Solutions of Homework at Week 10

May 26th, 2021

Chapter 16

$\boxed{\textbf{E18-3}}$ (a) The time for a particular point to move from maximum displacement to zero displacement is one-quarter of a period; the point must then go to maximum negative displacement, zero displacement, and finally maximum positive displacement to complete a cycle. So the period is 4(178 ms) = 712 ms.

(b) The frequency is $f = 1/T = 1/(712\times10^{-3}\text{s}) = 1.40$ Hz.

(c) The wave-speed is $v = f\lambda = (1.40 \text{ Hz})(1.38\,\text{m}) = 1.93\,\text{m/s}$.

E18-4 Use Eq. 18-9, except let $f = 1/T$:

$$y = (0.0213\text{ m})\cos 2\pi\left(\frac{x}{(0.114\,\text{m})} - (385\text{ Hz})t\right) = (0.0213\text{ m})\cos\left[(55.1\,\text{rad/m})x - (2420\,\text{rad/s})t\right].$$

E18-7 (a) $y_\text{m} = 0.060\,\text{m}$.

(b) $\lambda = (2\pi\text{ rad})/(2.0\pi\text{ rad/m}) = 1.0\,\text{m}$.

(c) $f = (4.0\pi\text{ rad/s})/(2\pi\text{ rad}) = 2.0$ Hz.

(d) $v = (4.0\pi\text{ rad/s})/(2.0\pi\text{ rad/m}) = 2.0\,\text{m/s}$.

(e) Since the second term is positive the wave is moving in the $-x$ direction.

(f) $u_y = y_\text{m}\omega = (0.060\,\text{m})(4.0\pi\text{ rad/s}) = 0.75\,\text{m/s}$.

E18-11 (a) $y_m = 0.05\,\text{m}$.

(b) $\lambda = (0.55\,\text{m}) - (0.15\,\text{m}) = 0.40\,\text{m}$.

(c) $v = \sqrt{F/\mu} = \sqrt{(3.6\,\text{N})/(0.025\,\text{kg/m})} = 12\,\text{m/s}$.

(d) $T = 1/f = \lambda/v = (0.40\,\text{m})/(12\,\text{m/s}) = 3.33\times10^{-2}\text{s}$.

(e) $u_y = y_\text{m}\omega = 2\pi y_\text{m}/T = 2\pi(0.05\,\text{m})/(3.33\times10^{-2}\text{s}) = 9.4\,\text{m/s}$.

(f) $k = (2\pi\text{ rad})/(0.40\,\text{m}) = 5.0\pi\text{ rad/m}$; $\omega = kv = (5.0\pi\text{ rad/m})(12\,\text{m/s}) = 60\pi\text{ rad/s}$. The phase angle can be found from

$$(0.04\text{ m}) = (0.05\text{ m})\cos(\phi),$$

or $\phi = 0.64$ rad. Then

$$y = (0.05\text{ m})\cos[(5.0\pi\,\text{rad/m})x + (60\pi\,\text{rad/s})t + (0.64\,\text{rad})].$$

P18-1 (a) $\lambda = v/f$ and $k = 360°/\lambda$. Then

$$x = (55°)\lambda/(360°) = 55(353\,\text{m/s})/360(493\text{ Hz}) = 0.109\,\text{m}.$$

(b) $\omega = 360° f$, so

$$\phi = \omega t = (360°)(493\text{ Hz})(1.12\times10^{-3}\text{s}) = 199°.$$

$\boxed{\textbf{E18-20}}$ Consider only the point $x = 0$. The displacement y at that point is given by

$$y = y_{\rm m1} \sin(\omega t) + y_{\rm m2} \sin(\omega t + \pi/2) = y_{\rm m1} \sin(\omega t) + y_{\rm m2} \cos(\omega t).$$

This can be written as

$$y = y_{\rm m}(A_1 \sin \omega t + A_2 \cos \omega t),$$

where $A_i = y_{{\rm m}i}/y_{\rm m}$. But if $y_{\rm m}$ is judiciously chosen, $A_1 = \cos\beta$ and $A_2 = \sin\beta$, so that

$$y = y_{\rm m} \sin(\omega t + \beta).$$

Since we then require $A_1^2 + A_2^2 = 1$, we must have

$$y_{\rm m} = \sqrt{(3.20 \text{ cm})^2 + (4.19 \text{ cm})^2} = 5.27 \text{ cm}.$$

E18-26 (a) $v = \sqrt{(152\,\text{N})/(7.16\times10^{-3}\text{kg/m})} = 146\,\text{m/s}$.
(b) $\lambda = (2/3)(0.894\,\text{m}) = 0.596\,\text{m}$.
(c) $f = v/\lambda = (146\,\text{m/s})/(0.596\,\text{m}) = 245$ Hz.

E18-27 (a) $y = -3.9$ cm.
(b) $y = (0.15\,\text{m}) \sin[(0.79 \text{ rad/m})x + (13 \text{ rad/s})t]$.
(c) $y = 2(0.15\,\text{m}) \sin[(0.79 \text{ rad/m})(2.3\,\text{m})] \cos[(13 \text{ rad/s})(0.16\,\text{s})] = -0.14\,\text{m}$.

P18-14 The direct wave travels a distance d from S to D. The wave which reflects off the original layer travels a distance $\sqrt{d^2 + 4H^2}$ between S and D. The wave which reflects off the layer after it has risen a distance h travels a distance $\sqrt{d^2 + 4(H+h)^2}$. Waves will interfere constructively if there is a difference of an integer number of wavelengths between the two path lengths. In other words originally we have

$$\sqrt{d^2 + 4H^2} - d = n\lambda,$$

and later we have destructive interference so

$$\sqrt{d^2 + 4(H+h)^2} - d = (n + 1/2)\lambda.$$

We don't know n, but we can subtract the top equation from the bottom and get

$$\sqrt{d^2 + 4(H+h)^2} - \sqrt{d^2 + 4H^2} = \lambda/2$$

$\boxed{\textbf{P18-15}}$ The wavelength is

$$\lambda = v/f = (3.00\times10^8\text{m/s})/(13.0\times10^6\text{Hz}) = 23.1\,\text{m}.$$

The direct wave travels a distance d from S to D. The wave which reflects off the original layer travels a distance $\sqrt{d^2+4H^2}$ between S and D. The wave which reflects off the layer one minute later travels a distance $\sqrt{d^2+4(H+h)^2}$. Waves will interfere constructively if there is a difference of an integer number of wavelengths between the two path lengths. In other words originally we have

$$\sqrt{d^2+4H^2} - d = n_1\lambda,$$

and then one minute later we have

$$\sqrt{d^2+4(H+h)^2} - d = n_2\lambda.$$

We don't know either n_1 or n_2, but we do know the difference is 6, so we can subtract the top equation from the bottom and get

$$\sqrt{d^2+4(H+h)^2} - \sqrt{d^2+4H^2} = 6\lambda$$

We could use that expression as written, do some really obnoxious algebra, and then get the answer. But we don't want to; we want to take advantage of the fact that h is small compared to d and H. Then the first term can be written as

$$\begin{aligned}
\sqrt{d^2+4(H+h)^2} &= \sqrt{d^2+4H^2+8Hh+4h^2}, \\
&\approx \sqrt{d^2+4H^2+8Hh}, \\
&\approx \sqrt{d^2+4H^2}\sqrt{1+\frac{8H}{d^2+4H^2}h}, \\
&\approx \sqrt{d^2+4H^2}\left(1+\frac{1}{2}\frac{8H}{d^2+4H^2}h\right).
\end{aligned}$$

Between the second and the third lines we factored out d^2+4H^2; that last line is from the binomial expansion theorem. We put this into the previous expression, and

$$\begin{aligned}
\sqrt{d^2+4(H+h)^2} - \sqrt{d^2+4H^2} &= 6\lambda, \\
\sqrt{d^2+4H^2}\left(1+\frac{4H}{d^2+4H^2}h\right) - \sqrt{d^2+4H^2} &= 6\lambda, \\
\frac{4H}{\sqrt{d^2+4H^2}}h &= 6\lambda.
\end{aligned}$$

Now what were we doing? We were trying to find the speed at which the layer is moving. We know H, d, and λ; we can then find h,

$$h = \frac{6(23.1\,\text{m})}{4(510\times10^3\text{m})}\sqrt{(230\times10^3\text{m})^2+4(510\times10^3\text{m})^2} = 71.0\,\text{m}.$$

The layer is then moving at $v = (71.0\,\text{m})/(60\,\text{s}) = 1.18\,\text{m/s}$.

Chapter 19

$\boxed{\textbf{E19-49}}$ (a) The frequency "heard" by the wall is

$$f' = f\frac{v + v_O}{v - v_S} = (438\ \text{Hz})\frac{(343\,\text{m/s}) + (0)}{(343\,\text{m/s}) - (19.3\,\text{m/s})} = 464\ \text{Hz}$$

(b) The wall then reflects a frequency of 464 Hz back to the trumpet player. Sticking with Eq. 19-44, the source is now at rest while the observer moving,

$$f' = f\frac{v + v_O}{v - v_S} = (464\ \text{Hz})\frac{(343\,\text{m/s}) + (19.3\,\text{m/s})}{(343\,\text{m/s}) - (0)} = 490\ \text{Hz}$$

$\boxed{\textbf{P19-19}}$ (a) We apply Eq. 19-44

$$f' = f\frac{v + v_O}{v - v_S} = (1030\ \text{Hz})\frac{(5470\ \text{km/h}) + (94.6\ \text{km/h})}{(5470\ \text{km/h}) - (20.2\ \text{km/h})} = 1050\ \text{Hz}$$

(b) The reflected signal has a frequency equal to that of the signal received by the second sub originally. Applying Eq. 19-44 again,

$$f' = f\frac{v + v_O}{v - v_S} = (1050\ \text{Hz})\frac{(5470\ \text{km/h}) + (20.2\ \text{km/h})}{(5470\ \text{km/h}) - (94.6\ \text{km/h})} = 1070\ \text{Hz}$$