Non-sterile immunity. How to model it?

Martin Treiber

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1 Problem statement

In Spring 2021, we thought that the new vaccinations offer a nearly perfect protection $1-\alpha = 95\,\%$ against infection, and even more against the more severe endpoints such as ICS or death). Moreover, we thought that the immunity is *sterile*, i.e., vaccinated people (even the infected ones) are not contagious. Then, the effective reproduction number R_{eff} in terms of the base reproduction number R_0 depends on the vaccination protection factor $1-\alpha$ and the percentage p_{vacc} of fully vaccinated people via

$$R_{\text{eff}} = R_0(1 - p_{\text{vacc}})$$

Particularly, at $100\,\%$ vaccination, we have $R_{\text{eff}} = 0$ since, per assumption, no vaccinated person can infect others. If infected vaccinated persons *can* infect, we would have $R_{\text{eff}} = R_0(1-p_{\text{vacc}}) + R_0\alpha p_{\text{vacc}}$ which, for $p_{\text{vacc}} = 1$ would be ddeep in the herd-immunity range. Furthermore, if $50\,\%$ were vaccinated, we would have, among the infected, an odds ratio

$$\#\text{not vaccinated} : \#\text{vaccinated} = 1/\alpha : 1 = 20 : 1,$$

regardless whether infected vaccinated persons are sterile or not.

However, it turned out (as of November 2021) that this is not the case. In fact, we have four reproduction rates:

- R_{00} for the infection path not vaccinated $\to$ not vaccinated
- R_{01} for the path not vaccinated $\to$ vaccinated
- R_{10} for the path vaccinated $\to$ not vaccinated
- R_{11} for the path vaccinated $\to$ vaccinated

Here, R_{00} denotes the average number of people that an infected not vaccinated person would infect during his/her desease if nobody were vaccinated, while the same person would infect an averaged number R_{01} of vaccinated persons if all others were vaccinated.

In the old assmuption of a sterile immunity, these would be the only reproduction numbers. If, however, the vaccinated persons are not sterile (with respect to contagions), an infected vaccinated person would infect (on average) R_{10} persons if everybody else were not vaccinated, and R_{11} persons if everybody else were vaccinated.

Hence, we have following problem statement:

1. *What is the effective reproduction rate R_{eff} as a function of the four R values and the percentage p_{vacc} of vaccinated people?*
2. *What is the odds ratio #not vaccinated : #vaccinated among the infected people as a function of the R_{ij} and p_{vacc}?*

References

[1] A. Einstein, *Zur Elektrodynamik bewegter Körper*, Annalen der Physik **17**, 891-921 (1905).

[2] A. Einstein, *Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?*, Annalen der Physik **18**, 639-641 (1905).