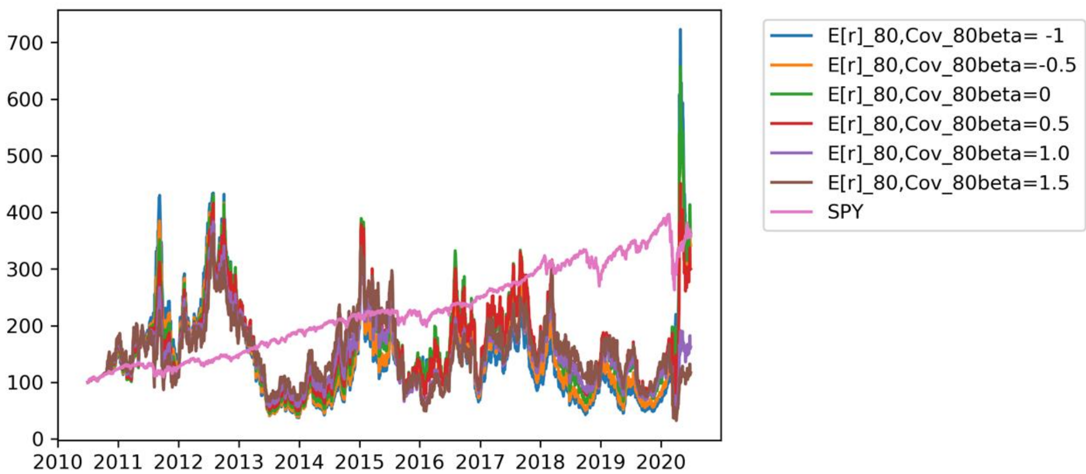
3. Compared with the benchmark SPY, the portfolio with the medium period of estimation provides a better result than the benchmark. Take target Beta = 0 as example:

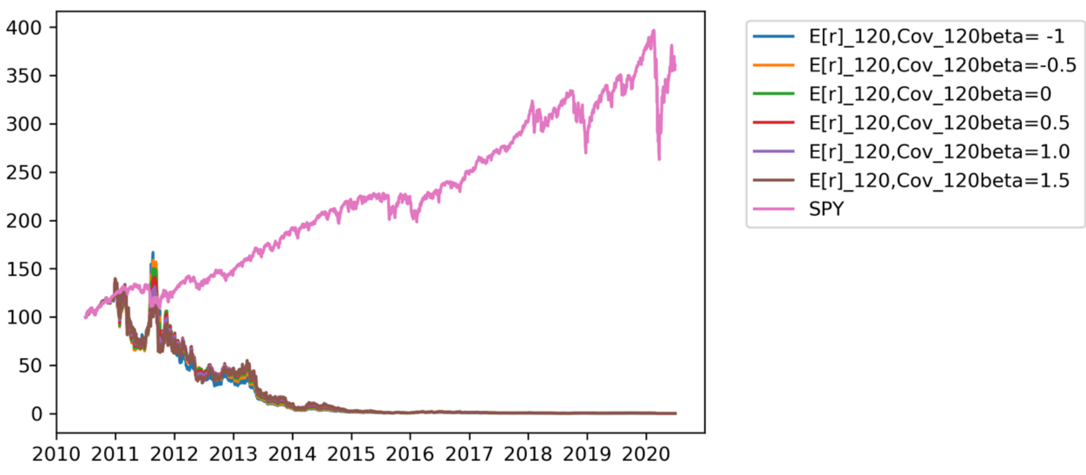
	E[r]_40,Cov_40	E[r]_80,Cov_80	E[r]_120,Cov_120	SPY
	beta=0	beta=0	beta=0	
cum_return	0.02620	0.36199	0.00013	0.35847
ari_mean_return	0.00105	0.00252	-0.00153	0.00057
geo_mean_return	-0.13242	0.12849	-0.65539	0.12752
modified_VaR	-134.18563	-151.18166	-122.13052	-24.95362
C_VaR	-192.12778	-217.02153	-164.27111	-42.13842

In conclusion, the portfolio with the medium period of estimation performs better than the benchmark after the crisis. The Sharpe Ratios of the above two are also similar.

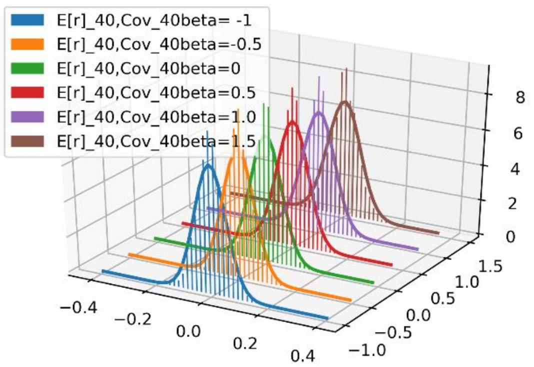
There are also graphs of the changes of Net Value and the distributions of return after the crisis:

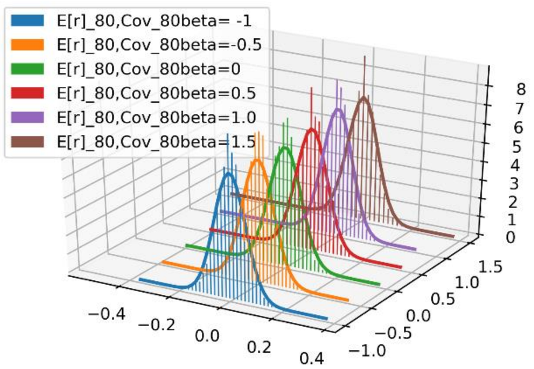


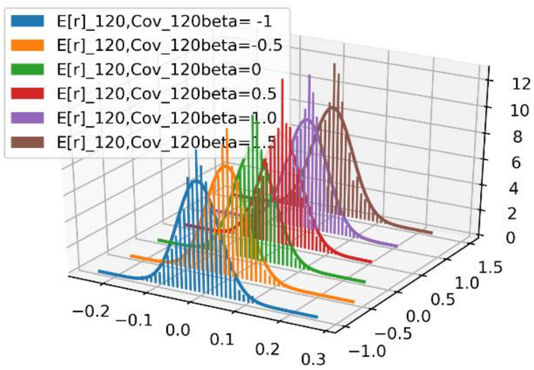




Daily return distributions:


E[r]_40,Cov_40beta= -1
E[r]_40,Cov_40beta=-0.5
E[r]_40,Cov_40beta=0
E[r]_40,Cov_40beta=0.5
E[r]_40,Cov_40beta=1.0
E[r]_40,Cov_40beta=1.5
−0.4 −0.2 0.0 0.2 0.4
−1.0 −0.5 0.0 0.5 1.0 1.5
0 2 4 6 8


E[r]_80,Cov_80beta= -1
E[r]_80,Cov_80beta=-0.5
E[r]_80,Cov_80beta=0
E[r]_80,Cov_80beta=0.5
E[r]_80,Cov_80beta=1.0
E[r]_80,Cov_80beta=1.5
−0.4 −0.2 0.0 0.2 0.4
−1.0 −0.5 0.0 0.5 1.0 1.5
0 1 2 3 4 5 6 7 8


E[r]_120,Cov_120beta= -1
E[r]_120,Cov_120beta=-0.5
E[r]_120,Cov_120beta=0
E[r]_120,Cov_120beta=0.5
E[r]_120,Cov_120beta=1.0
E[r]_120,Cov_120beta=1.5
−0.2 −0.1 0.0 0.1 0.2 0.3
−1.0 −0.5 0.0 0.5 1.0 1.5
0 2 4 6 8 10 12

3.4 The Whole Period

The Whole Period																			
	E[r]_40,Cov_40						E[r]_80,Cov_80						E[r]_120,Cov_120						SPY
	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5	
cum_return	0.20033	0.19225	0.16710	0.13158	0.09373	0.06031	0.00001	0.00002	0.00002	0.00002	0.00002	0.00002	0.00083	0.00156	0.00264	0.00400	0.00546	0.00670	0.21255
ari_mean_return	0.00198	0.00205	0.00213	0.00220	0.00228	0.00235	-0.00145	-0.00132	-0.00120	-0.00107	-0.00095	-0.00083	-0.00083	-0.00069	-0.00055	-0.00041	-0.00027	-0.00013	0.00040
geo_mean_returr	0.07371	0.07064	0.06019	0.04238	0.01711	-0.01573	-0.65168	-0.62838	-0.61282	-0.60322	-0.59946	-0.60170	-0.33483	-0.28780	-0.24882	-0.21778	-0.19466	-0.17941	0.07812
daily_min_return	-107.17109	-101.48980	-95.80537	-92.36145	-102.54156	-112.72385	-72.85756	-67.99318	-63.17322	-65.65130	-75.73503	-85.84640	-65.53676	-64.44402	-63.32936	-66.31298	-72.42103	-78.50652	-27.35590
drawdown	-0.57098	-0.58409	-0.59799	-0.62106	-0.67623	-0.72612	-0.40996	-0.43600	-0.47768	-0.52573	-0.57081	-0.61308	-0.58311	-0.56072	-0.53739	-0.53164	-0.57595	-0.61946	-0.24948
vol	14.38540	14.75158	15.22261	15.78913	16.44126	17.16925	11.97558	12.16497	12.46936	12.87639	13.37803	13.96239	7.91198	7.53093	7.39319	7.51187	7.87517	8.45136	3.28180
sharpe_ratio	0.00014	0.00014	0.00014	0.00014	0.00014	0.00014	-0.00012	-0.00011	-0.00010	-0.00008	-0.00007	-0.00006	-0.00010	-0.00009	-0.00007	-0.00006	-0.00003	-0.00002	0.00012
skewness	-0.24574	-0.25284	-0.25815	-0.26026	-0.25851	-0.25291	-0.04255	-0.05681	-0.06656	-0.06949	-0.06455	-0.05232	-0.47751	-0.39465	-0.38265	-0.44333	-0.53564	-0.61100	-0.03319
kurtosis	2.14019	2.16024	2.33184	2.65067	3.09701	3.64165	1.57721	1.61353	1.75817	2.03207	2.44246	2.97780	7.16417	6.22168	5.95062	6.16193	6.56028	7.00678	15.02476
modified_VaR	-148.84741	-152.74049	-157.40917	-162.68824	-168.44588	-174.58416	-125.37787	-127.37117	-130.29735	-133.91170	-138.05005	-142.56323	-82.96415	-78.66125	-77.12718	-78.70354	-83.07988	-89.48272	-27.41192
C_VaR	-206.29459	-213.41562	-221.91389	-231.29456	-242.27331	-254.57991	-168.81658	-173.29600	-179.27165	-186.38083	-194.03761	-202.82164	-120.26664	-113.47800	-110.64954	-111.74531	-117.94073	-128.22337	-51.52115

As shown above, we find that:

1. Under the same period of comparison, as beta increases, the return decreases significantly but the variance increases. Take a look-back period of 40 days as an instance:

	E[r]_40,Cov_40					
	beta= -1	beta=-0.5	beta=0	beta=0.5	beta=1.0	beta=1.5
cum_return	0.20033	0.19225	0.16710	0.13158	0.09373	0.06031
ari_mean_return	0.00198	0.00205	0.00213	0.00220	0.00228	0.00235
geo_mean_return	0.07371	0.07064	0.06019	0.04238	0.01711	-0.01573
modified_VaR	-148.84741	-152.74049	-157.40917	-162.68824	-168.44588	-174.58416
C_VaR	-206.29459	-213.41562	-221.91389	-231.29456	-242.27331	-254.57991

2. Under the same target Beta, the portfolio with the shortest period of estimation has the highest return. The variances of the three cases are similar. Take target Beta = -1 as example:

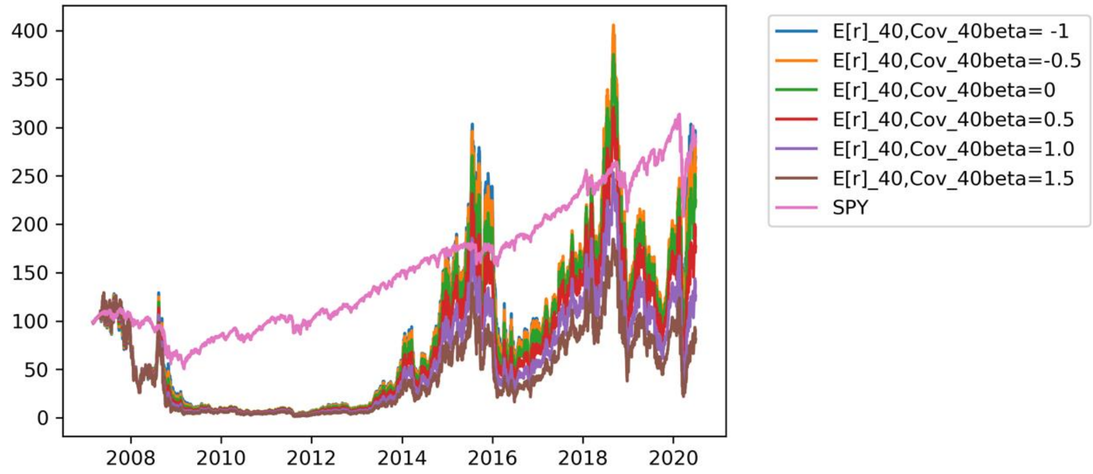
	E[r]_40,Cov_40	E[r]_80,Cov_80	E[r]_120,Cov_120
	beta= -1	beta= -1	beta= -1
cum_return	0.20033	0.00001	0.00083
ari_mean_return	0.00198	-0.00145	-0.00083
geo_mean_returr	0.07371	-0.65168	-0.33483
modified_VaR	-148.84741	-125.37787	-82.96415
C_VaR	-206.29459	-168.81658	-120.26664

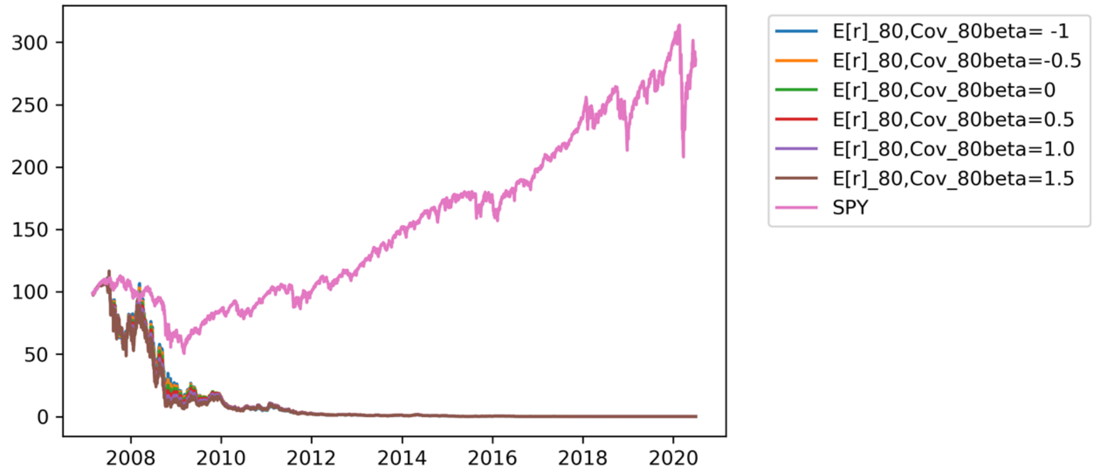
3. Compared with the benchmark SPY, the portfolio with the shortest period of estimation provides the closest result to the benchmark. Take target Beta = -1 as example:

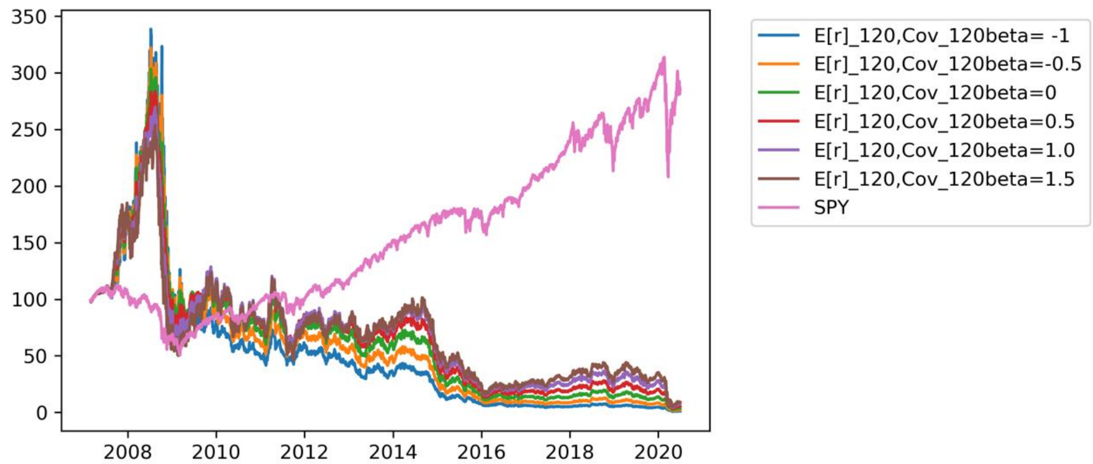
	E[r]_40,Cov_40	E[r]_80,Cov_80	E[r]_120,Cov_120	SPY
	beta= -1	beta= -1	beta= -1	
cum_return	0.20033	0.00001	0.00083	0.212547
ari_mean_return	0.00198	-0.00145	-0.00083	0.000399
geo_mean_return	0.07371	-0.65168	-0.33483	0.078115
modified_VaR	-148.84741	-125.37787	-82.96415	-27.4119
C_VaR	-206.29459	-168.81658	-120.26664	-51.5211

In conclusion, the portfolio with the shortest period of estimation has a similar performance to the benchmark. Compared to their Sharpe Ratio, the portfolio with the shortest period of estimation has a better chance to make profits.

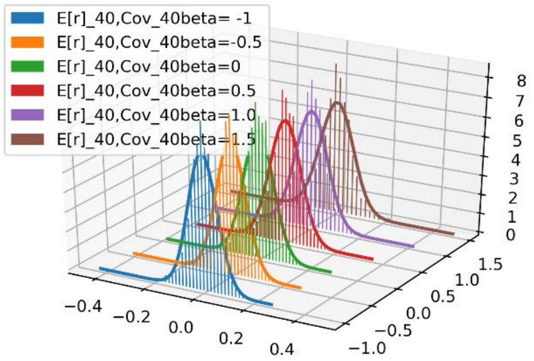
There are also graphs of the changes of Net Value and the distributions of return of the whole period:

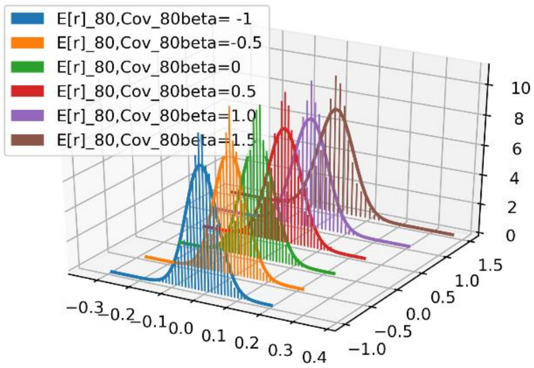


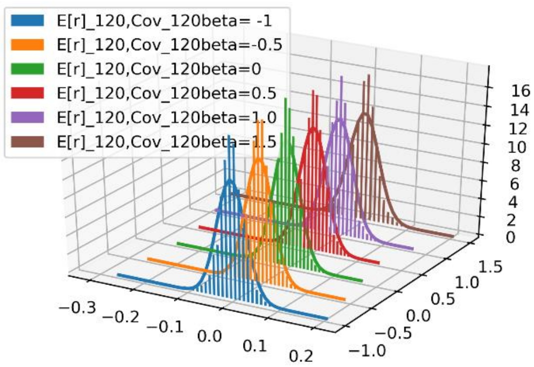




Daily return distributions:







4. Conclusion:

By the result discussed and graph shown in the last section, we found that in the same period, the return of the portfolio would become very different if we change the estimation periods of expected return and variance matrix. However, the change would not be so significant if we modify the target beta for the portfolio.

Based on result shown above, we know that there is no such a perfect estimator that perform well for all periods. In other word, given different market situations, we should use different estimators to build different portfolios. For this project, our recommendations would be:

Before crisis: use portfolio with P_{120}^{120} & $\beta_T^m = 0.5$.
During crisis: use portfolio with P_{80}^{80} & $\beta_T^m = -1$.
After crisis: use portfolio with P_{40}^{40} & $\beta_T^m = 0.5$.

We choose our recommend strategy by considering factors of large Sharpe-ratio, low modified VaR, CVaR and Max 10 days drawdown.

5. Appendix

Code description:

Our code for the project consists of 7 major parts:
Part (1): import historical data of index and factor.

Part (2): define a function to compute weight for the portfolio. Given index data, factor data, target beta, and previous weight, it would return a new weight. The mathematic equation used include:

$$\begin{cases} \max\limits_{\omega \in \mathbb{R}^n} \rho^T \omega - \lambda(\omega - \omega_p)^T Q(\omega - \omega_p) \\ \sum\limits_{i=1}^{n} \beta_i^m \omega_i = \beta_T^m \\ \sum\limits_{i=1}^{n} \omega_i = 1, \quad -2 \le \omega_i \le 2, \end{cases}$$

$$\rho_i = r_f + \beta_i^3(\rho_M - r_f) + b_i^s \rho_{SMB} + b_i^v \rho_{HML} + \alpha_i$$

$$R_t = \begin{bmatrix} r_{1t}^e \\ r_{2t}^e \\ \dots \\ r_{nt}^e \end{bmatrix}, B = \begin{bmatrix} \beta_1' \\ \beta_2' \\ \dots \\ \beta_n' \end{bmatrix} = \begin{bmatrix} \beta_1^3 & b_1^s & b_1^v \\ \vdots & \ddots & \vdots \\ \beta_n^3 & b_n^s & b_n^v \end{bmatrix}$$

$$R_t = \alpha + \mathbf{B}\mathbf{f}_t + \varepsilon_t \text{ for } i = 1,2,\dots,\mathrm{T}$$

$$cov(R_t) = \mathbf{B}\Omega_f\mathbf{B}' + D = \mathbf{B}\Omega_f\mathbf{B}^T + diag(\sigma_1^2, \dots, \sigma_n^2)$$

Part (3): define a function to perform the back-test and do the rebalance work every week. Given index data, factor data, look-back period, and target beta, it returns the list of returns of the portfolio.

Part (4): define a function to compute the performance table required in the project. Given the returns and principal to invest, it returns a performance table.

Part (5): define a function to get return table and performance table.

Part (6): define a function to draw graphs.

Part (7): split data into three periods run all result.

To get all result in this paper, open and run the file “project.ipynb” in the folder “code_graphs_data”. The source data (factor data & index data) & grpahs are also included in that folder.