Sec. 10.1 1, 3-9

1. Draw graph models, stating the type of graph (from Table 1) used, to represent airline routes where every day there are four flights from Boston to Newark, two flights from Newark to Boston, three flights from Newark to Miami, two flights from Miami to Newark, one flight from Newark to Detroit, two flights from Detroit to Newark, three flights from Newark to Washington, two flights from Washington to Newark, and one flight from Washington to Miami, with

a) an edge between vertices representing cities that have a flight between them (in either direction).

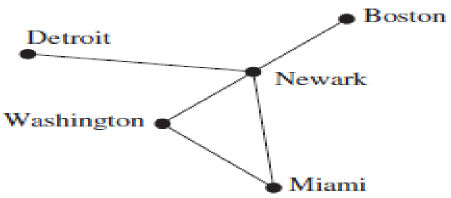
b) an edge between vertices representing cities for each flight that operates between them (in either direction).

c) an edge between vertices representing cities for each flight that operates between them (in either direction), plus a loop for a special sightseeing trip that takes off and lands in Miami.

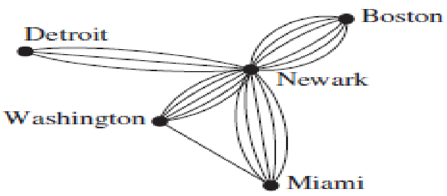
d) an edge from a vertex representing a city where a flight starts to the vertex representing the city where it ends.

e) an edge for each flight from a vertex representing a city where the flight begins to the vertex representing the city where the flight ends.

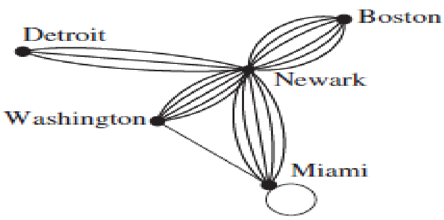
1. a)



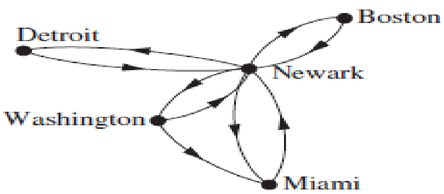
b)



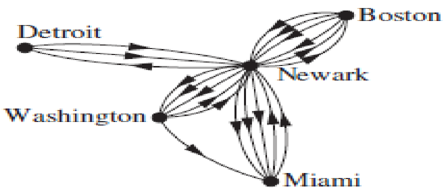
c)



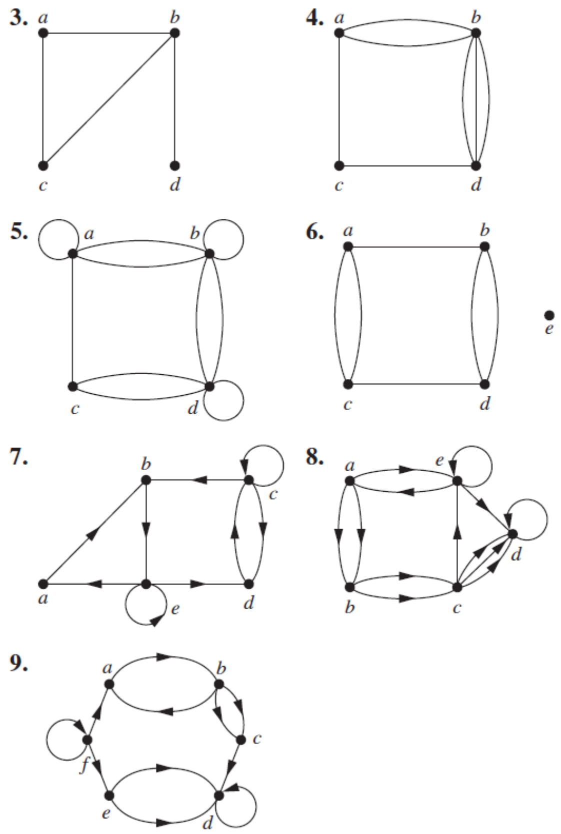
d)



e)



For Exercises 3–9, determine whether the graph shown has directed or undirected edges, whether it has multiple edges, and whether it has one or more loops. Use your answers to determine the type of graph in Table 1 this graph is.



3. simple 4. Multigraph 5. Pseudograph 6. Multigraph 7. Directed graph 8. Directed multigraph 9. Directed multigraph

3. Simple graph **5.** Pseudograph **7.** Directed graph

4. This is a multigraph; the edges are undirected, and there are no loops, but there are parallel edges.

6. This is a multigraph; the edges are undirected, and there are no loops, but there are parallel edges.

8. This is a directed multigraph; the edges are directed, and there are parallel edges.

9. Directed multigraph

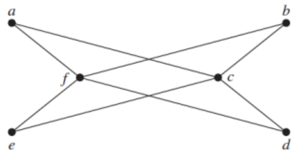
第7版 Sec. 10.2 5, 24, 25, 42(b, f, h), 53, 60

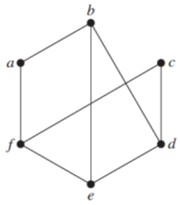
第8版 Sec. 10.2 5, 24, 25, 44(b, f, h), 55, 62

5. Can a simple graph exist with 15 vertices each of degree five?

No，由定理 1 可知矛盾

In Exercises 21–25 determine whether the graph is bipartite. You may find it useful to apply Theorem 4 and answer the question by determining whether it is possible to assign either red or blue to each vertex so that no two adjacent vertices are assigned the same color.

24. 

25. 

24. This is the complete bipartite graph $K_{2,4}$. The vertices in the part of size 2 are c and f, and the vertices in the part of size 4 are a, b, d, and e.

25. Not bipartite

44. Determine whether each of these sequences is graphic. For those that are, draw a graph having the given degree sequence.

a) 5, 4, 3, 2, 1, 0 **b)** 6, 5, 4, 3, 2, 1 **c)** 2, 2, 2, 2, 2, 2
d) 3, 3, 3, 2, 2, 2 **e)** 3, 3, 2, 2, 2, 2 **f)** 1, 1, 1, 1, 1, 1
g) 5, 3, 3, 3, 3, 3 **h)** 5, 5, 4, 3, 2, 1

44. **a)** Since the number of odd-degree vertices has to be even, no graph exists with these degrees. Another reason no such graph exists is that the vertex of degree 0 would have to be isolated but the vertex of degree 5 would have to be adjacent to every other vertex, and these two statements are contradictory.

b) Since the number of odd-degree vertices has to be even, no graph exists with these degrees. Another reason no such graph exists is that the degree of a vertex in a simple graph is at most 1 less than the number of vertices.

c) A 6-cycle is such a graph. (See picture below.)

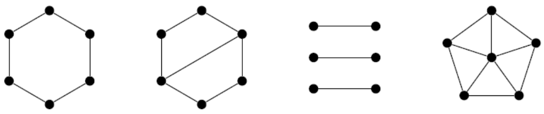
d) Since the number of odd-degree vertices has to be even, no graph exists with these degrees.

e) A 6-cycle with one of its diagonals added is such a graph. (See picture below.)

f) A graph consisting of three edges with no common vertices is such a graph. (See picture below.)

g) The 5-wheel is such a graph. (See picture below.)

h) Each of the vertices of degree 5 is adjacent to all the other vertices. Thus there can be no vertex of degree 1. So no such graph exists.



(c) (e) (f) (g)

55. For which values of n are these graphs regular?

a) K_n **b)** C_n **c)** W_n **d)** Q_n

55. **a)** For all $n \geq 1$ **b)** For all $n \geq 3$ **c)** For $n = 3$ **d)** For all $n \geq 0$ **57.** 5

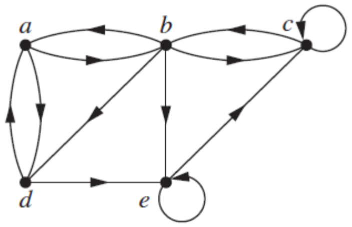
62. If G is a simple graph with 15 edges and $\overline{G}$ has 13 edges, how many vertices does G have?

62. The given information tells us that $G \cup \overline{G}$ has 28 edges. However, $G \cup \overline{G}$ is the complete graph on the number of vertices n that G has. Since this graph has $n(n-1)/2$ edges, we want to solve $n(n-1)/2 = 28$. Thus $n = 8$.

第7版 Sec. 10.3 8, 15, 17, 34-37

第8版 Sec. 10.3 8, 15, 17, 38-41

4.



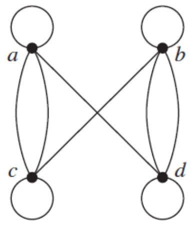
8. Represent the graph in Exercise 4 with an adjacency matrix.

8. This is similar to Exercise 7.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

In Exercises 13–15 represent the given graph using an adjacency matrix.

15.

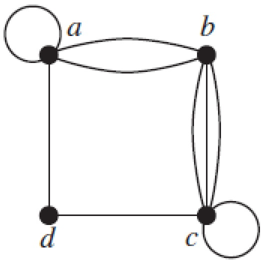


15. $\begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & 1 & 2 \\ 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$

In Exercises 16–18 draw an undirected graph represented by the given adjacency matrix.

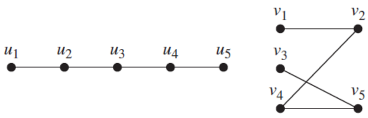
17. $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$

17.

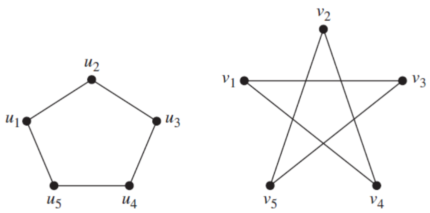


In Exercises 38–48 determine whether the given pair of graphs is isomorphic. Exhibit an isomorphism or provide a rigorous argument that none exists. For additional exercises of this kind, see Exercises 3–5 in the Supplementary Exercises.

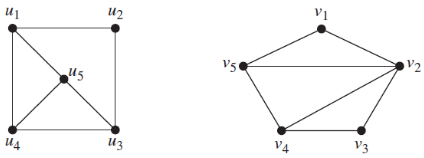
38.



39.



40.

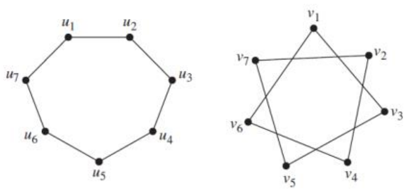


38. These graphs are isomorphic, since each is a path with five vertices. One isomorphism is $f(u_1) = v_1$, $f(u_2) = v_2$, $f(u_3) = v_4$, $f(u_4) = v_5$, and $f(u_5) = v_3$.

39. Isomorphic

40. These graphs are not isomorphic. The second has a vertex of degree 4, whereas the first does not.

41.



41. Isomorphic

Sec. 10.4 27(e), 28, 29, 62

27. Find the number of paths from a to e in the directed graph in Exercise 2 of length

a) 2. **b)** 3. **c)** 4. **d)** 5. **e)** 6. **f)** 7.

e) 5

***28.** Show that every connected graph with n vertices has at least $n - 1$ edges.

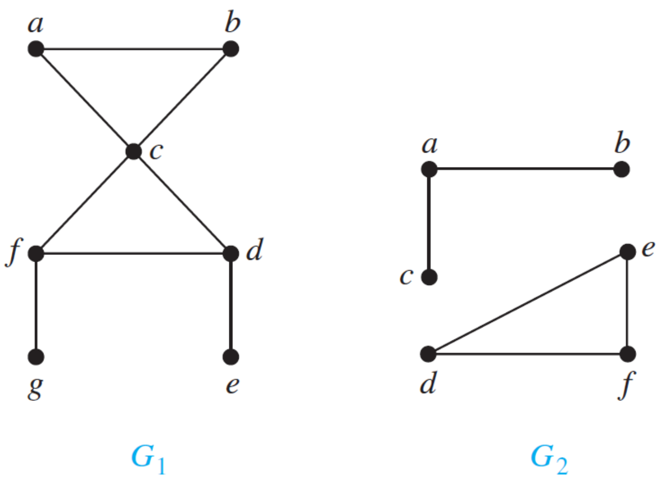
28. We show this by induction on n. For $n = 1$ there is nothing to prove. Now assume the inductive hypothesis, and let G be a connected graph with $n+1$ vertices and fewer than n edges, where $n \geq 1$. Since the sum of the degrees of the vertices of G is equal to 2 times the number of edges, we know that the sum of the degrees is less than $2n$, which is less than $2(n+1)$. Therefore some vertex has degree less than 2. Since G is connected, this vertex is not isolated, so it must have degree 1. Remove this vertex and its edge. Clearly the result is still connected, and it has n vertices and fewer than $n-1$ edges, contradicting the inductive hypothesis. Therefore the statement holds for G, and the proof is complete.

29. Let $G = (V, E)$ be a simple graph. Let R be the relation on V consisting of pairs of vertices (u, v) such that there is a path from u to v or such that $u = v$. Show that R is an equivalence relation.

b) 0 **c)** 27 **d)** 0 **27. a)** 1 **b)** 0 **c)** 2 **d)** 1 **e)** 5 **f)** 3 **29.** R is reflexive by definition. Assume that $(u, v) \in R$; then there is a path from u to v. Then $(v, u) \in R$ because there is a path from v to u, namely, the path from u to v traversed backward. Assume that $(u, v) \in R$ and $(v, w) \in R$; then there are paths from u to v and from v to w. Putting these two paths together gives a path from u to w. Hence, $(u, w) \in R$. It follows that R is transitive. **31.** c **33.** b, c, e, i **35.** If a vertex is pen-

61. Explain how Theorem 2 can be used to determine whether a graph is connected.

62. Use Exercise 61 to show that the graph G_1 in Figure 2 is connected whereas the graph G_2 in that figure is not connected.



62. The adjacency matrix of G is as follows:

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

We compute A^2 and A^3, obtaining

$$A^2 = \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 2 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 4 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 3 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad A^3 = \begin{bmatrix} 2 & 3 & 5 & 2 & 1 & 2 & 1 \\ 3 & 2 & 5 & 2 & 1 & 2 & 1 \\ 5 & 5 & 4 & 6 & 1 & 6 & 1 \\ 2 & 2 & 6 & 2 & 3 & 5 & 1 \\ 1 & 1 & 1 & 3 & 0 & 1 & 1 \\ 2 & 2 & 6 & 5 & 1 & 2 & 3 \\ 1 & 1 & 1 & 1 & 1 & 3 & 0 \end{bmatrix}.$$

Already every off-diagonal entry in A^3 is nonzero, so we know that there is a path of length 3 between every pair of distinct vertices in this graph. Therefore the graph G is connected.

On the other hand, the adjacency matrix of H is as follows:

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 \end{bmatrix}$$

We compute A^2 through A^5, obtaining the following matrices:

$$A^2 = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 1 \\ 0 & 0 & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 & 1 & 2 \end{bmatrix} \qquad A^3 = \begin{bmatrix} 0 & 2 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 3 & 3 \\ 0 & 0 & 0 & 3 & 2 & 3 \\ 0 & 0 & 0 & 3 & 3 & 2 \end{bmatrix}$$

$$A^4 = \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 5 & 5 \\ 0 & 0 & 0 & 5 & 6 & 5 \\ 0 & 0 & 0 & 5 & 5 & 6 \end{bmatrix} \qquad A^5 = \begin{bmatrix} 0 & 4 & 4 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 10 & 11 & 11 \\ 0 & 0 & 0 & 11 & 10 & 11 \\ 0 & 0 & 0 & 11 & 11 & 10 \end{bmatrix}$$

If we compute the sum $A + A^2 + A^3 + A^4 + A^5$ we obtain

$$\begin{bmatrix} 6 & 7 & 7 & 0 & 0 & 0 \\ 7 & 3 & 3 & 0 & 0 & 0 \\ 7 & 3 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 20 & 21 & 21 \\ 0 & 0 & 0 & 21 & 20 & 21 \\ 0 & 0 & 0 & 21 & 21 & 20 \end{bmatrix}.$$

There is a 0 in the $(1, 4)$ position, telling us that there is no path of length at most 5 from vertex a to vertex d. Since the graph only has six vertices, this tells us that there is no path at all from a to d. Thus the fact that there was a 0 as an off-diagonal entry in the sum told us that the graph was not connected.