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Apprenticeship Report M2 Quantitative Finance

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1 Introduction

The purpose of this thesis is to present one of the main assignments I was given during my apprenticeship period as part of my Master 2 in Quantitative Finance and Insurance at the Aix-Marseille School of Economics and Ecole Centrale Méditerranée.

There is a wide range of players in the Global Financial Services (GFS) sector, which encompasses essential and varied financial activities. Firstly, investment banking, which brings together major financial institutions. Their activities include investment advice, securities issuance and trading, corporate finance, mergers and acquisitions, and risk management. Their clients include corporations, financial institutions, governments, investment funds, and some high-net-worth individuals. Then there is asset management, where the major players specialize in managing investment portfolios. Their role is to invest in a variety of assets such as equities, bonds, commodities, etc., to provide investment advice, and to create investment products such as mutual funds and/or ETFs. Their clients are institutional investors such as pension funds, insurance companies, and sovereign wealth funds, as well as individual investors.

Retail banking and wealth management is another area where the major banks offer retail banking and wealth management services. They provide current accounts, cards, loans, and wealth management services to individual customers. Their customers include individuals, high-net-worth individuals, and small businesses. Companies that offer online trading platforms and brokerage services provide brokerage services. Their primary role is to facilitate the buying and selling of stocks, bonds, mutual funds, and other financial products for individual investors and active traders. Insurance companies specialize in providing insurance policies for protection against various risks, such as life insurance, health insurance, home insurance, car insurance, travel insurance, etc. Their clients include individuals, families, businesses, and institutions. Finally, the major commercial banks offer treasury and cash management services. They provide treasury and cash management services to businesses and financial institutions. These services include cash flow management, payment management, foreign exchange risk management, and the provision of transactional banking services.

Natixis Investment Managers (NIM) is one of the leading asset management companies, with the distinction of being a multi-affiliated asset manager, i.e., made up of several management companies. A subsidiary of the Natixis group and therefore an integral part of the Banque Populaire Caisse d'Epargne group, NIM offers a range of diversified solutions covering different asset classes, management strategies, and investment vehicles. Natixis Investment Managers is also committed to the development of sustainable finance, offering strategies and products dedicated to an ESG approach. Created in 2018 within Natixis Investment Managers, the Natixis Investment Managers SOLUTION department aims to develop and offer investment solutions by mobilizing the possibilities offered by Natixis Investment Managers' multi-affiliate structure, fore example through MIROVA.

It was in this department that I did a one-year work-study placement as a quantitative analyst in financial engineering & Quantitative Research Team. During this period, I was able to contribute to the smooth running of the department through several assignments. One of my tasks was to explore a research paper on the implementation of a systematic allocation strategy for equity factors.

So the point is to understand how, and to what extent, this tool is robust enough to make a significant contribution to the business of an asset manager specialising in fund allocation management?

I will begin by providing an overview of the company's organization and the objectives of my apprenticeship. Next, I will delve into the intricacies of the dynamic allocation model, detailing its construction and key components. Finally, I will present a comprehensive analysis and explanation of the results obtained from our testing and experimentation.

2 Environment, context, and objective of the apprenticeship

2.1 Company presentation

Natixis Investment Managers Solutions is a department of Natixis Investment Managers that capitalizes on the varied expertise of 22 independent affiliated asset management companies around the world. With teams based in Paris, London, Singapore, and Boston, the Solution department specializes in portfolio construction and allocation as well as the structured management of multi-class assets. Natixis Investment Managers Solutions' approach is to provide a wide range of expertise and integrated services, putting clients at the center of the investment process. From discovery to design and construction, this approach focuses on creating customized solutions that meet the specific needs of each client.

Natixis Investment Managers Solutions offerings and services include:

Market Strategies: This service provides a comprehensive analysis of expected returns to help clients define their investment strategy in key markets using macroeconomic data and asset allocations.

ESG Solutions: This service focuses on ESG (environmental, social, and governance) and climate change initiatives, offering comprehensive support throughout the ESG journey, from assessment to investment and reporting.

A Portfolio Clarity Tool: This is primarily a high-end independent analysis to help clients better understand the risks and diversification of their portfolios. It also offers ESG assessments, forward-looking climate scenarios, and personalized advice.

Advice: This offering provides expert advice based on an in-depth understanding of each client's needs, including the construction of bespoke model portfolios focused on risk control and alpha generation.

Multi-asset portfolio management: The multi-asset allocation team offers investment solutions in multi-asset class mandates, combining long-term strategic allocation, medium-term tactical allocation, and bottom-up selection.

NIM Solutions is a multi-asset manager specialising in fund of funds allocation management, based on a Benchmark allocation. By combining market strategies, expertise in affiliated funds and specific technical tools, NIM Solution is able to advise and respond to clients' requests.
The structure of the management company is not the simplest, however it is important to understand how the department works. To do this, we first need to integrate the way the different teams that make up NIM Solution are organized. It is also crucial to define the role of the Financial Engineering and Quantitative Research team.

2.2 Company activities and existing tools

The organisation is based on a number of interdependent divisions, each of which plays a specific role in creating investment solutions tailored to customers' needs. Here is a detailed description of each division and its role within the company:

ESG (Environment, Social and Governance): The ESG team's mission is to help management make a gradual transition towards a more ESG-compliant management approach. It also defines rules and criteria for assessing the ESG quality of funds. The ESG team works closely with the other teams to integrate ESG considerations into fund selection and management.

Multi-Asset Investment Management : The Multi-Asset Investment Management team is responsible for managing different types of funds for a variety of clients. It uses a diversified asset allocation approach to optimise returns and manage risk. This team works with the ESG team to ensure that investment decisions incorporate ESG criteria.

Fund Research : The Fund Research team is responsible for analysing and evaluating the funds of affiliated management companies. It carries out an in-depth analysis to recommend or not these funds to management team or the ESG team. This team plays a key role in the fund selection process, ensuring that the funds are of the highest quality and that they meet our clients' investment objectives.

Advisory : The role of the Advisory team is to advise and sell Natixis Investment Managers Solutions products to existing and potential clients. This team works closely with the other divisions to understand clients' needs and provide them with customised solutions that meet their investment objectives.

Financial Engineering & Quantitative Research: Financial Engineering & Quantitative Research team focuses on the design and financial engineering of investment products and solutions. It creates technical tools, participates in calls for tender and client meetings, and ensures that the solutions proposed are technically robust and adapted to clients' needs. Part of the team is also working on research to develop models and tools.

Global Market Strategy : The Global Market Strategy team provides guidance and analysis on the financial markets. This team guides portfolio managers in their market views and helps them to make informed decisions based on current economic and financial conditions.

All these teams interact and work closely together to offer a comprehensive range of investment solutions. Coordination between the ESG, Multi-Asset Investment Management, Fund Research, and Advisory teams ensures an integrated approach where ESG criteria are taken into account in the selection and management of funds. The Financial Engineering & Quantitative Research team ensures the construction and technical implementation of solutions, while the Global Market Strategy division provides strategic guidance to support investment decisions. Finally, the Business Management team plays a key role in the overall organization and coordination of these activities, ensuring that the necessary software and access are in place to support the entire process.

As an integral part of the Financial Engineering & Quantitative Research team, I had to immerse myself in the processes and tools inherent in the role of quantitative analyst. Within the financial engineering team based in Paris, we have access to a number of software and tools that are essential to our day-to-day activities. These have been selected for their effectiveness and their ability to meet our specific needs.

Bloomberg is an essential tool for our team, providing access to a wealth of financial data and functionality. We take full advantage of the Bloomberg API on Excel, which opens up a wide range of possibilities for our quantitative analysis and asset management decision-making. Using the BDH (Bloomberg Data History) and BQL (Bloomberg Query Language) functions, we can retrieve historical data on a wide variety of financial instruments, such as equities, bonds, currencies, indices, commodities, and derivatives. Using this historical data, we can carry out retrospective analyses and identify trends or key market events. This historical information is essential for assessing the past performance of assets and investment strategies. The Bloomberg API gives us the ability to perform complex queries to obtain accurate real-time and historical data. This allows us to react quickly to market events or client requests to make decisions. In addition, Bloomberg provides advanced technical and graphical analysis to help us better understand market trends and identify signals.

Natixis Investment Managers Solutions in-house developed QuantX software. It was designed to meet the team's specific needs in terms of quantitative portfolio management. It is used by the Paris-based team, in particular by quantitative analysts in financial engineering, for the benefit of portfolio managers. This multifunctional software is built in the C# programming language and can be used in three different ways. Firstly, as a database, QuantX stores the daily history of numerous assets. In addition to this, it also records the history of all model portfolio allocations, making it possible to track investment decisions back to the creation of the portfolio. In particular, the software fetches and stores a large number of indices from Bloomberg. QuantX has a graphical interface that allows users to view the history of a large number of assets. Users can also graph rolling indicators of a time series and quickly build allocations and portfolios. Members of Natixis Investment Managers Solutions use this functionality to view and extract data from Excel as well as to produce charts.

QuantX also offers a library of classes that can be used in other programs compatible with the Microsoft environment, notably Microsoft Excel. This means that it is possible to use QuantX in Excel through existing functions. For example, the "qtxHistory" function can be used to retrieve the entire history of an asset with a given frequency and currency directly in Excel cells without having to go through Quantx. What's more, it can also be used in VBA (Visual Basic for Applications). Within the team, QuantX is mainly used for historical data analysis, performance attribution, and portfolio construction. Its interface and advanced functionalities make it a valuable tool for quantitative management and research.

2.3 Objectives of the apprenticeship

During my work-study placement, I carried out a variety of assignments, including regular tasks, one-off tasks, and background work. The daily and monthly tasks were essential to the smooth running of the business.

Among my responsibilities, I was in charge of the performance assignment of the management team's model portfolios, which play a crucial role in portfolio managers' investment decisions. Each model portfolio follows a specific strategy designed to meet specific investment goals, in line with a benchmark allocation defined as follows:

- The first model portfolio adopts a beta diversification strategy aimed at distributing investments across a wide range of asset classes, sectors, and geographical regions. This reduces exposure to certain asset-specific risks and aims to stabilize overall performance. The interest is to reduce exposure to market fluctuations while maintaining market exposure in order to benefit from long-term positive movements, with a composition close to 50% equity and 50% fixed income.
- The second model portfolio adopts a flexible strategy, offering greater flexibility in asset allocation decisions based on changing market conditions. Portfolio managers can adjust asset weights to a predefined margin of maneuver, allowing them to take advantage of trends and price changes observed in different market segments with a composition close to 50% equity and 50% fixed income.

These two model portfolios, although different in their strategies, must have similar investment universes in order to ensure a coherent investment direction. They serve as benchmarks for managers,

who cannot go against the positions decided in the model portfolios.
At the end of each month, a meeting of the Cross-Asset Team (CAT) brings together senior managers, strategists, and financial engineers to share their market beliefs and discuss potential adjustments for the allocation of the model portfolios. These meetings take into account macroeconomic perspectives, sectoral developments, geopolitical developments, and risks specific to certain assets.

Another regular task was to update the benchmarks for funds allocated on a risk-parity basis. The aim of this strategy is to maximize risk diversification within a portfolio and to recalibrate the allocation each month in order to control portfolio risk over time. As associated with one of these funds, I also had to produce a quarterly report on the contribution to the performance of the fund's components.

I was also in charge of carrying out fund allocation backtests for the retirement savings of potential corporate clients. The aim was to simulate the return on retirement savings for different investor profiles. The Prudent, Equilibre, and Dynamic profiles each have different fund allocations. To do this, we calculate what their profit would have been if they had invested in our funds in X year of retirement, with a fixed annual contribution, for each saver's profile.

In general, the fund tasks I was given focused on the analysis of technical indicators and graphical curves, the study of research papers on systematic allocation and portfolio design, or the in-depth study of specific issues.
As a result, I will build my thesis on a study of a research article published in 2022 on the issue of dynamic allocation within a multi-factor portfolio based on economic regimes. The goal of this research is to create a macroeconomic tool that will provide a systematic allocation to a portfolio comprised of factors of the US stock market.

2.4 Presentation and mission statement

Dohyoung Kwon's study, on which my thesis is based, aims to develop a method for enhancing the performance of a multifactor portfolio by taking into account variations in economic regimes. Economic cycles, characterized by phases of growth, slowdown, or recession, have a significant impact on the performance of financial assets. The author therefore proposes a Factor Rotation Method that aims at adapting portfolio composition to these different economic phases. He suggests using a variety of financial factors that economists E. Fama and K. French identified in their work on asset valuation. These factors provide information that influences equity returns and, therefore, the returns on an equity portfolio. The strategy is based on the construction of an initial allocation based on these factors and defined criteria. Next, the author constructs a dynamic approach based on identifying the positioning of the economy in its cycle, using macroeconomic variables to build a signal on which the allocation rotation is based. Once the economic phases have been found, the factor rotation strategy changes the weighting of the different factors in the initial portfolio to improve returns while actively managing the risks that come with each economic regime.

The main contributions of this study are multi-facted. Firstly, it proposes a systematic and innovative approach to the construction and management of multifactor portfolios. Secondly, by taking economic cycles into account, this strategy aims to improve portfolio resilience in the face of market fluctuations. In addition, it enables asset managers to better understand the opportunities and risks associated with different economic regimes, thereby enhancing decision-making and the overall effectiveness of portfolio management.

The construction of a multifactor portfolio that adapts its allocation according to the macroeconomic situation can have several uses in asset management. It can be a way for managers to have an indicator of the macroeconomic impact on their exposure to the equity market. This portfolio can also be useful in the construction of a fund made up of ETFs that tracks the performance of the factors in question. Finally, it can also be used as a benchmark.

3 Execution of the Mission

Factor-based investing experienced significant development, particularly in the period following the subprime crisis, especially among pension funds and institutional managers. The interest stems from the possibility of exposure to risk factors, which offers a means of diversification while generating returns. Despite the advantages of Factor Investing, there is no consensus on management strategy. There is still debate as to whether a dynamic allocation is better than a static one, given the benefits of diversification.

As mentioned above, the aim of this project is to implement a dynamic rotation strategy for factor exposure based on changes in economic regimes. Several studies have been carried out on this subject. In particular, the work of Harvey (1989), Ferson and Harvey (1991), Ilmanen et al. (2014), Hodges et al. (2017), and Amenc et al. (2019) showed that factor premiums are highly cyclical and closely linked to macroeconomic conditions. Other more recent works have sought to put certain quantitative allocation methods into practice, such as Bruno Taillardat et al. (2018) and Alessio de Longis et al. (2019), who have carried out a dynamic allocation with reference to macroeconomic regimes.

3.1 Understanding the Essentials

Our objective is to enhance an initial allocation strategy by tailoring it to the prevailing economic conditions, thereby harnessing the cyclical nature of the factors. To achieve this, we begin by constructing an initial allocation using a quantitative strategy. Subsequently, we aim to incorporate economic perspectives into this allocation. To accomplish this task effectively, we rely on various knowledge and mathematical tools to identify economic regimes and construct portfolios.
In particular, it needs to understand what factors there are and how they can be used. Understanding the different quantitative management models that have been used for these factors But also how economic regimes are identified and with what tools. Then I'll go into detail about how the final dynamic rotation model was constructed.

3.1.1 Fundamental financial metrics

In finance, diverse metrics gauge risk, a critical focus for investors and asset managers. The primary aim is to comprehensively analyze risk types and levels while optimizing returns. H. Markowitz's Modern Portfolio Theory emphasizes this complexity by demonstrating the complex relationship between risk and return.
In most financial contexts, the key objective is to maximize profits while minimizing acceptable risk. Striking the optimal balance between return and risk is essential in investment strategy. Metrics related to risk and performance are vital in portfolio management and asset allocation.
Some metrics permit individuals and institutions to construct portfolios aligned with their financial goals, risk tolerance, and investment horizon by assessing both risk and performance.

The Expected Return

The expected return is one of the first criteria we use to assess the effectiveness of a portfolio strategy. This statistic is generated from historical data research and serves as a core pillar in analyzing investment strategies.

$$E\left(R_p\right)=E\left(\sum_{i=1}^{n} w_i r_i\right)=\sum_{i=1}^{n} w_i E\left(r_i\right)$$

Where:
$w_i =$ asset weight i
$r_i =$ asset return i
$R_p =$ portfolio return

It reflects the average or mean return that an investment might expect based on prior performance. However, we have to keep in mind that predicted returns are necessarily linked to prior data, making them historical averages rather than future guarantees.
It is crucial to emphasize that, while predicted return provides useful information, it should not be considered in isolation. To get a holistic picture of an investment's potential, investors must analyze it alongside other essential criteria such as risk measurements.

The Volatility

Volatility is an important statistic for both portfolio managers and investors. It gives useful information about the risk associated with an asset or a portfolio of assets.
When applied to a single asset, it represents a statistical assessment of the item's price movements over a certain time period. It is the standard deviation of the asset's returns over that time period. A higher standard deviation indicates greater price volatility and, as a result, greater risk.
Calculating volatility is equally important when working with a portfolio of many assets. Portfolio volatility considers not just the volatility of individual assets but also their correlations or covariances with one another.

$$V\left(R_p\right)=\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j \operatorname{cov}\left(w_i, w_j\right)=W^T \sum W$$

$$\sigma\left(R_p\right)=\sqrt{V\left(R_p\right)}=\left(W^T \Sigma W\right)^{\frac{1}{2}}$$

Where:
$w_i =$ asset weight i
$w_j =$ asset weight j
$\operatorname{cov}\left(x_i, x_j\right) =$ covariance of assets i and j

A common thing is to annualize expected returns as well as volatility, which is a crucial step in financial analysis, and it depends on the frequency of the data being used.
If we have daily returns, you can simply multiply the average daily return by the number of trading days in a year (252 days). If we're working with monthly returns, we multiply the average monthly return by the number of months in a year (12 months). For quarterly data, we multiply the average quarterly return by the number of quarters in a year (4 quarters). When it comes to volatility, the logic remains the same, with one adjustment: instead of a simple multiplication by the number of units in a year, we apply the square root.

The Covariance

Covariance is an important statistic in finance, especially when we want to assess both the volatility of individual portfolio components and the links, or correlations, between them.
The covariance between two assets, denoted as $\mathrm{Cov}(A, B)$, is calculated as follows:

$$\mathrm{Cov}(A, B) = \frac{1}{n-1} \sum_{i=1}^{n} \left(A_i - \bar{A}\right) \left(B_i - \bar{B}\right)$$

Where:
- n is the number of data points or observations.
- A_i and B_i are the returns of Asset A and Asset B at time i.
- $\bar{A}$ and $\bar{B}$ are the means (average returns) of Asset A and Asset B, respectively.

Covariance is important in assessing the risk associated with individual assets in a portfolio. It provides useful insights into the overall risk profile of the portfolio.
Furthermore, covariance serves as the foundation for comprehending correlations. The degree to which two or more assets move with respect to each other is shown by correlation.
This knowledge, in turn, influences asset allocation, risk management, and the creation of well-balanced, diverse portfolios.

The Sharpe Ratio

The Sharpe Ratio is a fundamental metric in finance, commonly used to assess the risk-adjusted performance of an investment portfolio or an asset. It is a key metric for portfolio managers and portfolio analysts.
The Sharpe Ratio, denoted as SR, is calculated as follows:

$$\mathrm{SR} \ = \frac{R_p - R_{rf}}{\sigma_p}$$

Where:
- $R_p =$ is the rate of return of portfolio p.
- $R_{rf} =$ is the risk free rate, also called monetary rate.
- $\sigma_p =$ is the volatility of portfolio p.

The Sharpe Ratio measures the excess return generated by a portfolio per unit of risk taken, with risk defined as the portfolio's volatility or standard deviation of returns.
A higher Sharpe Ratio indicates better risk-adjusted performance, as the portfolio generates more return for each unit of risk incurred. Conversely, a lower Sharpe ratio suggests a less favorable risk-return trade-off. It plays a pivotal role in portfolio optimization and decision-making, guiding investors toward portfolios that align with their risk tolerance and return objectives. We can also decompose the Sharpe ratio of a portfolio to study how its components contribute to the Sharpe.

The asset-sharpe ratio has the same properties; however, the risk-free rate is not involved here since the risk is intrinsic to the asset.

$$ASR = \frac{r_i}{\sigma_i}$$

Where:
- $R_i =$ is the risk free rate of asset i
- $\sigma_i =$ is the volatility of asset i

The Sharpe Ratio aids in identifying whether an investment provides an adequate return, considering the level of risk involved.

The Risk Contribution

The risk contribution is a crucial concept to understand. Referring to the portfolio volatility calculation, we can see that the covariance matrix and weight vectors may be used to measure the portfolio's overall risk. We may calculate the marginal risk of the asset i and the risk contribution by applying this formula as a function of the asset weight i.

$$MRC_i = \frac{\partial \sigma_p}{\partial w_i}, \quad RC_i = w_i \frac{\partial \sigma_P}{\partial w_i}, \quad PRC_i = \frac{w_i}{\sigma_P(w)} \frac{\partial \sigma_P}{\partial w_i}$$

Where:
- σ_P is the global portfolio risk.
- w_i is the weight of the asset i.
- MRC_i is the Marginal Risk Contribution of the asset i to the portfolio P.
- RC_i is the Absolute Risk Contribution of the asset i to the portfolio P.
- PRC_i is the Percentage Risk Contribution of the asset i to the portfolio P.

The Tracking Error

Tracking error is a critical metric in finance, particularly when evaluating the performance of an investment portfolio relative to its benchmark index or portfolio.
Tracking error, denoted as TE, is calculated as follows:

$$\text{TE} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (P_i - B_i)^2}$$

Where:
- n is the number of data points or observations.
- P_i represents the returns of the portfolio at time i.
- B_i represents the returns of the benchmark index at time i.

Tracking error quantifies the variability in returns between the portfolio and its benchmark. It measures the portfolio manager's success in mimicking the benchmark's performance. A lower tracking error suggests that the portfolio closely follows the benchmark, while a higher tracking error indicates deviations from the benchmark. The Tracking Error, can also be decomposed in order to get the contribution of the components to the TE.

Maximum Drawdown (MDD)

Maximum Drawdown (MDD) is a crucial indicator in finance that assesses the highest negative deviation or loss an investment portfolio or asset suffers from its peak value within a certain time.
One formula to calculate Maximum Drawdown by using portfolio returns is as follows:

$$\text{MDD} = \min_{i,j:j<i} \left(\frac{P_i - P_j}{P_i} \right) - 1$$

Where:
- P_i represents the portfolio cumulative return at time i.
- P_j represents the portfolio cumulative return at time j.
- The min function finds the maximum down-slide value over all possible pairs of i and j where j is lower than i.

Maximum Drawdown estimates the biggest negative deviation from an investment's peak return or value over a certain time period. It assists investors and portfolio managers in understanding the worst-case scenario for losses. A lower Maximum Drawdown implies a more unstable investment with more downside risk, while a greater Maximum Drawdown closer to zero indicates less risk and the possibility of lower losses.

3.1.2 The Factors

A factor, in the investment context, refers to a specific characteristic of financial assets that can explain their performance over and above overall market performance. Factors are variables or attributes that have been identified as having a significant and systematic influence on asset returns over an extended period of time.
Factors may be based on economic fundamentals, financial ratios, valuation indicators, or market trends. They represent sources of risk and return inherent in the different assets in a portfolio.

The subprime crisis and the Internet bubble of 2000 prompted investors to turn to asset classes that offered better risk diversification than index strategies. Indeed, capitalization-weighted indices were dominated by stocks in the sectors affected by these bubbles, leading to higher levels of risk and potential losses.

Factors	**Description**
Size	The SMB (Small Minus Big) factor calculates the difference between the average return of the 30% small-cap equity portfolios and the average return of the 30% large-cap equity portfolios. This factor measures the effect of market capitalisation on equity returns.
Value	The HML (High Minus Low) factor calculates the difference between the average return of the two value equity portfolios and the average return of the two growth equity portfolios. This factor measures the effect of equity valuation on returns.
Momentum	The WML (Winner Minus Loser) factor calculates the weighted average of the returns of winning portfolios and subtracts the average of the returns of losing portfolios. This factor measures the tendency of equities to continue their past performance in the short term.
Profitability	The RMW (Robust Minus Weak) factor calculates the difference between the average return of the two equity portfolios with robust profitability and the average return of the two equity portfolios with weak profitability. This factor measures the effect of efficiency on returns.
Investment	The CMA (Conservative Minus Aggressive) factor calculates the difference between the average return of the two equity portfolios with a conservative profile and the average return of the two equity portfolios with an aggressive profile. This factor measures the effect of corporate investment on returns.

Table 1: Equity Factors

Low-risk approaches have naturally attracted significant interest among investors as they aim to produce a more balanced portfolio in terms of risk. This includes portfolios in which the risk contribution of different assets is equalized, others in which overall portfolio volatility is minimized, and another category that seeks to maximize portfolio diversification.

3.1.3 Portfolio optimization models

Portfolio construction and optimization methods stem from the work of Markowitz, who studied the relationship between return and risk, seeking to minimize the level of risk in a portfolio for a given level of return. Or conversely, to maximize the portfolio's return for a given level of risk. Thus, if, for a large number of risk levels, we maximize the return on a portfolio, we obtain the portfolio's efficient frontier. Many other models with different constraints and objective functions are based on this model.

Generally speaking, a portfolio optimization model is a tool used in finance to construct investment portfolios that meet specific objectives and strategies. The main objective of an optimization model is to find the optimal combination of assets that maximizes expected returns while minimizing risk, given different constraints and investor preferences. The table below presents different portfolio optimization models and their specific characteristics. Each model is associated with a mathematical optimization problem that defines the objective function (to be maximized or minimized) and the constraints governing the composition of the portfolio.

Model	Optimization Problem	Strengths	Weaknesses
Equally Weighted	$w_i = \frac{1}{N}$ $N =$ Total assets	1. No assumptions about portfolio components. 2. Minimizes concentration in the portfolio by assigning equal weights to all assets.	1. No risk management. 2. Ignores differences in asset risk, return, and correlation.
Minimum Variance	$\min_w w\Sigma w'$ s.t. $\sum_{i=1}^{N} w_i = 1$	1. Does not rely on specific assumptions about returns. 2. Minimizes the global variance of the portfolio, considering correlation.	1. No risk management. 2. Ignores differences in asset risk and return potential.
Maximum Sharpe Ratio	$\max_w \frac{R_p - R_f}{\sigma_p}$ s.t. $\sum_{i=1}^{N} w_i = 1$	1. Considers risk-return trade-off, also called efficient portfolio in the sense of Sharpe. 2. Provides a benchmark for comparison.	1. Sensitive to input data and parameter estimation. 2. Ignores other risk metrics.
Maximum Diversification	$\max_w \frac{\sigma^T w}{\sqrt{w^T \Sigma w}}$ s.t. $\sum_{i=1}^{N} w_i = 1$	1. Enhances risk reduction through diversification. 2. Attractive for risk-averse investors.	1. Performance dependent on asset selection and estimation. 2. May not consider expected returns.
Naive Risk Parity	$\min_w \sum_{i=1}^{N} \left(\frac{w_i}{\sigma_i}\right)^2$ s.t. $\sum_{i=1}^{N} w_i = 1$	1. Inverse relationship between asset weights and volatilities. 2. Requires only asset volatilities.	1. Ignores correlations between components. 2. Ignores differences in expected returns.
Equally Risk Contribution	$\min_w w^T \sum w$ st. $\frac{w_i(\Sigma w)_i}{\sqrt{w'\Sigma w}} = \frac{w_j(\Sigma w)_j}{\sqrt{w'\Sigma w}}$	1. Considers asset volatilities and correlations. 2. Minimizes risk concentration.	1. More complex implementation. 2. Ignores differences in expected returns.
Mean Variance	$\min_w \delta \frac{1}{2} w'\Sigma w$ s.t. $w'\mu = c$ $w'1 = 1$	1. Optimal risk-return trade-off. 2. Considers both expected returns and risk.	1. Sensitive to input data and parameter estimation. 2. Assumes normal distribution of returns.
Minimum Correlation	$\min_w \sum_{i,j=1}^{N} w_i w_j \rho_{ij}$ s.t. $\sum_{i=1}^{N} w_i = 1$	1. Reduces portfolio correlation. 2. Diversification benefits.	1. Sensitive to estimation errors and changes in correlation structure. 2. Does not explicitly consider expected returns.
Black-Litterman	$\mu = (\tau\Sigma)^{-1}(\tau\Sigma\pi + P^T Q)$ $\Sigma = (\tau\Sigma)^{-1} + P^T \Omega P$ $w^* = (\delta\Sigma)^{-1}\mu$	1. Incorporates investor views and market equilibrium. 2. Enhances the Mean-Variance model. 3. Flexible and adaptive.	1. Requires estimation of covariance matrix and investor's views. 2. Complexity and implementation challenges. 3. Sensitive to choice of market equilibrium and investor's views.

Table 2: Models of Portfolio Construction

The models presented include classic approaches such as equipondation, minimum variance, and maximum Sharpe ratio, as well as more sophisticated approaches such as maximum diversification and equal contribution to risk. Each model has its own strengths and weaknesses in terms of simplicity, risk reduction, consideration of expected returns, diversification, and sensitivity to input data.

The value of portfolio optimization models lies in their ability to provide objective, mathematically-based recommendations for portfolio construction. These models help investors make informed decisions by balancing return and risk according to their specific preferences and constraints. They are used by portfolio managers, institutional investors, and individuals to meet a variety of investment objectives. For example, these models can be used to perform a one-off rebalancing of a portfolio's allocation based on historical data for the portfolio's components.

3.1.4 Principal component analysis

Factor models are statistical models used to describe the structure of observed data, assuming the existence of latent variables or factors. They attempt to capture the relationships between variables by means of reduced-dimension factors. For instance, factor analysis (FA) considers that a number of underlying factors influence the observed variables. These models are useful for reducing the data, identifying patterns, and understanding the hidden structure of the data.

In factor analysis, the observed variables are modeled as linear combinations of latent factors and a noise term. The key idea is that the variables share a common variance due to the underlying factors as well as a variance specific to each one. The objective is to estimate the factor loadings that determine the relationships between the latent factors and the observed variables.

$$x = \Lambda f + \epsilon$$

where:

- x is a d-dimensional vector of observed variables.
- Λ is a $d \times q$ factor loading matrix, where q is the number of latent factors.
- f is a q-dimensional vector of latent factors.
- ϵ is a d-dimensional vector of unique noise terms.

Principal Component Analysis (PCA) is a specific technique in the field of factorial models. It is a dimensionality reduction method that aims to transform the original variables into a new set of variables (principal components) that are orthogonal (uncorrelated) and capture the maximum variance present in the data.

It is carried out in several stages, as follows:

1. **Standardization:** Given a data matrix $\mathbf{X}$ with n observations and d variables, standardize the data by subtracting the mean and dividing by the standard deviation for each variable.
2. **Covariance Matrix:** Compute the $d \times d$ covariance matrix $\mathbf{S}$ of the standardized data.
3. **Eigenvalue Decomposition:** Calculate the eigenvalues (λ) and eigenvectors ($\mathbf{v}$) of the covariance matrix $\mathbf{S}$. Sort the eigenvectors by their corresponding eigenvalues in descending order.
4. **Principal Components:** The i-th principal component is given by $\mathbf{PC}_i = \mathbf{X}\mathbf{v}_i$, where $\mathbf{v}_i$ is the i-th eigenvector.
5. **Projection:** Project the original data onto the new principal component axes: $\mathbf{Y} = \mathbf{X}\mathbf{V}$, where $\mathbf{V}$ is a matrix of the top k eigenvectors.

PCA possesses several properties:

Variance Maximization: PCA aims to maximize the variance of the data projected onto new axes (principal components). The first principal component captures the most variance, then the second, and so on.

Orthogonality: The principal components are orthogonal to each other, meaning they are not correlated. This property simplifies interpretation and analysis.

Dimensionality Reduction: PCA reduces the dimensionality of data while retaining as much information (variance) as possible. It allows visualization of high-dimensional data and identification of key features.

Data Reconstruction: PCA enables data reconstruction using a subset of principal components. The original data can be approximately reconstructed from the projected data.

Noise Reduction: High-dimensional noise tends to concentrate in directions with low variance. Focusing on the highest principal components can effectively reduce noise.

N.B. : As you will see in this study, our goal was to capture the shared variance among our variables primarily resulting from their strong correlations, rather than specifically aiming to reduce dimensionality.

3.1.5 L1 Trend Filtering

There are a large number of filtering methods for time series. Each filtering method has specific properties but always has the same objective, which is to provide additional information about the data. Depending on the information you want to highlight, certain models are preferable. The best-known methods include moving averages, the Kalman filter, and seasonal decomposition filters, but we're going to focus more specifically on the Hodrick and Prescott filter (HP filter), which separates a time series into two components: the trend and the cycle. This method has been widely used by economists to analyze countries' long-term economic cycles and their underlying trends.

The Hodrick-Prescott filter is based on the fundamental idea that economic time series are made up of two elements: a structural trend reflecting long-term developments, and a cyclical component representing short-term economic fluctuations around this trend. This decomposition makes it possible to isolate long-term economic movements from shorter, potentially noisier variations.
The Hodrick-Prescott (HP) filter is a popular method for separating a time series into its trend and cyclical components. It is commonly used in economics and finance to decompose a time series into a long-term trend and short-term fluctuations.

The HP filter equation can be written as follows:

$$y_t = \tau_t + c_t$$

Where:
y_t is the observed time series data at time t.
τ_t represents the trend component at time t.
c_t is the cyclical component at time t.

The goal of the HP filter is to estimate the trend component τ_t and the cyclical component c_t in such a way that the sum of their variances is minimized while satisfying the following two conditions:

1. Smoothing Condition: This condition aims to minimize the variability in the estimated trend component. It is achieved by minimizing the sum of squared second differences of the estimated trend component, which can be written as:

$$\sum_{t=2}^{T-1}[(\tau_{t+1} - 2\tau_t + \tau_{t-1})^2]$$

2. Trend Deviation: This condition enforces the trend to deviate as little as possible from the observed data. It can be expressed as:

$$\sum_{t=1}^{T}(\tau_t - y_t)^2$$

Where λ is a smoothing parameter that controls the trade-off between fitting the data and smoothing the trend. Once we have estimated the trend component, we can compute the cyclical component c_t, as the difference between the observed data and the estimated trend.

The HP filter solves the optimization problem by finding the values of τ_t that minimize the combined objective of the smoothing condition and trend deviation.
By applying the HP filter, the time series is decomposed into its trend and cyclical components, which can provide insights into the long-term behavior and short-term fluctuations of the data.

$$\min_{\tau_t} \sum_{t=1}^{T}(y_t - \tau_t)^2 + \lambda \sum_{t=2}^{T-1}[(\tau_{t+1} - 2\tau_t + \tau_{t-1})^2]$$

The concept of the HP Filter served as inspiration for Kim et al. (2009) to develop a distinct filtering technique known as L1 trend filtering. Building upon the principles of the HP Filter, they introduced the L1 trend filtering method, which shares certain foundational ideas in the structure.

The L1 trend filtering problem can be formulated as follows:

$$(1/2) \sum_{t=1}^{n} (y_t - x_t)^2 + \lambda \sum_{t=2}^{n-1} |x_{t-1} - 2x_t + x_{t+1}|,$$

which can be written in matrix form as,

$$(1/2)\|y - x\|_2^2 + \lambda \|Dx\|_1$$

where:
y is the input signal.
x_t is the estimated trend signal to be determined.
λ is the regularization parameter controlling the trade-off between fidelity to the data and sparsity of the estimated trend.
D is the finite difference matrix, representing the first-order differences between adjacent elements of x_t.
$\|\cdot\|_2$ denotes the L2 (Euclidean) norm, measuring the quadratic difference between vectors.
$\|\cdot\|_1$ denotes the L1 norm, measuring the absolute values of elements in a vector.

The L1 trend filtering method has a penalty term that is based on the absolute values of first differences (rather than squared second differences) of the estimated trend component. This penalty encourages sparsity in the first differences of the trend component.

One of the main objectives of those methods is to separate a time series into its trend and cyclical components. The trend component represents the long-term or underlying behavior of the data, while the cyclical component captures shorter-term fluctuations or deviations from the trend. The goals of the filters are also to reduce or remove noise and random variations in the data and smooth trends to make them more interpretable and less sensitive to short-term fluctuations.
The L1 filter is different from the HP filter, and the choice between these methods depends on the specific characteristics of the time series and the signal we want to extract. When the HP gives smooth trend estimates by penalizing the curvature of the trend component, the L1 filter will be less sensitive to outliers in the data and look like a piece-wise linear function.

3.1.6 Vector Auto-Regressive (VAR)

A VAR model describes the evolution of a set of k variables, called endogenous variables, over time. These variables are collected in a $(k \times 1)$ vector, y_t. The vector is modeled as a linear function of its previous values. The components of the vector are referred to as $y_{i,t}$, representing the observations at time t for the ith variable.
VAR models are characterized by their order, which refers to the number of earlier time periods the model will consider. A lag represents the value of a variable in a previous time period. So, in general, a pth-order VAR refers to a VAR model that includes lags for the last p time periods. A pth-order VAR model can be written as:

$$y_t = c + A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + e_t,$$

Here, the variables of the form y_{t-i} indicate the value of the variable i time periods earlier, and they are called the "ith lag" of y_t, it can be write as $y_t = (yt_1, yt_2, \ldots, y_t k)$. The vector c is a k-vector of constants serving as the intercept of the model. A_i is a time-invariant $(k \times k)$ matrix, and e_t is a k-vector of error terms.

We can rewrite the model equation in the form of a multivariate regression as :

$$y_t = \Psi^\top x_t + \epsilon_t,$$

where $\Psi = (A_1, A_2, \ldots, A_p, C)^\top$ is a $(Kp + L) \times K$ matrix of coefficients, $x_t = \left(y_{t-1}^\top, \ldots, y_{t-p}^\top, d_t^\top\right)^\top$ is a $(Kp + L) \times 1$ vector of regressors.

The error terms must satisfy three conditions:
$\mathrm{E}(e_t) = 0$, Every error term has a mean of zero.
$\mathrm{E}(e_t e_t') = \Omega$, The contemporaneous covariance matrix of error terms is a $k \times k$ positive-semidefinite matrix denoted Ω.
$\mathrm{E}(e_t e_{t-k}') = 0$,For any non-zero k, there is no correlation across time. In particular, there is no serial correlation in individual error terms.

The Vector Auto-Regressive (VAR) model serves as a valuable tool for modeling the dynamics of a set of endogenous variables over time. Its ability to capture lagged effects and relationships between variables makes it a powerful instrument in time series analysis and forecasting.

3.2 Model construction

Having defined the necessary tools we need, we can now focus on the construction of the dynamic allocation model. As a reminder, allocation rotation is based on economic regimes identified using a macroeconomic indicator.

3.2.1 Macro-Indicator's construction

To create the Macro-indicator, we selected five macroeconomic variables that provide information specific to the US economy.

- The first index gauges the spread between 10-year US Treasury bond yields and FED interest rates. A wide yield spread may signal market forecasts of an upcoming economic slowdown, while a narrow spread may indicate market expectations of economic growth. This index measures how the market sees the Federal Reserve's monetary policy and its possible influence on the larger economy. A broad spread may signal an economic recession, whereas a narrow spread may signal growth.
- A second index compares the rates on corporate bonds rated Baa by Moody's to those on 10-year US Treasury bonds. This spread represents the market's perception of corporate debt default risk. A big spread between Baa-rated corporate bonds and Treasuries may indicate concerns about firms' and the entire economy's financial stability. A narrower spread, on the other hand, may imply greater trust in business creditworthiness.
- The four-week moving average of the first jobless claims index filed by people who have recently been laid off. It indicates the health of the labor market, with a spike in first-time unemployment claims indicating economic turmoil and a likely labor market downturn. A decline in claims, on the other hand, indicates that the economy and labor market are improving.
- The total number of new privately owned residence permits issued, which is a leading economic activity indicator. An increase in the number of house permits implies increasing demand for real estate, which has a knock-on effect on other sectors such as construction, raw materials, and the labor market.
- Finally, the Volatility Index assesses the implied volatility of the United States stock market. When the 'fear index' rises, it indicates increased anticipation of stock market turmoil; when it falls, it indicates decreased investor confidence. A lower VIX, on the other hand, indicates higher confidence in the economy and financial markets.

For each variable, we choose a monthly frequency because the data are widely available and are commonly used in macroeconomic research. Monthly data also provides a good balance between capturing short-term fluctuations in the economy and avoiding excessive noise in the data.

Then, by standardizing and performing a Principal Component Analysis (PCA) on the five macroeconomic variables selected, we can synthesize the information contained in these variables into a more comprehensible macroeconomic indicator. The aim is to identify the main trends and relationships between these variables while reducing their dimensionality, thus facilitating their interpretation. In this context, PCA is used to extract the underlying factors that explain the correlation observed between the different variables. The first factor to be extracted will be the one that captures the greatest proportion of the total variance in the data. In other words, it is the eigenvector associated with the highest eigenvalue. This first factor represents a linear combination of the original variables that best synthesizes the common variations observed in these variables.

Figure 1: Macro-Indicator and major events since 1990

This macroeconomic indicator has several advantages. Firstly, it reduces the complexity of the analysis by condensing the information into a single significant factor. Secondly, it can be interpreted more easily than the original variables since it represents a general trend that synthesizes the joint evolution of the five variables. Thirdly, it allows us to track movements and fluctuations in the US macroeconomic environment more intuitively.

There are several methods for identifying economic regimes on the basis of the indicator. One of the best-known methods for identifying economic regimes is the Markov Switching Model, which is used to model time series that show changes in regime or behavior over time. This model is based on the principles of the Markov chain, a probabilistic theory that describes transitions between different states in a stochastic process. In essence, the Markov regime-switching model assumes that observed data are generated by a number of regimes or unobservable states and that transitions between these regimes follow a specific probability distribution. Each regime can be characterized by different statistical parameters. Although this model is efficient, it is more accurate in identifying binary regimes, such as volatility or trendiness. But as the number of regimes increases, the model becomes less relevant.

To identify our economic regimes, we relied on simple transformations of our macroeconomic indicators. This method has been widely used in several studies, as shown by Vliet and Blitz (2011). To do this, we used the L1 norm filtering method on our indicator to provide us with a short-term trend signal. Thus, by combining the trend with the level of our indicator, we will be able to identify the position of the economy in its cycle using an identification rule.

Figure 2: Filtration of the Macro-Indicator

3.2.2 Regimes

For our model, we chose four economic regimes for theoretical reasons, and empirical evidence suggesting four regimes, as well as comparisons with similar studies, reinforce this choice. Indeed, the criteria of interpretability and the stability of results depend on the number of regimes. This selection therefore maintains a balance between capturing significant variation and avoiding excessive complexity. The regimes are as follows: Recovery, Expansion, Slowdown, and Contraction.

The four economic regimes are defined based on the following approach:

- Recovery: The recovery phase denotes an economic period in which activity is below average but on an upward trend. This stage is distinguished by an increase in economic activity that indicates a favorable shift in the economic environment. During a rebound, companies frequently regain confidence, employment rates climb, and consumer spending begins to increase.
- Expansion: The expansion regime denotes a time in which economic activity exceeds the average and continues to accelerate. This stage exhibits a strong and consistent growth trajectory with growing momentum. Expansions are distinguished by increasing economic activity, higher industrial production, positive consumer and corporate confidence, and an overall favorable economic environment.
- Slowdown: During the slowdown period, economic activity stays above average but begins to decline. This stage denotes a slowing of economic growth, indicating a transition from fast expansion to a more moderate pace. A slowdown does not always indicate that a recession is rapidly approaching; rather, it indicates that the economy is transitioning.
- Contraction: The contraction regime is a difficult economic period in which economic activity swiftly falls and decelerates significantly. This stage emphasizes a reduction in economic performance, which is frequently connected with economic downturns or recessions.

Since the macro-indicator serves as a representative measure of the US economy and is standardized with a mean value of zero. This standardized representation enables the assessment of the economy's health in relation to the mean, determined by the polarity of the indicator. Additionally, the slope of the trend line associated with the macro-indicator, filtered through the L1 filter, offers insights into the underlying momentum of the economy.

In particular, the concurrent positivity of both the momentum and the macro-indicator means an expansionary phase in the economy. Conversely, the simultaneous negativity of these indicators points to a phase of economic contraction. When the macro-indicator is negative and the momentum remains positive, the economy indicates a stage of recovery. Similarly, in cases where the macro-indicator retains positivity but the momentum turns negative, it reflects a period of economic slowdown. We encode the economic regimes to enhance visualization as follows:

Expansion: 2, Slowdown: 3, Contraction: 0, Recovery: 1.

Economic regimes are efficiently detected using our indicator, which is supplemented with signals produced by the applied filter. However, it is important to note that recognizing the complete economic cycle is not always simple. For example, if we concentrate our focus on the years 2002-2009, right before the start of the second Gulf War, we see a pattern in which the US economy moved from a contraction phase to a recovery phase. Following that, it entered a growth phase, only to revert to a contraction phase due to the subprime mortgage crisis.

Figure 3: Regime identification through Macro-Indicator signals

When we turn our focus to the transition matrix, we may delve deeper into the dynamics and relationships of different economic regimes. This matrix gives us a systematic framework for analyzing how the economy transitions between stages, revealing insights on the patterns and frequency of transitions between recession, recovery, growth, and slowdown phases.

Table 3: Transition Matrix (in %)

From \To	**Contraction**	**Recovery**	**Expansion**	**Slowdown**
Contraction	72.73	15.15	6.06	6.06
Recovery	2.83	83.02	14.15	0.00
Expansion	1.91	5.10	87.26	5.73
Slowdown	16.22	0.00	2.70	81.08

The transition matrix, as expected, has the highest probability along its diagonal, showing that economic conditions tend to remain unchanged from one period to the next. This trend shows that each economic regime has maintained a degree of stability over time.

As in our last example, the matrix largely corresponds to our predictions. Changes from a slowdown to a contraction state, as well as changes from contraction to recovery and recovery to expansion, are more likely. These data support the notion that economic situations frequently follow a logical development according to the normal phases of economic cycles.

3.2.3 Factors

Fama and French's research has methodically demonstrated the significance of these factors in explaining the cross-sectional volatility of average stock returns. In their fundamental study, "Fama and French (1992)," they proposed the Three-Factor Model, questioning the standard Capital Asset Pricing Model (CAPM)'s sufficiency in explaining stock returns. They found that integrating two more components, size (SMB) and book-to-market (value) equity (HML), into the model offered a more realistic picture of the variances in stock returns.
Fama and French (1993) expanded on their findings by exploring deeper into the roles of size and value components in explaining stock performance. Their empirical findings supported the notion that these variables might effectively explain the variations in average returns among diverse portfolios.
Following that, in Carhart (1997), Carhart proposed momentum as a fourth element. The momentum factor (UMD) was added to the Three-Factor Model to account for the tendency of past winning stocks to maintain their excellent performance in the future.
Fama and French (2015 and 2016) revised their criteria based on these basic findings. Following these efforts, the five-factor model was developed, which incorporated market risk, size, value, profitability, and investment as variables for understanding stock returns.
The five equity - size, value, momentum, profitability, and investment - were chosen based on their widespread use and substantial documentation in academic research. Notably, the persistent occurrence of favorable long-term premiums linked with each aspect highlights their importance in developing effective investing strategies.
The data for these factors, encompassing the period from January 1990 until August 2023, were obtained from Kenneth French's data library. This extensive time span provides a comprehensive overview of how these factors have performed across time.

Table 4: Historical characteristics of factor returns for January 1990–August 2023.

	Size	Value	Profitability	Investment	Momentum
Descriptive Statistics					
Mean (%)	1.305	1.597	4.291	3.072	6.357
Std Dev (%)	9.635	12.159	8.235	9.438	16.025
Sharpe Ratio	0.135	0.131	0.521	0.325	0.396
Skewness	0.223	0.544	0.247	0.945	−0.213
Kurtosis	4.431	3.738	9.373	4.368	8.349
Factor Correlations					
Size	1.000	−0.041	−0.427	−0.113	0.084
Value	−0.041	1.000	0.315	0.792	−0.251
Profitability	−0.427	0.315	1.000	0.319	−0.059
Investment	−0.113	0.792	0.319	1.000	−0.068
Momentum	0.084	−0.251	−0.059	−0.068	1.000

Table 3 provides a detailed overview of the descriptive statistics for the individual factor returns over the entire observation period. Most factors exhibited relatively consistent annual returns, ranging from approximately 1.31% to 6.36%, with the momentum factor standing out as the highest, delivering an annualized return of 6.36%. However, it also had the highest volatility at 16.03%, double that of the investment factor. Remarkably, the momentum factor displayed the best Risk-Adjusted Return due to its robust absolute performance. In contrast, the investment factor's risk-adjusted performance was aided by its lower volatility. The skewness and kurtosis metrics reveal substantial disparities in the distribution of returns among the factors. Notably, both the momentum and profitability factors display leptokurtic distributions, indicating a higher likelihood of extreme events. The momentum factor, in particular, exhibits significant negative skewness, implying a greater probability of tail events.

These findings align with the correlations and demonstrate that most factors have moderate to weak correlations ranging from -0.427 to 0.792. This suggests diversification benefits when combining these diverse factors. An exception is observed between the value and investment factors, displaying a strong positive correlation of 0.792, consistent with the observation that high-value companies often exhibit low-investment characteristics (Fama and French, 2015). Additionally, the size factor exhibits the lowest average correlation, indicating that its relatively lower absolute and Risk-Adjusted Returns are offset by its diversification benefits.

Table 5: Factor-return characteristics across economic regimes.

	Economic Regime			
	Recovery	Expansion	Slowdown	Contraction
Annualized Return (%)				
Size	−3.15	8.71	0.93	−5.35
Value	3.43	2.52	−0.18	2.20
Profitability	19.47	−0.90	0.50	8.18
Investment	12.65	2.72	−7.53	3.58
Momentum	21.34	−4.34	8.89	2.61
Annualized Volatility (%)				
Size	10.36	8.17	10.17	9.33
Value	16.71	9.25	11.11	11.52
Profitability	9.62	6.97	8.32	9.90
Investment	14.16	6.32	10.53	14.63
Momentum	16.77	15.64	11.42	10.48
Sharpe Ratio				
Size	0.19	−0.08	−0.35	−0.29
Value	−0.06	−0.04	−0.15	−0.11
Profitability	0.12	0.18	0.19	0.52
Investment	0.01	−0.11	0.19	0.23
Momentum	0.03	−0.04	0.36	0.69

The factors exhibit diverse behaviors across different economic regimes. Notably, market volatility tends to rise during contractionary regimes characterized by rapid economic downturns. During such periods, the profitability factor stands out, demonstrating its effectiveness in mitigating risk. This factor benefits companies with lower leverage and predictable earnings, making it a valuable addition to portfolios during economic contractions.

These different patterns in the returns of each component, which show how they respond to changes in the economy, provide a strong basis for the development of factor rotation strategies that are in tune with economic regimes. These strategies leverage the temporal dynamics of factor performance to dynamically adjust portfolio exposures. Consequently, they enhance returns while effectively managing risks in response to shifting economic circumstances.

3.3 Portfolio construction

The core concept behind Dynamic Multi-Factor Portfolio Allocation is to adjust the initial allocation based on the prevailing identified economic regime, guided by the conditional expected returns of the factors within these regimes. This approach aims to enhance portfolio performance by aligning investments with the anticipated returns of factors specific to the current economic regime.

3.3.1 Black-Litterman Model

The Black-Litterman model stands as a comprehensive framework designed to formulate a well-informed portfolio allocation strategy that explicitly considers the perspectives associated with various economic regimes. By integrating regime-specific views, this model ensures that the resulting allocation is finely tuned to the dynamic shifts in market conditions, providing a more robust and adaptable investment approach.

The Black-Litterman (BL) model developed by Fischer Black and Robert Litterman combines Capital Asset Pricing Theory (CAPM) with Bayesian statistics and Markowitz's modern portfolio theory (Mean-Variance Optimization) to produce efficient estimates of the portfolio weights.
The Modern Portfolio Theory (MPT) introduced by Harry Markowitz changed the assumption that the risk and return relationship of a portfolio was linear with the notion that the diversification of a portfolio can inherently decrease the risk of a portfolio.
The MPT has various limitations, including the dependence on historical data, which leads to erroneous forecasts in volatile markets, the focus on a few stocks, and the lack of investor insights that may improve optimization.

1. **Market Equilibrium:**
   Portfolio Expected Returns (μ):

   $$\mu = w_1 E(R_1) + w_2 E(R_2) + \ldots + w_n E(R_n)$$

   Portfolio Variance (σ_p^2):

   $$\sigma_p^2 = w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \ldots + w_n^2\sigma_n^2 + 2\sum_{i<j} w_i w_j \sigma_{ij}$$

   Covariance Matrix (Σ):

   $$\Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \ldots & \sigma_{1n} \\ \sigma_{21} & \sigma_2^2 & \ldots & \sigma_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \ldots & \sigma_n^2 \end{bmatrix}$$

   Mean-variance optimization:

   $$\max_w \; w^T\mu - \frac{\delta}{2} w^T \Sigma w$$

   $$\text{subject to:} \quad w^T.1 = 1$$

   Implied Equilibrium Returns (π):

   $$\mu^* = \pi = \delta \Sigma w$$

where π is the vector of implied equilibrium returns, δ is the Risk Aversion Parameter, Σ is the covariance matrix of asset returns, and w is the vector of portfolio weights.

2. **Investor Views:**

The Pick Matrix (P) is used in the Black-Litterman model to represent the assets in our portfolio that are involved in each of the investor's views. For a portfolio with n assets and k views, the Pick Matrix has dimensions $k \times n$. It is defined as:

$$P = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kn} \end{bmatrix}$$

where:
P : Pick Matrix
p_{ij} : Element of the matrix corresponding to the ith view and jth asset, which can take value 1 which correspond to a to a bullish position or -1 which correspond to a bearish position.

Views Vector (Q):

$$Q = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_n \end{bmatrix}$$

Confidence Matrix (Ω):

$$\Omega = \text{diag}(P'\Sigma P)$$

Where Ω is a diagonal matrix of variance for corresponding views.

3. **Posterior Expected Returns:**
Posterior Expected Returns (μ_{BL}):

$$\mu_{\text{BL}} = (\tau\Sigma)^{-1}(\tau\Sigma\pi + P^T Q)$$

where μ_{BL} is the vector of posterior expected returns, τ is the scaling factor, Σ is the covariance matrix of asset returns, π is the vector of implied equilibrium returns, P is the matrix of portfolio weights, and Q is the vector of investor views.

4. **Posterior Covariance Matrix:**
Posterior Covariance Matrix (Σ_{BL}):

$$\Sigma_{\text{BL}} = (\tau\Sigma)^{-1} + P^T \Omega P$$

where Σ_{BL} is the posterior covariance matrix, τ is the scaling factor, Σ is the covariance matrix of asset returns, P is the matrix of portfolio weights, and Ω is the matrix of view uncertainties.

5. **Weights vector:**
Using the formula of the implied equilibrium return,

$$\mu = \delta\Sigma w$$

The final weights can be deduce as following :

$$w^* = (\delta\Sigma_{BL})^{-1}\mu_{BL}$$

where μ and Σ are the expected returns and covariance matrix of the assets, δ is the Risk Aversion Parameter, μ_{BL} and Σ_{BL} are the expected returns and covariance matrix generated by the Black-Litterman model, and w is the vector of portfolio weights.

3.3.2 Regime-based Dynamic Factor Portfolio

We can use the Black-Litterman model as a methodology to construct an allocation that incorporates economic regimes views of factors. This approach enables the integration of regime-specific viewpoints, enhancing the precision and adaptability of the portfolio allocation strategy.
To construct a dynamic factor portfolio, we will first assume that the view vector will be the historical expected return of the factors conditionally to the regimes. Then we will make the assumption that our initial allocation is optimal in a mean-variance way. So we can set the weight vector in the implied equilibrium return π using the formula:

$$\pi = \delta \Sigma w_{init},$$

where :
δ : represents the Risk Aversion Parameter.
Σ : denotes the covariance matrix of factor returns.
w_{init} : signifies the initial portfolio weights.

The weight vectors w_{init} here are set in input as the result of a calculation from another strategy by assuming that this allocation is optimal in a mean-variance sense. Without making this assumption, we couldn't set the weight vector in the implied equilibrium formula.

To derive the posterior expected returns, we compute them as a combination of the implied expected returns and the regime-based view vector Q is utilized:

$$\mu = (1 - \kappa)\pi + \kappa Q,$$

Here, $\kappa \in [0, 1]$ serves as the confidence parameter for the macroeconomics's active views, determining the level of engagement in implementing the dynamic strategy. The value of κ is set at 0.5 as a reference point for this parameter.
Then, we set the portfolio weights for the multi-factor dynamic regime-based portfolio using mean-variance optimization with the posterior perspective of expected factor returns. These portfolio weights adjust in response to changes in regime-based perspectives, aligning with the relevant conditional expected return vector for each regime.

3.3.3 Model Implementation with Various Strategies

We decide to use several initial portfolios with different strategies to construct the multi-factor dynamic regime-based portfolio. We select five portfolio strategies: equally weighted (EW), minimum variance (MV), maximum sharpe ratio (MS), maximum diversification (MD), and risk parity (RP).
The objective of creating many portfolios with various initial methods is twofold. For starters, it enables a comparison of the performance measures of the original allocation computations themselves. Furthermore, it allows for a comparison of regime-based portfolios and initial methods.

The Equally Weighted (EW) portfolio assigns an equal weight to each asset in the portfolio. It provides a simple approach and is easy to implement. The EW portfolio can be useful when there's no specific information available about the assets' expected returns and covariances.

The Minimum Variance (MV) portfolio aims to minimize the portfolio's overall variance, taking into consideration the covariance structure of the assets. It seeks to achieve the most diversified portfolio by taking into account the covariance matrix of the components.

The Maximum Sharpe Ratio (MS) or efficient portfolio, seeks to achieve the highest Risk-Adjusted Return by optimizing the trade-off between risk and return. It divides the excess return (the difference between asset return and risk-free rate) by the asset's volatility.

The Maximum Diversification (MD) portfolio aims to maximize the diversification benefits by allocating weights based on the inverse of assets' volatilities. It seeks to minimize the impact of any single asset's extreme movements on the portfolio's overall performance.

The Risk Parity (RP) portfolio allocates weights to assets in a way that each asset contributes equally to the portfolio's total risk, usually measured by volatility. It aims to provide a more balanced risk allocation across different assets.

Each method was calculated using a two-year rolling window of historical data. Because many of these techniques rely on both volatility and return measurements, using a 24-month rolling window allows us to examine medium-term prior swings while preventing long-term historical data from having an undue influence on our calculations. We also define a rule of weight concentration for some portfolios to avoid overexposure to a single component. We set the rule such that the allocation on a factor can't exceed more than 50% for the maximum diversification, the maximum Sharpe ratio, and the minimum variance portfolio. Finally, we constrain all our strategies to be long only, which means that they are not able to have a short position.

3.3.4 Out of sample test

It is crucial to understand the importance of evaluating the performance of this strategy. To do so, two commonly used methods are the out-of-sample test and the backtest. These methods enable us to assess the relevance and robustness of allocation rotation.

In our case, we didn't use the backtesting method. It is a retrospective method that involves applying an allocation strategy to historical data. This allows us to see how the strategy would have performed in the past. While this can give initial indications of the strategy's effectiveness, there is an inherent risk. However, strategies that perform well on historical data may underperform or even be completely ineffective under different economic or market conditions. This is known as the Overfitting problem, where a strategy is overoptimized for specific data and does not generalize well to new data.

This is where the out-of-sample test comes in. This method involves testing the strategy on a period of data that has not been used to develop the strategy. In other words, it simulates how the strategy would behave in the future, using data that was not 'seen' when the strategy was designed. The out-of-sample test is extremely important for assessing the strategy's ability to adapt to changing contextual conditions.
To achieve this, we divided the dataset into two distinct periods. The initial span encompassed 22 years, spanning from January 1, 1990, to January 2010, during which we calibrated our model. Leveraging historical data preceding this period, we conducted calculations to devise a regime-based allocation strategy for each subsequent month.

To obtain our regime for the next month, we aggregated data from each component variable over the same timeframe, commencing from January 1990 until the most recent available data point (t). We then proceeded to estimate the data for the subsequent time point, t+1. Knowing that the indicator is stationary and looking at the Auto-Correlation Function (ACF) and Partial Auto-Correlation Function (PACF) of our indicator, we can say that there is auto-correlation. We also know that all the Macro-Variables we used to compute the Macro-Indicator are correlated.

To make these predictions using a recursive method, we have employed a multivariate approach based on the Vector Auto-Regressive Model (VAR model). As explened in the previous section, the VAR model encapsulates the temporal dynamics of a set of k variables, referred to as endogenous variables. These variables are amalgamated into a $kx1$ vector denoted y_t.

In our specific case, we employ the VAR model of order 1 to capture the intricate relationships and temporal dynamics within our dataset with one step in the past. This model leverages five carefully chosen Macro-Variables, which form the basis of our macro-indicator, alongside the observed data of this indicator.
For estimation of VAR parameters from the observed time series data $y_1, y_2, \ldots, y_T$, we define data matrices as $Y = (y_{p+1}, y_{p+2}, \ldots, y_T)^\top \in \mathbb{R}^{N\times K}$, $X = (x_{p+1}, x_{p+2}, \ldots, x_T)^\top \in \mathbb{R}^{N\times(Kp+L)}$, with $N = T - p$.
Then, we can write in a matrix form as

$$Y = X\Psi + E \in \mathbb{R}^{N\times K},$$

with $E = (\epsilon_{p+1}, \ldots, \epsilon_T)^\top$ and $N = T - p$.

In this particular forecasting circumstance, the VAR(1) model is an excellent choice. This is due to two major factors: the strong correlation and causality between the macro-variable and the macroeconomic indicator, as well as the existence of auto-correlation within the indicator itself.

The VAR(1) model, or Vector Auto-Regressive Model of Order 1, is used to reflect the interdependence of numerous variables across time. It aids us in analyzing the links between numerous input factors and the macroeconomic indicator in our scenario.

After estimating the macro-indicator, we apply the L1 trend filtering method to the one-step-ahead forecast to identify the regime for the next month.

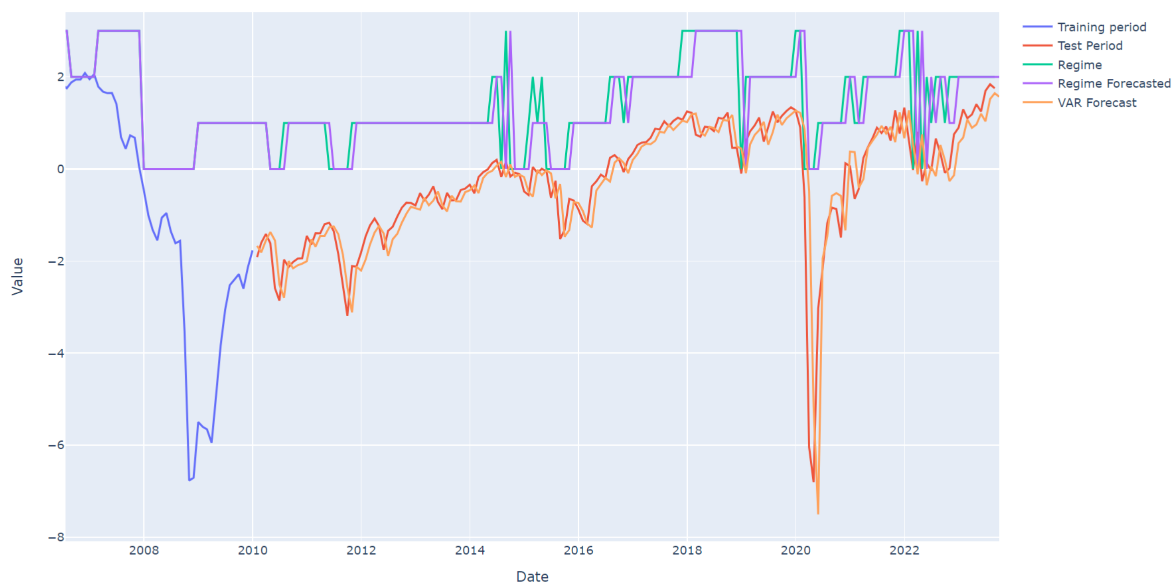

Figure 4: Out of Sample Test on regime identification

In the chart above, the regimes Recovery (1), Expansion (2), Slowdown (3), and Contraction (0) are encode as previously in 3.2.2.

We can observe some lag between the projected regimes over time and the actual regimes observed. This lag is a natural result of autoregressive models that use historical data and allows for the inclusion of the most recent observed values. Furthermore, it becomes clear that the anticipated regimes occasionally have fewer transitions than the real ones. Another significant observation concerns regime forecasting: when the indicator is close to zero, noise is generated, resulting in a decrease in switching accuracy.

	Predicted Regime				
Actual Regime	0	1	2	3	Metrics
0	55	5	1	5	Sensitivity 0.87
1	10	92	4	0	Specificity 0.96
2	2	13	141	2	Accuracy 0.94
3	3	0	9	65	Precision 0.89

Table 6: Confusion Matrix

Since we have an estimated regime, we can apply our model to get our new exposure on our factors according to the regimes.
As previously stated, we used two-year historical data to conduct the out-of-sample tests. We used a rolling window method for each of the initial allocations. This method was specifically designed to reduce the impact of long-term past data on our computations.
For the regime-based portfolio allocations, however, we used a different approach. We started with a two-year historical dataset, but instead of using a rolling window, we used an expanding window technique. The reasoning behind this decision was to use a largest historical dataset to construct our conditional anticipated factor returns. We used an extending frame to guarantee that our estimates took into account all of the available historical data for more robustness.

It's important to remember that each portfolio in our analysis was re-balanced on a monthly basis. We performed a full recalculation of all our tools and techniques at the end of each month. So, whether it's the macroeconomic indicator, our filter, our estimation model, or our allocation, all of these parameters are recalculated on a monthly schedule. This technique enabled us to first reassess our strategy and then generate a fresh allocation based on the most recent information and market conditions in an accurate way.
To compare our performances we will use two type of benchmarks, the performance of the S&P 500 Index and the initial portfolio strategies.

4 Results

In this section, we will analyse the results of our study on dynamic allocation portfolios rotation. The proposed approach claims the potential to improve the performance of a multi-factor portfolio, by taking into account variations in economic regimes. To recall, the Factor Rotation Method based on identifying economics, suggest that the dynamic approach can adapt portfolio composition to different economic regimes, resulting in higher returns with moderate risk.

We can now examine the results of these strategies after carefully analyzing them using a recursive algorithm across an extensive eleven-year dataset in an out-of-sample situation. This analysis aims to assess and compare the effectiveness of the different strategies in order to highlight both their benefits and drawbacks.

4.1 Primary Strategies Results

Our initial emphasis is on the analysis of the primary strategies themselves. In order to determine these strategies' relative position inside each of their individual components, we want to know how they have performed in terms of risk and return over the temporal frame.
This provides us with insightful information about how various strategies behave over time testing, enabling a thorough assessment of their effectiveness and positioning.

Several crucial characteristics in the figure above catch our eye. The red line depicts the portfolio's global efficient frontier, which reflects all alternative allocations that optimize returns for a given level of risk. It is important to point out that this frontier is not practically feasible in a forward-looking way. Nonetheless, it is a useful tool for assessing the original strategies' success in terms of risk and return. These computations were done on a yearly basis.

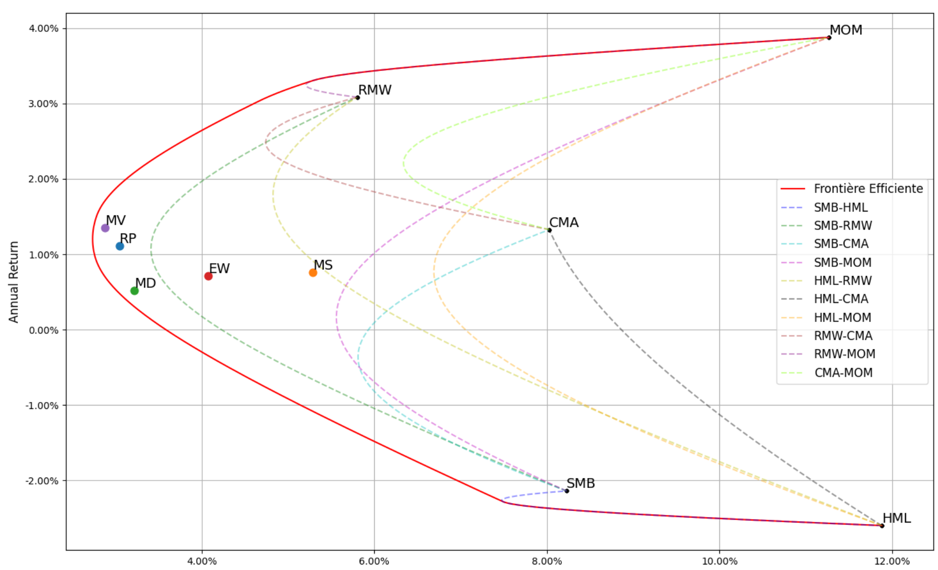

Figure 5: Position of our factors strategies

A close inspection reveals the location of our factors as well as their contributions to risk and return. Consider the Value factor (HML), which has a negative annualized return and contributes significantly to volatility. This implies that the Value factor did not provide beneficial results between January 2010 and August 2023.
The Size (SML) component, on the other hand, has a negative annualized return but substantially reduced volatility. The remaining three elements all provide favorable results. Momentum stands out for its high risk and tremendous reward. Profitability (RMW) has the best risk-return ratio, with the lowest volatility and the most consistent returns.

When we look at the different strategies, we can see that they all have a positive annualized return with a reasonable amount of risk. MS and EW have more volatility, which is consistent with the nature of these approaches. However, whereas MS should theoretically give larger returns, a deeper examination of the cumulative return (Figure 6) indicates a significant decline. This may be attributed to the MS strategy's early 2017 heavy investment in the HML component, which later saw a major drop during that time.
RP and MV are two techniques that stand out in terms of risk-return ratio. MV, in particular, appears as the approach with the best returns for the least amount of risk.

Table 7: Performance Metrics of initial strategies

Metrics	RP	MS	MD	EW	MV	$SP500$
Annualized Return (%)	1.11	0.26	0.52	0.71	1.35	11.09
Annualized Volatility (%)	3.05	5.39	3.22	4.08	2.88	14.89
Sharpe Ratio	0.36	0.05	0.16	0.17	0.47	0.74
Max Drawdown (%)	-9.74	-20.65	-11.42	-15.04	-7.31	-24.77
Skewness	0.24	0.07	0.02	0.41	0.29	-0.43
Kurtosis	1.37	0.95	1.89	2.81	0.61	0.80

In order to get a good insights into the characteristics of each portfolio strategy, we can look a the performance metrics presented in Table 7. These metrics reveal their returns, risks, and other important characteristics, highlighting their relative strengths and weaknesses First, we can observe that the risks and returns are fully coherent to the observations we made in Figure 5. We have included the SP500 in this table to provide a benchmark for comparison. As expected, the SP500 boasts the highest annual return with a higher volatility,which is in line with conventional wisdom.

All portfolios fall below 0.5 except for the SP500 when we assess the Sharpe ratios, a measure of risk-adjusted returns. Notably, the MV and RP portfolios stand out with the next highest Sharpe ratios, implying a more favorable trade-off between risk and return. Turning our attention to maximum drawdown (MDD), we observe relatively high values for the MS and EW portfolios, although it's worth noting that the SP500 experiences the most significant drawdown. In stark contrast, the MV and RP portfolios demonstrate remarkably low MDDs, each well above -10%.

Considering the asymmetry of returns, we find positive values for all portfolios except the SP500. This observation is especially pronounced for the EW and MV portfolios. Lastly, it's evident that the distribution of returns for all portfolios is leptokurtic, indicating a concentration of returns with thinner tails. This characteristic indicates a more pronounced peak around the mean and indicates that extreme returns are less likely compared to a normal distribution.

4.2 Regime based strategies Result

Now that we have a good insight into how the primary portfolios perform independently, we can study how the regime-based view we add to those strategies contributes to increasing or decreasing the results. Observing the Table 8, the metrics types remain the same as before, but add the tracking errors, which had been calculated using the primary portfolios.

Table 8: Performance Metrics for B-L Portfolio

Metric	BL_{RP}	BL_{MS}	BL_{MD}	BL_{EW}	BL_{MV}	$SP500$
Annualized Return (%)	3.87	3.30	3.78	3.92	3.62	11.09
Annualized Volatility (%)	5.79	5.92	5.81	5.88	5.80	14.89
Sharpe Ratio	0.67	0.56	0.65	0.67	0.62	0.74
Max Drawdown (%)	-7.36	-8.93	-7.21	-8.02	-7.36	-24.77
Tracking Error (%)	5.16	3.33	5.21	5.54	5.32	-
Skewness	-0.30	-0.24	-0.28	-0.20	-0.29	-0.43
Kurtosis	1.52	1.15	1.47	1.53	1.50	0.80

Across all strategies, the B-L Portfolio demonstrates favorable annualized returns, ranging from 3.30% to 3.92%. Volatility levels are relatively low, ranging from 5.79% to 5.92%. These strategies have managed to generate robust returns with low risk compared to the SP500, which has a volatility rate of 14.89%. This becomes evident when examining the Sharpe ratios, which are significantly higher than those of the initial portfolio and are closer to the SP500's Sharpe ratio. These ratios range from 0.56 to 0.67, and the regime-based views complement the RP strategy.

Regarding the maximum drawdown (MDD) metrics, all the portfolio strategies perform much better than before, with each of them experiencing drawdowns above -9%, in stark contrast to the SP500's MDD. This indicates that the B-L Portfolio strategies have displayed resilience in challenging market conditions. As a reminder, tracking errors are essential for assessing how closely a portfolio mimics its benchmark. If the value is negative (positive), it means that the portfolio underperforms (outperforms) its benchmark. Here, we can observe how the regime strategies outperform their primary portfolio allocations, with an average tracking error of around 4.9.

The B-L Portfolio strategies exhibit a negative skew in the returns distribution, with values ranging from -0.20 to -0.30, indicating near-symmetry. This suggests that, on average, the strategy returns tend to be closer to zero, with larger positive returns occurring somewhat less frequently. Finally, upon examining the kurtosis of each portfolio, it is evident that they all have leptokurtic distributions, characterized by smaller standard deviations of returns.

4.3 Global Comment

In this section, we provide a brief overview and commentary on the results obtained from our analysis of both the primary and regime-based portfolios. Our initial focus was on the primary strategies, and we delved into understanding how they performed individually in terms of risk and return. The graphical representation in Figure 5 allowed us to assess their relative positions and contributions to the portfolio's risk-return profile. We observed that while some factors, such as the Value factor (HML), had negative annualized returns and added significantly to portfolio volatility, others like the Size (SML) component displayed lower volatility despite negative returns. Momentum stood out with high risk and reward, while Profitability (RMW) showed the best risk-return ratio. When we examined the different portfolio strategies, we noticed that each of them delivered positive annualized returns with a moderate level of risk. When assessing Sharpe ratios, a measure of risk-adjusted returns, we observed that all portfolios fell below 0.5. Their maximum drawdown (MDD), were high values for the MS and EW portfolios, even if it's worth less that the SP500' drawdown. In contrast, the strategies with risk management demonstrated a low MDDs. The portfolios distribution shows that extreme returns were less likely compared to a normal distribution. Transitioning to the regime-based strategies (Table 8), we observed that the B-L Portfolio consistently delivered favorable annualized returns, coupled with relatively low volatility levels. These strategies demonstrated robust risk-return trade-off compared to the SP500, which exhibited a higher volatility. In term of sharpe ratios, the B-L Portfolio strategies were even more favorable when compared to the initial portfolios, as well as maximum drawdown (MDD). An other marker of the performance, is tracking errors revealed that the regime-based views consistently outperformed their initial portfolio allocations. Finally, the distributions of returns shows thinner tails, further substantiating the observation that extreme returns were less likely compared to a normal distribution.

5 Conclusion

5.1 Conclusion on the Model

Finally, this research investigates the robustness and potential contributions of a dynamic multi-factor portfolio allocation technique, specifically in the context of asset management with an emphasis on fund allocation. The main question is whether this strategy can help an asset manager who specializes in fund allocation management.

Through this study based on the Dohyoung Kwon works, we try to improve the performance of multifactor portfolios by incorporating economic regimes into the allocation construction. Economic cycles, which in our model we identified as expansion, slowdown, contraction, and recovery, have a significant influence on financial asset performance and risk. We presented an allocation rotation approach based on equity factors, discovered and used by economists Fama, French, and Carhat, to modify portfolio composition based on various economic regimes. This strategy takes a systematic and unique approach to multi-factor portfolio development, with the goal of increasing portfolio resilience in the face of market swings.

The study investigates the usefulness of several portfolio design strategies, such as equally weighted, minimum variance, maximum sharpe ratio, maximum diversification, and risk parity, in the context of regime-based allocation. These strategies are evaluated for adaptability to changing economic conditions using an out-of-sample methodology. By comparing the result of the dynamic allocation based on the regimes to the initial strategies mentioned, we observed a significant improvement in terms of performance. Indeed, on average, the regime-based portfolios outperformed, with good management of risk over time.

Constructing multifactor portfolios tailored to macroeconomic conditions has numerous valuable applications within the realm of asset management. They can serve as benchmarks for portfolio managers. Being fully quantitative, they allow for the addition of constraints to the model. Portfolio managers can compare their performance against these multifactor portfolios, providing a clear reference point for evaluating their strategies.

Multifactor portfolios can be incorporated into diversification pockets within investment funds through the use of Exchange-Traded Funds (ETFs). This approach enhances diversification and provides investors with exposure to various factors while maintaining a systematic and rules-based approach.

They can also be employed as picking indicators to guide portfolio managers in determining how they want to be exposed to factors, funds, stocks, or ETFs based on the prevailing economic regime. By analyzing macroeconomic conditions, portfolio managers can make informed decisions about their allocations.

While the study provides encouraging insights into the potential benefits of dynamic multi-factor portfolio allocation, it also admits to a number of limitations that we will see in the next section.

5.2 Improvement and Limitation

In the parts that follow, we explore the elements, variables, and models used in our research, as well as the numerous assumptions, considerations, and constraints that support this study.

A number of assumptions form the foundation of this study, with the Macro-indicator serving as the key instrument. It is the foundation for measuring the economy's position within its cycle, and it serves as the foundation for developing our allocation strategy. One key assumption we've made concerns the five Macro-Variables we've chosen to forecast the economy. While these variables do provide useful information, it is crucial to recognize the presence of other leading indexes that are commonly employed by traders and financial strategists. Manufacturing Demand, Consumer Spending, and other indicators display correlations with each other and can reveal extra information when submitted to PCA.

Furthermore, the simplicity with which these variables may be accessed inside the huge US economy, owing to easily available data from the Federal Reserve Economic Data website, is a luxury. The situation is different in smaller or developing economies, where some variables may not be easily available. Constraints on data availability limit the model's generalizability and applicability. Concerning factors, a similar limitation develops. Not all nations benefit from easily measured and accessible equity factor returns. Calculating these components becomes necessary in such instances.

On the portfolio side, we implemented regime-based allocation using well-established portfolio models. However, it is critical to note that alternative models may be equally applicable in different settings. We made an important assumption while building the regime-based allocation using the Black-Litterman model. We assumed that any allocation determined from model portfolios is efficient in terms of mean-variance. The mean-variance approach, in essence, implies that investors are risk-averse by nature and want to maximize anticipated return given a certain risk level or reduce risk given an expected return level. By using weights from other models, we assume that these allocations have the same qualities, which may not be true in practice.

Another critical element to examine is the use of initial allocations and the S&P 500 as a benchmark. The selection of a benchmark is an important and extensively studied part of asset allocation and fund management. In our case, we might be able to develop benchmarks that are more closely aligned with regime-based allocation techniques. However, it is critical to understand that there is no one-size-fits-all standard. Client preferences, fund objectives, portfolio components, and other variables all influence benchmark selection.
We can also mention the fact that there are now a huge number of factors that have different characteristics than those we selected. We made our selection based on the fact that they are mainly used in the literature and not very closely correlated. So we can imagine picking other kinds of factors to construct a quantitative portfolio with a regime-based allocation.

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7 Glossary

Glossary

Auto-Correlation Function (ACF) A statistical tool used to assess the correlation of a variable with its past values in a time series..

Benchmark A benchmark is a standard or measurement that may be used to examine a portfolio's allocation, risk, and return. Individual funds and investment portfolios will often have benchmarks defined for standard analysis..

Dynamic Multi-Factor Portfolio Allocation A strategy that adjusts portfolio composition based on changing economic conditions, leveraging multiple financial factors to optimize returns and manage risks..

Factor Investing An investment strategy that focuses on specific factors or characteristics of assets, such as value, size, profitability, and momentum, to achieve desired portfolio outcomes..

Factor Rotation Method A technique that involves changing the weighting of various financial factors in a portfolio based on economic phase identification..

Macro-Variables Economic indicators or variables that provide insights into the overall economic health and conditions of a region or country..

Overfitting The phenomenon where an investment strategy is overly optimized for historical data and may not perform well in new or changing market conditions..

Partial Auto-Correlation Function (PACF) A statistical tool that measures the correlation between a variable and its lagged values while controlling for the influence of intervening values..

Risk Aversion Parameter A measure of an investor's aversion to risk, which influences their preference for riskier or safer investments. There is no consensus about his value since it fully depend of investors..

Risk-Adjusted Return A measure of investment performance that considers both returns and the level of risk taken to achieve those returns, often assessed using metrics like the Sharpe ratio..

8 Appendix

8.1 Encountered Challenges and Made Choices

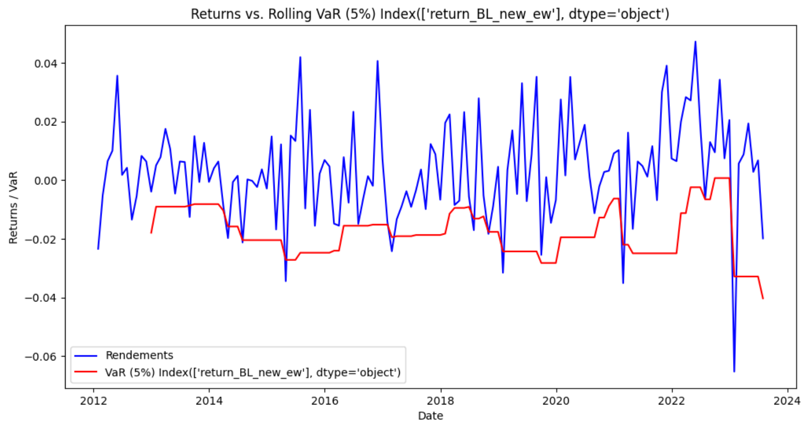
During the course of this study, several challenges were encountered, and various decisions were made to address them. This section highlights some of the key challenges and the choices made in response.

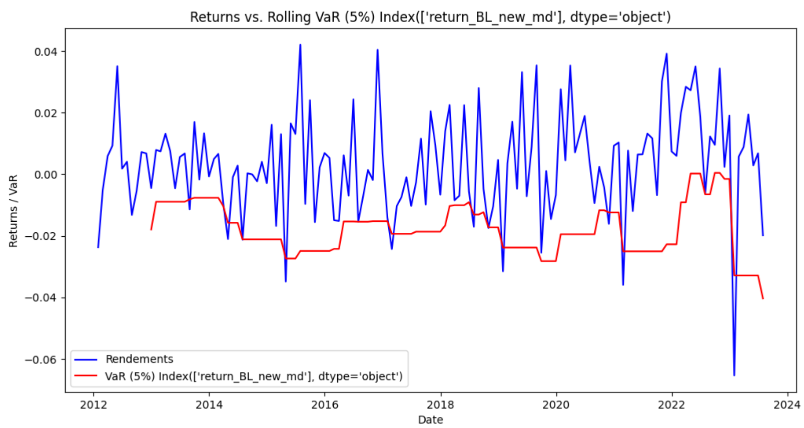
One of the first obstacles addressed in this investigation was overcoming data issues. It was difficult to collect macroeconomic variables that were not only important but also coordinated in terms of frequency and historical depth. While data sources such as FRED gave access to these variables, the frequency with which they were available varied—daily, weekly, or monthly. To overcome this, one intermediate approach was to perform the research on a weekly basis and use cubic interpolation on the monthly data. However, by presuming that cubic interpolation correctly reflected the underlying structures of the data, this technique introduced biases into the monthly data. This, in turn, had an impact on the data acquired by Principal Component Analysis (PCA). There were also practical concerns with transaction costs in portfolio allocation and noise in regime forecasts, which sometimes resulted in frequent regime flips, resulting in disappointing outcomes.

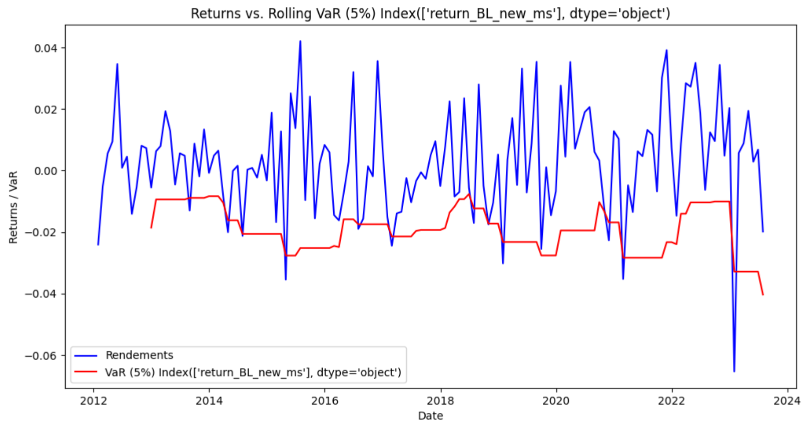
Another difficult task was selecting an adequate prediction model for the out-of-sample testing. The selection of a predictive model was crucial for evaluating the model's performance without access to future data. Initially, an AR(1) model was used, with the assumption that the best estimate at a particular time (t+1) was purely dependent on the observed value at time t. Extensive testing, however, demonstrated that the AR(1) model provided lagging forecasts, especially when the macro-indicator neared zero. Recognizing the causality and correlation between macro-variables and the indicator, a VAR model was used to collect extra data, resulting in decreased delays and noise. Following that, an effort was made to improve prediction accuracy using a VAR Shrink model that combined the VAR model with ridge penalization. While promising in terms of accuracy increase, applying the VAR Shrink model proved exceedingly difficult but provided a possible option for advancement.

A final challenge revolved around the selection of primary portfolio strategies. Initially, the intention was to test the regime-based allocation against a single risk parity portfolio. However, in order to demonstrate the significance of regime-based allocation across various quantitative portfolios based on the selected factors, a more diversified set of portfolio models was needed. These models not only differed in their risk management approaches but also exhibited variations in return management and concentration. Consequently, five primary portfolio strategies were chosen to provide a comprehensive evaluation of the regime-based model's performance across a range of initial strategies.

8.2 Additional Graphics



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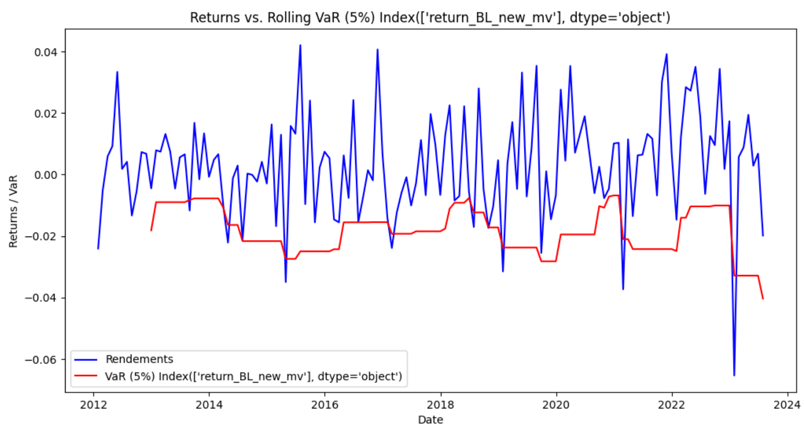
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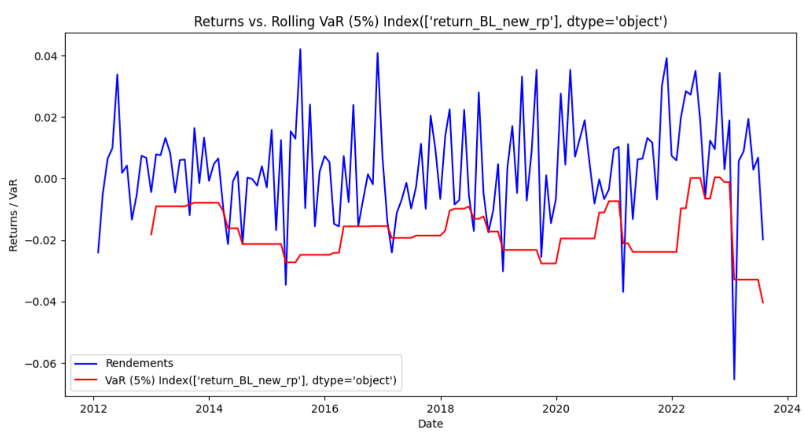
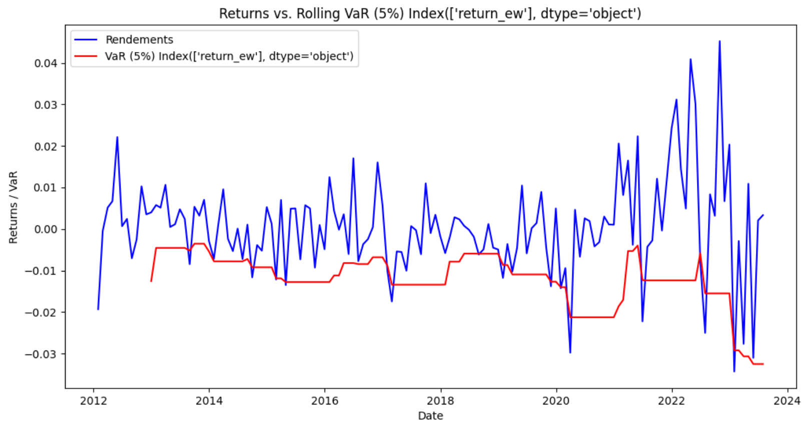
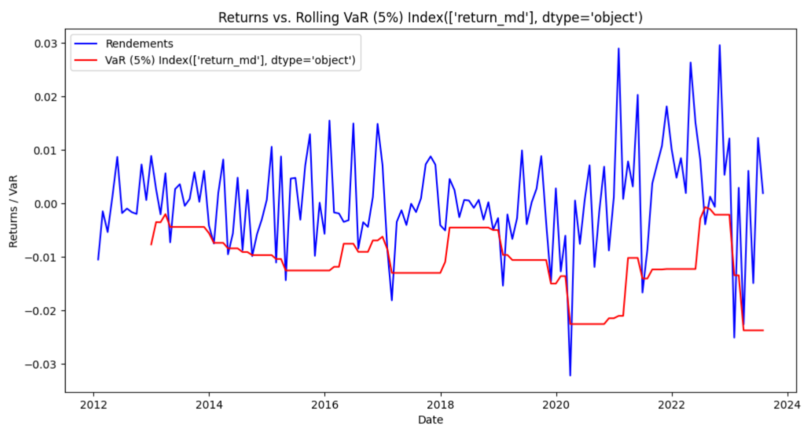
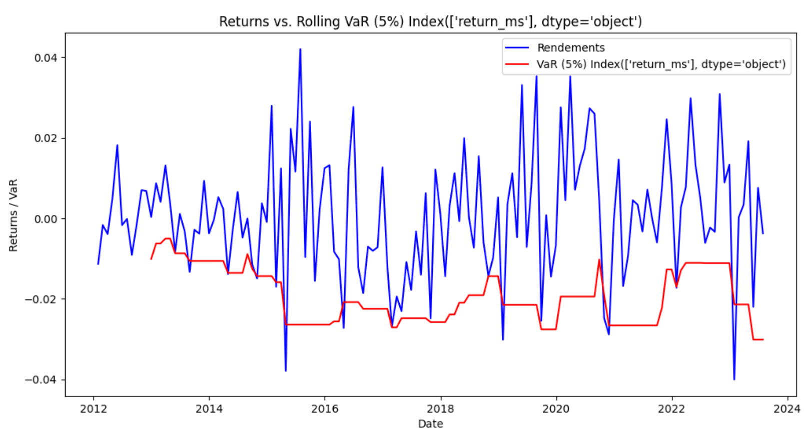
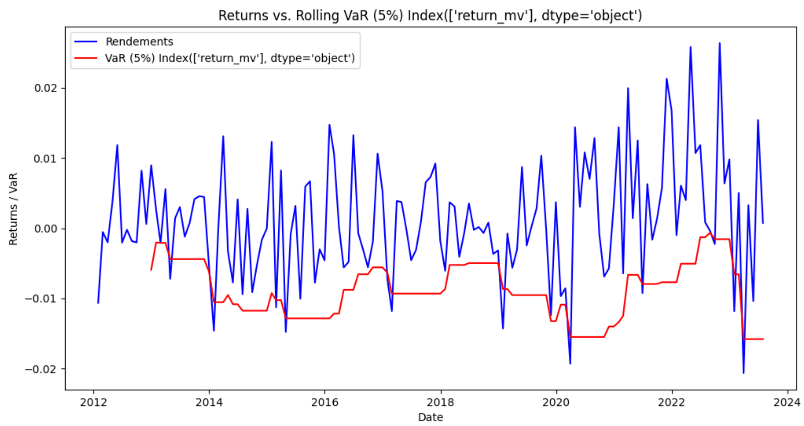
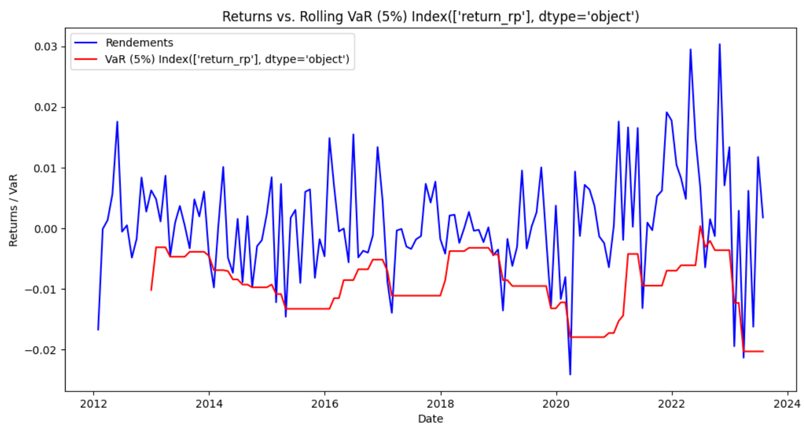

Returns vs. Rolling VaR (5%) Index(['return_BL_new_rp'], dtype='object')
Returns / VaR
Date
Rendements
VaR (5%) Index(['return_BL_new_rp'], dtype='object')

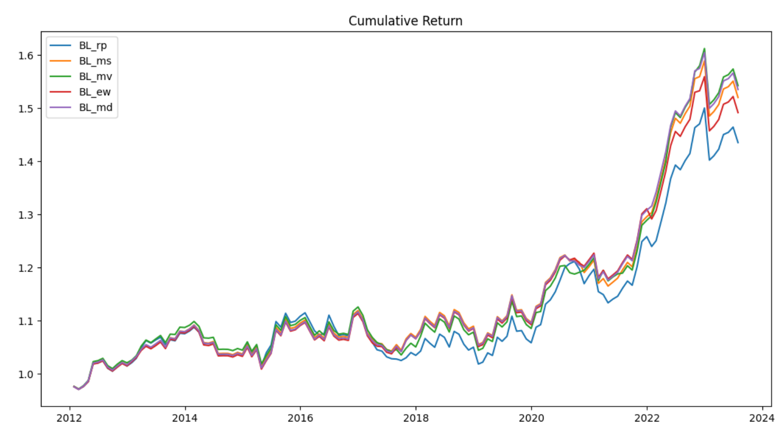

Returns vs. Rolling VaR (5%) Index(['return_ew'], dtype='object')
Rendements
VaR (5%) Index(['return_ew'], dtype='object')
Returns / VaR
Date

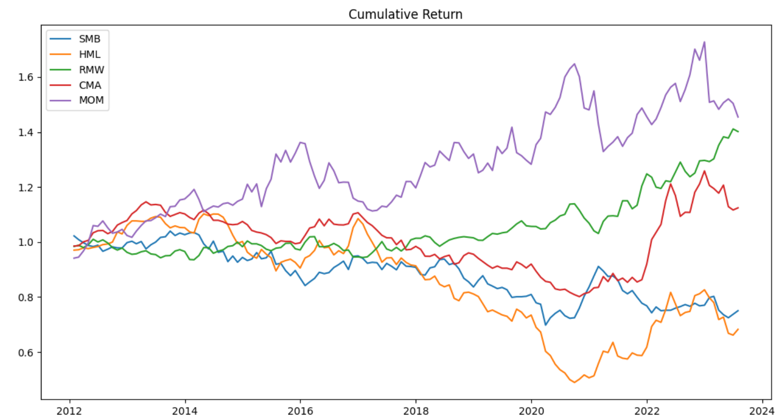

Returns vs. Rolling VaR (5%) Index(['return_md'], dtype='object')
Rendements
VaR (5%) Index(['return_md'], dtype='object')
Returns / VaR
Date

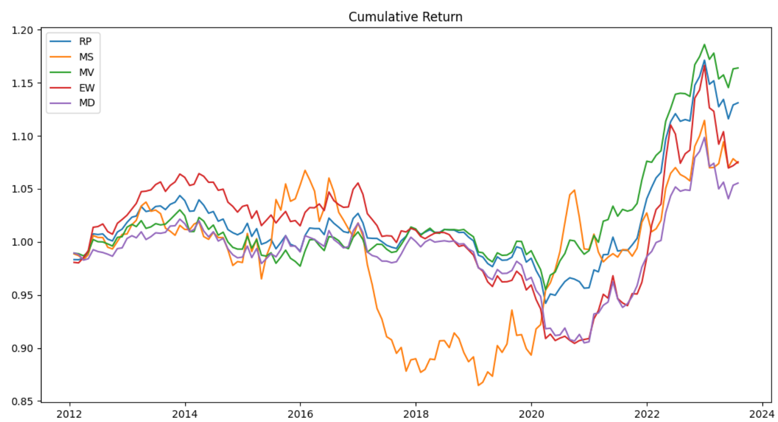

Returns vs. Rolling VaR (5%) Index(['return_ms'], dtype='object')
Rendements
VaR (5%) Index(['return_ms'], dtype='object')
Returns / VaR
Date


Returns vs. Rolling VaR (5%) Index(['return_mv'], dtype='object')
Rendements
VaR (5%) Index(['return_mv'], dtype='object')
Returns / VaR
Date


Returns vs. Rolling VaR (5%) Index(['return_rp'], dtype='object')
Rendements
VaR (5%) Index(['return_rp'], dtype='object')
Returns / VaR
Date


Cumulative Return
BL_rp
BL_ms
BL_mv
BL_ew
BL_md


Cumulative Return
SMB
HML
RMW
CMA
MOM
1.6
1.4
1.2
1.0
0.8
0.6
2012
2014
2016
2018
2020
2022
2024


Cumulative Return
RP
MS
MV
EW
MD
1.20
1.15
1.10
1.05
1.00
0.95
0.90
0.85
2012
2014
2016
2018
2020
2022
2024