浙江大学 2020－2021 学年春夏学期

《离散数学理论基础》课程模拟试卷

课程号：____21121780_____，开课学院：___计算机科学与技术__

考试试卷：√A 卷、B 卷（请在选定项上打√）

考试形式：√闭、开卷（请在选定项上打√），允许带___________入场

考试日期：2021 年 0 月 0 日 00:00-00:00，考试时间：90 分钟

诚信考试，沉着应考，杜绝违纪。

考生姓名：______________学号：______________ 所属院系：_____________

授课教师：_____________ ________________________

题序	一	二	三	四	五	六	总 分
得分							
评卷人							

1. (20%) Determine whether the following statements are true or false

(1) (√) The total degrees of vertices are the same as the total degrees of regions in every planar graph.

(2) (√) A simple graph is connected if and only if it has a spanning tree.

(3) (×) Let R and S be binary relations on nonempty set A. If R and S are anti-symmetric, so is $R \circ S$.

(4) (√) A weakly connected directed graph with $deg^{+}(v)=deg^{-}(v)$ for every vertex v in the graph is strongly connected.

(5) (×) There is a tree with degrees 3, 2, 2, 2, 1, 1, 1, 1, 1.

(6) (×) Every $K_{m,n}$ has a Hamilton circuit, where m>1 and n>1.

(7) (√) $K_{2,3}$ is a planar graph.

(8) (√) If G = (V,E) is a simple connected non-planar undirected graph, then |V|+|E| ≥ 15.

(9) (√) The chromatic number of a simple connected non-bipartite undirected graph is no less than 3.

(10)(×) If both planar graphs G_1 and planar graphs G_2 each have v vertices, e edges, and r regions, these two graphs are isomorphic.

2. (28%) Fill in the blanks

(1) A tournament is a simple directed graph such that if u and v are distinct vertices in the graph, exactly one of (u,v) and (v,u) is an edge of the graph. Assume all vertices are labeled. There are ___$2^{n(n-1)/2}$___different tournaments with n vertices.

(2) Give a formula for the coefficient of x^k in the expression of $(x-1/x)^{100}$ where k is an integer ___$\binom{100}{(100-k)/2}(-1)^{(100-k)/2}$___

Hint: By Binomial Theorem.

$$\binom{100}{j}x^{100-j}(-1/x)^j = \binom{100}{j}x^{100-2j}(-1)^j$$

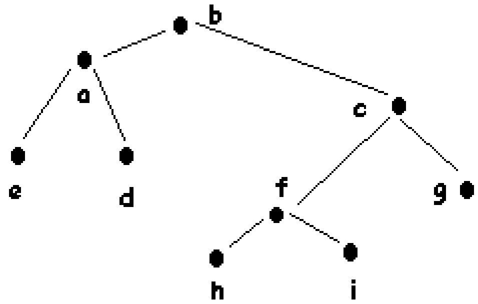
for all integers k such that $k = 100 - 2j$, the coefficient is $\binom{100}{(100-k)/2}(-1)^{(100-k)/2}$.

(3) Determine true for false: the following statements are respectively logically equivalent ___true, true___

$\neg(p \oplus q) \Leftrightarrow (p \Leftrightarrow q)$,

$[\neg p \Rightarrow (q \Rightarrow r)] \Leftrightarrow [q \Rightarrow (p \vee r)]$

(4) Show the post order traversals of the tree below: ___e d a h i f g c b___



(5) Calculate the value of the general coefficient a_n in the power series expansion

$$\frac{2-3x}{1-10x+21x^2}=\sum_{n=0}^{\infty}a_n x^n$$

Answer: $a_n=-\frac{3}{4}\cdot 3^n+\frac{11}{4}\cdot 7^n$

(6) If by snooping, you know that the letters of the password are "qaafwwwgbbbhvvj". How many tests must be tested 15!/(2!2!3!3!)

(7) Find the smallest partial order relation on {1, 2, 3, 4, 5} that contains (1, 1), (3, 2) and (1, 3), (3, 4). Answer: {(1, 1), (2, 2), (3, 3), (4, 4), (5,5) , (1, 2), (1, 3), (1, 4), (3, 2), (3, 4)}

(8) How many numbers must be selected from the set {1, 3, 5, 7, 9, 11, 13, 15} to guarantee that at least one pair of these numbers add up to 16? _ _Answer: pigeonhole principle, add up to 16: {1,15}, {3,13}, {5,11}, and {7,9}. Select five numbers.

(9) Arrange the functions below in a list so each is big-O of the next one in the list: n^3+88n^2+3, $\log\log n$, $n\log n$, $\log(n^2+1)$, $\log 2^n$, $n^2\log n$, 9999

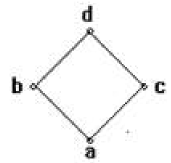
Answer: 9999, $\log\log n$, $\log(n^2+1)$, $\log 2^n$, $n\log n$, $n^2\log n$, n^3+88n^2+3

(10) If G is a planar connected graph with 20 vertices, each of degree 4, then G has ___22_____ regions.

(11) Every full 3-ary tree of height 2 has at least ____7____ vertices and at most ____13____ vertices.

(12) Let A={a, b, c, d}, the Hasse diagram of partial relation R on A is illustrated in Fig. 1, then |R|=_9__. Hint: (a,a)(b,b)(c,c)(d,d)(a,b)(a,c)(a,d)(b,d)(c,d)

Fig.1



(13) Suppose W is a weighted graph (See Fig. 2). The length of the shortest path between a and z is ____15________

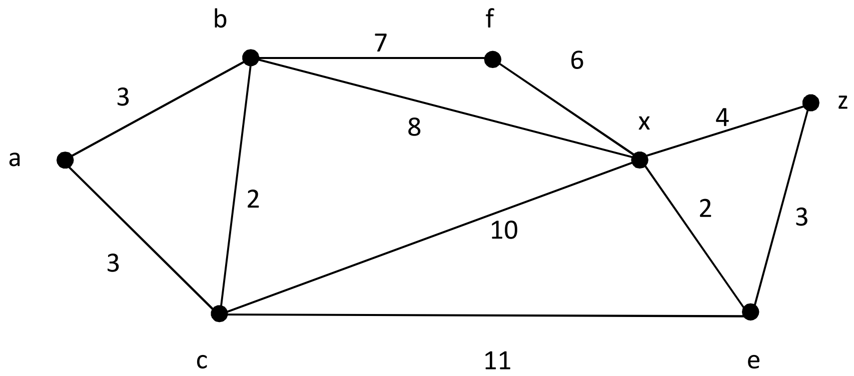
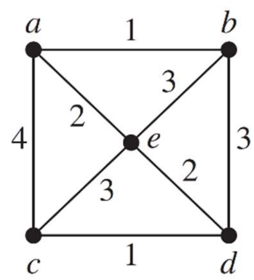
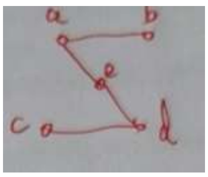


Fig. 2

(14) In the k × n grid (graph), the length of every edge is 1, counting the number of shortest paths from the bottom left corner to the top right corner. (Note that the distance between these two points is k+n−2.) C(k+n−2,k-1) （或C(k+n−2,n-1)）

3. (10%) Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph 5.



Answer: 

weight=6

4. (15%) A machine that inserts letters into envelopes goes haywire and inserts letters randomly into envelopes. What is the probability that in a group of 100 letters

(1)no letter is put into the correct envelope?

(2)exactly one letter is put into the correct envelope?

(3)exactly 98 letters are put into the correct envelopes?

(4)exactly 99 letters are put into the correct envelopes?

(5)all letters are put into the correct envelopes?

Answer: (1) $1-\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\cdots+(-1)^{100}\frac{1}{100!} = \frac{1}{2!}-\frac{1}{3!}+\cdots+(-1)^{100}\frac{1}{100!}$

(2) $C_{100}^{1} \times 99! \times (1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^{99}\frac{1}{99!}) \div (100!)$

$= \frac{1}{2!} - \frac{1}{3!} + \cdots + (-1)^{99}\frac{1}{99!}$

(3) C(100,2)/(100!)

(4) 0

(5) 1/(100!)

5. (12%) Find numbers A, B and C such that $a_n = n2^n + 2$ is a solution to the recursion $a_n = Aa_{n-1} + Ba_{n-2} + Ca_{n-3}$

$a_n = n\cdot 2^n + 2\cdot 1^n$ ，2为二重根．1为一重根

特征根方程： $0 = \lambda^3 - A\lambda^2 - B\lambda - C = (\lambda-2)^2(\lambda-1)$

$= \lambda^3 - 5\lambda^2 + 8\lambda - 4$

$$\begin{cases} A = 5 \\ B = -8 \\ C = 4 \end{cases}$$

6. (15%) G is a directed graph(See Fig. 3).

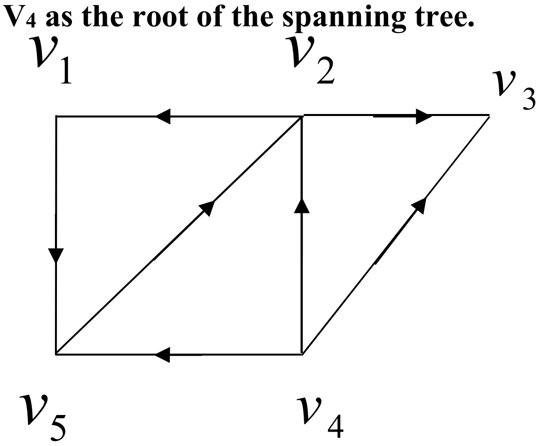
(1) Find the number of different paths of length 4.

(2) Find the strongly connected components of the graph G.

(3) Determine if G has Euler circuit/path or Hamilton circuit/path. If yes, give a path or circuit; otherwise, give the reason.

(4) Find the chromatic number of the underlying undirected graph of the directed graph G.

(5) Find the spanning tree for the underlying undirected graph of the directed graph G. Choose V4 as the root of the spanning tree.



(1) 关系矩阵

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \qquad A^4 = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

答案：6.

2）{v1,v2,v5},{v3},{v4}

3）No Euler circuit exists, deg+(v4) =3 deg-(v4)= 0

No Euler path exists, deg+(v4) =3 deg-(v4)= 0

No Hamilton circuit exists, because it is not strongly connected

No Hamilton path exists. If a Hamilton path exists, it should begins at v4 and ends at v3. V2 should immediately preceding to V3. But it is impossible to place v1 and v5 correctly in the path.

4）3. V1,V2,V5 must have different colors. {V1,V4}-Red, {V3,v5}-Green, {V2}-blue

5）For example: