Solutions of Homework at Week 2

March 8th, 2020

Chapter 3

$\boxed{\textbf{E3-3}}$ Assuming constant acceleration we can find the average speed during the interval from Eq. 2-27

$$v_{\mathrm{av},x} = \frac{1}{2}(v_x + v_{0x}) = \frac{1}{2}\big((5.8\times10^6\ \mathrm{m/s}) + (0)\big) = 2.9\times10^6\ \mathrm{m/s}.$$

From this we can find the time spent accelerating from Eq. 2-22, since $\Delta x = v_{\mathrm{av},x}\Delta t$. Putting in the numbers $\Delta t = 5.17\times10^{-9}$s. The acceleration is then

$$a_x = \frac{\Delta v_x}{\Delta t} = \frac{(5.8\times10^6\ \mathrm{m/s}) - (0)}{(5.17\times10^{-9}\mathrm{s})} = 1.1\times10^{15}\mathrm{m/s^2}.$$

The net force on the electron is from Eq. 3-5,

$$\sum F_x = ma_x = (9.11\times10^{-31}\mathrm{kg})(1.1\times10^{15}\mathrm{m/s^2}) = 1.0\times10^{-15}\ \mathrm{N}.$$

E3-6 53 km/hr is 14.7 m/s. The average speed while decelerating is $v_{\mathrm{av}} = 7.4$ m/s. The time of deceleration is $t = x/v_{\mathrm{av}} = (0.65\ \mathrm{m})/(7.4\ \mathrm{m/s}) = 8.8\times10^{-2}$ s. The deceleration is $a = \Delta v/t = (-14.7\ \mathrm{m/s})/(8.8\times10^{-2}\ \mathrm{s}) = -17\times10^2\mathrm{m/s^2}$. The force is $F = ma = (39\mathrm{kg})(1.7\times10^2\mathrm{m/s^2}) = 6600$ N.

E3-14 (a) $W = mg = (75.0\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 736\,\mathrm{N}$.
(b) $W = mg = (75.0\,\mathrm{kg})(3.72\,\mathrm{m/s^2}) = 279\,\mathrm{N}$.
(c) $W = mg = (75.0\,\mathrm{kg})(0\,\mathrm{m/s^2}) = 0\,\mathrm{N}$.
(d) The mass is 75.0 kg at all locations.

P3-10 (a) The acceleration of the two blocks is $a = F/(m_1 + m_2)$ The net force on block 2 is from the force of contact, and is

$$P = m_2 a = Fm_2/(m_1 + m_2) = (3.2\,\mathrm{N})(1.2\,\mathrm{kg})/(2.3\,\mathrm{kg} + 1.2\,\mathrm{kg}) = 1.1\,\mathrm{N}.$$

(b) The acceleration of the two blocks is $a = F/(m_1 + m_2)$ The net force on block 1 is from the force of contact, and is

$$P = m_1 a = Fm_1/(m_1 + m_2) = (3.2\,\mathrm{N})(2.3\,\mathrm{kg})/(2.3\,\mathrm{kg} + 1.2\,\mathrm{kg}) = 2.1\,\mathrm{N}.$$

Not a third law pair, eh?

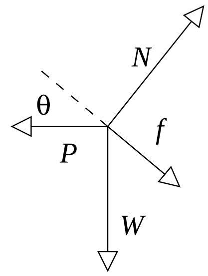
Chapter 5

E5-22 (a) The static friction between A and the table must be equal to the weight of block B to keep A from sliding. This means $m_B g = \mu_s(m_A + m_C)g$, or $m_c = m_B/\mu_s - m_A = (2.6\ \mathrm{kg})/(0.18) - (4.4\ \mathrm{kg}) = 10$ kg.

(b) There is no up/down motion for block A, so $f = \mu_k N = \mu_k m_A g$. The net force on the system containing blocks A and B is $F = W_B - f = m_B g - \mu_k m_A g$; the acceleration of this system is then

$$a = g\frac{m_B - \mu_k m_A}{m_A + m_B} = (9.\ \mathrm{m/s^2})\frac{(2.6\,\mathrm{kg}) - (0.15)(4.4\,\mathrm{kg})}{(2.6\,\mathrm{kg}) + (4.4\,\mathrm{kg})} = 2.7\,\mathrm{m/s^2}.$$

$\boxed{\textbf{E5-23}}$ There are four forces on the block— the force of gravity, $W = mg$; the normal force, N; the horizontal push, P, and the force of friction, f. Choose the coordinate system so that components are either parallel (x-axis) to the plane or perpendicular (y-axis) to it. $\theta = 39°$. Refer to the figure below.



The magnitudes of the x components of the forces are $W_x = W \sin\theta$, $P_x = P\cos\theta$ and f; the magnitudes of the y components of the forces are $W_y = W\cos\theta$, $P_y = P\sin\theta$.

(a) We consider the first the case of the block moving up the ramp; then f is directed down. Newton's second law for each set of components then reads as

$$\begin{aligned}\sum F_x &= P_x - f - W_x = P\cos\theta - f - W\sin\theta = ma_x,\\ \sum F_y &= N - P_y - W_y = N - P\sin\theta - W\cos\theta = ma_y\end{aligned}$$

Then the second equation is easy to solve for N

$$N = P\sin\theta + W\cos\theta = (46\text{ N})\sin(39°) + (4.8\text{ kg})(9.8\text{ m/s}^2)\cos(39°) = 66\text{ N}.$$

The force of friction is found from $f = \mu_k N = (0.33)(66\text{ N}) = 22$ N. This is directed down the incline while the block is moving up. We can now find the acceleration in the x direction.

$$\begin{aligned}ma_x &= P\cos\theta - f - W\sin\theta,\\ &= (46\,\text{N})\cos(39°) - (22\,\text{N}) - (4.8\,\text{kg})(9.8\,\text{m/s}^2)\sin(39°) = -16\,\text{N}.\end{aligned}$$

So the block is slowing down, with an acceleration of magnitude 3.3 m/s^2.

(b) The block has an initial speed of $v_{0x} = 4.3$ m/s; it will rise until it stops; so we can use $v_y = 0 = v_{0y} + a_y t$ to find the time to the highest point. Then $t = (v_y - v_{0y})/a_y = -(-4.3\text{ m/s})/(3.3$ m/s^2 $= 1.3$ s. Now that we know the time we can use the other kinematic relation to find the distance

$$y = v_{0y}t + \frac{1}{2}a_y t^2 = (4.3\text{ m/s})(1.3\text{ s}) + \frac{1}{2}(-3.3\text{ m/s}^2)(1.3\text{ s})^2 = 2.8\text{ m}$$

(c) When the block gets to the top it *might* slide back down. But in order to do so the frictional force, which is now directed up the ramp, must be sufficiently small so that $f + P_x \leq W_x$. Solving for f we find $f \leq W_x - P_x$ or, using our numbers from above, $f \leq -6$ N. Is this possible? No, so the block will not slide back down the ramp, *even if the ramp were frictionless,* while the horizontal force is applied.

E5-29 This problem is similar to Sample Problem 5-7, except now there is friction which can act on block B. The relevant equations are now for block B

$$N - m_B g\cos\theta = 0$$

and

$$T - m_B g\sin\theta \pm f = m_B a,$$

where the sign in front of f depends on the direction in which block B is moving. If the block is moving up the ramp then friction is directed down the ramp, and we would use the negative sign. If the block is moving down the ramp then friction will be directed up the ramp, and then we will use the positive sign. Finally, if the block is stationary then friction we be in such a direction as to make $a = 0$.

For block A the relevant equation is

$$m_A g - T = m_A a.$$

Combine the first two equations with $f = \mu N$ to get

$$T - m_B g \sin\theta \pm \mu m_B g \cos\theta = m_B a.$$

Take some care when interpreting friction for the static case, since the static value of μ yields the maximum possible static friction force, which is not necessarily the actual static frictional force.

Combine this last equation with the block A equation,

$$m_A g - m_A a - m_B g \sin\theta \pm \mu m_B g \cos\theta = m_B a,$$

and then rearrange to get

$$a = g\frac{m_A - m_B \sin\theta \pm \mu m_B \cos\theta}{m_A + m_B}.$$

For convenience we will use metric units; then the masses are $m_A = 13.2\,\mathrm{kg}$ and $m_B = 42.6\,\mathrm{kg}$. In addition, $\sin 42^\circ = 0.669$ and $\cos 42^\circ = 0.743$.

(a) If the blocks are originally at rest then

$$m_A - m_B \sin\theta = (13.2\,\mathrm{kg}) - (42.6\,\mathrm{kg})(0.669) = -15.3\,\mathrm{kg}$$

where the negative sign indicates that block B would slide downhill if there were no friction.

If the blocks are originally at rest we need to consider static friction, so the last term can be as large as

$$\mu m_B \cos\theta = (.56)(42.6\,\mathrm{kg})(0.743) = 17.7\,\mathrm{kg}.$$

Since this quantity is larger than the first static friction would be large enough to stop the blocks from accelerating if they are at rest.

(b) If block B is moving up the ramp we use the negative sign, and the acceleration is

$$a = (9.81\,\mathrm{m/s^2})\frac{(13.2\,\mathrm{kg}) - (42.6\,\mathrm{kg})(0.669) - (.25)(42.6\,\mathrm{kg})(0.743)}{(13.2\,\mathrm{kg}) + (42.6\,\mathrm{kg})} = -4.08\,\mathrm{m/s^2}.$$

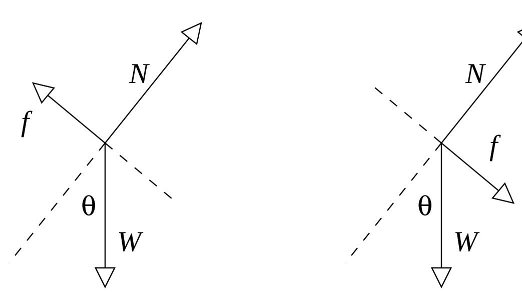
where the negative sign means down the ramp. The block, originally moving up the ramp, will slow down and stop. Once it stops the static friction takes over and the results of part (a) become relevant.

(c) If block B is moving down the ramp we use the positive sign, and the acceleration is

$$a = (9.81\,\mathrm{m/s^2})\frac{(13.2\,\mathrm{kg}) - (42.6\,\mathrm{kg})(0.669) + (.25)(42.6\,\mathrm{kg})(0.743)}{(13.2\,\mathrm{kg}) + (42.6\,\mathrm{kg})} = -1.30\,\mathrm{m/s^2}.$$

E5-41 There are three forces to consider: the normal force of the road on the car N; the force of gravity on the car W; and the frictional force on the car f. The acceleration of the car in circular motion is toward the center of the circle; this means the *net* force on the car is horizontal, toward the center. We will arrange our coordinate system so that r is horizontal and z is vertical. Then the components of the normal force are $N_r = N\sin\theta$ and $N_z = N\cos\theta$; the components of the frictional force are $f_r = f\cos\theta$ and $f_z = f\sin\theta$.

The direction of the friction depends on the speed of the car. The figure below shows the two force diagrams.



The turn is designed for 95 km/hr, at this speed a car should require *no* friction to stay on the road. Using Eq. 5-17 we find that the banking angle is given by

$$\tan\theta_b = \frac{v^2}{rg} = \frac{(26 \text{ m/s})^2}{(210 \text{ m})(9.8 \text{ m/s}^2)} = 0.33,$$

for a bank angle of $\theta_b = 18^\circ$.

(a) On the rainy day traffic is moving at 14 m/s. This is slower than the rated speed, so any frictional force must be directed up the incline. Newton's second law is then

$$\begin{aligned}\sum F_r &= N_r - f_r = N\sin\theta - f\cos\theta = \frac{mv^2}{r},\\ \sum F_z &= N_z + f_z - W = N\cos\theta + f\sin\theta - mg = 0.\end{aligned}$$

We can substitute $f = \mu_s N$ to find the minimum value of μ_s which will keep the cars from slipping. There will then be two equations and two unknowns, μ_s and N. Solving for N,

$$N\left(\sin\theta - \mu_s\cos\theta\right) = \frac{mv^2}{r} \text{ and } N\left(\cos\theta + \mu_s\sin\theta\right) = mg.$$

Combining,

$$\left(\sin\theta - \mu_s\cos\theta\right) mg = \left(\cos\theta + \mu_s\sin\theta\right)\frac{mv^2}{r}$$

Rearrange,

$$\mu_s = \frac{gr\sin\theta - v^2\cos\theta}{gr\cos\theta + v^2\sin\theta}.$$

We know all the numbers. Put them in and we'll find $\mu_s = 0.22$

(b) Now the frictional force will point the other way, so Newton's second law is now

$$\begin{aligned}\sum F_r &= N_r + f_r = N\sin\theta + f\cos\theta = \frac{mv^2}{r},\\ \sum F_z &= N_z - f_z - W = N\cos\theta - f\sin\theta - mg = 0.\end{aligned}$$

The bottom equation can be rearranged to show that

$$N = \frac{mg}{\cos\theta - \mu_s\sin\theta}.$$

This can be combined with the top equation to give

$$mg\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta} = \frac{mv^2}{r}.$$

We can solve this final expression for v using all our previous numbers and get $v = 35$ m/s. That's about 130 km/hr.

E5-49 The force only has an x component, so we can use Eq. 5-19 to find the velocity.

$$v_x = v_{0x} + \frac{1}{m}\int_0^t F_x\,dt = v_0 + \frac{F_0}{m}\int_0^t (1 - t/T)\ dt$$

Integrating,

$$v_x = v_0 + a_0\left(t - \frac{1}{2T}t^2\right)$$

Now put this expression into Eq. 5-20 to find the position as a function of time

$$x = x_0 + \int_0^t v_x \, dt = \int_0^t \left(v_{0x} + a_0 \left(t - \frac{1}{2T} t^2 \right) \right) \, dt$$

Integrating,

$$x = v_0 T + a_0 \left(\frac{1}{2} T^2 - \frac{1}{6T} T^3 \right) = v_0 T + a_0 \frac{T^2}{3}.$$

Now we can put $t = T$ into the expression for v.

$$v_x = v_0 + a_0 \left(T - \frac{1}{2T} T^2 \right) = v_0 + a_0 T/2.$$

P5-2 (a) Since the pulley is massless, $F = 2T$. The largest value of T that will allow block 2 to remain on the floor is $T \leq W_2 = m_2 g$. So $F \leq 2(1.9\text{ kg})(9.8\text{ m/s}^2) = 37\text{ N}$.

(b) $T = F/2 = (110\,\text{N})/2 = 55\,\text{N}$.

(c) The net force on block 1 is $T - W_1 = (55\,\text{N}) - (1.2\,\text{kg})(9.8\,\text{m/s}^2) = 43\,\text{N}$. This will result in an acceleration of $a = (43\,\text{N})/(1.2\,\text{kg}) = 36\,\text{m/s}^2$.

$\boxed{\textbf{P5-7}}$ There are four forces on the broom: the force of gravity $W = mg$; the normal force of the floor N; the force of friction f; and the applied force from the person P (the book calls it F). Then

$$\begin{aligned} \sum F_x &= P_x - f = P \sin\theta - f = m a_x, \\ \sum F_y &= N - P_y - W = N - P\cos\theta - mg = m a_y = 0 \end{aligned}$$

Solve the second equation for N,

$$N = P\cos\theta + mg.$$

(a) If the mop slides at constant speed $f = \mu_k N$. Then

$$P\sin\theta - f = P\sin\theta - \mu_k \left(P\cos\theta + mg \right) = 0.$$

We can solve this for P (which was called F in the book);

$$P = \frac{\mu mg}{\sin\theta - \mu_k \cos\theta}.$$

This is the force required to push the broom at constant speed.

(b) Note that P becomes negative (or infinite) if $\sin\theta \leq \mu_k \cos\theta$. This occurs when $\tan\theta_c \leq \mu_k$. If this happens the mop stops moving, to get it started again you must overcome the static friction, but this is impossible if $\tan\theta_0 \leq \mu_s$

$\boxed{\textbf{P5-9}}$ To hold up the smaller block the frictional force between the larger block and smaller block must be as large as the weight of the smaller block. This can be written as $f = mg$. The normal force of the larger block on the smaller block is N, and the frictional force is given by $f \leq \mu_s N$. So the smaller block won't fall if $mg \leq \mu_s N$.

There is only one horizontal force on the large block, which is the normal force of the small block on the large block. Newton's third law says this force has a magnitude N, so the acceleration of the large block is $N = Ma$.

There is only one horizontal force on the two block system, the force F. So the acceleration of this system is given by $F = (M + m)a$. The two accelerations are equal, otherwise the blocks won't stick together. Equating, then, gives $N/M = F/(M + m)$.

We can combine this last expression with $mg \leq \mu_s N$ and get

$$mg \leq \mu_s F \frac{M}{M + m}$$

or

$$F \geq \frac{g(M + m)m}{\mu_s M} = \frac{(9.81\,\text{m/s}^2)(88\,\text{kg} + 16\,\text{kg})(16\,\text{kg})}{(0.38)(88\,\text{kg})} = 490\,\text{N}$$

P5-18 The net force on the cube is $F = mv^2/r$. The speed is $2\pi r\omega$. (Note that we are using ω in a non-standard way!) Then $F = 4\pi^2 mr\omega^2$. There are three forces to consider: the normal force of the funnel on the cube N; the force of gravity on the cube W; and the frictional force on the cube f. The acceleration of the cube in circular motion is toward the center of the circle; this means the *net* force on the cube is horizontal, toward the center. We will arrange our coordinate system so that r is horizontal and z is vertical. Then the components of the normal force are $N_r = N\sin\theta$ and $N_z = N\cos\theta$; the components of the frictional force are $f_r = f\cos\theta$ and $f_z = f\sin\theta$.

The direction of the friction depends on the speed of the cube; it will point up if ω is small and down if ω is large.

(a) If ω is small, Newton's second law is

$$\begin{aligned}\sum F_r &= N_r - f_r = N\sin\theta - f\cos\theta = 4\pi^2 mr\omega^2,\\ \sum F_z &= N_z + f_z - W = N\cos\theta + f\sin\theta - mg = 0.\end{aligned}$$

We can substitute $f = \mu_s N$. Solving for N,

$$N(\cos\theta + \mu_s\sin\theta) = mg.$$

Combining,

$$4\pi^2 r\omega^2 = g\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}.$$

Rearrange,

$$\omega = \frac{1}{2\pi}\sqrt{\frac{g}{r}\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}}.$$

This is the minimum value.

(b) Now the frictional force will point the other way, so Newton's second law is now

$$\begin{aligned}\sum F_r &= N_r + f_r = N\sin\theta + f\cos\theta = 4\pi^2 mr\omega^2,\\ \sum F_z &= N_z - f_z - W = N\cos\theta - f\sin\theta - mg = 0.\end{aligned}$$

We've swapped + and - signs, so

$$\omega = \frac{1}{2\pi}\sqrt{\frac{g}{r}\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}}$$

is the maximum value.