Solutions of Homework at Week 13

June 8th, 2021

Chapter 20

$\boxed{\textbf{E20-5}}$ We can apply $\Delta x = v\Delta t$ to find the time the particle existed before it decayed. Then

$$\Delta t = \frac{x}{v}\frac{(1.05\times10^{-3}\,\mathrm{m})}{(0.992)(3.00\times10^{8}\,\mathrm{m/s})} = 3.53\times10^{-12}\,\mathrm{s}.$$

The *proper lifetime* of the particle is

$$\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (3.53\times10^{-12}\,\mathrm{s})\sqrt{1-(0.992)^2} = 4.46\times10^{-13}\,\mathrm{s}.$$

E20-8 $\Delta t = \Delta t_0/\sqrt{1-u^2/c^2} = (26\text{ ns})\sqrt{1-(0.99)^2} = 184$ ns. Then

$$L = v\Delta t = (0.99)(3.00\times10^{8}\mathrm{m/s})(184\times10^{-9}\mathrm{s}) = 55\,\mathrm{m}.$$

P20-1 (a) $\gamma = 2$, so $v = \sqrt{1-1/(2)^2} = 0.866c$.

(b) $\gamma = 2$.

P20-4

$\gamma = 1/\sqrt{1-(0.60)^2} = 1.25$. Use the equations from Table 20-2.

$\Delta t = (1.25)\left[\left(4.0\times10^{-12}\mathrm{s}\right) - (0.60)\left(3.0\times10^{3}\mathrm{m}\right)/\left(3.00\times10^{8}\mathrm{m/s}\right)\right] = -7.5\times10^{-6}\mathrm{s}$

E20-13 (a) $\gamma = 1/\sqrt{1-(0.380)^2} = 1.081$. Then

$$\begin{aligned} x' &= (1.081)[(3.20\times10^{8}\mathrm{m}) - (0.380)(3.00\times10^{8}\mathrm{m/s})(2.50\,\mathrm{s})] = 3.78\times10^{7}\mathrm{m},\\ t' &= (1.081)[(2.50\,\mathrm{s}) - (3.20\times10^{8}\mathrm{m})(0.380)/(3.00\times10^{8}\mathrm{m/s})] = 2.26\,\mathrm{s}. \end{aligned}$$

(b) $\gamma = 1/\sqrt{1-(0.380)^2} = 1.081$. Then

$$\begin{aligned} x' &= (1.081)[(3.20\times10^{8}\mathrm{m}) - (-0.380)(3.00\times10^{8}\mathrm{m/s})(2.50\,\mathrm{s})] = 6.54\times10^{8}\mathrm{m},\\ t' &= (1.081)[(2.50\,\mathrm{s}) - (3.20\times10^{8}\mathrm{m})(-0.380)/(3.00\times10^{8}\mathrm{m/s})] = 3.14\,\mathrm{s}. \end{aligned}$$

E20-19 (a) $\gamma = 1/\sqrt{1-(0.8)^2} = 5/3$.

$$\begin{aligned} v'_x &= \frac{v_x}{\gamma(1-uv_y/c^2)} = \frac{3(0.8c)}{5[1-(0)]} = \frac{12}{25}c,\\ v'_y &= \frac{v_y-u}{1-uv_y/c^2} = \frac{(0)-(0.8c)}{1-(0)} = -\frac{4}{5}c. \end{aligned}$$

Then $v' = c\sqrt{(-4/5)^2+(12/25)^2} = 0.933c$ directed $\theta = \arctan(-12/20) = 31°$ East of South.

(b) $\gamma = 1/\sqrt{1-(0.8)^2} = 5/3$.

$$\begin{aligned} v'_x &= \frac{v_x-u}{1-uv_x/c^2} = \frac{(0)-(0.8c)}{1-(0)} = -\frac{4}{5}c,\\ v'_y &= \frac{v_y}{\gamma(1-uv_x/c^2)} = \frac{3(0.8c)}{5[1-(0)]} = \frac{12}{25}c. \end{aligned}$$

Then $v' = c\sqrt{(4/5)^2+(12/25)^2} = 0.933c$ directed $\theta = \arctan(20/12) = 59°$ West of North.

E20-23 The length of the ship as measured in the "certain" reference frame is

$$L = L_0\sqrt{1 - v^2/c^2} = (358\,\mathrm{m})\sqrt{1 - (0.728)^2} = 245\,\mathrm{m}.$$

In a time Δt the ship will move a distance $x_1 = v_1\Delta t$ while the micrometeorite will move a distance $x_2 = v_2\Delta t$; since they are moving toward each other then the micrometeorite will pass then ship when $x_1 + x_2 = L$. Then

$$\Delta t = L/(v_1 + v_2) = (245\,\mathrm{m})/[(0.728 + 0.817)(3.00\times 10^8\mathrm{m/s})] = 5.29\times 10^{-7}\,\mathrm{s}.$$

This answer is the time measured in the "certain" reference frame. We can use Eq. 20-21 to find the time as measured on the ship,

$$\Delta t = \frac{\Delta t' + u\Delta x'/c^2}{\sqrt{1 - u^2/c^2}} = \frac{\left(5.29\times 10^{-7}\ \mathrm{s}\right) + (0.728c)(131\ \mathrm{m})/c^2}{\sqrt{1 - (0.728)^2}} = 1.23\times 10^{-6}\ \mathrm{s}.$$

E20-24 (a) $\gamma = 1/\sqrt{1 - (0.622)^2} = 1.28$.
(b) $\Delta t = (183\,\mathrm{m})/(0.622)(3.00\times 10^8\mathrm{m/s}) = 9.81\times 10^{-7}\mathrm{s}$. On the clock, however,

$$\Delta t' = \Delta t/\gamma = (9.81\times 10^{-7}\mathrm{s})/(1.28) = 7.66\times 10^{-7}\mathrm{s}.$$

E20-34 (a) If $K = E - mc^2 = 2mc^2$, then $E = 3mc^2$, so $\gamma = 3$, and

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(3)^2} = 0.943c.$$

(b) If $E = 2mc^2$, then $\gamma = 2$, and

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(2)^2} = 0.866c.$$

E20-41 Work is change in energy, so

$$W = mc^2/\sqrt{1 - (v_\mathrm{f}/c)^2} - mc^2/\sqrt{1 - (v_\mathrm{i}/c)^2}.$$

(a) Plug in the numbers,

$$W = (0.511\ \mathrm{MeV})(1/\sqrt{1 - (0.19)^2} - 1/\sqrt{1 - (0.18)^2}) = 0.996\ \mathrm{keV}.$$

(b) Plug in the numbers,

$$W = (0.511\ \mathrm{MeV})(1/\sqrt{1 - (0.99)^2} - 1/\sqrt{1 - (0.98)^2}) = 1.05\ \mathrm{MeV}.$$