Magnetic Properties of Materials

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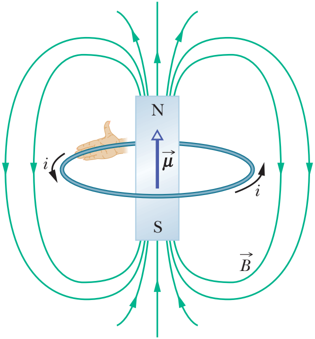
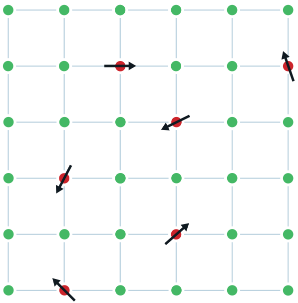
Lecture 10

Bar Magnet vs Current-Carrying Coil

- A magnet has two poles, and is known as a magnetic dipole. It produces a magnetic field.
- A current-carrying coil also has a magnetic dipole moment. When placed in a magnetic field, the coil will tend to rotate, just like a bar magnet or a magnetic dipole placed in the field.



Spin Glasses (Nobel Prize, 2021)



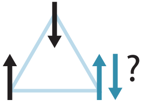
Spin glass

A spin glass is a metal alloy where iron atoms, for example, are randomly mixed into a grid of copper atoms. Each iron atom behaves like a small magnet, or spin, which is affected by the other magnets around it. However, in a spin glass they are frustrated and have difficulty choosing which direction to point. Using his studies of spin glass, Parisi developed a theory of disordered and random phenomena that covers many other complex systems.

- Iron
- Copper

. . . because all these fields include problems that are directly related to frustration.

[Discoveries of] spin glasses were so deep that they not only influenced physics, but also mathematics, biology, neuroscience and machine learning, . . .



Frustration

When one spin points upward and the other downward, the third one cannot satisfy them both at the same time, because neighbouring spins want to point in different directions. How do the spins find an optimal orientation? Giorgio Parisi is a master at answering these questions for many different materials and phenomena.

Outline

- The Magnetic Dipole Moment
- Magnetic Field of a Current-Carrying Coil
- Magnetic Materials

The Magnetic Dipole

- A magnetic dipole can have different origins. It can be
  - a current-carrying coil,
  - a permanent magnet,
  - a rotating sphere of charge, such as Earth, or
  - a subatomic particle.
- A complete description of magnetism needs quantum mechanics. But for current purposes, we can model magnetic dipoles by current loops, and explain some magnetic phenomena.

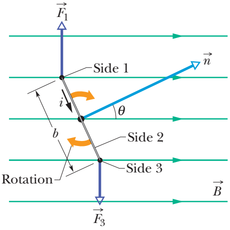
- First, we can assign a **magnetic dipole moment** $\vec{\mu}$ for a magnetic dipole. For example, the dipole moment for a current-carrying coil is

$$\vec{\mu} = Ni\vec{A}.$$

where N is the number of turns, i is the current in each turn, and $\vec{A}$ is the area vector for the coil.

- An external $\vec{B}$ will rotate the coil with total torque $\vec{\tau} = \vec{\mu} \times \vec{B}$. Introducing θ measured from $\vec{B}$, we have

$$\tau = -\mu B \sin\theta = -\frac{\partial}{\partial\theta}(-\mu B \cos\theta).$$

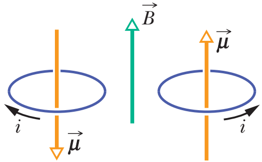


- Therefore, we can define the energy of a dipole, which depends on its orientation in the field, as

$$U_B = -\vec{\mu} \cdot \vec{B} = -\mu B \cos\theta.$$

It tends to line up with the magnetic field.

The magnetic moment vector attempts to align with the magnetic field.



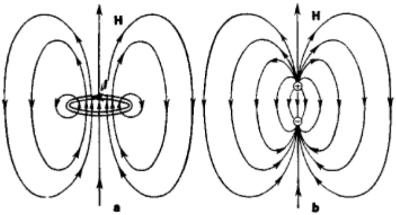
- We assume that the current is not changing, so *the magnitude of the dipole moment is fixed*. This, however, is tricky, because we are not discussing the energy required to keep the current flowing.

Comparison to an Electric Dipole

$$\vec{\mu} = Ni\vec{A}, \quad \vec{p} = q\vec{d}$$

$$\tau_B = \vec{\mu} \times \vec{B}, \quad \tau_E = \vec{p} \times \vec{E}$$

$$U_B = -\vec{\mu} \cdot \vec{B}, \quad U_E = -\vec{p} \cdot \vec{E}$$



- A significant difference between the electric and magnetic field lines is that electric field lines start on positive charges and end on negative charges, whereas magnetic field lines always form close loops.
- Next, we consider the magnetic field generated by a magnetic dipole.

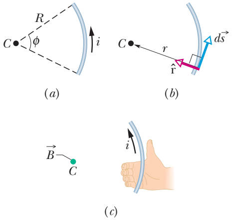
Magnetic Field of a Circular Arc of Wire

- From the Biot-Savart law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\vec{r}}{r^3} = \frac{\mu_0}{4\pi}\frac{id\vec{s}\times\hat{r}}{R^2} = \frac{\mu_0 i\hat{B}}{4\pi R}d\phi.$$

- Integrating $d\phi$, we have

$$B = \frac{\mu_0 i\phi}{4\pi R}.$$



The right-hand rule reveals the field's direction at the center.

Figure 1: A wire in the shape of a circular arc with center C carries current i. Note that $ds = Rd\phi$.

- Note that this equation gives us the magnetic field only at the center of curvature of a circular arc of current. At the center of a single-loop coil, or a full circle of current,

$$B = \frac{\mu_0 i}{2R}.$$

- The magnetic dipole moment of the coil is $\mu = iA = i\pi R^2$. So we have

$$B = \frac{\mu_0}{2\pi}\frac{\mu}{R^3}$$

at the center of the coil.

The Field of a Magnetic Dipole

- It is not difficult to guess that both electric and magnetic dipole fields decay as r^{-3} when we are far from the dipole.
- We now demonstrate this by calculating the field along the whole axis of the dipole.

- We have already learned that at the center of a single-loop coil with a magnetic dipole moment $\mu = iA = i\pi R^2$,

$$B = \frac{\mu_0}{4\pi}\frac{i(2\pi R)}{R^2} = \frac{\mu_0}{2\pi}\frac{\mu}{R^3}.$$

- Meanwhile, recall that the electric field at an arbitrary point along the axis of an electric dipole with moment p is

$$\vec{E} = \frac{1}{2\pi\epsilon_0}\frac{\vec{p}}{z^3},$$

for large z.

- An intelligent guess for the magnetic field at axial points far from the loop ($z \gg R$) is, then,

$$\vec{B} = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}.$$

- Therefore, an intelligent guess for the magnetic field at an axial point is

$$B(z) = \frac{\mu_0}{2\pi} \frac{\mu}{(R^2 + z^2)^{3/2}},$$

  which smoothly interpolates the results at $z = 0$ and at large z. We will prove it in the following.
- Note that the problem does not have enough symmetry to make Ampere's law useful; so we must turn to the law of Biot and Savart.

- Consider a point P on the central axis of the loop, a distance z from its plane.
- From the symmetry, the vector sum of all the perpendicular components $d\vec{B}$ due to all the loop elements $d\vec{s}$ is zero. This leaves only the axial (parallel) components

$$dB_{\parallel} = \frac{\mu_0}{4\pi} \frac{ids}{r^2} \cos\alpha.$$

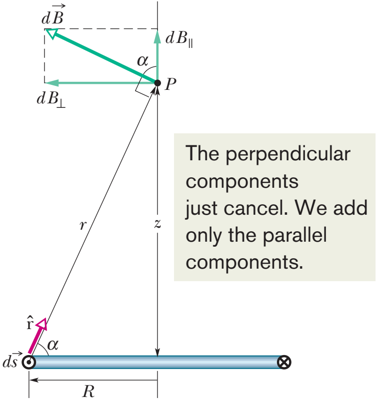


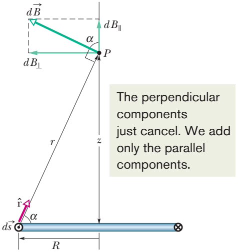
Figure 2: The back half of a circular loop of radius R carrying a current i.

- With $r^2 = R^2 + z^2$, we have

$$\cos\alpha = \frac{R}{r} = \frac{R}{\sqrt{R^2 + z^2}}.$$

Therefore,

$$B = \int dB_{\parallel} = \frac{\mu_0 i R}{4\pi (R^2 + z^2)^{3/2}} \int ds.$$



- Because $\int ds$ is simply the circumference $2\pi R$ of the loop, we reach the desired relation, with $\mu = i\pi R^2$,

$$B(z) = \frac{\mu_0}{2\pi} \frac{\mu}{(R^2 + z^2)^{3/2}}.$$

Magnetic Materials

- A magnetic material can be regarded as a collection of magnetic dipole moments (of atomic origin), each with a north and a south pole. They respond to an external magnetic field, they generate magnetic field, and thus they interact with each other.
- Depending on the magnetic dipole moments of the atoms and on the interactions among the atoms, magnetic properties of the materials can be classified into paramagnetism, diamagnetism, ferromagnetism, among others.

Paramagnetism

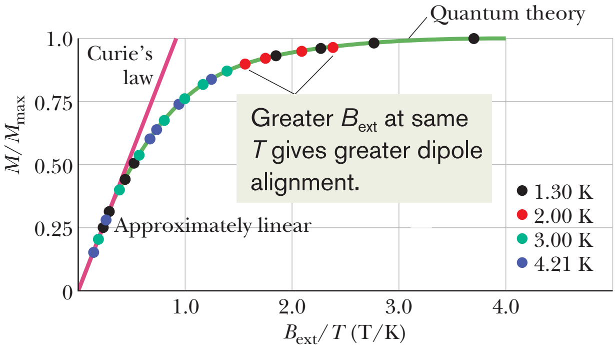
- Paramagnetism occurs in materials whose atoms have permanent magnetic dipole moments $\vec{\mu}$.
- In the absence of an external magnetic field, these atomic dipole moments are randomly oriented, and the net magnetic dipole moment of the material is zero.
- In an external magnetic field $\vec{B}_{\text{ext}}$, the magnetic dipole moments tend to line up with the field, which gives the sample a net magnetic dipole moment.

- We can define a vector quantity **magnetization** $\vec{M}$ as the net magnetic dipole moment per unit volume.
- In 1895 Pierre Curie discovered experimentally that

$$M = C\frac{B_{\rm ext}}{T},$$

  where T is the temperature in kelvins. This is known as **Curie's law**, and C is called the **Curie constant**.
- Increasing $B_{\rm ext}$ tends to align the atomic dipole moments in a sample and thus to increase M.
- Increasing T tends to disrupt the alignment via thermal agitation and thus to decrease M.

- The law is actually an approximation that is valid only when the ratio B_{ext}/T is not too large.

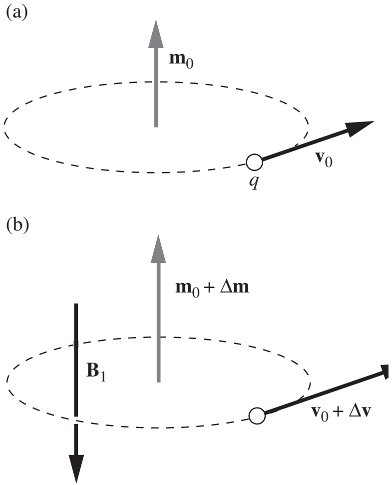


- In a sufficiently strong $\vec{B}_{\mathrm{ext}}$, all dipoles in a sample of N atoms and a volume V line up with $\vec{B}$, hence $\vec{M}$ saturates at $M_{\max} = N\mu/V$.

- A paramagnetic solid containing N atoms per unit volume, each atom having a magnetic dipole moment $\vec{\mu}$, with energy U being $-\vec{\mu} \cdot \vec{B}$.
- Suppose the direction of $\vec{\mu}$ can be only parallel or antiparallel to an externally applied magnetic field $\vec{B}$ (this will be the case if $\vec{\mu}$ is due to the spin of a single electron).
- The fraction of atoms whose dipole moment is parallel to $\vec{B}$ is proportional to $e^{-U/k_B T} = e^{\mu B/k_B T}$ and the fraction of atoms whose dipole moment is antiparallel to $\vec{B}$ is proportional to $e^{-\mu B/k_B T}$.
- The magnetization is therefore $e^{\mu B/k_B T} - e^{-\mu B/k_B T} \propto B/T$ for small B/T.

Diamagnetism

- Paramagnetic substances are always *attracted* by a magnet, while diamagnetic substances are *repelled* by a strong magnet.
- Diamagnetism occurs in all materials, but the weak effect is only observable in materials having atomic dipole moments of zero.
- Such a material can be modeled by equal numbers of electrons orbiting counterclockwise or clockwise. An external magnetic field will either accelerate or decelerate these electrons, leading to a net magnetic dipole moment.

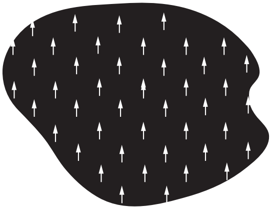


- The change in the magnetic moment vector is opposite to the direction of $\vec{B}_1$, for both directions of motion.

Ferromagnetism

- A ferromagnet has strong, permanent magnetism. What distinguishes ferromagnets from paramagnets is that there is a strong interaction between neighboring atoms.

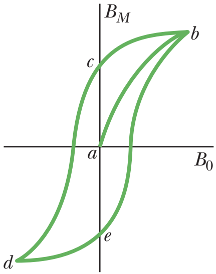
$$U = -J \sum_{\langle ij \rangle} \vec{\mu}_i \cdot \vec{\mu}_j - \sum_i \vec{\mu}_i \cdot \vec{B}.$$



- The interaction keeps the dipole moments of atoms aligned even when the magnetic field is removed.
- Iron, cobalt, nickel, gadolinium, dysprosium, and alloys containing these elements exhibit ferromagnetism.

Hysteresis Curve

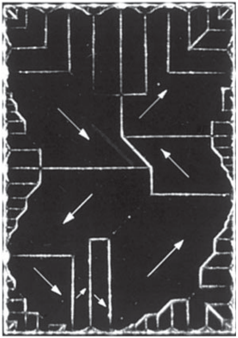
- Magnetization curves for ferromagnetic materials are not retraced; the lack of retraceability is called **hysteresis**, and the curve *bcdeb* is called a **hysteresis loop**.



- Note that at points c and e the iron core is magnetized, even though there is no external magnetic field; this is the familiar phenomenon of permanent magnetism.
- This memory of magnetic materials is essential for the magnetic storage of information.

Ferromagnetic Domains

- Why isn't every piece of iron a naturally strong magnet?
    - A ferromagnetic specimen, in its normal state, is made up of many **magnetic domains**.
    - The domains are so oriented that their external magnetic effects are largely cancelled.



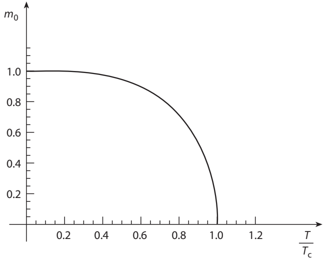
- An external magnetic field leads to a growth in size of the domains, and a shift of the orientation of the dipoles within a domain.

Paramagnet to Ferromagnet Phase Transition

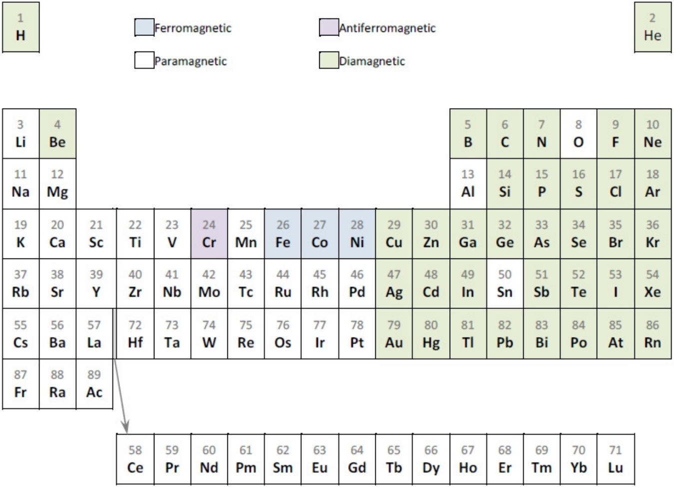
- Admittedly, there is always some neighboring interaction in magnetic materials, whose energy can be written as

$$U = -J \sum_{\langle ij \rangle} \vec{\mu}_i \cdot \vec{\mu}_j - \sum_i \vec{\mu}_i \cdot \vec{B}.$$

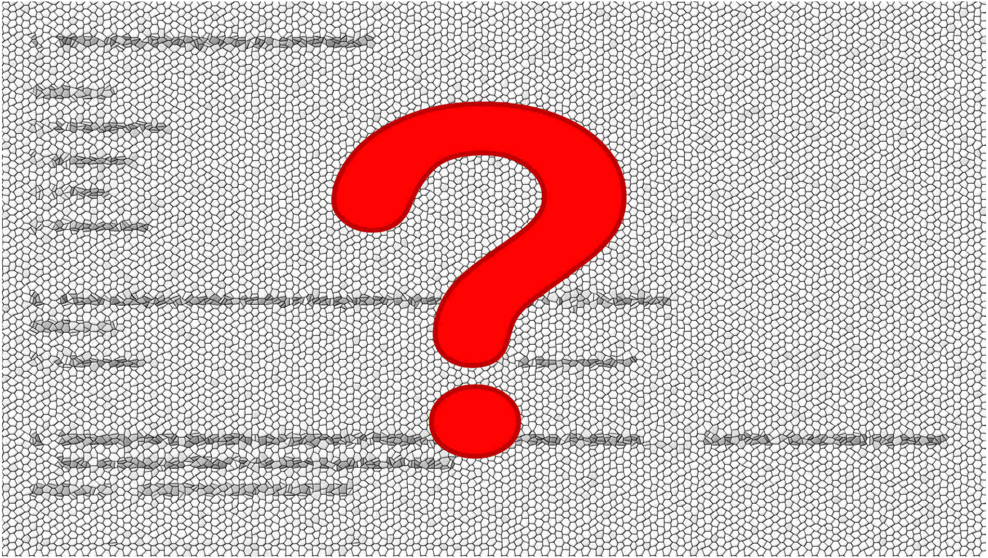
- Whether a material with permanent magnetic dipole moments is paramagnetic or ferromagnetic depends on temperature T or, more precisely, the ratio T/J.



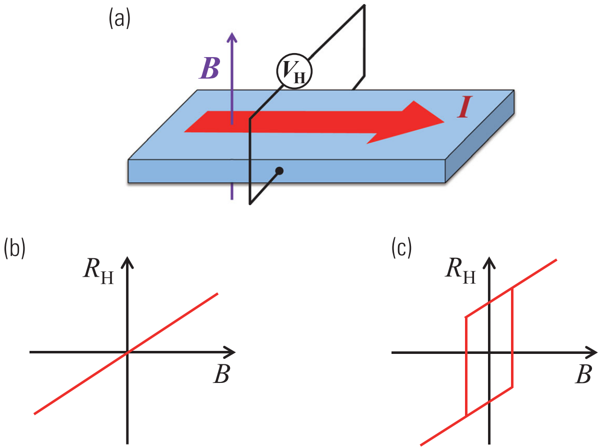
Classification of Magnetic Materials



Quiz 10-1

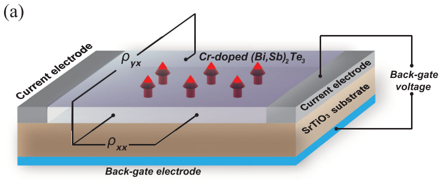


Anomalous Hall Effect (1880)

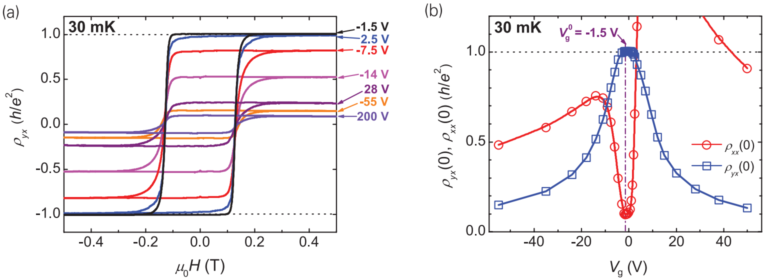


Ordinary Hall Effect **Anomalous Hall Effect**

Quantum Anomalous Hall Effect



C.-Z. Chang et al., Science 340, 167 (2013).



Summary

- Magnetic dipole vs. electric dipole

$$\vec{\mu} = Ni\vec{A}, \quad \vec{p} = q\vec{d}$$

$$\tau_B = \vec{\mu} \times \vec{B}, \quad \tau_E = \vec{p} \times \vec{E}$$

$$U_B = -\vec{\mu} \cdot \vec{B}, \quad U_E = -\vec{p} \cdot \vec{E}$$

$$\vec{B} = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}, \quad \vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\vec{p}}{z^3}$$

- Classification of magnetism: paramagnetism, diamagnetism, ferromagnetism, ...

Reading

Halliday, Resnick & Krane:

- Chapter 35: Magnetic Properties of Materials