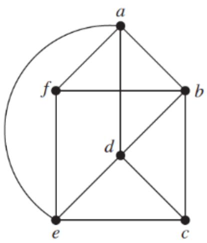
Sec. 10.5 4, 6, 31, 34, 38, 41

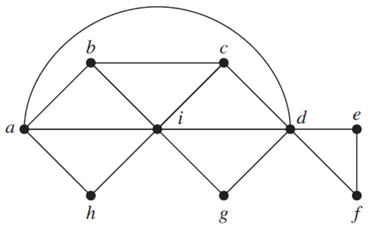
In Exercises 1–8 determine whether the given graph has an Euler circuit. Construct such a circuit when one exists. If no Euler circuit exists, determine whether the graph has an Euler path and construct such a path if one exists.

4.



4. This graph has no Euler circuit, since the degree of vertex c (for one) is odd. There is an Euler path between the two vertices of odd degree. One such path is $f, a, b, c, d, e, f, b, d, a, e, c$.

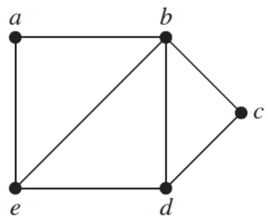
6.



6. This graph has no Euler circuit, since the degree of vertex b (for one) is odd. There is an Euler path between the two vertices of odd degree. One such path is $b, c, d, e, f, d, g, i, d, a, h, i, a, b, i, c$.

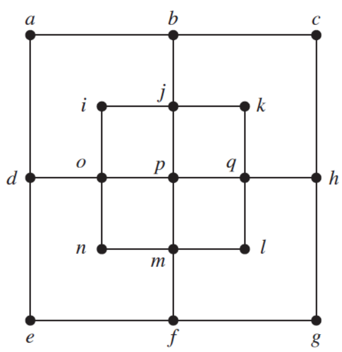
In Exercises 30–36 determine whether the given graph has a Hamilton circuit. If it does, find such a circuit. If it does not, give an argument to show why no such circuit exists.

31.



31. a, b, c, d, e, a is a Hamilton circuit.

34.



34. This graph has no Hamilton circuit. If it did, then certainly the circuit would have to contain edges $\{d, a\}$ and $\{a, b\}$, since these are the only edges incident to vertex a. By the same reasoning, the circuit would have to contain the other six edges around the outside of the figure. These eight edges already complete a circuit, and this circuit omits the nine vertices on the inside. Therefore there is no Hamilton circuit.

38. Does the graph in Exercise 31 have a Hamilton path? If so, find such a path. If it does not, give an argument to show why no such path exists.

38. This graph has the Hamilton path a, b, c, d, e.

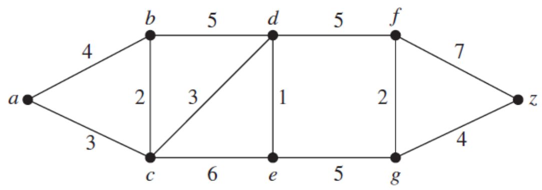
***41.** Does the graph in Exercise 34 have a Hamilton path? If so, find such a path. If it does not, give an argument to show why no such path exists.

ton path. **41.** No Hamilton path exists. There are eight vertices of degree 2, and only two of them can be end vertices of a path. For each of the other six, their two incident edges must be in the path. It is not hard to see that if there is to be a Hamilton path, exactly one of the inside corner vertices must be an end, and that this is impossible. **43.** *a, b, c, f, i, h,*

Sec. 10.6 3, 17a), 26

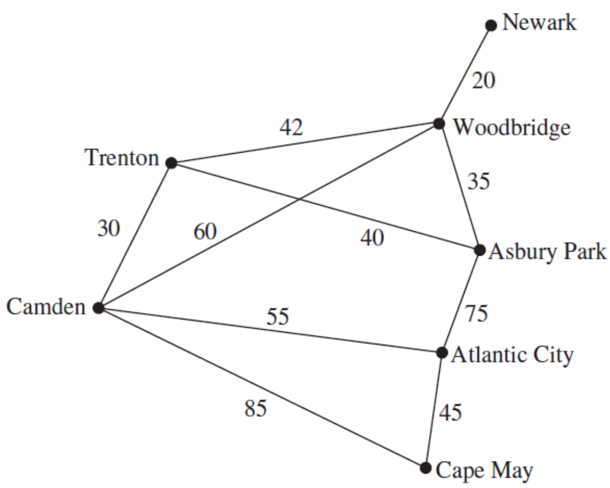
In Exercises 2–4 find the length of a shortest path between a and z in the given weighted graph.

3.



3. 16 = 3+3+1+5+4

17. The weighted graphs in the figures here show some major roads in New Jersey. Part (a) shows the distances between cities on these roads; part (b) shows the tolls.

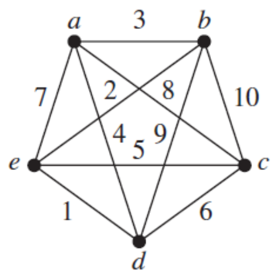


(a)

a) Find a shortest route in distance between Newark and Camden, and between Newark and Cape May, using these roads.

a) Via Woodbridge, via Woodbridge and Camden

26. Solve the traveling salesperson problem for this graph by finding the total weight of all Hamilton circuits and determining a circuit with minimum total weight.



26. The following table shows the twelve different Hamilton circuits and their weights:

Circuit	Weight
a-b-c-d-e-a	$3+10+6+1+7=27$
a-b-c-e-d-a	$3+10+5+1+4=23$
a-b-d-c-e-a	$3+9+6+5+7=30$
a-b-d-e-c-a	$3+9+1+5+8=26$
a-b-e-c-d-a	$3+2+5+6+4=20$
a-b-e-d-c-a	$3+2+1+6+8=20$
a-c-b-d-e-a	$8+10+9+1+7=35$
a-c-b-e-d-a	$8+10+2+1+4=25$
a-c-d-b-e-a	$8+6+9+2+7=32$
a-c-e-b-d-a	$8+5+2+9+4=28$
a-d-b-c-e-a	$4+9+10+5+7=35$
a-d-c-b-e-a	$4+6+10+2+7=29$

Thus we see that the circuits a-b-e-c-d-a and a-b-e-d-c-a (or the same circuits starting at some other point but traversing the vertices in the same or exactly opposite order) are the ones with minimum total weight.