Solutions of Homework at Week 9

May 18th, 2021

Chapter 15

E15-7 $\Delta p = (1060\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(1.83\,\text{m}) = 1.90\times10^4\text{Pa}.$

E15-25 (a) The pressure on the top surface is $p = p_0 + \rho g L/2$. The downward force is

$$\begin{aligned} F_\text{t} &= (p_0 + \rho g L/2)L^2, \\ &= \left[(1.01\times10^5\text{Pa}) + (944\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.608\,\text{m})/2\right](0.608\,\text{m})^2 = 3.84\times10^4\text{N}. \end{aligned}$$

(b) The pressure on the bottom surface is $p = p_0 + 3\rho g L/2$. The upward force is

$$\begin{aligned} F_\text{b} &= (p_0 + 3\rho g L/2)L^2, \\ &= \left[(1.01\times10^5\text{Pa}) + 3(944\,\text{kg/m}^3)(9.81\,\text{m/s}^2)(0.608\,\text{m})/2\right](0.608\,\text{m})^2 = 4.05\times10^4\text{N}. \end{aligned}$$

(c) The tension in the wire is given by $T = W + F_\text{t} - F_\text{b}$, or

$$T = (4450\,\text{N}) + (3.84\times10^4\text{N}) - (4.05\times10^4\text{N}) = 2350\,\text{N}.$$

(d) The buoyant force is

$$B = L^3\rho g = (0.608^3)(944\,\text{kg/m}^3)(9.81\,\text{m/s}^2) = 2080\,\text{N}.$$

$\boxed{\textbf{E15-35}}$ The force required is just the surface tension times the circumference of the circular patch. Then

$$F = (0./072\,\text{N/m})2\pi(0.12\,\text{m}) = 5.43\times10^{-2}\text{N}.$$

$\boxed{\textbf{P15-11}}$ We can start with Eq. 15-11, except that we'll write our distance in terms of r instead if y. Into this we can substitute our expression for g,

$$g = g_0\frac{R^2}{r^2}.$$

Substituting, then integrating,

$$\begin{aligned} \frac{dp}{p} &= -\frac{g\rho_0}{p_0}dr, \\ \frac{dp}{p} &= -\frac{g_0\rho_0R^2}{p_0}\frac{dr}{r^2}, \\ \int_{p_0}^{p}\frac{dp}{p} &= -\int_R^r\frac{g_0\rho_0R^2}{p_0}\frac{dr}{r^2}, \\ \ln\frac{p}{p_0} &= \frac{g_0\rho_0R^2}{p_0}\left(\frac{1}{r}-\frac{1}{R}\right) \end{aligned}$$

If $k = g_0\rho_0R^2/p_0$, then

$$p = p_0e^{k(1/r-1/R)}.$$

P15-17 When the beaker is half filled with water it has a total mass exactly equal to the maximum amount of water it can displace. The total mass of the beaker is the mass of the beaker plus the mass of the water inside the beaker. Then

$$\rho_{\rm w}(m_{\rm g}/\rho_{\rm g} + V_{\rm b}) = m_{\rm g} + \rho_{\rm w}V_{\rm b}/2,$$

where $m_{\rm g}/\rho_{\rm g}$ is the volume of the glass which makes up the beaker. Rearrange,

$$\rho_{\rm g} = \frac{m_{\rm g}}{m_{\rm g}/\rho_{\rm w} - V_{\rm b}/2} = \frac{(0.390\,{\rm kg})}{(0.390\,{\rm kg})/(1000\,{\rm kg/m^3}) - (5.00\times10^{-4}\,{\rm m^3})/2} = 2790\,{\rm kg/m^3}.$$

Chapter 16

E16-16 Assume streamlined flow, then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ (p_1 - p_2)/\rho + g(y_1 - y_2) &= \frac{1}{2}v_2^2. \end{aligned}$$

Then upon rearranging

$$v_2 = \sqrt{2\,[(2.1)(1.01\times10^5{\rm Pa})/(660{\rm kg/m^3}) + (9.81\,{\rm m/s^2})(53.0\,{\rm m})]} = 40.1\,{\rm m/s}.$$

$\boxed{\textbf{E16-19}}$ (a) There are three forces on the plug. The force from the pressure of the water, $F_1 = P_1 A$, the force from the pressure of the air, $F_2 = P_2 A$, and the force of friction, F_3. These three forces must balance, so $F_3 = F_1 - F_2$, or $F_3 = P_1 A - P_2 A$. But $P_1 - P_2$ is the pressure difference between the surface and the water 6.15 m below the surface, so

$$\begin{aligned} F_3 &= \Delta P A = -\rho g y A, \\ &= -(998\,{\rm kg/m^3})(9.81\,{\rm m/s^2})(-6.15\,{\rm m})\pi(0.0215\,{\rm m})^2, \\ &= 87.4\,{\rm N} \end{aligned}$$

(b) To find the volume of water which flows out in three hours we need to know the volume flow rate, and for that we need both the cross section area of the hole and the speed of the flow. The speed of the flow can be found by an application of Bernoulli's equation. We'll consider the horizontal motion only— a point just inside the hole, and a point just outside the hole. These points are at the same level, so

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

Combine this with the results of Pascal's principle above, and

$$v_2 = \sqrt{2(p_1 - p_2)/\rho} = \sqrt{-2gy} = \sqrt{-2(9.81\,{\rm m/s^2})(-6.15\,{\rm m})} = 11.0\,{\rm m/s}.$$

The volume of water which flows out in three hours is

$$V = Rt = (11.0\,{\rm m/s})\pi(0.0215\,{\rm m})^2(3\times3600\,{\rm s}) = 173\,{\rm m^3}.$$

$\boxed{\textbf{E16-25}}$ The larger pipe has a radius $r_1 = 12.7$ cm, and a cross sectional area of $A_1 = \pi r_1^2$. The speed of the fluid flow at this point is v_1. The smaller pipe has a radius of $r_2 = 5.65$ cm, a cross sectional area of $A_2 = \pi r_2^2$, and a fluid speed of v_2. Then

$$A_1 v_1 = A_2 v_2 \text{ or } r_1^2 v_1 = r_2^2 v_2.$$

Now Bernoulli's equation. The two pipes are at the same level, so $y_1 = y_2$. Then

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 &= p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2, \\ p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2. \end{aligned}$$

Combining this with the results from the equation of continuity,

$$\begin{aligned} p_1 + \frac{1}{2}\rho v_1^2 &= p_2 + \frac{1}{2}\rho v_2^2, \\ v_1^2 &= v_2^2 + \frac{2}{\rho}\left(p_2 - p_1\right), \\ v_1^2 &= \left(v_1 \frac{r_1^2}{r_2^2}\right)^2 + \frac{2}{\rho}\left(p_2 - p_1\right), \\ v_1^2\left(1 - \frac{r_1^4}{r_2^4}\right) &= \frac{2}{\rho}\left(p_2 - p_1\right), \\ v_1^2 &= \frac{2\left(p_2 - p_1\right)}{\rho\left(1 - r_1^4/r_2^4\right)}. \end{aligned}$$

It may look a mess, but we can solve it to find v_1,

$$v_1 = \sqrt{\frac{2(32.6\times10^3\text{Pa} - 57.1\times10^3\text{Pa})}{(998\,\text{kg/m}^3)(1 - (0.127\,\text{m})^4/(0.0565\,\text{m})^4)}} = 1.41\,\text{m/s}.$$

The volume flow rate is then

$$R = Av = \pi(0.127\text{ m})^2(1.41\text{ m/s}) = 7.14\times10^{-3}\text{m}^3/\text{s}.$$

That's about 71 liters/second.

$\boxed{\textbf{P16-3}}$ (a) Apply Torricelli's law (Exercise 16-14): $v = \sqrt{2gh}$. The speed v is a horizontal velocity, and serves as the initial horizontal velocity of the fluid "projectile" after it leaves the tank. There is no initial vertical velocity.

This fluid "projectile" falls through a vertical distance $H - h$ before splashing against the ground. The equation governing the time t for it to fall is

$$-(H - h) = -\frac{1}{2}gt^2,$$

Solve this for the time, and $t = \sqrt{2(H - h)/g}$. The equation which governs the horizontal distance traveled during the fall is $x = v_x t$, but $v_x = v$ and we just found t, so

$$x = v_x t = \sqrt{2gh}\,\sqrt{2(H - h)/g} = 2\sqrt{h(H - h)}.$$

(b) How many values of h will lead to a distance of x? We need to invert the expression, and we'll start by squaring both sides

$$x^2 = 4h(H - h) = 4hH - 4h^2,$$

and then solving the resulting quadratic expression for h,

$$h = \frac{4H \pm \sqrt{16H^2 - 16x^2}}{8} = \frac{1}{2}\left(H \pm \sqrt{H^2 - x^2}\right).$$

For values of x between 0 and H there are two real solutions, if $x = H$ there is one real solution, and if $x > H$ there are no real solutions.

If h_1 is a solution, then we can write $h_1 = (H + \Delta)/2$, where $\Delta = 2h_1 - H$ could be positive or negative. Then $h_2 = (H + \Delta)/2$ is also a solution, and

$$h_2 = (H + 2h_1 - 2H)/2 = h_1 - H/2$$

is also a solution.

(c) The farthest distance is $x = H$, and this happens when $h = H/2$, as we can see from the previous section.

P16-4 (a) Apply Torricelli's law (Exercise 16-14): $v = \sqrt{2g(d + h_2)}$, assuming that the liquid remains in contact with the walls of the tube until it exits at the bottom.

(b) The speed of the fluid in the tube is everywhere the same. Then the pressure difference at various points are only functions of height. The fluid exits at C, and assuming that it remains in contact with the walls of the tube the pressure difference is given by $\Delta p = \rho(h_1 + d + h_2)$, so the pressure at B is

$$p = p_0 - \rho(h_1 + d + h_2).$$

(c) The lowest possible pressure at B is zero. Assume the flow rate is *so* slow that Pascal's principle applies. Then the maximum height is given by $0 = p_0 - \rho g h_1$, or

$$h_1 = (1.01\times10^5\text{Pa})/[(9.81\,\text{m/s}^2)(1000\,\text{kg/m}^3)] = 10.3\,\text{m}.$$

Chapter 17

$\boxed{\textbf{E17-9}}$ If the drive wheel rotates at 193 rev/min then

$$\omega = (193\text{ rev/min})(2\pi\text{ rad/rev})(1/60\text{ s/min}) = 20.2\text{ rad/s},$$

then $v_\text{m} = \omega x_\text{m} = (20.2\text{ rad/s})(0.3825\,\text{m}) = 7.73\,\text{m/s}$.

E17-32 The spring will extend until the force from the spring balances the weight, or when $Mg = kh$. The frequency of this system is then

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{Mg/h}{M}} = \frac{1}{2\pi}\sqrt{\frac{g}{h}},$$

which is the frequency of a pendulum of length h. The mass of the bob is irrelevant.

$\boxed{\textbf{P17-11}}$ Conservation of momentum for the bullet block collision gives $mv = (m + M)v_\text{f}$ or

$$v_\text{f} = \frac{m}{m+M}v.$$

This v_f will be equal to the maximum oscillation speed v_m. The angular frequency for the oscillation is given by

$$\omega = \sqrt{\frac{k}{m+M}}.$$

Then the amplitude for the oscillation is

$$x_\text{m} = \frac{v_\text{m}}{\omega} = v\frac{m}{m+M}\sqrt{\frac{m+M}{k}} = \frac{mv}{\sqrt{k(m+M)}}.$$

P17-13 The initial energy stored in the spring is $k{x_\mathrm{m}}^2/2$. When the cylinder passes through the equilibrium point it has a translational velocity v_m and a rotational velocity $\omega_\mathrm{r} = v_\mathrm{m}/R$, where R is the radius of the cylinder. The total kinetic energy at the equilibrium point is

$$\frac{1}{2}m{v_\mathrm{m}}^2 + \frac{1}{2}I{\omega_\mathrm{r}}^2 = \frac{1}{2}\left(m + \frac{1}{2}m\right){v_\mathrm{m}}^2.$$

Then the kinetic energy is 2/3 translational and 1/3 rotational. The total energy of the system is

$$E = \frac{1}{2}(294\,\mathrm{N/\,m})(0.239\,\mathrm{m})^2 = 8.40\,\mathrm{J}.$$

(a) $K_\mathrm{t} = (2/3)(8.40\,\mathrm{J}) = 5.60\,\mathrm{J}$.
(b) $K_\mathrm{r} = (1/3)(8.40\,\mathrm{J}) = 2.80\,\mathrm{J}$.
(c) The energy expression is

$$\frac{1}{2}\left(\frac{3m}{2}\right)v^2 + \frac{1}{2}kx^2 = E,$$

which leads to a standard expression for the period with $3M/2$ replacing m. Then $T = 2\pi\sqrt{3M/2k}$.

P17-16 (a) The rotational inertia of the pendulum about the pivot is

$$(0.488\,\mathrm{kg})\left(\frac{1}{2}(0.103\,\mathrm{m})^2 + (0.103\,\mathrm{m} + 0.524\,\mathrm{m})^2\right) + \frac{1}{3}(0.272\,\mathrm{kg})(0.524\,\mathrm{m})^2 = 0.219\,\mathrm{kg}\cdot\mathrm{m}^2.$$

(b) The center of mass location is

$$d = \frac{(0.524\,\mathrm{m})(0.272\,\mathrm{kg})/2 + (0.103\,\mathrm{m} + 0.524\,\mathrm{m})(0.488\,\mathrm{kg})}{(0.272\,\mathrm{kg}) + (0.488\,\mathrm{kg})} = 0.496\,\mathrm{m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{(0.219\mathrm{kg}\cdot\mathrm{m}^2)/(0.272\,\mathrm{kg} + 0.488\,\mathrm{kg})(9.81\,\mathrm{m/s}^2)(0.496\,\mathrm{m})} = 1.53\,\mathrm{s}.$$

P17-20 Let x be the distance from the center of mass to the first pivot point. Then the period is given by

$$T = 2\pi\sqrt{\frac{I + Mx^2}{Mgx}}.$$

Solve this for x by expressing the above equation as a quadratic:

$$\left(\frac{MgT^2}{4\pi^2}\right)x = I + Mx^2,$$

or

$$Mx^2 - \left(\frac{MgT^2}{4\pi^2}\right)x + I = 0$$

There are *two* solutions. One corresponds to the first location, the other the second location. Adding the two solutions together will yield L; in this case the discriminant of the quadratic will drop out, leaving

$$L = x_1 + x_2 = \frac{MgT^2}{M4\pi^2} = \frac{gT^2}{4\pi^2}.$$

Then $g = 4\pi^2 L/T^2$.