Solutions of Homework at Week 3

March 13th, 2020

Chapter 6

E6-9 The change in momentum of the ball is $\Delta p = (mv) - (-mv) = 2mv = 2(0.058\text{ kg})(32\text{ m/s}) = 3.7$ kg · m/s. The impulse is the area under a force - time graph; for the trapezoid in the figure this area is $J = F_{\text{max}}(2\text{ ms} + 6\text{ ms})/2 = (4\text{ ms})F_{\text{max}}$. Then $F_{\text{max}} = (3.7\text{ kg}\cdot\text{m/s})/(4\text{ ms}) = 930\text{ N}$.

E6-14 (a) $p = mv = (2.14\times10^{-3}\text{kg})(483\text{ m/s}) = 1.03\text{ kg}\cdot\text{m/s}$.

(b) The impulse imparted to the wall in one second is ten times the above momentum, or $J = 10.3\,\text{kg}\cdot\text{m/s}$. The average force is then $F_{\text{av}} = (10.3\,\text{kg}\cdot\text{m/s})/(1.0\,\text{s}) = 10.3\,\text{N}$.

(c) The average force for each individual particle is $F_{\text{av}} = (1.03\,\text{kg}\cdot\text{m/s})/(1.25\times10^{-3}\,\text{s}) = 830\,\text{N}$.

$\boxed{\textbf{P6-7}}$ The weight of the marbles in the box after a time t is $mgRt$ because Rt is the number of marbles in the box.

The marbles fall a distance h from rest; the time required to fall this distance is $t = \sqrt{2h/g}$, the speed of the marbles when they strike the box is $v = gt = \sqrt{2gh}$. The momentum each marble imparts on the box is then $m\sqrt{2gh}$. If the marbles strike at a rate R then the force required to stop them is $Rm\sqrt{2gh}$.

The reading on the scale is then

$$W = mR(\sqrt{2gh} + gt).$$

This will give a numerical result of

$$(4.60\times10^{-3}\text{kg})(115\,\text{s}^{-1})\left(\sqrt{2(9.81\,\text{m/s}^2)(9.62\,\text{m})} + (9.81\,\text{m/s}^2)(6.50\,\text{s})\right) = 41.0\,\text{N}.$$

P6-16 There will always be at least two collisions. The balls are a, b, and c from left to right. After the first collision between a and b one has

$$v_{b,1} = v_0 \text{ and } v_{a,1} = 0.$$

After the first collision between b and c one has

$$v_{c,1} = 2mv_0/(m+M) \text{ and } v_{b,2} = (m-M)v_0/(m+M).$$

(a) If $m \ge M$ then ball b continue to move to the right (or stops) and there are no more collisions.

(b) If $m < M$ then ball b bounces back and strikes ball a which was at rest. Then

$$v_{a,2} = (m-M)v_0/(m+M) \text{ and } v_{b,3} = 0.$$

$\boxed{\textbf{P6-19}}$ (a) The speed of the bullet after leaving the first block but before entering the second can be determined by momentum conservation.

$$\begin{aligned}
P_{\text{f}} &= P_{\text{i}}, \\
p_{\text{f,bl}} + p_{\text{f,bu}} &= p_{\text{i,bl}} + p_{\text{i,bu}}, \\
m_{\text{bl}}v_{\text{f,bl}} + m_{\text{bu}}v_{\text{f,bu}} &= m_{\text{bl}}v_{\text{i,bl}} + m_{\text{bu}}v_{\text{i,bu}}, \\
(1.78\text{kg})(1.48\,\text{m/s}) + (3.54\times10^{-3}\text{kg})(1.48\,\text{m/s}) &= (1.78\text{kg})(0) + (3.54\times10^{-3}\text{kg})v_{\text{i,bu}},
\end{aligned}$$

which has solution $v_{\text{i,bl}} = 746\,\text{m/s}$.

(b) We do the same steps again, except applied to the first block,

$$\begin{aligned}
P_\mathrm{f} &= P_\mathrm{i},\\
p_\mathrm{f,bl} + p_\mathrm{f,bu} &= p_\mathrm{i,bl} + p_\mathrm{i,bu},\\
m_\mathrm{bl} v_\mathrm{f,bl} + m_\mathrm{bu} v_\mathrm{f,bu} &= m_\mathrm{bl} v_\mathrm{i,bl} + m_\mathrm{bu} v_\mathrm{i,bu},\\
(1.22\mathrm{kg})(0.63\,\mathrm{m/s}) + (3.54\times 10^{-3}\mathrm{kg})(746\,\mathrm{m/s}) &= (1.22\mathrm{kg})(0) + (3.54\times 10^{-3}\mathrm{kg}) v_\mathrm{i,bu},
\end{aligned}$$

which has solution $v_\mathrm{i,bl} = 963\,\mathrm{m/s}$.

P6-21 (a) For an object with initial speed v and deceleration $-a$ which travels a distance x before stopping, the time t to stop is $t = v/a$, the average speed while stopping is $v/2$, and $d = at^2/2$. Combining, $v = \sqrt{2ax}$. The deceleration in this case is given by $a = \mu_\mathrm{k} g$.

Then just after the collision

$$v_A = \sqrt{2(0.130)(9.81\,\mathrm{m/s^2})(8.20\,\mathrm{m})} = 4.57\,\mathrm{m/s},$$

while

$$v_B = \sqrt{2(0.130)(9.81\,\mathrm{m/s^2})(6.10\,\mathrm{m})} = 3.94\,\mathrm{m/s},$$

(b) $v_0 = [(1100\,\mathrm{kg})(4.57\,\mathrm{m/s}) + (1400\mathrm{kg})(3.94\,\mathrm{m/s})]/(1400\,\mathrm{kg}) = 7.53\,\mathrm{m/s}$.

Chapter 7

E7-9 It takes the man $t = (18.2\text{ m})/(2.08\text{ m/s}) = 8.75$ s to walk to the front of the boat. During this time the center of mass of the system has moved forward $x = (4.16\text{ m/s})(8.75\text{ s}) = 36.4$ m. But in walking forward to the front of the boat the man shifted the center of mass by a distance of (84.4 kg) (18.2 m)/(84.4 kg + 425 kg) = 3.02 m, so the boat only traveled 36.4 m − 3.02 m = 33.4 m.

E7-13 The center of mass should lie on the perpendicular bisector of the rod of mass $3M$. We can view the system as having two parts: the heavy rod of mass $3M$ and the two light rods each of mass M. The two light rods have a center of mass at the center of the square.

Both of these center of masses are located along the vertical line of symmetry for the object. The center of mass of the heavy bar is at $y_\mathrm{h,cm} = 0$, while the *combined* center of mass of the two light bars is at $y_\mathrm{l,cm} = L/2$, where down is positive. The center of mass of the system is then at

$$y_\mathrm{cm} = \frac{2My_\mathrm{l,cm} + 3My_\mathrm{h,cm}}{2M + 3M} = \frac{2(L/2)}{5} = L/5.$$

P7-6 The center of mass will be located along symmetry axis. Call this the x axis. Then

$$\begin{aligned}
x_\mathrm{cm} &= \frac{1}{M}\int x\,dm,\\
&= \frac{4}{\pi R^2}\int_0^R \int_0^{\sqrt{R^2-x^2}} x\,dy\,dx,\\
&= \frac{4}{\pi R^2}\int_0^R x\sqrt{R^2 - x^2}\,dx,\\
&= \frac{4}{\pi R^2} R^3/3 = \frac{4R}{3\pi}.
\end{aligned}$$

P7-7 (a) The components of the shell velocity with respect to the cannon are

$$v'_x = (556\,\mathrm{m/s})\cos(39.0^\circ) = 432\,\mathrm{m/s} \text{ and } v'_y = (556\,\mathrm{m/s})\sin(39.0^\circ) = 350\,\mathrm{m/s}.$$

The vertical component with respect to the ground is the same, $v_y = v'_y$, but the horizontal component is found from conservation of momentum:

$$M(v_x - v'_x) + m(v_x) = 0,$$

so $v_x = (1400\,\mathrm{kg})(432\,\mathrm{m/s})(70.0\,\mathrm{kg} + 1400\,\mathrm{kg}) = 411\,\mathrm{m/s}$. The resulting speed is $v = 540\,\mathrm{m/s}$.

(b) The direction is $\theta = \arctan(350/411) = 40.4^\circ$.

P7-10 (a) The thrust must be at least equal to the weight, so

$$dm/dt = (5860\,\mathrm{kg})(9.81\,\mathrm{m/s^2})/(1170\,\mathrm{m/s}) = 49.1\,\mathrm{kg/s}.$$

(b) The net force on the rocket will need to be $F = (5860\text{ kg})(18.3\text{ m/s}^2) = 107000$ N. Add this to the weight to find the thrust, so

$$dm/dt = [107000\,\mathrm{N} + (5860\,\mathrm{kg})(9.81\,\mathrm{m/s^2})]/(1170\,\mathrm{m/s}) = 141\,\mathrm{kg/s}$$