Solutions of Homework at Week 5

April 13th, 2021

Chapter 8

E8-5 (a) Integrate.

$$\omega_z = \omega_0 + \int_0^t \left(4at^3 - 3bt^2\right) dt = \omega_0 + at^4 - bt^3$$

(b) Integrate, again.

$$\Delta\theta = \int_0^t \omega_z dt = \int_0^t \left(\omega_0 + at^4 - bt^3\right) dt = \omega_0 t + \frac{1}{5}at^5 - \frac{1}{4}bt^4$$

E8-19 (a) We are given $\phi = 42.3$ rev= 266 rad, $\omega_{0z} = 1.44$ rad/s, and $\omega_z = 0$. Assuming a uniform deceleration, the average angular velocity during the interval is

$$\omega_{\mathrm{av},z} = \frac{1}{2}\left(\omega_{0z} + \omega_z\right) = 0.72 \text{ rad/s}.$$

Then the time taken for deceleration is given by $\phi = \omega_{\mathrm{av},z} t$, so $t = 369$ s.

(b) The angular acceleration can be found from Eq. 8-6,

$$\begin{aligned}
\omega_z &= \omega_{0z} + \alpha_z t, \\
&(0) = (1.44\,\mathrm{rad/s}) + \alpha_z(369\,\mathrm{s}), \alpha_z \\
&= -3.90 \times 10^{-3}\,\mathrm{rad/s}^2.
\end{aligned}$$

(c) We'll solve Eq. 8-7 for t,

$$\begin{aligned}
\phi &= \phi_0 + \omega_{0z} t + \frac{1}{2}\alpha_z t^2, \\
(133 \text{ rad}) &= (0) + (1.44 \text{ rad/s})t + \frac{1}{2}(-3.9 \times 10^{-3} \text{ rad/s}^2)t^2, \\
0 &= -133 + (1.44\,\mathrm{s}^{-1})t - (-1.95 \times 10^{-3}\mathrm{s}^{-2})t^2.
\end{aligned}$$

Solving this quadratic expression yields two answers: $t = 108$ s and $t = 630$ s.

E8-33 (a) The object is "slowing down", so $\vec{\alpha} = (-2.66 \text{ rad/s}^2)\hat{\mathbf{k}}$. We know the direction because it is rotating about the z axis and we are given the direction of $\vec{\omega}$. Then from Eq. 8-19, $\vec{\mathbf{v}} = \vec{\omega} \times \vec{\mathbf{R}} = (14.3 \text{ rad/s})\hat{\mathbf{k}} \times [(1.83 \text{ m})\hat{\mathbf{j}} + (1.26 \text{ m})\hat{\mathbf{k}}]$. But only the cross term $\hat{\mathbf{k}} \times \hat{\mathbf{j}}$ survives, so $\vec{\mathbf{v}} = (-26.2\,\mathrm{m/s})\hat{\mathbf{i}}$.

(b) We find the acceleration from Eq. 8-21,

$$\begin{aligned}
\vec{\mathbf{a}} &= \vec{\alpha} \times \vec{\mathbf{R}} + \vec{\omega} \times \vec{\mathbf{v}}, \\
&= (-2.66 \text{ rad/s}^2)\hat{\mathbf{k}} \times [(1.83\,\mathrm{m})\hat{\mathbf{j}} + (1.26\,\mathrm{m})\hat{\mathbf{k}}] + (14.3 \text{ rad/s})\hat{\mathbf{k}} \times (-26.2\,\mathrm{m/s})\hat{\mathbf{i}}, \\
&= (4.87\,\mathrm{m/s}^2)\hat{\mathbf{i}} + (-375\,\mathrm{m/s}^2)\hat{\mathbf{j}}.
\end{aligned}$$

$\boxed{\textbf{P8-9}}$ (a) There are 500 teeth (and 500 spaces between these teeth); so disk rotates $2\pi/500$ rad between the outgoing light pulse and the incoming light pulse. The light traveled 1000 m, so the elapsed time is $t = (1000\,\text{m})/(3\times10^8\,\text{m/s}) = 3.33\times10^{-6}$s.

Then the angular speed of the disk is $\omega_z = \phi/t = 1.26\times10^{-2}$ rad)/(3.33×10^{-6}s) = 3800 rad/s.

(b) The linear speed of a point on the edge of the would be

$$v_T = \omega R = (3800\,\text{rad/s})(0.05\,\text{m}) = 190\,\text{m/s}.$$

P8-11 (a) The final angular speed is ω_o = (130 cm/s)/(5.80 cm) = 22.4 rad/s.

(b) The recording area is $\pi({R_\text{o}}^2 - {R_\text{i}}^2)$, the recorded track has a length l and width w, so

$$l = \frac{\pi[(5.80\text{ cm})^2 - (2.50\text{ cm}^2)]}{(1.60\times10^{-4}\text{ cm})} = 5.38\times10^5\text{ cm}.$$

(c) Playing time is $t = (5.38\times10^5\text{ cm})/(130\text{ cm/s}) = 4140\,\text{s}$, or 69 minutes.

Chapter 9

$\boxed{\textbf{E9-17}}$ The diagonal distance from the axis through the center of mass and the axis through the edge is $h = \sqrt{(a/2)^2 + (b/2)^2}$, so

$$I = I_{cm} + Mh^2 = \frac{1}{12}M\left(a^2+b^2\right) + M\left((a/2)^2 + (b/2)^2\right) = \left(\frac{1}{12} + \frac{1}{4}\right)M\left(a^2+b^2\right).$$

Simplifying, $I = \frac{1}{3}M\left(a^2+b^2\right)$.

$\boxed{\textbf{P9-13}}$ (a) From Eq. 9-15, $I = \int r^2 dm$ about some axis of rotation when r is measured from that axis. If we consider the x axis as our axis of rotation, then $r = \sqrt{y^2+z^2}$, since the distance to the x axis depends only on the y and z coordinates. We have similar equations for the y and z axes, so

$$\begin{aligned} I_x &= \int \left(y^2+z^2\right) dm, \\ I_y &= \int \left(x^2+z^2\right) dm, \\ I_z &= \int \left(x^2+y^2\right) dm. \end{aligned}$$

These three equations can be added together to give

$$I_x + I_y + I_z = 2\int \left(x^2+y^2+z^2\right) dm,$$

so if we *now* define r to be measured from the origin (which is not the definition used above), then we end up with the answer in the text.

(b) The right hand side of the equation is integrated over the entire body, regardless of how the axes are defined. So the integral should be the same, no matter how the coordinate system is rotated.

P9-14 (a) Since the shell is spherically symmetric $I_x = I_y = I_z$, so $I_x = (2/3) \quad r^2 dm = (2R^2/3) \int dm = 2MR^2/3$.

(b) Since the solid ball is spherically symmetric $I_x = I_y = I_z$, so

$$I_x = \frac{2}{3} \int r^2 \frac{3Mr^2\,dr}{R^3} = \frac{2}{5} MR^2.$$

E9-38 (a) $\alpha = 2\theta/t^2$.

(b) $a = \alpha R = 2\theta R/t^2$.

(c) T_1 and T_2 are *not* equal. Instead, $(T_1 - T_2)R = I\alpha$. For the hanging block $Mg - T_1 = Ma$. Then

$$T_1 = Mg - 2MR\theta/t^2,$$

and

$$T_2 = Mg - 2MR\theta/t^2 - 2(I/R)\theta/t^2.$$

$\boxed{\textbf{E9-39}}$ Apply a kinematic equation from chapter 2 to find the acceleration:

$$\begin{aligned} y &= v_{0y}t + \frac{1}{2}a_y t^2, \\ a_y &= \frac{2y}{t^2} = \frac{2(0.765\text{ m})}{(5.11\text{ s})^2} = 0.0586\text{ m/s}^2 \end{aligned}$$

Closely follow the approach in Sample Problem 9-10. For the heavier block, $m_1 = 0.512$ kg, and Newton's second law gives

$$m_1 g - T_1 = m_1 a_y,$$

where a_y is positive and *down*. For the lighter block, $m_2 = 0.463$ kg, and Newton's second law gives

$$T_2 - m_2 g = m_2 a_y,$$

where a_y is positive and *up*. We do know that $T_1 > T_2$; the net force on the pulley creates a torque which results in the pulley rotating toward the heavier mass. That net force is $T_1 - T_2$; so the rotational form of Newton's second law gives

$$(T_1 - T_2)\,R = I\alpha_z = Ia_T/R,$$

where $R = 0.049$ m is the radius of the pulley and a_T is the tangential acceleration. But this acceleration is equal to a_y, because everything— both blocks and the pulley— are moving together.

We then have *three* equations and *three* unknowns. We'll add the first two together,

$$\begin{aligned} m_1 g - T_1 + T_2 - m_2 g &= m_1 a_y + m_2 a_y, \\ T_1 - T_2 &= (g - a_y)m_1 - (g + a_y)m_2, \end{aligned}$$

and then combine this with the third equation by substituting for $T_1 - T_2$,

$$\begin{aligned} (g - a_y)m_1 - (g + a_y)m_2 &= Ia_y/R^2, \\ \left[\left(\frac{g}{a_y} - 1\right)m_1 - \left(\frac{g}{a_y} + 1\right)m_2\right]R^2 &= I. \end{aligned}$$

Now for the numbers:

$$\begin{aligned} \left(\frac{(9.81\,\text{m/s}^2)}{(0.0586\,\text{m/s}^2)} - 1\right)(0.512\,\text{kg}) - \left(\frac{(9.81\,\text{m/s}^2)}{(0.0586\,\text{m/s}^2)} + 1\right)(0.463\,\text{kg}) &= 7.23\,\text{kg}, \\ (7.23\,\text{kg})(0.049\,\text{m})^2 &= 0.0174\,\text{kg}\cdot\text{m}^2. \end{aligned}$$

E9-42 (a) The angular acceleration is derived in Sample Problem 9-13,

$$\alpha = \frac{g}{R_0}\frac{1}{1+I/(MR_0^2)} = \frac{(981\text{ cm/s}^2)}{(0.320\text{ cm})}\frac{1}{1+(0.950\,\text{kg}\cdot\text{cm}^2)/[(0.120\,\text{kg})(0.320\text{ cm})^2]} = 39.1\text{ rad/s}^2.$$

The acceleration is $a = \alpha R_0 = (39.1\text{ rad/s}^2)(0.320\text{ cm}) = 12.5\text{ cm/s}^2$.

(b) Starting from rest, $t = \sqrt{2x/a} = \sqrt{2(134\text{ cm})/(12.5\text{ cm/s}^2)} = 4.63\,\text{s}$.

(c) $\omega = \alpha t = (39.1\text{ rad/s}^2)(4.63\,\text{s}) = 181\text{ rad/s}$. This is the same as 28.8 rev/s.

(d) The yo-yo accelerates toward the ground according to $y = at^2 + v_0 t$, where *down* is positive. The time required to move to the end of the string is found from

$$t = \frac{-v_0+\sqrt{v_0^2+4ay}}{2a} = \frac{-(1.30\,\text{m/s})+\sqrt{(1.30\,\text{m/s})^2+4(0.125\,\text{m/s}^2)(1.34\,\text{m})}}{2(0.125\,\text{m/s}^2)} = 0.945\,\text{s}$$

The initial rotational speed was $\omega_0 = (1.30\,\text{m/s})/(3.2\times10^{-3}\text{m}) = 406\text{ rad/s}$. Then

$$\omega = \omega_0 + \alpha t = (406\text{ rad/s}) + (39.1\text{ rad/s}^2)(0.945\,\text{s}) = 443\text{ rad/s},$$

which is the same as 70.5 rev/s.

P9-16 (a) Another simple ratio will suffice:

$$\frac{dm}{\pi r^2 dz} = \frac{M}{(4/3)\pi R^3} \text{ or } dm = \frac{3M(R^2-z^2)}{4R^3}dz.$$

(b) $dI = r^2 dm/2 = [3M(R^2-z^2)^2/8R^3]dz$.

(c) There are a few steps to do here:

$$\begin{aligned} I &= \int_{-R}^{R}\frac{3M(R^2-z^2)^2}{8R^3}dz,\\ &= \frac{3M}{4R^3}\int_0^R (R^4-2R^2z^2+z^4)dz,\\ &= \frac{3M}{4R^3}(R^5-2R^5/3+R^5/5) = \frac{2}{5}MR^2. \end{aligned}$$

$\boxed{\textbf{P9-21}}$ This problem is equivalent to Sample Problem 9-11, except that we have a sphere instead of a cylinder. We'll have the same two equations for Newton's second law,

$$Mg\sin\theta - f = Ma_{\text{cm}} \text{ and } N - Mg\cos\theta = 0.$$

Newton's second law for rotation will look like

$$-fR = I_{\text{cm}}\alpha.$$

The conditions for accelerating without slipping are $a_{\text{cm}} = \alpha R$, rearrange the rotational equation to get

$$f = -\frac{I_{\text{cm}}\alpha}{R} = -\frac{I_{\text{cm}}(-a_{\text{cm}})}{R^2},$$

and then

$$Mg\sin\theta - \frac{I_{\text{cm}}(a_{\text{cm}})}{R^2} = Ma_{\text{cm}},$$

$$a_{\rm cm} = g\frac{\sin\theta}{1+\beta}$$

For the sphere, $a_{\rm cm} = 5/7g\sin\theta$.

(a) If $a_{\rm cm} = 0.133g$, then $\sin\theta = 7/5(0.133) = 0.186$, and $\theta = 10.7°$.

(b) A frictionless block has no rotational properties; in this case $\beta = 0$! Then $a_{\rm cm} = g\sin\theta = 0.186g$.

P9-22 (a) There are three forces on the cylinder: gravity W and the tension from each cable T. The downward acceleration of the cylinder is then given by $ma = W - 2T$.

The ropes unwind according to $\alpha = a/R$, but $\alpha = \tau/I$ and $I = mR^2/2$. Then

$$a = \tau R/I = (2TR)R/(mR^2/2) = 4T/m.$$

Combining the above, $4T = W - 2T$, or $T = W/6$.

(b) $a = 4(mg/6)/m = 2g/3$.

P9-25 This problem is equivalent to Sample Problem 9-11, except that we have an unknown rolling object. We'll have the same two equations for Newton's second law,

$$Mg\sin\theta - f = Ma_{\rm cm} \text{ and } N - Mg\cos\theta = 0.$$

Newton's second law for rotation will look like

$$-fR = I_{\rm cm}\alpha.$$

The conditions for accelerating without slipping are $a_{\rm cm} = \alpha R$, rearrange the rotational equation to get

$$f = -\frac{I_{\rm cm}\alpha}{R} = -\frac{I_{\rm cm}(-a_{\rm cm})}{R^2},$$

and then

$$Mg\sin\theta - \frac{I_{\rm cm}(a_{\rm cm})}{R^2} = Ma_{\rm cm},$$

and solve for $a_{\rm cm}$. Write the rotational inertia as $I = \beta MR^2$, where $\beta = 2/5$ for a sphere, $\beta = 1/2$ for a cylinder, and $\beta = 1$ for a hoop. Then, upon some mild rearranging, we get

$$a_{\rm cm} = g\frac{\sin\theta}{1+\beta}$$

Note that a is largest when β is smallest; consequently the sphere wins. Neither M nor R entered into the final equation.

Chapter 10

$\boxed{\textbf{E10-13}}$ An impulse of 12.8 N·s will change the linear momentum by 12.8 N·s; the stick starts from rest, so the final momentum *must* be 12.8 N·s. Since $p = mv$, we then can find $v = p/m = (12.8$ N·s$)/(4.42$ kg$) = 2.90$ m/s.

Impulse is a vector, given by $\int \vec{\mathbf{F}}dt$. We can take the cross product of the impulse with the displacement vector $\vec{\mathbf{r}}$ (measured from the axis of rotation to the point where the force is applied) and get

$$\vec{\mathbf{r}} \times \int \vec{\mathbf{F}}dt \approx \int \vec{\mathbf{r}} \times \vec{\mathbf{F}}dt,$$

The two sides of the above expression are only equal if $\vec{\mathbf{r}}$ has a constant magnitude *and* direction. This won't be true, but if the force is of sufficiently short duration then it hopefully won't change much. The right hand side is an integral over a torque, and will equal the change in angular momentum of the stick.

The exercise states that the force is perpendicular to the stick, then $|\vec{\mathbf{r}} \times \vec{\mathbf{F}}| = rF$, and the "torque impulse" is then $(0.464\text{ m})(12.8\text{ N·s}) = 5.94$ kg·m/s. This "torque impulse" is equal to the change in the angular momentum, but the stick started from rest, so the final angular momentum of the stick is 5.94 kg·m/s.

But how fast is it rotating? We can use Fig. 9-15 to find the rotational inertia about the center of the stick: $I = \frac{1}{12}ML^2 = \frac{1}{12}(4.42\text{ kg})(1.23\text{ m})^2 = 0.557\text{ kg·m}^2$. The angular velocity of the stick is $\omega = l/I = (5.94\text{ kg·m/s})/(0.557\text{ kg·m}^2) = 10.7$ rad/s.

E10-15 From Exercise 8 we can immediately write

$$I_1(\omega_1 - \omega_0)/r_1 = I_2(\omega_2 - 0)/r_2,$$

but we also have $r_1\omega_1 = -r_2\omega_2$. Then

$$\omega_2 = -\frac{r_1 r_2 I_1 \omega_0}{r_1^2 I_2 + r_2^2 I_1}$$

E10-25 Conservation of angular momentum:

$$(m_\text{m}k^2 + m_\text{g}R^2)\omega = m_\text{g}R^2(v/R),$$

so

$$\omega = \frac{(44.3\text{kg})(2.92\text{m/s})(1.22\text{m})^2/(1.22\text{m})}{(176\text{kg})(0.916\text{m})^2 + (44.3\text{kg})(1.22\text{m})^2} = 0.739\text{rad/s}$$

$\boxed{\textbf{P10-3}}$ Assume that the cue stick strikes the ball horizontally with a force of constant magnitude F for a time Δt. Then the magnitude of the change in linear momentum of the ball is given by $F\Delta t = \Delta p = p$, since the initial momentum is zero.

If the force is applied a distance x above the center of the ball, then the magnitude of the torque about a horizontal axis through the center of the ball is $\tau = xF$. The change in angular momentum of the ball is given by $\tau\Delta t = \Delta l = l$, since initially the ball is not rotating.

For the ball to roll without slipping we need $v = \omega R$. We can start with this:

$$\begin{aligned} v &= \omega R, \\ \frac{p}{m} &= \frac{lR}{I}, \\ \frac{F\Delta t}{m} &= \frac{\tau \Delta t R}{I}, \\ \frac{F}{m} &= \frac{xFR}{I}. \end{aligned}$$

Then $x = I/mR$ is the condition for rolling without sliding from the start. For a solid sphere, $I = \frac{2}{5}mR^2$, so $x = \frac{2}{5}R$.

P10-8 (a) $l = I\omega_0 = (1/2)MR^2\omega_0$.

(b) The initial speed is $v_0 = R\omega_0$. The chip decelerates in a time $t = v_0/g$, and during this time the chip travels with an average speed of $v_0/2$ through a distance of

$$y = v_{\text{av}}t = \frac{v_0}{2}\frac{v_0}{g} = \frac{R^2\omega^2}{2g}.$$

(c) Loosing the chip won't change the angular velocity of the wheel.

$\boxed{\textbf{P10-11}}$ The cockroach initially has an angular speed of $\omega_{\text{c,i}} = -v/r$. The rotational inertia of the cockroach about the axis of the turntable is $I_{\text{c}} = mR^2$. Then conservation of angular momentum gives

$$\begin{aligned}
l_{\text{c,i}} + l_{\text{s,i}} &= l_{\text{c,f}} + l_{\text{s,f}},\\
I_{\text{c}}\omega_{\text{c,i}} + I_{\text{s}}\omega_{\text{s,i}} &= I_{\text{c}}\omega_{\text{c,f}} + I_{\text{s}}\omega_{\text{s,f}},\\
-mR^2v/r + I\omega &= (mR^2 + I)\omega_{\text{f}},\\
\omega_{\text{f}} &= \frac{I\omega - mvR}{I + mR^2}.
\end{aligned}$$