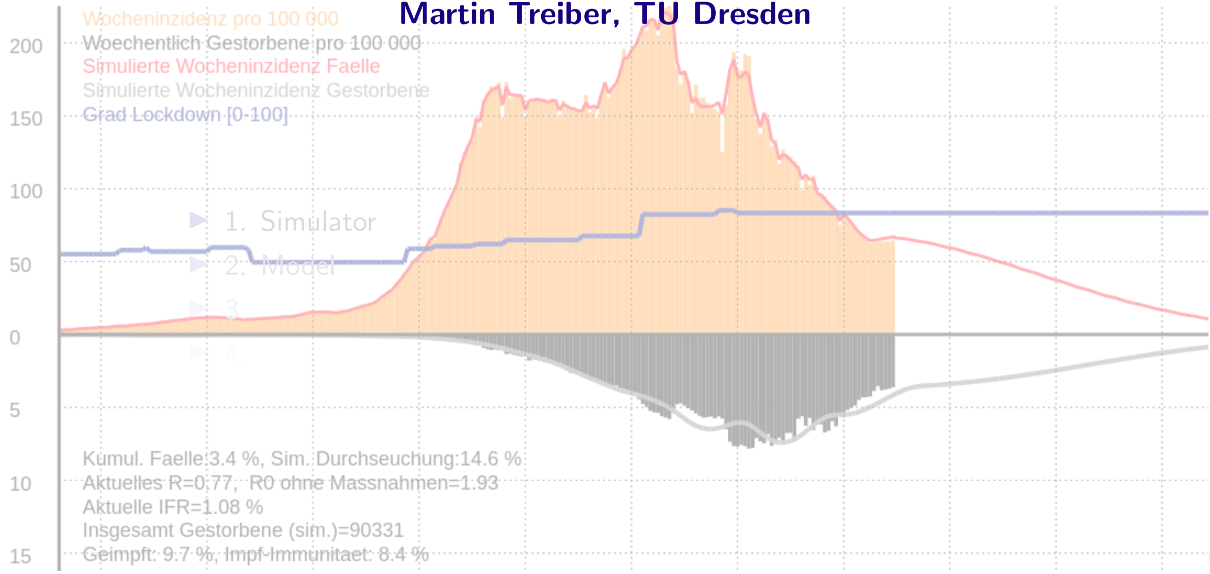
Covid-19 Mutation Dynamics

mit `traffic-simulation.de`

Martin Treiber, TU Dresden



1. Simulator
2. Model
3.
4.

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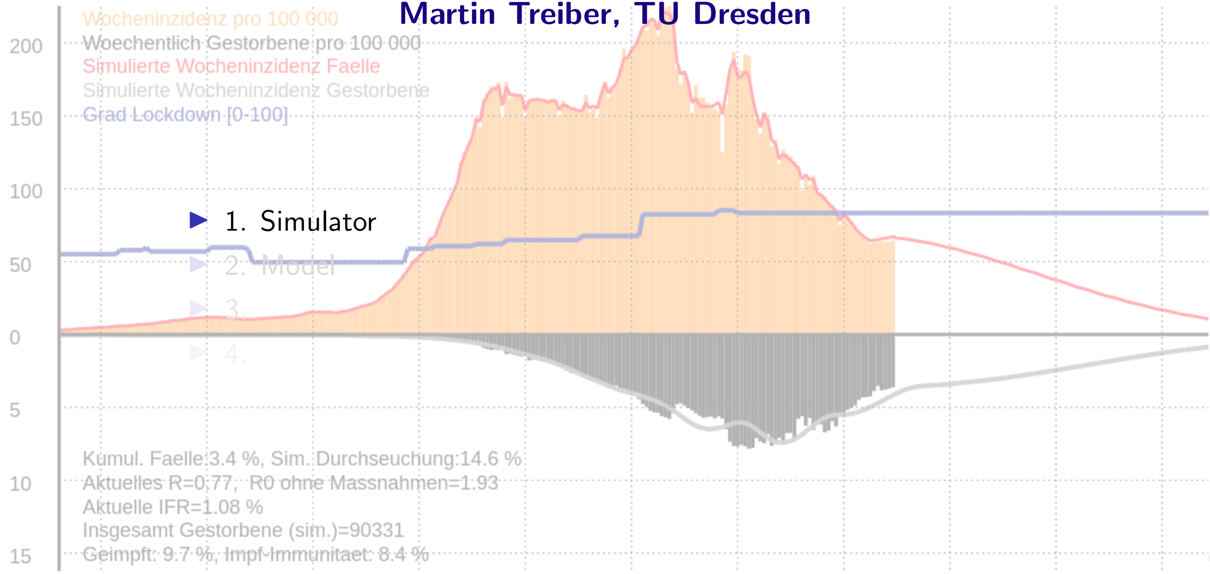

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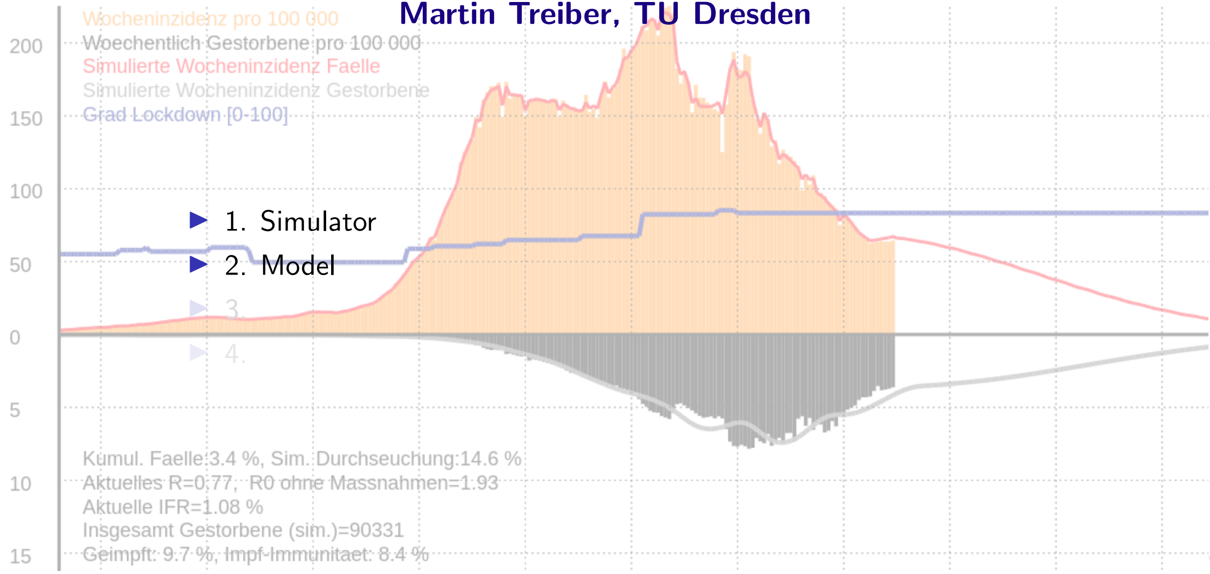

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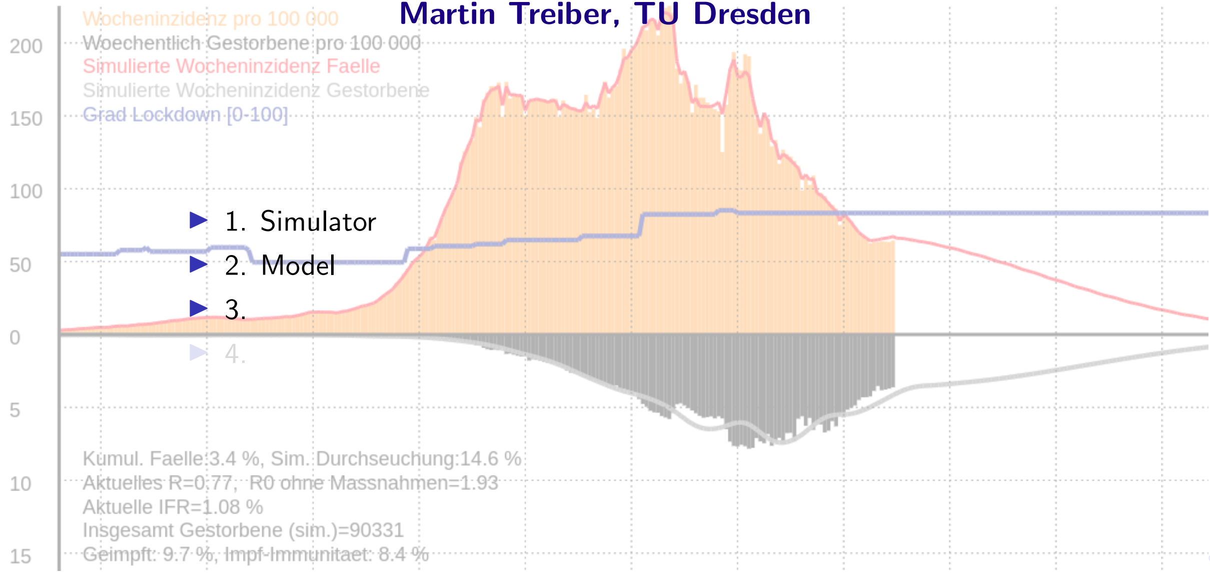

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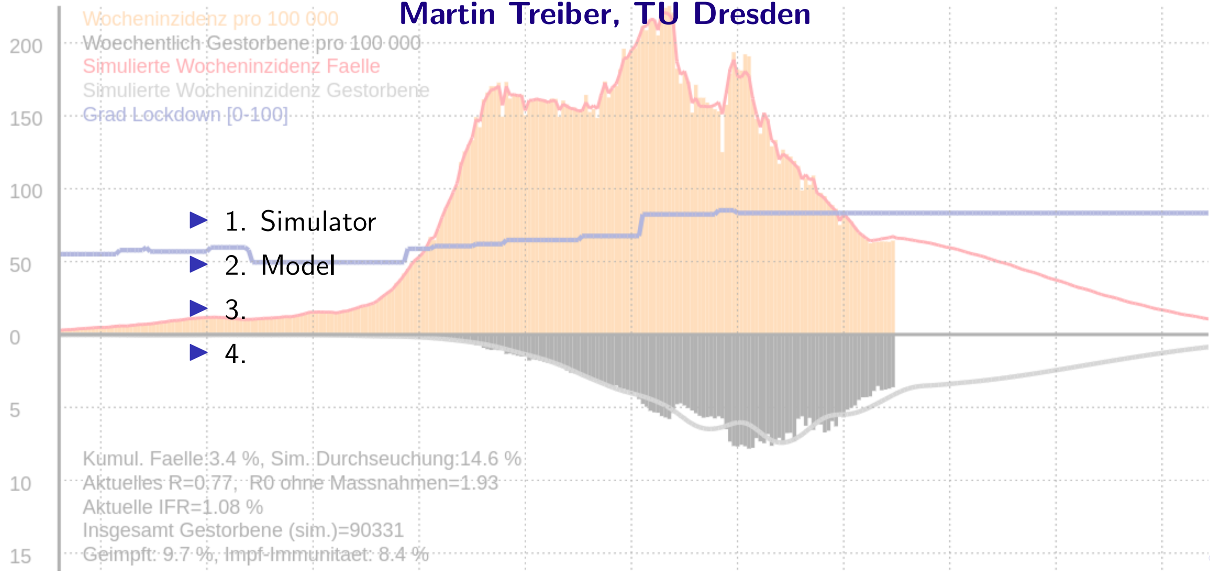

Macroscopic Dynamics of new Virus Strains

- **Reproduction period τ**: average time interval between two infection generations
- **Actively infected #persons x** in the 'E' or 'I' states
- **Base reproduction numbers R_0^{wild} and R_0^{mut}** of the wild type and the mutated strain (e.g., B.1.1.7), respectively
- **Penetration rate p** of the mutated strain

Dynamics without immunity and measures:

$$x(t+\tau) = R_0 x \stackrel{!}{=} (1-p)R_0^{\text{wild}} x + pR_0^{\text{mut}} x$$

$$\Rightarrow R_0 = (1-p)R_0^{\text{wild}} + pR_0^{\text{mut}}$$

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Macroscopic Dynamics of new Virus Strains II

Dynamics of the **percentage** p of the new strain:

$$p(t+\tau) = \frac{x^{\text{mut}}(t+\tau)}{x^{\text{wild}}(t+\tau)} = \frac{pR_0^{\text{mut}}}{(1-p)R_0^{\text{wild}}}$$

Dynamics of the **odds ratio** $y = p/(1-p)$:

$$y(t+\tau) = \frac{R_0^{\text{mut}}}{R_0^{\text{wild}}}\, y(t)$$

Formulation as a **differential equation**:

$$\frac{\mathrm{d}y}{\mathrm{d}t} \approx \frac{y(t+\tau)-y(t)}{\tau} = \frac{1}{\tau}\left(\frac{R_0^{\text{mut}}}{R_0^{\text{wild}}} - 1\right) y \stackrel{!}{=} r_y y \tag{1}$$

resulting **replacement dynamics**:

$$p = \frac{y}{1+y}, \quad y(t) = y_0 e^{r_y t}, \quad \boxed{p(t) = \frac{y_0 e^{r_y t}}{1+y_0 e^{r_y t}} = \frac{p_0 e^{r_y t}}{1+p_0\left(e^{r_y t}-1\right)}}$$

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Parameters from Data

Given:

- Observed strain fractions p_1 and p_2 at the times t_1 and t_2
- Observed or simulated base reproduction number R_0 at time t

Wanted: dynamic parameters r_y, R_0^{wild} and R_0^{mut}, and all the rest

Solution:

- use the relation $y(t) = y_0 e^{r_y(t-t_0)}$ to estimate the growth rate r_y:

$$r_y \approx (\ln y(t_2) - \ln y(t_1))/(t_2 - t_1)$$

- Use Relation (1) to estimate the reproduction number ratio:

$$R_0^{\text{mut}}/R_0^{\text{wild}} \approx \tau r_y + 1$$

- Use the observed/simulated total base reproduction number to estimate the R_0's individually:

$$R_0^{\text{wild}} = \frac{R_0(t)}{1 + p(t)\tau r_y}, \quad R_0^{\text{mut}} = (\tau r_y + 1) R_0^{\text{wild}}$$

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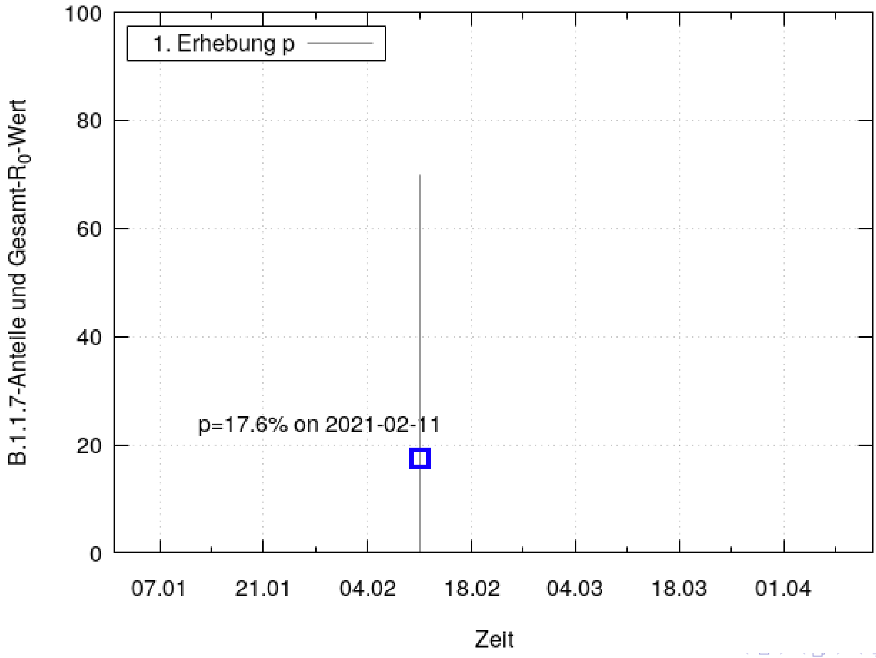
Example Germany, 2021-03-12



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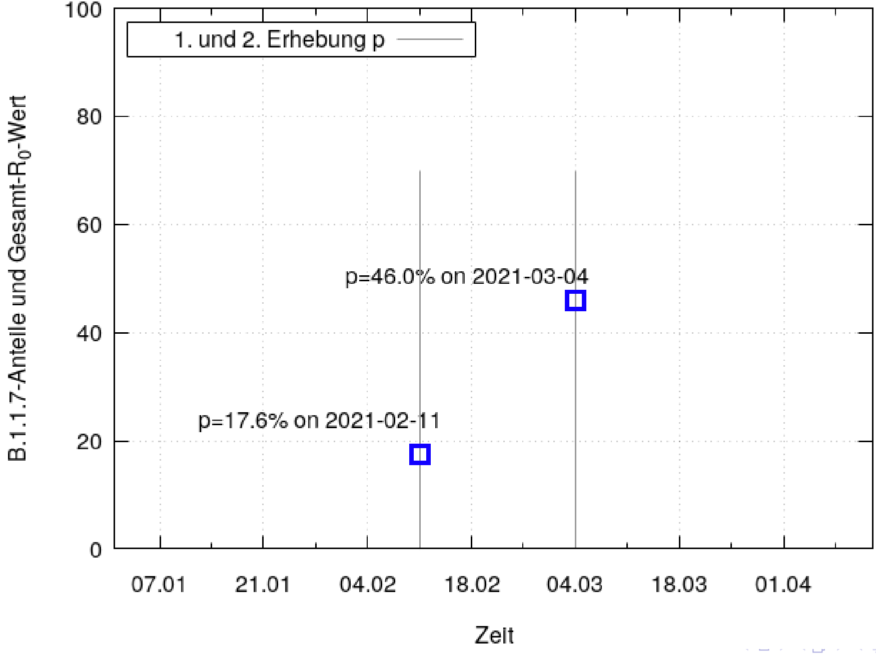

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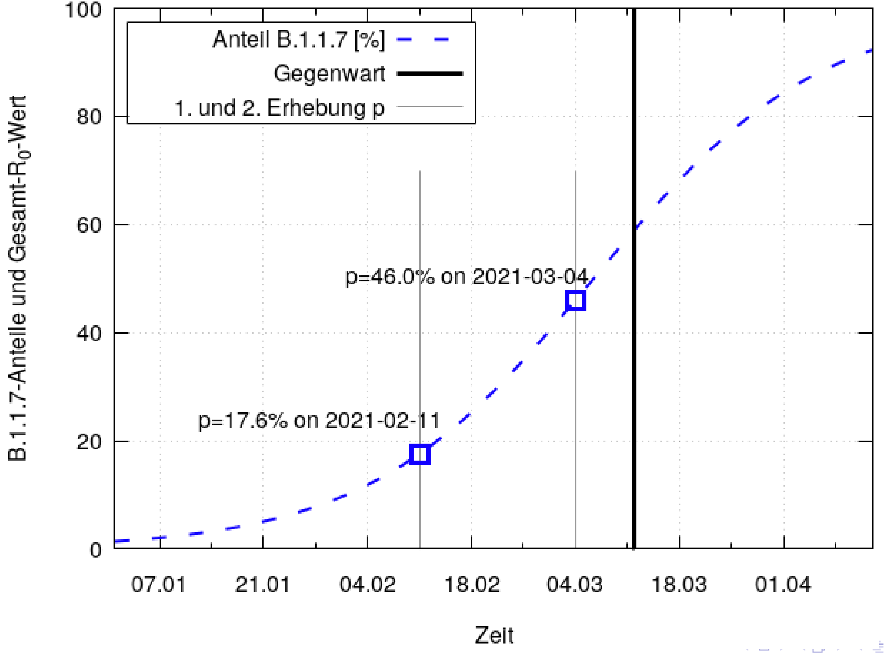

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