Solutions of Homework at Week 7

April 28th, 2021

P8-4 (a) The period is time per complete rotation, so $\omega = 2\pi/T$.

(b) $\alpha = \Delta\omega/\Delta t$, so

$$\begin{aligned}
\alpha &= \left(\frac{2\pi}{T_0+\Delta T} - \frac{2\pi}{T_0}\right)/(\Delta t), \\
&= \frac{2\pi}{\Delta t}\left(\frac{-\Delta T}{T_0(T_0+\Delta T)}\right), \\
&\approx \frac{2\pi}{\Delta t}\frac{-\Delta T}{T_0^2}, \\
&= \frac{2\pi}{(3.16\times 10^7\text{s})}\frac{-(1.26\times 10^{-5}\text{s})}{(0.033\,\text{s})^2} = -2.30\times 10^{-9}\text{rad/s}^2.
\end{aligned}$$

(c) $t = (2\pi/0.033\,\text{s})/(2.30\times 10^{-9}\text{rad/s}^2) = 8.28\times 10^{10}\text{s}$, or 2600 years.
(d) $2\pi/T_0 = 2\pi/T - \alpha t$, or

$$T_0 = \Big(1/(0.033\,\text{s}) - (-2.3\times 10^{-9}\text{rad/s}^2)(3.0\times 10^{10}\,\text{s})/(2\pi)\Big)^{-1} = 0.024\,\text{s}.$$

P9-17 The *rotational* acceleration will be given by $\alpha_z = \sum \tau/I$.

The torque about the pivot comes from the force of gravity on each block. This forces will both originally be at right angles to the pivot arm, so the net torque will be $\sum\tau = mgL_2 - mgL_1$, where clockwise as seen on the page is positive.

The rotational inertia about the pivot is given by $I = \sum m_n r_n^2 = m(L_2^2 + L_1^2)$. So we can now find the rotational acceleration,

$$\alpha = \frac{\sum\tau}{I} = \frac{mgL_2 - mgL_1}{m(L_2^2+L_1^2)} = g\frac{L_2 - L_1}{L_2^2 + L_1^2} = 8.66\ \text{rad/s}^2.$$

The linear acceleration is the tangential acceleration, $a_T = \alpha R$. For the left block, $a_T = 1.73\ \text{m/s}^2$; for the right block $a_T = 6.93\ \text{m/s}^2$.

P10-7 We assume the bowling ball is solid, so the rotational inertia will be $I = (2/5)MR^2$ (see Figure 9-15).

The normal force on the bowling ball will be $N = Mg$, where M is the mass of the bowling ball. The kinetic friction on the bowling ball is $F_f = \mu_k N = \mu_k Mg$. The magnitude of the net torque on the bowling ball while skidding is then $\tau = \mu_k MgR$.

Originally the angular momentum of the ball is zero; the final angular momentum will have magnitude $l = I\omega = Iv/R$, where v is the final translational speed of the ball.

(a) The time requires for the ball to stop skidding is the time required to change the angular momentum to l, so

$$\Delta t = \frac{\Delta l}{\tau} = \frac{(2/5)MR^2 v/R}{\mu_k MgR} = \frac{2v}{5\mu_k g}.$$

Since we don't know v, we can't solve this for Δt. But the same time through which the angular momentum of the ball is increasing the linear momentum of the ball is decreasing, so we also have

$$\Delta t = \frac{\Delta p}{-F_f} = \frac{Mv - Mv_0}{-\mu_k Mg} = \frac{v_0 - v}{\mu_k g}.$$

Combining,

$$\begin{aligned}
\Delta t &= \frac{v_0 - v}{\mu_k g}, \\
&= \frac{v_0 - 5\mu_k g\Delta t/2}{\mu_k g},
\end{aligned}$$

$$\begin{aligned}
2\mu_k g\Delta t &= 2v_0 - 5\mu_k g\Delta t,\\
\Delta t &= \frac{2v_0}{7\mu_k g},\\
&= \frac{2(8.50\,\mathrm{m/s})}{7(0.210)(9.81\,\mathrm{m/s^2})} = 1.18\,\mathrm{s}.
\end{aligned}$$

(d) Use the expression for angular momentum and torque,

$$v = 5\mu_k g\Delta t/2 = 5(0.210)(9.81\,\mathrm{m/s^2})(1.18\,\mathrm{s})/2 = 6.08\,\mathrm{m/s}.$$

(b) The acceleration of the ball is $F/M = -\mu g$. The distance traveled is then given by

$$\begin{aligned}
x &= \frac{1}{2}at^2 + v_0 t,\\
&= -\frac{1}{2}(0.210)(9.81\,\mathrm{m/s^2})(1.18\,\mathrm{s})^2 + (8.50\,\mathrm{m/s})(1.18\,\mathrm{s}) = 8.60\,\mathrm{m},
\end{aligned}$$

(c) The angular acceleration is $\tau/I = 5\mu_k g/(2R)$. Then

$$\begin{aligned}
\theta &= \frac{1}{2}\alpha t^2 + \omega_0 t,\\
&= \frac{5(0.210)(9.81\,\mathrm{m/s^2})}{4(0.11\,\mathrm{m})}(1.18\,\mathrm{s})^2 = 32.6 \text{ rad} = 5.19 \text{ revolutions}.
\end{aligned}$$

P11-20 (a) $W_\mathrm{g} = -(0.263\,\mathrm{kg})(9.81\,\mathrm{m/s^2})(-0.118\,\mathrm{m}) = 0.304\,\mathrm{J}$.

(b) $W_\mathrm{s} = -\frac{1}{2}(252\,\mathrm{N/m})(-0.118\,\mathrm{m})^2 = -1.75\,\mathrm{J}$.

(c) The kinetic energy just before hitting the block would be $1.75\,\mathrm{J} - 0.304\,\mathrm{J} = 1.45\,\mathrm{J}$. The speed is then $v = \sqrt{2(1.45\,\mathrm{J})/(0.263\,\mathrm{kg})} = 3.32\,\mathrm{m/s}$.

(d) Doubling the speed quadruples the initial kinetic energy to 5.78 J. The compression will then be given by

$$-5.78\,\mathrm{J} = -\frac{1}{2}(252\,\mathrm{N/m})y^2 - (0.263\,\mathrm{kg})(9.81\,\mathrm{m/s^2})y,$$

with solution $y = 0.225\,\mathrm{m}$.

P12-17 The function needs to fall off at infinity in both directions; an exponential envelope would work, but it will need to have an $-x^2$ term to force the potential to zero on *both* sides. So we propose something of the form

$$U(x) = P(x)e^{-\beta x^2}$$

where $P(x)$ is a polynomial in x and β is a positive constant.

We proposed the *polynomial* because we need a symmetric function which has two zeroes. A quadratic of the form $\alpha x^2 - U_0$ would work, it has two zeroes, a minimum at $x = 0$, and is symmetric.

So our *trial* function is

$$U(x) = \left(\alpha x^2 - U_0\right)e^{-\beta x^2}.$$

$$\frac{dU}{dx} = 2\alpha x e^{-\beta x^2} - 2\left(\alpha x^2 - U_0\right)\beta x e^{-\beta x^2}.$$

Setting this equal to zero leaves two possibilities,

$$\begin{aligned}
x &= 0,\\
2\alpha - 2\left(\alpha x^2 - U_0\right)\beta &= 0.
\end{aligned}$$

The first equation is trivial, the second is easily rearranged to give

$$x = \pm\sqrt{\frac{\alpha + \beta U_0}{\beta\alpha}}$$

These are the points $\pm x_1$. We can, if we wanted, try to find α and β from the picture, but you might notice we have one equation, $U(x_1) = U_1$ and two unknowns. It really isn't very illuminating to take this problem much farther, but we could.

(b) The force is the derivative of the potential; this expression was found above.

(c) As long as the energy is *less* than the two peaks, then the motion would be oscillatory, trapped in the well.