$\boxed{\textbf{P2-11}}$ (a) The average velocity is displacement divided by change in time,

$$v_{\rm av} = \frac{(2.0\ {\rm m/s}^3)(2.0\ {\rm s})^3 - (2.0\ {\rm m/s}^3)(1.0\ {\rm s})^3}{(2.0\ {\rm s}) - (1.0\ {\rm s})} = \frac{14.0\ {\rm m}}{1.0\ {\rm s}} = 14.0\ {\rm m/s}.$$

The average acceleration is the change in velocity. So we need an expression for the velocity, which is the time derivative of the position,

$$v = \frac{dx}{dt} = \frac{d}{dt}(2.0\ {\rm m/s}^3)t^3 = (6.0\ {\rm m/s}^3)t^2.$$

From this we find average acceleration

$$a_{\rm av} = \frac{(6.0\ {\rm m/s}^3)(2.0\ {\rm s})^2 - (6.0\ {\rm m/s}^3)(1.0\ {\rm s})^2}{(2.0\ {\rm s}) - (1.0\ {\rm s})} = \frac{18.0\ {\rm m/s}}{1.0\ {\rm s}} = 18.0\ {\rm m/s}^2.$$

(b) The instantaneous velocities can be found directly from $v = (6.0\ {\rm m/s}^2)t^2$, so $v(2.0\ {\rm s}) = 24.0$ m/s and $v(1.0\ {\rm s}) = 6.0$ m/s. We can get an expression for the instantaneous acceleration by taking the time derivative of the velocity

$$a = \frac{dv}{dt} = \frac{d}{dt}(6.0\ {\rm m/s}^3)t^2 = (12.0\ {\rm m/s}^3)t.$$

Then the instantaneous accelerations are $a(2.0\ {\rm s}) = 24.0\ {\rm m/s}^2$ and $a(1.0\ {\rm s}) = 12.0\ {\rm m/s}^2$

(c) Since the motion is monotonic we expect the average quantities to be somewhere between the instantaneous values at the endpoints of the time interval. Indeed, that is the case.

$\boxed{\textbf{P4-27}}$ The velocity of the police car with respect to the ground is $\vec{\mathbf{v}}_{pg} = -76{\rm km/h}\hat{\mathbf{i}}$. The velocity of the motorist with respect the ground is $\vec{\mathbf{v}}_{mg} = -62\ {\rm km/h}\hat{\mathbf{j}}$.

The velocity of the motorist with respect to the police car is given by solving

$$\vec{\mathbf{v}}_{mg} = \vec{\mathbf{v}}_{mp} + \vec{\mathbf{v}}_{pg},$$

so $\vec{\mathbf{v}}_{mp} = 76{\rm km/h}\hat{\mathbf{i}} - 62\ {\rm km/h}\hat{\mathbf{j}}$. This velocity has magnitude

$$v_{mp} = \sqrt{(76{\rm km/h})^2 + (-62\ {\rm km/h})^2} = 98\ {\rm km/h}.$$

The direction is

$$\theta = \arctan(-62\ {\rm km/h})/(76{\rm km/h}) = -39^\circ,$$

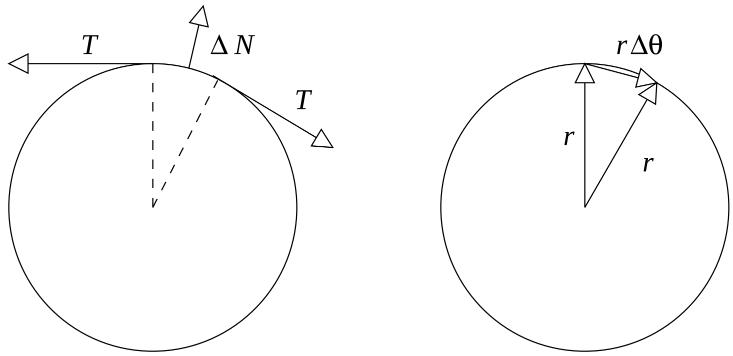
but that is relative to $\hat{\mathbf{i}}$. We want to know the direction relative to the line of sight. The line of sight is

$$\alpha = \arctan(57\ {\rm m})/(41\ {\rm m}) = -54^\circ$$

relative to $\hat{\mathbf{i}}$, so the answer must be 15°.

$\boxed{\textbf{P5-11}}$ The rope wraps around the dowel and there is a contribution to the frictional force Δf from each small segment of the rope where it touches the dowel. There is also a normal force ΔN at each point where the contact occurs. We can find ΔN much the same way that we solve the circular motion problem.

In the figure on the left below we see that we can form a triangle with long side T and short side ΔN. In the figure on the right below we see a triangle with long side r and short side $r\Delta\theta$. These triangles are similar, so $r\Delta\theta/r = \Delta N/T$.



Now $\Delta f = \mu \Delta N$ and $T(\theta) + \Delta f \approx T(\theta + \Delta\theta)$. Combining, and taking the limit as $\Delta\theta \to 0$, $dT = df$

$$\int \frac{1}{\mu}\frac{dT}{T} = \int d\theta$$

integrating both sides of this expression,

$$\begin{aligned} \int \frac{1}{\mu}\frac{dT}{T} &= \int d\theta, \\ \frac{1}{\mu} \ln T|_{T_1}^{T_2} &= \pi, \\ T_2 &= T_1 e^{\pi\mu}. \end{aligned}$$

In this case T_1 is the weight and T_2 is the downward force.

P6-9 (a) It takes a time $t_1 = \sqrt{2h/g}$ to fall $h = 6.5$ ft. An object will be moving at a speed $v_1 = gt_1 = \sqrt{2hg}$ after falling this distance. If there is an inelastic collision with the pile then the two will move together with a speed of $v_2 = Mv_1/(M+m)$ after the collision.

If the pile then stops within $d = 1.5$ inches, then the time of stopping is given by $t_2 = d/(v_2/2) = 2d/v_2$.

For inelastic collisions this corresponds to an average force of

$$F_{\text{av}} = \frac{(M+m)v_2}{t_2} = \frac{(M+m)v_2^2}{2d} = \frac{M^2 v_1^2}{2(M+m)d} = \frac{(gM)^2}{g(M+m)}\frac{h}{d}.$$

Note that we multiply through by g to get weights. The numerical result is $F_{\text{av}} = 130$ t.

(b) For an elastic collision $v_2 = 2Mv_1/(M+m)$; the time of stopping is still expressed by $t_2 = 2d/v_2$, but we now know F_{av} instead of d. Then

$$F_{\text{av}} = \frac{mv_2}{t_2} = \frac{mv_2^2}{2d} = \frac{2M^2 m v_1^2}{(M+m)^2 d} = \frac{4(gM)^2(gm)}{g^2(M+m)^2}\frac{h}{d}$$

or

$$d = \frac{4(gM)^2(gm)}{g^2(M+m)^2}\frac{h}{F_{\text{av}}} \quad ,$$

which has a numerical result of $d = 0.88$ inches.

But wait! The weight, which just had an elastic collision, "bounced" off of the pile, and then hit it again. This drives the pile deeper into the earth. The weight hits the pile a second time with a speed of $v_3 = (M-m)/(M+m)v_1$; the pile will (in this second elastic collision) then have a speed of $v_4 = 2M(M+m)v_3 = [(M-m)/(M+m)]v_2$. In other words, we have an infinite series of distances traveled by the pile, and if $\alpha = [(M-m)/(M+m)] = 0.71$, the depth driven by the pile is

$$d_\mathrm{f} = d(1 + \alpha^2 + \alpha^4 + \alpha^6 \cdots) = \frac{d}{1-\alpha^2},$$

or $d = 1.8$ inches.

P7-3 This is a glorified Atwood's machine problem. The total mass on the right side is the mass per unit length times the length, $m_r = \lambda x$; similarly the mass on the left is given by $m_l = \lambda(L-x)$. Then

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = \frac{\lambda x - \lambda(L-x)}{\lambda x + \lambda(L-x)} g = \frac{2x - L}{L} g$$

which solves the problem. The acceleration is in the direction of the side of length x if $x > L/2$.