Final Review

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December 29, 2021

Disclaimer

This review serves as a guideline for your preparation for the final exam. It covers key ideas, formulas, and methods on the main topics of General Physics II, as we have discussed in this semester. It does not, however, include everything in the final exam as a subset. On the other hand, the review also contains much information that is not of practical use in the final exam. I hold no responsibility or liability for the scope of this review being too broad or too narrow.

Topics We Have Covered

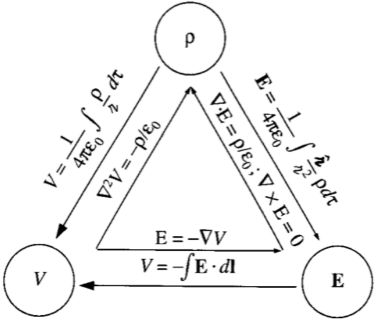
- Electrostatics
- Magnetostatics
- Electrodynamics
- DC and AC Circuits
- Electromagnetic Waves
- Geometrical Optics
- Interference, Diffraction, and Polarization
- The Old Quantum Theory
- Quantum Mechanics
- Electromagnetism in Matter

Electrostatics

- [L01] Understand how charge responds to electric field.

$$\vec{F} = q\vec{E}.$$

- [L01-04] Understand the relation between charge (or charge density), electric field, and electric potential. Learn to obtain the other two once one of them is known.

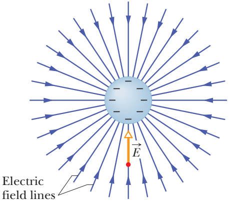


- The curl of electric field (electrostatic field only)

$$\oint \vec{E} \cdot d\vec{s} = 0 \quad \leftrightarrow \quad V_f - V_i = -\int_i^f \vec{E} \cdot d\vec{s}$$

$$\vec{E} = -\nabla V \quad \Rightarrow \quad \nabla \times \vec{E} = 0$$

- The divergence of electric field (Gauss' law)



$$\oint_S \vec{E} \cdot d\vec{A} = \Phi_E = \frac{q_{\text{enc}}}{\epsilon_0} = \frac{1}{\epsilon_0} \int_V \rho dV$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \quad \Rightarrow \quad \nabla^2 V \equiv \nabla \cdot \nabla V = -\frac{\rho}{\epsilon_0}$$

Magnetostatics

- [L8] Understand how charge (current) responds to magnetic field and the Hall effect.

$$\vec{F}_B = q\vec{v} \times \vec{B}, \quad \vec{F}_B = i\vec{L} \times \vec{B}$$

$$R_H = \frac{E}{BJ} = \frac{1}{nq}$$

- [L9] Understand how steady current gives rise to magnetic field. Learn to apply the Biot-Savart law.

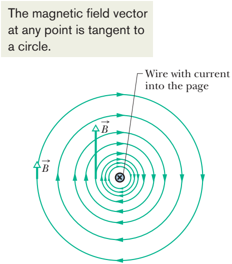
$$d\vec{B} = \frac{\mu_0}{4\pi} \frac{i d\vec{s} \times \hat{r}}{r^2}$$

- [L9] Learn to apply Ampere's law in cases with symmetry.

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}} \quad \text{or} \quad \nabla \times \vec{B} = \mu_0 \vec{J}$$

- [L10] Understand Gauss' law for magnetism and the absence of magnetic monopole.

$$\oint \vec{B} \cdot d\vec{A} = 0 \quad \text{or} \quad \nabla \cdot \vec{B} = 0$$



Electrodynamics

- [L11] Understand Faraday's law in the context of unification of electricity and magnetism.

$$\oint \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int \vec{B} \cdot d\vec{A} \quad \text{or} \quad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- [L14] Understand why and how Maxwell modified Ampere's law

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \vec{J}_d = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t},$$

where $\vec{J}_d = \epsilon_0 \partial \vec{E} / \partial t$ is the displacement current.

- [L14] Write down Maxwell's equations (integral and differential forms) and explain what the meaning is of each equation.

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0} \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0 \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- Understand, qualitatively, how Maxwell's equation leads to the electromagnetic wave equation.

Electromagnetic Waves

- [L14] Write down the wave equation for each component of the electric and magnetic field of an electromagnetic wave in vacuum

$$\nabla^2 \vec{E} = \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2}, \qquad \nabla^2 \vec{B} = \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2}.$$

- Electromagnetic waves transport energy (and momentum).

$$\vec{S} = \left(\frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0} \right) c\hat{k}.$$

- In addition to the wave equation, electromagnetic waves satisfy the following constraints.
  - Electromagnetic waves are transverse:

$$\hat{k} \cdot \vec{E} = \hat{k} \cdot \vec{B} = 0.$$

  - $\vec{E}$ is always perpendicular to $\vec{B}$:

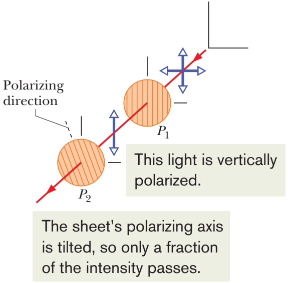
$$\vec{B} = \frac{1}{c}(\hat{k} \times \vec{E}).$$

- [L15] Write down the general solution to the wave equations.

Polarization

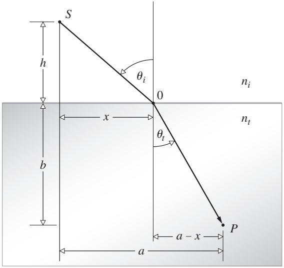
- [L20] Understand polarization as the superposition of coherent waves. Distinguish unpolarized (or randomly polarized), partially polarized, and polarized light. Distinguish linearly polarized and circularly polarized light.
- [L20] Understand the function of polarizing sheets. Learn to calculate the relative intensity of light transmitted by a polarizing sheet.

$$I_1 = I_0/2, \quad I_2 = I_1 \cos^2\theta$$



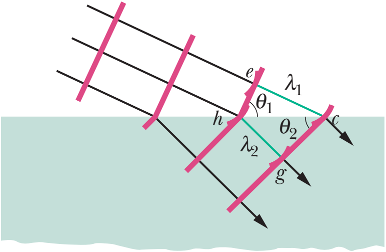
Fermat's Principle of Least Time

- The actual path between two points taken by a beam of light is the one that is traversed in the least time.



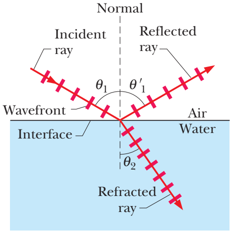
Huygens' Principle

- All points on a wavefront serve as point sources of spherical secondary wavelets. After a time t, the new position of the wavefront will be that of a surface tangent to these secondary wavelets.



Geometrical Optics

- [L15] Learn to calculate the propagation of light rays upon an interface.
  - Law of reflection: $\theta_1 = \theta_1'$.
  - Law of refraction: $n_1 \sin\theta_1 = n_2 \sin\theta_2$.



- [L16] Write down the focal length for plane mirrors, spherical mirrors, and for thin lenses.
  - For plane mirrors, $f = \infty$.
  - For spherical mirrors, $f = r/2$.
  - For thin lenses,

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

- [L16] Understand how to relate the object distance, the image distance, and the focal length:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f},$$

  if the rays are sufficiently close to the central axis through the plane mirrors, spherical mirrors, or thin lenses.
- Calculate the lateral magnification

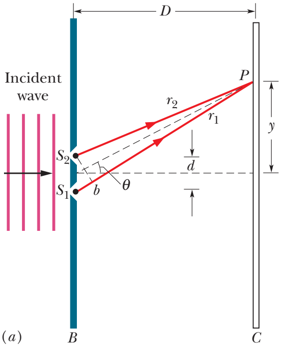
$$m = -i/p.$$

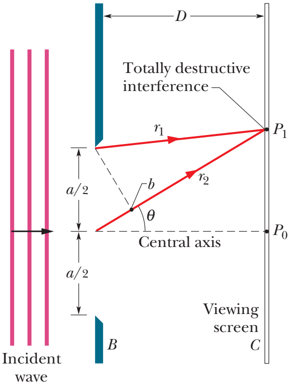
- Understand (not memorize) the sign of various quantities. Distinguish real and virtual images.

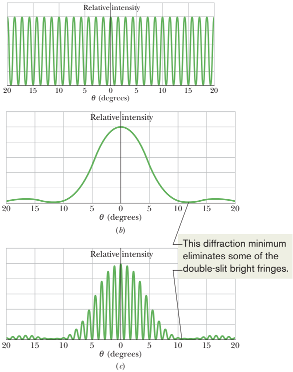
Interference, Diffraction, and Grating

- [L17] Understand the general superposition principle of waves. Learn to use the phasor addition or complex method to study superposition.
- Compare the analyses of [L17] the double-slit interference, [L18] the single-slit diffraction, [L19] the double-slit diffraction, and [L19] the diffraction grating. Understand the interference pattern in each case and how to determined the locations of bright and dark fringes.
- Understand how we can resolve two adjacent features: Rayleigh's criterion.

- Understand how to derive the interference and diffraction patterns based on pictures like these.





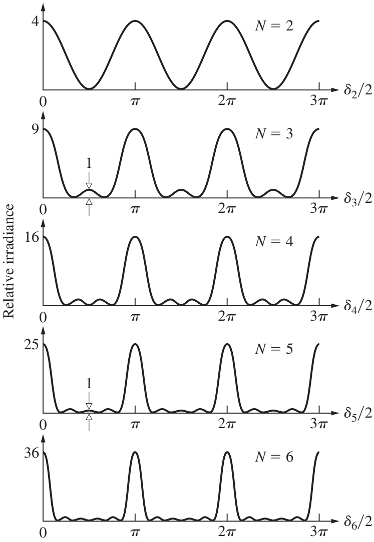


$$I(\theta) = I_{\max} \cos^2 \beta \left(\frac{\sin \alpha}{\alpha} \right)^2$$

- In the case of grating, we have $N \gg 1$ slits, but the detail of each narrow slit can be neglected.
- The phase difference between adjacent slits is

$$\delta_N = \frac{2\pi}{\lambda} d \sin\theta.$$

- As N increases, the normalized intensity (irradiance) evolves with sharper and sharper peaks.



The Old Quantum Theory

- The old quantum theory explains how the classical laws were modified to account for phenomena in atoms and solids by Planck, Einstein, Bohr and other contributors. We emphasized the following three stories:
  - [L21] Einstein's idea of light quanta and its application in the photoelectric effect.
  - [L22] The hypothesis of matter wave by de Broglie.
  - [L25] Bohr's theory of the hydrogen atom and the consequent understanding of the hydrogen spectrum.

- Einstein's photon

$$E = hf = \hbar\omega, \quad p = hf/c = h/\lambda = \hbar k$$

- De Broglie's matter wave: $\lambda = h/p$
- Bohr's hydrogen atom

$$L = n\hbar, \ \text{for } n = 1, 2, 3, \ldots$$

$$E_n = -\frac{E_R}{n^2}, \quad E_R = \frac{me^4/(4\pi\epsilon_0)^2}{2\hbar^2} = 13.6 \text{ eV}$$

$$r_n = n^2 a_B, \quad a_B = \frac{\hbar^2}{me^2/(4\pi\epsilon_0)} = 0.529 \text{ Å}$$

"All of my meagre efforts go toward killing off and suitably replacing the concept of the orbital path which one cannot observe."

`--Heisenberg, letter to Pauli, 1925`

"I knew of [Heisenberg's] theory, of course, but I felt discouraged, not to say repelled, by the methods of transcendental algebra, which appeared difficult to me, and by the lack of visualizability."

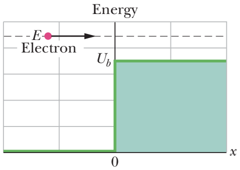
`--Schroedinger in 1926`

Quantum Mechanics

- Understand [L22] Heisenberg's uncertainty principle and [L25] how to use it to estimate the ground state energy of a quantum system.
- Applications of Schroedinger's equation:
  - [L23] Potential step
  - [L23] Potential barrier
  - [L24] Infinite potential well (1D, 2D, and 3D)
  - One should be able to understand [L22] the probabilistic interpretation of wave function and how to use probability density to calculate physics quantities.
- Keep in mind [L27] the Pauli exclusion principle in the case of a system of multiple electrons.

- The general procedure (for a piecewise potential):
  1. Separate the space into several regions (in which the potential is a constant) and write down the general solution with unknown coefficients for Schroedinger's equation in each region.
  2. Match the general solutions at the boundaries between neighboring regions.
  3. In a propagation problem, calculate the ratios of the coefficients in the general solutions. One can further calculate the reflection and transmission coefficients based on the ratios.
  4. In a confinement problem, find out the conditions on momentum, hence energy, that allow a bound state. Normalized the wave function if needed.

- Region 1 ($x < 0$): $k = \sqrt{2mE}/\hbar$



$$\psi_1 = Ae^{ikx} + Be^{-ikx}$$

- Region 2 ($x > 0$): $k_b = \sqrt{2m(E - U_b)}/\hbar$

$$\psi_2 = Ce^{ik_b x}$$

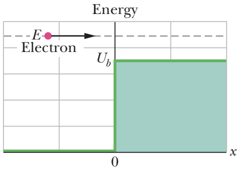
$$A + B = C \text{ (matching of values)}$$

$$Ak - Bk = Ck_b \text{ (matching of slopes)}$$

$$R = 1 - T = \frac{|B|^2}{|A|^2} \text{ (reflection coefficient)}$$

- What happens when we have $E < U_b$?

$$\psi_2 = Ce^{ik_b x} \quad \rightarrow \quad Ce^{-\kappa_b x}$$



where

$$\kappa_b = \sqrt{2m(U_b - E)}/\hbar = ik_b.$$

- The two types of solutions are closely related to each other.
  - They are the solutions of the same equation.
  - They represent classically allowed and forbidden solutions.
  - They are propagating waves versus localized waves (not standing waves!).

- From Heisenberg, Physics and Beyond, Arnold J. Pomerans, trans. (New York: Harper, 1971), p. 63.
    - [Heisenberg:] "We cannot observe electron orbits inside the atom. . . Now, since a good theory must be based on directly observable magnitudes, I thought it more fitting to restrict myself to these, treating them, as it were, as representatives of the electron orbits."
    - "But you don't seriously believe," Einstein protested, "that none but observable magnitudes must go into a physical theory?"
    - "Isn't that precisely what you have done with relativity?" I asked in some surprise. . .
    - "Possibly I did use this kind of reasoning," Einstein admitted, "but it is nonsense all the same. . . .In reality the very opposite happens. It is the theory which decides what we can observe."

Electromagnetism in Matter

- This is a subject that we touched upon from time to time. One needs to understand the following aspects:
  - Properties of conductors, including [L2] excess charge distribution and surface field, [L3] electric potential, and [L5] electric current and resistivity.
  - [L06] Polarization of dielectrics.
  - [L10] Current carrying coil as a magnetic dipole.
  - [L10] Paramagnetism, diamagnetism, and ferromagnetism.
  - [L26] Orbital and spin magnetic dipole moments, which give rise to macroscopic magnetism.

- Magnetic dipole versus electric dipole

$$\vec{\mu} = Ni\vec{A}, \quad \vec{p} = q\vec{d}$$

$$\tau_B = \vec{\mu} \times \vec{B}, \quad \tau_E = \vec{p} \times \vec{E}$$
$$U_B = -\vec{\mu} \cdot \vec{B}, \quad U_E = -\vec{p} \cdot \vec{E}$$

- Magnetic dipole moment $\vec{\mu}$ from the angular momentum $\vec{J}$ of an electron

$$\vec{\mu} = -g\frac{e}{2m}\vec{J},$$

where $g = 1$ or 2 for orbital and spin angular momentum.