Discrete Mathematics Quiz 2   2020-5-28

Name _____   Student Number _____

1、 (3%) Find the next larger permutation in lexicographic order after 1375642.

Answer: 1376245

(3%) Find the next largest 4-combinations of the set $\{1, 2, 3, 4, 5, 6, 7, 8\}$ after $\{1, 2, 3, 8\}$.

Answer: $\{1, 2, 4, 5\}$

2、 How many relations are there on a set with n elements that are

(1) (4%) both symmetric and antisymmetric?

Answer: 2^n

(2) (8%) neither symmetric nor antisymmetric?

Answer: $2^{n^2} - (2^{n(n+1)/2} + 2^n \, 3^{n(n-1)/2} - 2^n)$

3、 (6%) a) How many ways are there to distribute 5 balls into 8 boxes, both the balls and boxes are labeled?   Answer: 8^5

(6%) b) How many ways are there to distribute 5 balls into 8 boxes, if each box must have at most one ball in it, and the balls are labeled but the boxes are unlabeled?   Answer: 1

(6%) c) How many ways are there to distribute 24 balls into 8 boxes, if the balls are unlabeled, but the boxes are labeled, and each box must have at least 1 balls in it?   Answer: C(8+16-1,16)

(6%) d) How many ways are there to distribute 5 balls into 8 boxes, if each box must have at most one ball in it, and the balls are unlabeled but the boxes are labeled?   Answer: $C(8, 5)$

(6%) e) How many ways are there to distribute 8 balls into 5 boxes, if no box can be left empty, both the balls and boxes are labeled?

Answer: 5^8 - C(5,1) 4^8 + C(5,2)3^8 - C(5,3) 2^8 + C(5,4) = 126000

4、 (6%) The coefficient of $x_1^2 x_2^3 x_3^2 x_4^3$ in the expansion of $(x_1 - 2x_2 + 3x_3 - x_4)^{10}$ is

Answer: （-2）^{3} * （3）^{2} * （-1）^{3} * 10！/（2！3！3！2！）

5、 (6%) Let $a_1 = 2$, $a_2 = 9$, and $a_n = 2a_{n-1} + 3a_{n-2}$ for $n \geq 3$. Then

Answer: $a_n = \frac{3}{4}(-1)^n + \frac{11}{12} 3^n$

6、 (6%) Suppose $|A| = |B| = |C| = 100$, $|A \cap B| = 60$, $|A \cap C| = 50$, $|B \cap C| = 40$, and $|A \cup B \cup C| = 175$. How many elements are in $A \cap B \cap C$?

Answer: 25

7、 (6%) Find the number of solutions of $x_1 + x_2 + x_3 = 10$, where x_1, x_2, x_3 are integers,

$x_1 \geq -3, x_2 \geq -4, x_3 \geq 5.$
Answer: C(3+12-1,12) = C(14,12)

8、(8%) Select 3 numbers from 1, 2， … , 100 such that the sum of these 3 numbers must be divided by 4. Then the total number of different selection ways is__
Answer: C(25,3) + 3*C(25,1)*C(25,2) + 25^3

9、(6%) If G(x) is the generating function for $a_0, a_1, a_2, a_3, \ldots$, describe in terms of G(x) the generating function for $a_0, 2a_1, 4a_2, 8a_3, 16a_4, \ldots$.
Answer:　G(2x)

10、(8 %) Use generating functions to solve the recurrence relation $a_k = 5a_{k-1} - 6a_{k-2}$ with initial conditions a_0 = 6 and a_1 = 30.

Answer: $a_k = 18 \cdot 3^k - 12 \cdot 2^k$

11、(6%)Assume that the characteristic equation for a homogeneous linear recurrence

relation with constant coefficients is (r −5) (r + 5)^2 = 0. Describe the form for the general

solution to the recurrence relation.

Answer: $a_n = c5^n + d(-5)^n + e \cdot n(-5)^n$