Discrete Mathematics Quiz 1 2020-5-12

Name ________________________ Student Number ________________________

Note: For each question, if you answer in Chinese, use the following scoring method.

If the original score <= 1, then the final score = 0;

else if the original score > 1, then the final score = the original score -1.

注意：对每一道大题，如果用中文作答，则原得分不超过1分扣到0分，原得分大于1分扣掉1分。

1、 (9%) Determine whether the following statements are true or false :

a) 1 + 3 = 5 if and only if 1 - 3 = 5. Answer: True

b) if A, B, and C are sets, then $A-(B \cap C) = (A-B) \cup (A-C)$. Answer: True

c) For all real numbers x and y, $\lfloor x-y \rfloor = \lfloor x \rfloor - \lfloor y \rfloor$ Answer: False

2、(9%) Suppose P(x, y) is a predicate and the universe for the variables x and y is {1, 2, 3}. Suppose P(1, 3), P(2, 1), P(2, 2), P(2, 3), P(3, 1), P(3, 2) are true, and P(x, y) is false otherwise. Determine whether the following statements are true or false :

a) $\forall x \exists y P(x,y)$ Answer: True

b) $\exists x \forall y P(x,y)$ Answer: True

c) $\forall y \exists x (x \le y \wedge P(x,y))$ Answer: False

3、(10%) (a) Write a proposition equivalent to $(p \vee \neg q)$ that uses only $p, q, \neg$, and the connective $\wedge$.

Answer: $\neg(\neg p \wedge q)$.

(b)Write a proposition equivalent to $p \vee q$ using only p, q, and the connective | (NAND).

Answer：(p | p) | (q | q)

4、(12%)There is a proposition formula with three variables p, q, and r that is true when at most one of the three variables is true, and false otherwise.

(a) Express the proposition formula in full disjunctive normal form.

Answer：$(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$

(b) Express the proposition formula in full conjunctive normal form.

Answer：$(\neg p \vee \neg q \vee r) \wedge (p \vee \neg q \vee \neg r) \wedge (\neg p \vee q \vee \neg r) \wedge (\neg p \vee \neg q \vee \neg r)$

注1：先注明p/q/r顺序，然后用m0,m1这种描述也是可以的

注2：全非那个漏了，给2分。

5、(10%) Prove that the distributive law $A_1 \cap (A_2 \cup \cdots \cup A_n) = (A_1 \cap A_2) \cup \cdots \cup (A_1 \cap A_n)$ is true for all $n > 2$.

Answer：The second form of mathematical induction is used.

(3%) $P(3)$ is true since it is the ordinary distributive law for intersection over union.

(7%) $P(3) \wedge \cdots \wedge P(n) \rightarrow P(n+1)$:

$$\begin{aligned} A_1 \cap (A_2 \cup \cdots \cup A_{n+1}) &= A_1 \cap \left((A_2 \cup \cdots \cup A_n) \cup A_{n+1}\right) = [A_1 \cap (A_2 \cup \cdots \cup A_n)] \cup (A_1 \cap A_{n+1}) \\ &= [(A_1 \cap A_2) \cup \cdots \cup (A_1 \cap A_n)] \cup (A_1 \cap A_{n+1}) \\ &= (A_1 \cap A_2) \cup \cdots \cup (A_1 \cap A_n) \cup (A_1 \cap A_{n+1}) \end{aligned}$$

注：不用归纳法证明，如果正确，也能得全分

6、(10%) Prove that between every two unequal rational numbers a/b and c/d，there are an infinite number of rational numbers.

Answer：If n=1, let a_1 = (a/b+c/d)/2. We get a rational number a_1.

If n=2, let a_2 = (a_1+c/d)/2. Thus we get another rational number a_2.

Supposing n=k, we can obtain the k^{th} rational number a_k = (a_{k-1}+c/d)/2.

Then n=k+1, Let a_{k+1} = (a_k + c/d)/2. We get the $(k+1)^{th}$ rational number.

Therefore, by the method of mathematical induction, we can get an infinite number of rational numbers between a/b and c/d:

$a/b<a_1<a_2<\cdots<a_k<a_{k+1}\cdots<c/d$ (If $a/b<c/d$)

Note: There many ways to construct this sequence, this is just one of them.

7、(10%) Suppose g: A→B and f: B → C where A = {1, 2, 3, 4}, B = {a, b, c}, C = {1, 5, 10}, and f and g are defined by g = {(1, b), (2, a), (3, a), (4, b)} and f = {(a, 10), (b, 5), (c, 1)}. Find f ○ g.

Answer: {(1,5), （2，10），（3，10），（4，5）}

全对得10分，漏一个得5分，其他情况得0分。

8、(10%) Build all the functions from A = {1,2} to B ={a,b} and point out which is bijection, and which is surjection .

Answer: (4%) **(a)** f(1)=a, f(2)=a; **(b)** f(1)=a, f(2)=b; **(c)** f(1)=b, f(2)=a; **(d)**f(1)=b, f(2)=b

(3%) Bijection: (b)(c)

(3%) Surjection: (b)(c)

9、(10%) Arrange the following functions in a list so each is big-O of the next one in the list:

$\log n^2$, $n^3 + 88n^2 + 3$, $\log\log n$, $n\log n$, $\log(n^2 + 1)$, $\log 2^n$, $n^2\log n$, 9999

Answer: 9999, $\log\log n$, $\log n^2$, $\log(n^2 + 1)$, $\log 2^n$, $n\log n$, $n^2\log n$, , $n^3 + 88n^2 + 3$

全对得10分，错或漏一个顺序得5分，其他情况得0分。

注1：$\log n^2$, $\log(n^2 + 1)$可以互换

注2：顺序全反，这次放水，给8分吧。如果期末考这种题目，不可能这样给分了，一定要吸取教训记住。

10、(10%) Suppose that the only paper money consists of 3-dollar bills and 10-dollar bills. Show that any dollar amount greater than 17 dollars could be made from a combination of these bills.

Answer: （3%）1. P(18): Eighteen dollars can be made using six 3-dollar bills.

（7%）2. P(k) → P(k+1). Suppose that k dollars can be formed for some k≥18. If at least two 10-dollars bills are used, replace them by seven 3-dollar bills to form k+1 dollars. Otherwise (that is, at most one 10-dollar bill is used), at least three 3-dollar bills are being used, and three of them can be replaced by one 10-dollar bill to form k+1 dollars.

注：不用归纳法证明，如果正确，也能得全分