Solutions of Homework at Week 1

E1-29 The definition of the meter was wavelengths per meter; the question asks for meters per wavelength, so we want to take the reciprocal. The definition is accurate to 9 figures, so the reciprocal should be written as $1/1,650,763.73 = 6.05780211 \times 10^{-7}\,\mathrm{m} = 605.780211\ \mathrm{nm}$.

E1-31 The easiest approach is to first solve Darcy's Law for K, and then substitute the known SI units for the other quantities. Then

$$K = \frac{VL}{AHt} \text{ has units of } \frac{\mathrm{m}^3\ (\mathrm{m})}{(\mathrm{m}^2)\,(\mathrm{m})\,(\mathrm{s})}$$

which can be simplified to m/s.

$\boxed{\textbf{P1-7}}$ Let the radius of the grain be given by r_g. Then the surface area of the grain is $A_g = 4\pi r_g^2$, and the volume is given by $V_g = (4/3)\pi r_g^3$.

If N grains of sand have a total surface area equal to that of a cube 1 m on a edge, then $NA_g = 6\ \mathrm{m}^2$. The total volume V_t of this number of grains of sand is NV_g. Eliminate N from these two expressions and get

$$V_t = NV_g = \frac{(6\ \mathrm{m}^2)}{A_g}V_g = \frac{(6\ \mathrm{m}^2)r_g}{3}.$$

Then $V_t = (2\ \mathrm{m}^2)(50 \times 10^{-6}\ \mathrm{m}) = 1 \times 10^{-4}\ \mathrm{m}^3$.

The mass of a volume V_t is given by

$$1 \times 10^{-4}\ \mathrm{m}^3 \left(\frac{2600\ \mathrm{kg}}{1\ \mathrm{m}^3}\right) = 0.26\ \mathrm{kg}.$$

P1-9 (a) The volume per particle is

$$(9.27 \times 10^{-26}\mathrm{kg})/(7870\,\mathrm{kg/m^3}) = 1.178 \times 10^{-28}\mathrm{m}^3.$$

The radius of the corresponding sphere is

$$r = \sqrt[3]{\frac{3(1.178 \times 10^{-28}\mathrm{m}^3)}{4\pi}} = 1.41 \times 10^{-10}\mathrm{m}.$$

Double this, and the spacing is 282 pm.

(b) The volume per particle is

$$(3.82 \times 10^{-26}\mathrm{kg})/(1013\,\mathrm{kg/m^3}) = 3.77 \times 10^{-29}\mathrm{m}^3.$$

The radius of the corresponding sphere is

$$r = \sqrt[3]{\frac{3(3.77 \times 10^{-29}\mathrm{m}^3)}{4\pi}} = 2.08 \times 10^{-10}\mathrm{m}.$$

Double this, and the spacing is 416 pm.

E2-9 (a) The magnitude of $\vec{\mathbf{a}}$ is $\overline{4.0^2 + (-3.0)^2} = 5.0$; the direction is $\theta = \tan^{-1}(-3.0/4.0) = 323°$.

(b) The magnitude of $\vec{\mathbf{b}}$ is $\sqrt{6.0^2 + 8.0^3} = 10.0$; the direction is $\theta = \tan^{-1}(6.0/8.0) = 36.9°$.

(c) The resultant vector is $\vec{\mathbf{a}} + \vec{\mathbf{b}} = (4.0 + 6.0)\hat{\mathbf{i}} + (-3.0 + 8.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{a}} + \vec{\mathbf{b}}$ is $\sqrt{(10.0)^2 + (5.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(5.0/10.0) = 26.6°$.

(d) The resultant vector is $\vec{\mathbf{a}} - \vec{\mathbf{b}} = (4.0 - 6.0)\hat{\mathbf{i}} + (-3.0 - 8.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{a}} - \vec{\mathbf{b}}$ is $\sqrt{(-2.0)^2 + (-11.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(-11.0/-2.0) = 260°$.

(e) The resultant vector is $\vec{\mathbf{b}} - \vec{\mathbf{a}} = (6.0 - 4.0)\hat{\mathbf{i}} + (8.0 - -3.0)\hat{\mathbf{j}}$. The magnitude of $\vec{\mathbf{b}} - \vec{\mathbf{a}}$ is $\sqrt{(2.0)^2 + (11.0)^2} = 11.2$; the direction is $\theta = \tan^{-1}(11.0/2.0) = 79.7°$.

$\boxed{\textbf{E2-17}}$ (a) Evaluate $\vec{\mathbf{r}}$ when $t = 2$ s.

$$\begin{aligned}
\vec{\mathbf{r}} &= [(2\ \text{m/s}^3)t^3 - (5\ \text{m/s})t]\hat{\mathbf{i}} + [(6\ \text{m}) - (7\ \text{m/s}^4)t^4]\hat{\mathbf{j}} \\
&= [(2\ \text{m/s}^3)(2\ \text{s})^3 - (5\ \text{m/s})(2\ \text{s})]\hat{\mathbf{i}} + [(6\ \text{m}) - (7\ \text{m/s}^4)(2\ \text{s})^4]\hat{\mathbf{j}} \\
&= [(16\ \text{m}) - (10\ \text{m})]\hat{\mathbf{i}} + [(6\ \text{m}) - (112\ \text{m})]\hat{\mathbf{j}} \\
&= [(6\ \text{m})]\hat{\mathbf{i}} + [-(106\ \text{m})]\hat{\mathbf{j}}.
\end{aligned}$$

(b) Evaluate:

$$\begin{aligned}
\vec{\mathbf{v}} = \frac{d\vec{\mathbf{r}}}{dt} &= [(2\ \text{m/s}^3)3t^2 - (5\ \text{m/s})]\hat{\mathbf{i}} + [-(7\ \text{m/s}^4)4t^3]\hat{\mathbf{j}} \\
&= [(6\ \text{m/s}^3)t^2 - (5\ \text{m/s})]\hat{\mathbf{i}} + [-(28\ \text{m/s}^4)t^3]\hat{\mathbf{j}}.
\end{aligned}$$

Into this last expression we now evaluate $\vec{\mathbf{v}}(t = 2\ \text{s})$ and get

$$\begin{aligned}
\vec{\mathbf{v}} &= [(6\ \text{m/s}^3)(2\ \text{s})^2 - (5\ \text{m/s})]\hat{\mathbf{i}} + [-(28\ \text{m/s}^4)(2\ \text{s})^3]\hat{\mathbf{j}} \\
&= [(24\ \text{m/s}) - (5\ \text{m/s})]\hat{\mathbf{i}} + [-(224\ \text{m/s})]\hat{\mathbf{j}} \\
&= [(19\ \text{m/s})]\hat{\mathbf{i}} + [-(224\ \text{m/s})]\hat{\mathbf{j}},
\end{aligned}$$

for the velocity $\vec{\mathbf{v}}$ when $t = 2$ s.

(c) Evaluate

$$\begin{aligned}
\vec{\mathbf{a}} = \frac{d\vec{\mathbf{v}}}{dt} &= [(6\ \text{m/s}^3)2t]\hat{\mathbf{i}} + [-(28\ \text{m/s}^4)3t^2]\hat{\mathbf{j}} \\
&= [(12\ \text{m/s}^3)t]\hat{\mathbf{i}} + [-(84\ \text{m/s}^4)t^2]\hat{\mathbf{j}}.
\end{aligned}$$

Into this last expression we now evaluate $\vec{\mathbf{a}}(t = 2\ \text{s})$ and get

$$\begin{aligned}
\vec{\mathbf{a}} &= [(12\,\text{m/s}^3)(2\,\text{s})]\hat{\mathbf{i}} + [-(84\,\text{m/s}^4)(2\,2)^2]\hat{\mathbf{j}} \\
&= [(24\,\text{m/s}^2)]\hat{\mathbf{i}} + [-(336\,\text{m/s}^2)]\hat{\mathbf{j}}.
\end{aligned}$$

E2-35 (a) Up to A $v_x > 0$ and is constant. From A to B v_x is decreasing, but still positive. From B to C $v_x = 0$. From C to D $v_x < 0$, but $|v_x|$ is decreasing.

(b) No. Constant acceleration would appear as (part of) a parabola; but it would be challenging to distinguish between a parabola and an almost parabola.

P2-9 (a) The average velocity during the time interval is $v_{\text{av}} = \Delta x/\Delta t$, or

$$v_{\text{av}} = \frac{(A + B(3\,\text{s})^3) - (A + B(2\,\text{s})^3)}{(3\,\text{s}) - (2\,\text{s})} = (1.50\ \text{cm/s}^3)(19\,\text{s}^3)/(1\,\text{s}) = 28.5\ \text{cm/s}.$$

(b) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(2\,\text{s})^2 = 18\ \text{cm/s}$.

(c) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(3\,\text{s})^2 = 40.5\ \text{cm/s}$.

(d) $v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(2.5\,\text{s})^2 = 28.1\ \text{cm/s}$.

(e) The midway position is $(x_{\text{f}} + x_{\text{i}})/2$, or

$$x_{\text{mid}} = A + B[(3\,\text{s})^3 + (2\,\text{s})^3)]/2 = A + (17.5\,\text{s}^3)B.$$

This occurs when $t = \sqrt[3]{(17.5\,\text{s}^3)}$. The instantaneous velocity at this point is

$$v = dx/dt = 3Bt^2 = 3(1.50\ \text{cm/s}^3)(\sqrt[3]{(17.5\,\text{s}^3)})^2 = 30.3\ \text{cm/s}.$$

P2-17 The runner covered a distance d_1 in a time interval t_1 during the acceleration phase and a distance d_2 in a time interval t_2 during the constant speed phase. Since the runner started from rest we know that the constant speed is given by $v = at_1$, where a is the runner's acceleration.

The distance covered during the acceleration phase is given by

$$d_1 = \frac{1}{2}at_1^2.$$

The distance covered during the constant speed phase can also be found from

$$d_2 = vt_2 = at_1t_2.$$

We want to use these two expressions, along with $d_1 + d_2 = 100$ m and $t_2 = (12.2 \text{ s}) - t_1$, to get

$$\begin{aligned} 100 \text{ m} &= d_1 + d_2 = \frac{1}{2}at_1^2 + at_1(12.2 \text{ s} - t_1), \\ &= -\frac{1}{2}at_1^2 + a(12.2 \text{ s})t_1, \\ &= -(1.40 \text{ m/s}^2)t_1^2 + (34.2 \text{ m/s})t_1. \end{aligned}$$

This last expression is quadratic in t_1, and is solved to give $t_1 = 3.40\,\text{s}$ or $t_1 = 21.0\,\text{s}$. Since the race only lasted 12.2 s we can ignore the second answer.

(b) The distance traveled during the acceleration phase is then

$$d_1 = \frac{1}{2}at_1^2 = (1.40 \text{ m/s}^2)(3.40\,\text{s})^2 = 16.2\,\text{m}.$$

E4-38 (a) $v = 2\pi r/T = 2\pi(6.37\times10^6\text{m})/(86400 \text{ s}) = 463 \text{ m/s}$. $a = v^2/r = (463 \text{ m/s})^2/(6.37\times10^6\text{m}) = 0.034 \text{ m/s}^2$.

(b) The net force on the object is $F = ma = (25.0 \text{ kg})(0.034 \text{ m/s}^2) = 0.85 \text{ N}$. There are two forces on the object: a force up from the scale (S), and the weight down, $W = mg = (25.0 \text{ kg})(9.80 \text{ m/s}^2) = 245 \text{ N}$. Then $S = W - F = 245 \text{ N} - 0.85 \text{ N} = 244 \text{ N}$.

E4-42 The horizontal component of the rain drop's velocity is 28 m/s. Since $v_x = v \sin\theta$, $v = (28 \text{ m/s})/\sin(64°) = 31 \text{ m/s}$.