Solutions of Homework at Week 6

April 21th, 2020

Chapter 11

$\boxed{\textbf{E11-37}}$ From Eq. 11-29, $K_\mathrm{i} = \frac{1}{2}I\omega_\mathrm{i}^2$. The object is a hoop, so $I = MR^2$. Then

$$K_\mathrm{i} = \frac{1}{2}MR^2\omega^2 = \frac{1}{2}(31.4\,\mathrm{kg})(1.21\,\mathrm{m})^2(29.6\,\mathrm{rad/s})^2 = 2.01\times 10^4\,\mathrm{J}.$$

Finally, the average power required to stop the wheel is

$$P = \frac{W}{t} = \frac{K_\mathrm{f} - K_\mathrm{i}}{t} = \frac{(0) - (2.01\times 10^4\,\mathrm{J})}{(14.8\,\mathrm{s})} = -1360\,\mathrm{W}.$$

$\boxed{\textbf{P11-13}}$ The work required to stretch the spring from x_i to x_f is given by

$$W = \int_{x_\mathrm{i}}^{x_\mathrm{f}} kx^3 dx = \frac{k}{4}{x_\mathrm{f}}^4 - \frac{k}{4}{x_\mathrm{i}}^4.$$

The problem gives

$$W_0 = \frac{k}{4}(l)^4 - \frac{k}{4}(0)^4 = \frac{k}{4}l^4.$$

We then want to find the work required to stretch from $x = l$ to $x = 2l$, so

$$\begin{aligned} W_{l\to 2l} &= \frac{k}{4}(2l)^4 - \frac{k}{4}(l)^4, \\ &= 16\frac{k}{4}l^4 - \frac{k}{4}l^4, \\ &= 15\frac{k}{4}l^4 = 15W_0. \end{aligned}$$

P11-22 (a) $\alpha = (-39.0\text{ rev/s})(2\pi\text{ rad/rev})/(32.0\,\mathrm{s}) = -7.66\text{ rad/s}^2$.

(b) The total rotational inertia of the system about the axis of rotation is

$$I = (6.40\,\mathrm{kg})(1.20\,\mathrm{m})^2/12 + 2(1.06\,\mathrm{kg})(1.20\,\mathrm{m}/2)^2 = 1.53\,\mathrm{kg}\cdot\mathrm{m}^2.$$

The torque is then $\tau = (1.53\,\mathrm{kg}\cdot\mathrm{m}^2)(7.66\text{ rad/s}^2) = 11.7\,\mathrm{N}\cdot\mathrm{m}$.

(c) $K = \frac{1}{2}(1.53\,\mathrm{kg}\cdot\mathrm{m}^2)(245\text{ rad/s})^2 = 4.59\times 10^4\,\mathrm{J}$.

(d) $\theta = \omega_\mathrm{av}t = (39.0\text{ rev/s}/2)(32.0\,\mathrm{s}) = 624\text{ rev}$.

(e) Only the loss in kinetic energy is independent of the behavior of the frictional torque.

P11-31 (a) Inelastic collision, so $v_\mathrm{f} = mv_\mathrm{i}/(m+M)$.

(b) $K = \frac{1}{2}mv^2 = p^2/2m$, so

$$\frac{\Delta K}{K_\mathrm{i}} = \frac{1/m - 1/(m+M)}{1/m} = \frac{M}{m+M}.$$

P11-32 Inelastic collision, so

$$v_\mathrm{f} = \frac{(1.88\,\mathrm{kg})(10.3\,\mathrm{m/s}) + (4.92\,\mathrm{kg})(3.27\,\mathrm{m/s})}{(1.88\,\mathrm{kg}) + (4.92\,\mathrm{kg})} = 5.21\,\mathrm{m/s}.$$

The loss in kinetic energy is

$$\Delta K = \frac{(1.88\,\mathrm{kg})(10.3\,\mathrm{m/s})^2}{2} + \frac{(4.92\,\mathrm{kg})(3.27\,\mathrm{m/s})^2}{2} - \frac{(1.88\,\mathrm{kg} + 4.92\,\mathrm{kg})(5.21\,\mathrm{m/s})^2}{2} = 33.7\,\mathrm{J}.$$

This change is because of work done on the spring, so

$$x = \sqrt{2(33.7\,\mathrm{J})/(1120\,\mathrm{N/m})} = 0.245\,\mathrm{m}$$

Chapter 12

$\boxed{\textbf{E12-11}}$ (a) The force constant of the spring is

$$k = F/x = mg/x = (7.94\,\text{kg})(9.81\,\text{m/s}^2)/(0.102\,\text{m}) = 764\,\text{N/m}.$$

(b) The potential energy stored in the spring is given by Eq. 12-8,

$$U = \frac{1}{2}kx^2 = \frac{1}{2}(764\,\text{N/m})(0.286\,\text{m} + 0.102\,\text{m})^2 = 57.5\,\text{J}.$$

(c) Conservation of energy,

$$\begin{aligned} K_\text{f} + U_\text{f} &= K_\text{i} + U_\text{i}, \\ \frac{1}{2}m{v_\text{f}}^2 + mgy_\text{f} + \frac{1}{2}k{x_\text{f}}^2 &= \frac{1}{2}m{v_\text{i}}^2 + mgy_\text{i} + \frac{1}{2}k{x_\text{i}}^2, \\ \frac{1}{2}(0)^2 + mgh + \frac{1}{2}k(0)^2 &= \frac{1}{2}(0)^2 + mg(0) + \frac{1}{2}k{x_\text{i}}^2. \end{aligned}$$

Rearranging,

$$h = \frac{k}{2mg}{x_\text{i}}^2 = \frac{(764\,\text{N/m})}{2(7.94\,\text{kg})(9.81\,\text{m/s}^2)}(0.388\,\text{m})^2 = 0.738\,\text{m}.$$

$\boxed{\textbf{E12-23}}$ There are *three* contributions to the kinetic energy: rotational kinetic energy of the shell (K_s), rotational kinetic energy of the pulley (K_p), and translational kinetic energy of the block (K_b). The conservation of energy statement is then

$$\begin{aligned} K_\text{s,i} + K_\text{p,i} + K_\text{b,i} + U_\text{i} &= K_\text{s,f} + K_\text{p,f} + K_\text{b,f} + U_\text{f}, \\ (0) + (0) + (0) + (0) &= \frac{1}{2}I_\text{s}{\omega_\text{s}}^2 + \frac{1}{2}I_\text{p}{\omega_\text{p}}^2 + \frac{1}{2}m{v_\text{b}}^2 + mgy. \end{aligned}$$

Finally, $y = -h$ and

$$\omega_\text{s}R = \omega_\text{p}r = v_\text{b}.$$

Combine all of this together, and our energy conservation statement will look like this:

$$0 = \frac{1}{2}\left(\frac{2}{3}MR^2\right)\left(\frac{v_\text{b}}{R}\right)^2 + \frac{1}{2}I_\text{p}\left(\frac{v_\text{b}}{r}\right)^2 + \frac{1}{2}m{v_\text{b}}^2 - mgh$$

which can be fairly easily rearranged into

$${v_\text{b}}^2 = \frac{2mgh}{2M/3 + I_\text{P}/r^2 + m}.$$

$\boxed{\textbf{P12-9}}$ Assume that $U_0 = K_0 = 0$. Then conservation of energy requires $K = -U$; consequently, $v = \sqrt{2g(-y)}$. If the ball *barely* swings around the top of the peg then the speed at the top of the loop is just fast enough so that the centripetal force is equal in magnitude to the weight,

$$mv^2/R = mg.$$

The energy conservation problem is then

$$\begin{aligned} mv^2 &= 2mg(L - 2(L-d)) = 2mg(2d - L) \\ mg(L-d) &= 2mg(2d-L), \\ d &= 3L/5. \end{aligned}$$

P12-13 The rotational inertia is

$$I = \frac{1}{3}ML^2 + ML^2 = \frac{4}{3}ML^2.$$

Conservation of energy is

$$\frac{1}{2}I\omega^2 = 3Mg(L/2),$$

so $\omega = \sqrt{9g/(4L)}$.

P12-14 The rotational speed of the sphere is $\omega = v/r$; the rotational kinetic energy is $K_{\mathrm{r}} = \frac{1}{2}I\omega^2 = \frac{1}{5}mv^2$.

(a) For the marble to stay on the track $mv^2/R = mg$ at the top of the track. Then the marble needs to be released from a point

$$mgh = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + 2mgR,$$

or $h = R/2 + R/5 + 2R = 2.7R$.

(b) Energy conservation gives

$$6mgR = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 + mgR,$$

or $mv^2/R = 50mg/7$. This corresponds to the horizontal force acting on the marble.