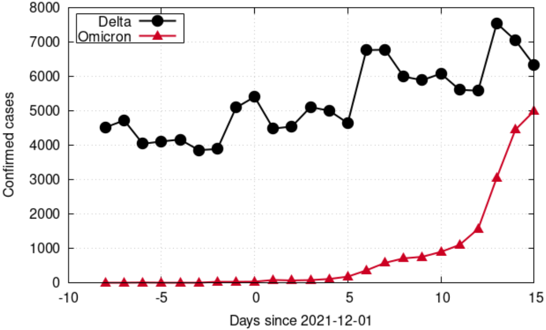
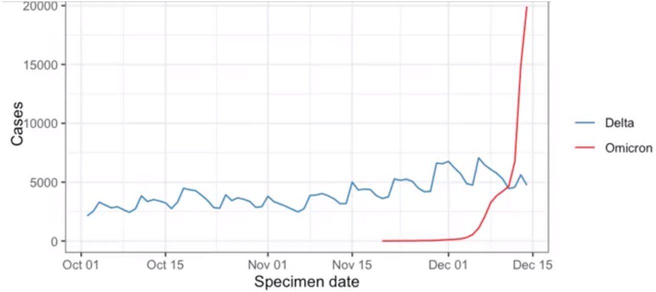
1. Delta and Omicron: Observations

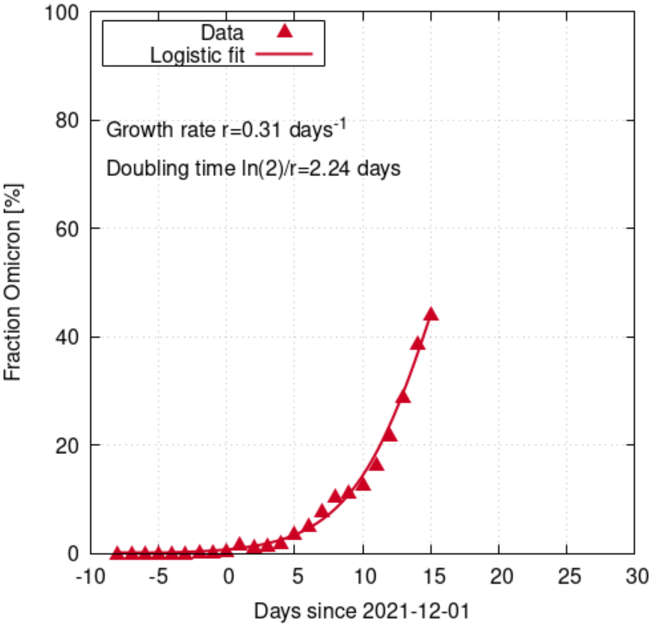




Data: UKHSA. SGTF proportions from daily update combined with case numbers from coronavirus.data.gov.uk.

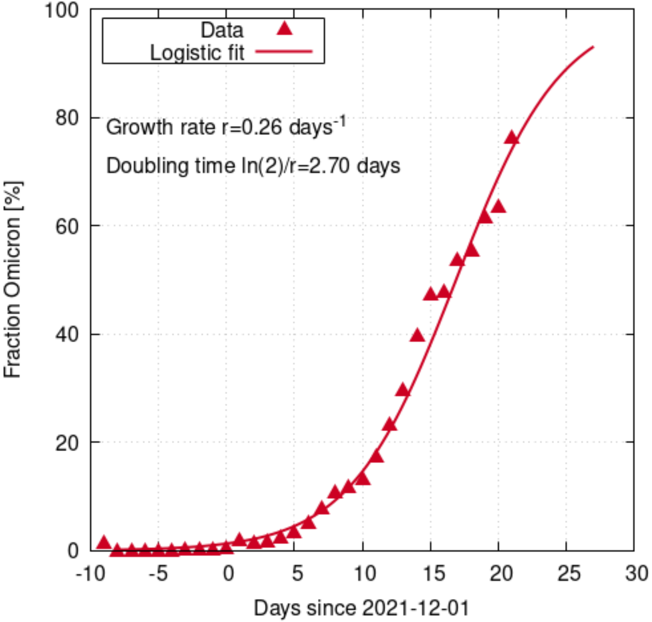
- Data from Danmark and London
- The Delta and Omicron variants coexist without directly affecting each other
- Indirect interaction via competing for common ressources, i,e., *first come, first served*

2. Rate r of the logistic growth of the Omicron share



- The share of Omicron can be well described by a logistic function with growth rate r

Update and: how to determine the rate r



- Data update.

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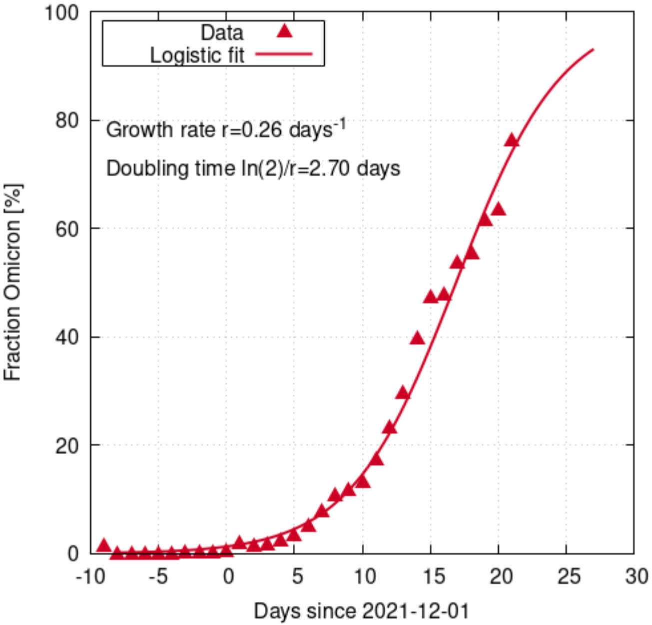

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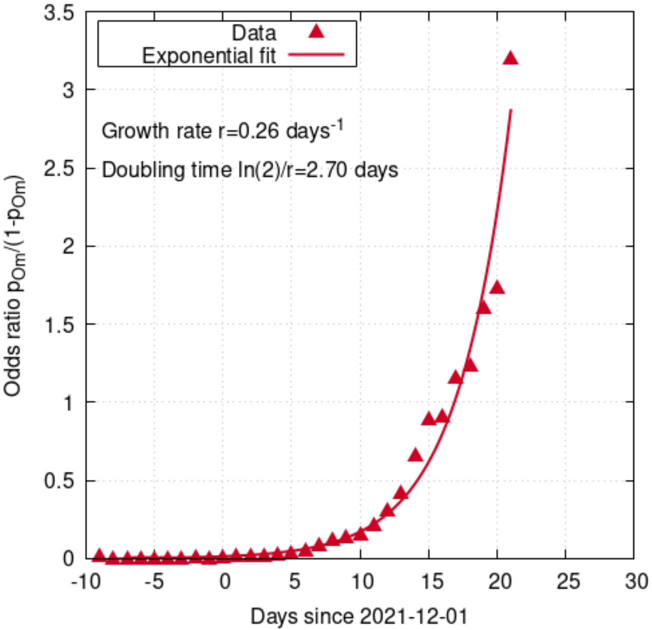

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- Transform the observed Omicron share p into the **odds ratio** $y = p/(1-p)$
- From Observation 1 (coexistence), it follows that the odds ratio grows exponentially:

$$y(t) = y_0 \, e^{rt}$$

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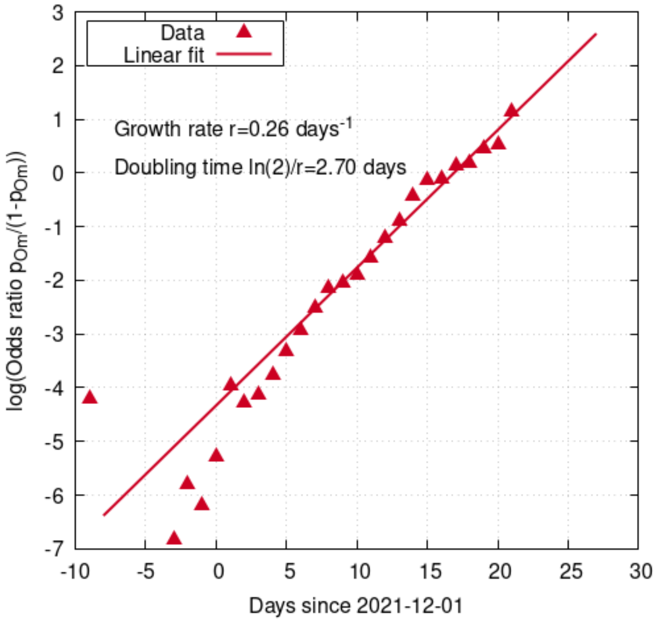

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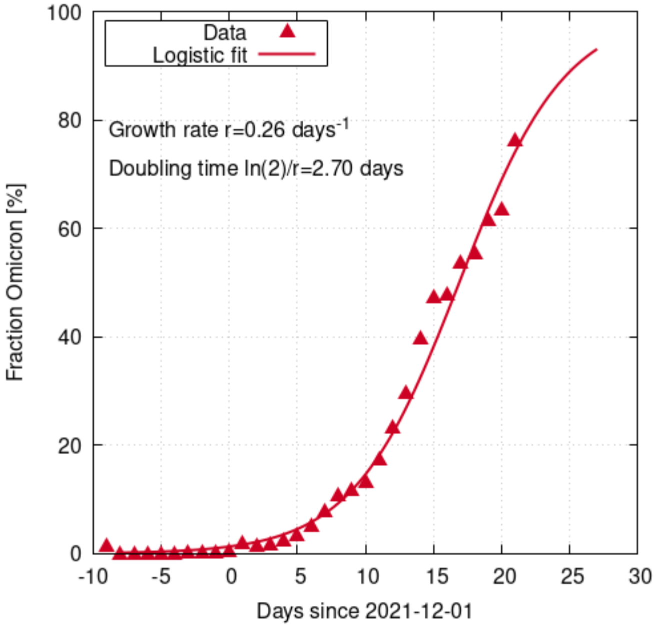

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- Transforming back gives the s-shaped predicted Omicron share (logistic function)

$$p(t) = \frac{y(t)}{1 + y(t)}$$

3. Relation between the logistic growth rate r and the reproduction numbers

Assumptions:

- Neither positive nor negative **cross effects**: Each variant acts on its own (using common ressources of susceptible humans)
- The Delta and Omicron variants have different **base reproduction numbers** R_{10} and R_{20} and different **generation times** T_1 and T_2, respectively (e.g., $R_{10} = 5$, $T_1 = 5\,\text{days}$, $T_2 = 4\,\text{days}$)
- The **immunities** I_1 and I_2 (including vaccinations and past infections) against Delta and Omicron are generally different
- The **reduction factors** f_m by isolation measures and the **seasonal factor** f_s are common
- All factors influencing the effective reproduction number R are multiplicative

$$\Rightarrow \begin{aligned} x_1(t_0+T_1) &= R_1 x_1(t_0) = R_{10}(1-I_1) f_m f_s x_1(t_0), \\ x_2(t_0+T_2) &= R_2 x_2(t_0) = R_{20}(1-I_2) f_m f_s x_2(t_0) \end{aligned} \qquad (1)$$

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Relation between the logistic growth rate r and R_1 and R_2 (ctned)

Assuming continuous infections (slowly varying rates), we can write (1) as

$$x_1(t) = x_1(0) R_1^{t/T_1} = x_1(0) \exp\left(\frac{t}{T_1} \ln R_1\right) \equiv x_1(0) \exp(r_1 t),$$

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Just use Relation (3) and insert the definitions of R_1 and R_2 from (1)

After some manipulations ...

$$R_{20} = \exp(rT_2)\, f_{\mathrm{m}}^{\gamma-1}\, f_{\mathrm{s}}^{\gamma-1}\, \frac{(R_{10}(1-I_1))^{\gamma}}{1-I_2}, \quad \gamma = \frac{T_2}{T_1} \tag{4}$$

- For equal generation times $T_1 = T_2 = T$, the measures and the seasonal effects drop out and r depends only on the past infection and vacination immunities (remains time dependent since the immunities change):

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Example: $e^{rT} = 3,\ R_{10} = 4,\ I_1 = 0.6,\ I_2 = 0.2,\ R_{20} = 3/2 R_{10} = 6$

- With neither immunities nor measures nor season effects but $T_2 = 0.5T_1$ ($\gamma = 0.5$), we have $R_{20} = e^{rT_2}\sqrt{R_{10}}$, e.g., for $e^{rT_2} = 2$ and $R_{01} = 4$, we have $R_{02} = R_{01} = 4$
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5. Effective infection growth rate and reproduction number

The **effective growth rate r_{eff}** of the infection dynamics (not to be confused with the logistic growth rate r of the Omicron shares p) comes directly from (2):

$$\dot{x} = \dot{x}_1 + \dot{x}_2 = r_1 x_1 + r_2 x_2 = \big[(1-p) r_1 + p r_2\big] x \equiv r_{\text{eff}} x$$

Effective reproduction number R_{eff} by association $r_{\text{eff}} \equiv \ln R_{\text{eff}} / T_1$:

$$\boxed{\ln R_{\text{eff}} = (1-p) \ln R_1 + \frac{p}{\gamma} \ln R_2} \qquad (5)$$

- Because $1/\gamma = T_1/T_2 > 1$, influence factors, e.g., measures, have a more sensitive effect on Omicron than om Delta: If $T_1/T_2 = 1/\gamma = 2$ and measures lead to a factor $1/\sqrt{2} \approx 0.7$ on Delta (R_1), they simultaneously lead to a factor $1/2$ on Omicron (R_2)
- If, at a certain time, the true Omicron share p, the effective reproduction number R_{eff}, and the logistic growth rate r are known (all three can be estimated), and the generation time ratio $\gamma = T_2/T_1$ as well as the total immunities I_1 and I_2 and the effects of the measures and the season at this time can be estimated, the Eqs (1), (4), and (5) allow for a simultaneous estimation of R_{10} and R_{20}

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6. Immunity I: vaccinations/boosters vs. Delta variant

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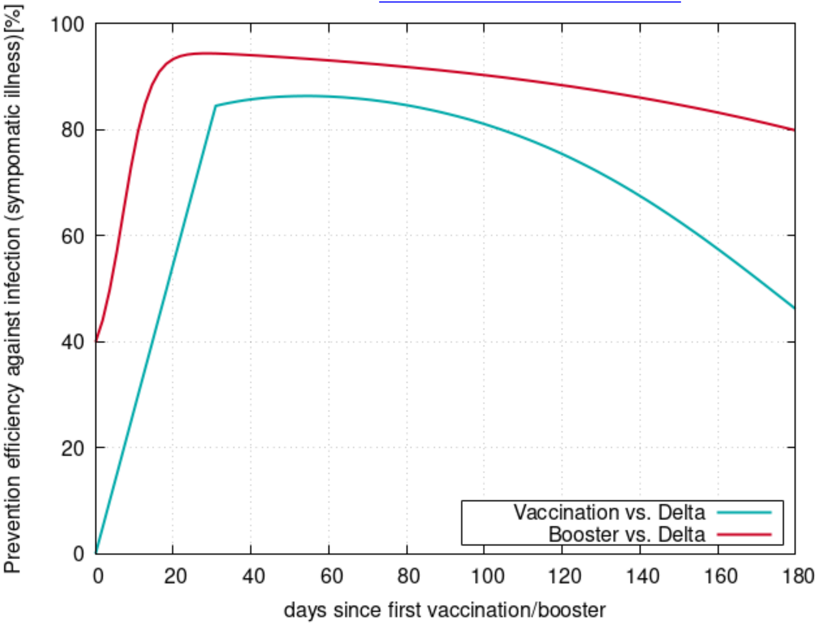
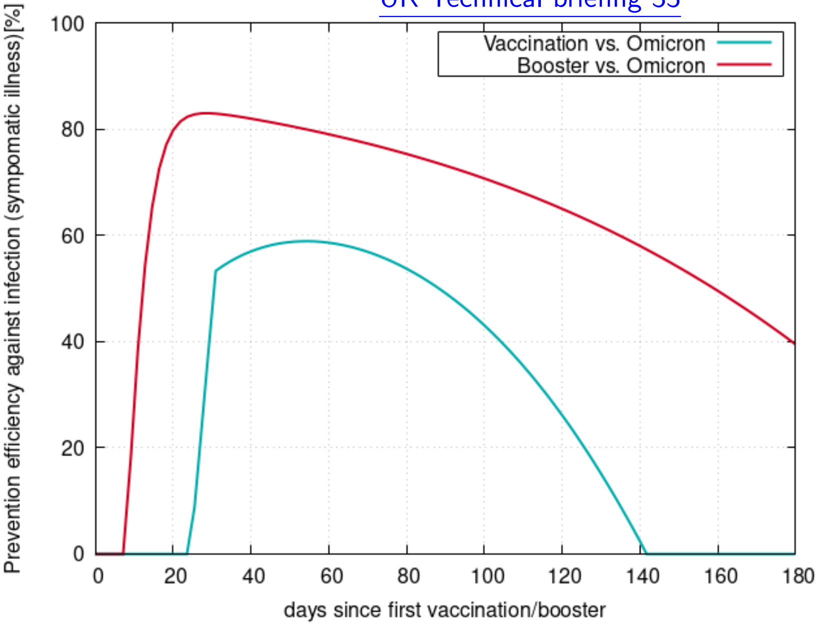


"First vaccinated-first boostered" principle

6. Immunity II: vaccinations/boosters vs. Omicron variant

UK Technical briefing 33



Only fresh full vaccinations or boosters help against Omicron

6. Immunity III: past infections

- 100 % immunity of Delta against Delta reinfections
- 100 % immunity of Omicron against Omicron reinfections
- 100 % no cross immunity (people can get both Delta and Omicron infections)

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7. Determining the population immunities in the simulator I: vaccinations

Here, I make following assumptions

- Vaccination efficiency curves $I_1^{\mathrm{v}}(\tau)$ and $I_2^{\mathrm{v}}(\tau)$ against Delta and Omega as shown,
- corresponding booster efficencies $I_1^{\mathrm{b}}(\tau)$ and $I_2^{\mathrm{b}}(\tau)$,
- *First vaccinated-first boostered*

Since the protection depends on the vaccination times, I sum up the different histories weighted with the past daily vaccination and booster rates $r_{t'}^v$ and $r_{t'}^b$ (fraction of the population per day):

$$I_1^{\mathrm{vacc}}(t) = \sum_{t'=t_v}^{t} r_{t'}^v I_1^{\mathrm{v}}(t-t') + \sum_{t'=t_b}^{t} r_{t'}^b I_1^{\mathrm{b}}(t-t')$$

where t_b is the time of the first booster shot, and t_v the oldest time of the first vaccination of any person who is not yet boostered.

The vaccination immunity I_2 is calculated in analogy.

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Determining the population immunities in the simulator II: infections and total

- Everybody can only be infected once with any variant but there is no cross immunity, so the immunity is just equal to the total percentage X_1 and X_2 of people infected with either variant:
$$I_1^x = X_1, \quad I_2^x = X_2$$
Notice: X_i is not just the cumulated number of cases divided by the population because any infection, whether detected or not detected, counts

- There is no correlation between vaccinations and infections:
$$1 - I_1 = (1 - I_1^v)(1 - I_1^x), \quad 1 - I_2 = (1 - I_2^v)(1 - I_2^x) \tag{6}$$

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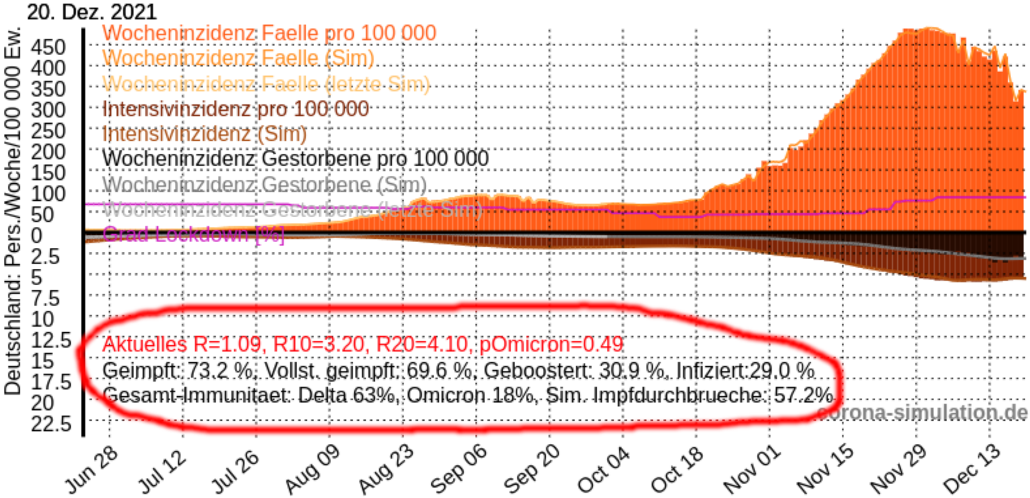
Simulation



All items I_1, I_2, p, R_{10}, R_{20}, f_{season} and $f_{\text{stringency}}$ are displayed in the simulation