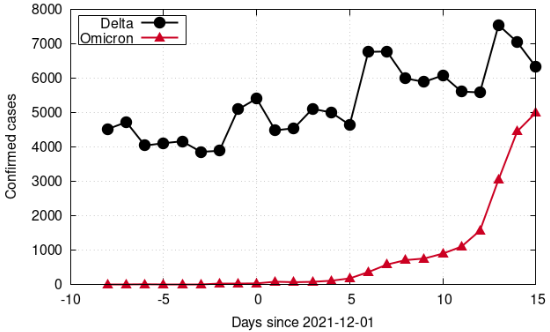
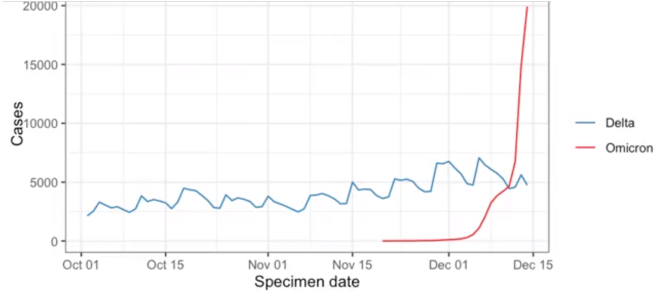
1. Delta and Omicron: Observations

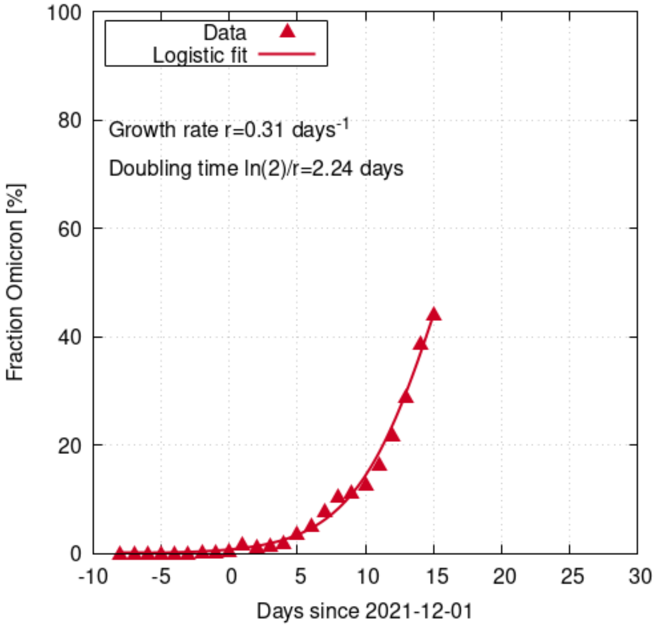




Data: UKHSA. SGTF proportions from daily update combined with case numbers from coronavirus.data.gov.uk.

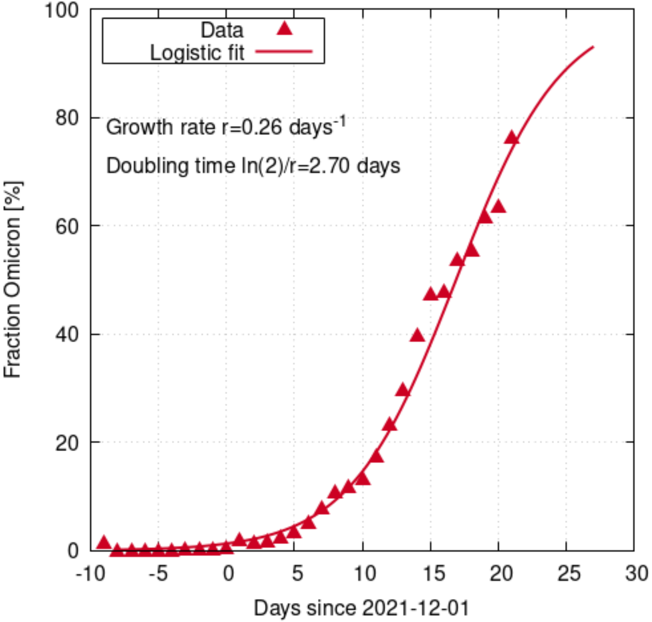
- Data from Danmark and London
- The Delta and Omicron variants coexist without directly affecting each other
- Indirect interaction via competing for common ressources, i,e., *first come, first served*

2. Rate r of the logistic growth of the Omicron share



- The share of Omicron can be well described by a logistic function with growth rate r

Update and: how to determine the rate r



- Data update.

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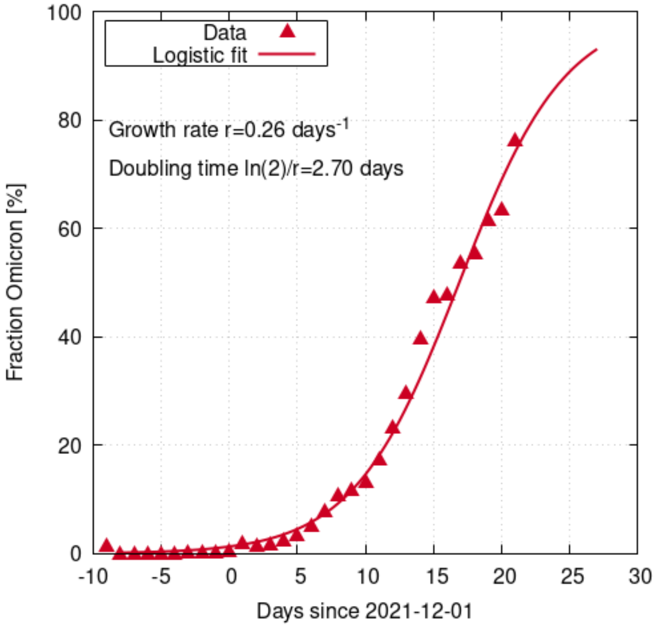

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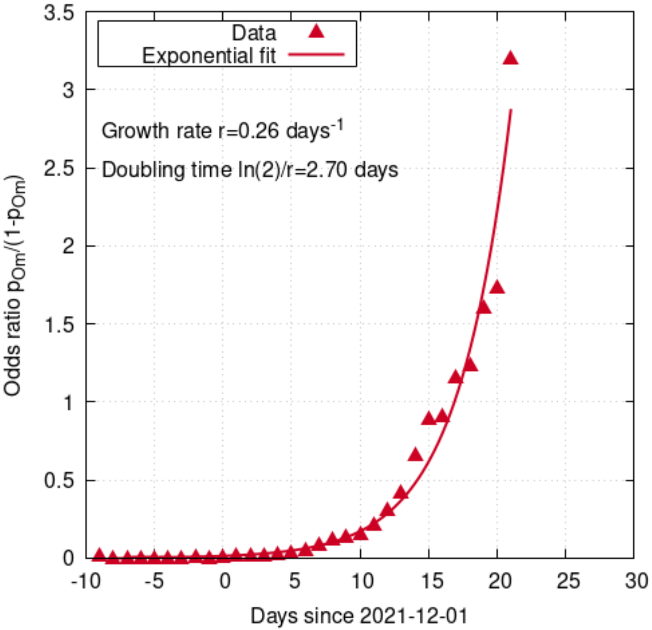

- Data update. How to get the curve?
- Transform the observed Omicron share p into the **odds ratio** $y = p/(1-p)$
- From Observation 1 (coexistence), it follows that the odds ratio grows exponentially:

$$y(t) = y_0 \, e^{rt}$$

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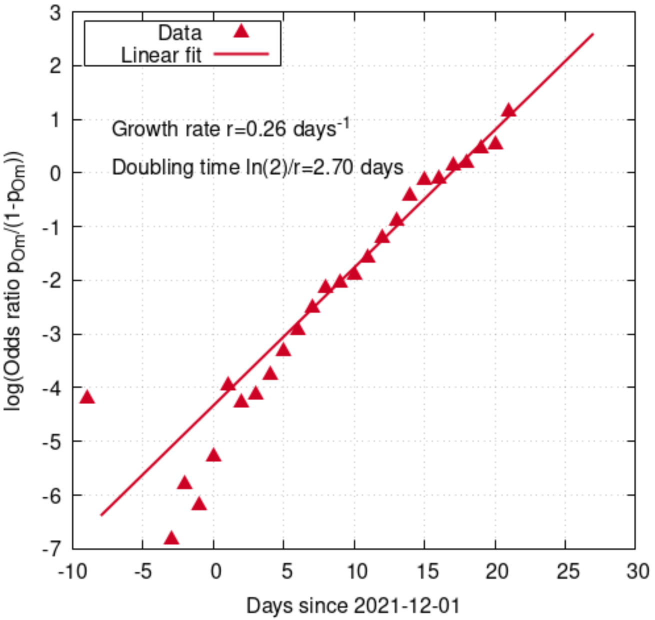

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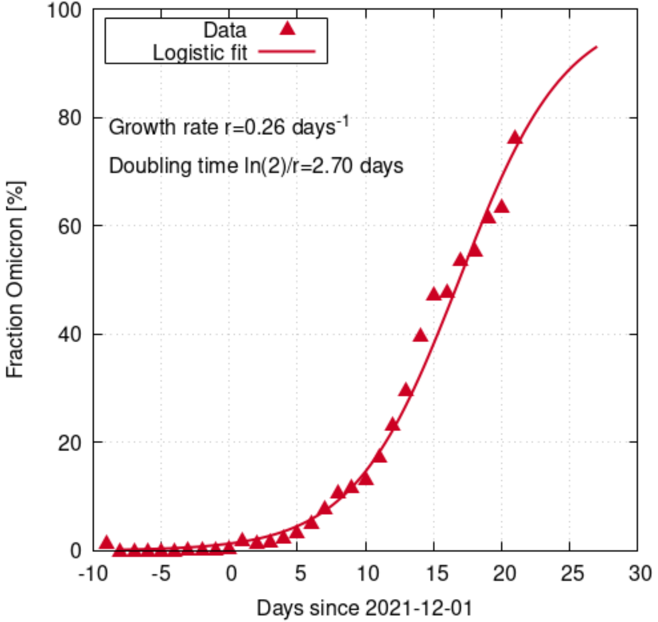

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- Transforming back gives the s-shaped predicted Omicron share (logistic function)

$$p(t) = \frac{y(t)}{1 + y(t)}$$

3. Relation between the logistic growth rate r and the reproduction numbers

Assumptions:

- Neither positive nor negative **cross effects**: Each variant acts on its own (using common ressources of susceptible humans)
- The Delta and Omicron variants have different **base reproduction numbers** R_{10} and R_{20} and different **generation times** T_1 and T_2, respectively (e.g., $R_{10} = 5$, $T_1 = 5\,\text{days}$, $T_2 = 4\,\text{days}$)
- The **immunities** I_1 and I_2 (including vaccinations and past infections) against Delta and Omicron are generally different
- The **reduction factors** f_m by isolation measures and the **seasonal factor** f_s are common
- All factors influencing the effective reproduction number R are multiplicative. Daily new infections x_1 (Delta) and x_2 (Omicron) develop according to following

Infection dynamics as $f(R_{01}, R_{02}, \text{factors})$:

$$\begin{aligned} x_1(t_0+T_1) &= R_1 x_1(t_0) = R_{10}(1-I_1) f_\text{m} f_\text{s} x_1(t_0), \\ x_2(t_0+T_2) &= R_2 x_2(t_0) = R_{20}(1-I_2) f_\text{m} f_\text{s} x_2(t_0) \end{aligned} \tag{1}$$

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Relation between the logistic growth rate r and R_1 and R_2 (ctned)

Assuming continuous infections (slowly varying rates), we can write (1) as

$$x_1(t) = x_1(0) {R_1}^{t/T_1} = x_1(0) \exp\left(\frac{t}{T_1} \ln R_1\right) \equiv x_1(0) \exp(r_1 t),$$

$x_2(t)$ likewise, so $\dot{x}_1 = r_1 x_1$ and $\dot{x}_2 = r_2 x_2$ defining the

$$\textbf{Effective infection growth rates:} \quad r_1 = \frac{\ln R_1}{T_1}, \quad r_2 = \frac{\ln R_2}{T_2} \qquad (2)$$

How does the odds $y = x_2/x_1$ evolve?

$$\begin{aligned} \dot{y} &= \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{x_2}{x_1}\right) = \frac{\dot{x}_2}{x_1} - \frac{x_2}{x_1^2}\dot{x}_1 \\ &= \frac{r_2 x_2}{x_1} - \frac{x_2}{x_1^2} r_1 x_1 = (r_2 - r_1) y \end{aligned}$$

$$\Rightarrow \quad \textbf{Omicron spreading rate:} \quad r = r_2 - r_1 = \frac{\ln R_2}{T_2} - \frac{\ln R_1}{T_1} \qquad (3)$$

Special case $T_1 = T_2 = T$: $r = 1/T \ln(R_2/R_1)$: the spreading rate is proportional to the *ratio* of the actual reproduction numbers

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4. Determining the Omicron base reproduction rate from the logistic growth rate

Just use Relation (3) and insert the dependence of the actual growth rates R_1 and R_2 as a function of the base reproduction numbers R_{01}, R_{02} and factors from (1)

After some manipulations ...

Observed Omicron R_{02}: $$R_{20} = e^{rT_2}(f_\mathrm{m} f_\mathrm{s})^{\gamma-1} \frac{(R_{10}(1-I_1))^\gamma}{1-I_2}, \quad \gamma = \frac{T_2}{T_1} \tag{4}$$

- For equal generation times $T_1 = T_2 = T$, the measures and the seasonal effects drop out and r depends only on the past infection and vacination immunities (remains time dependent since the immunities change):

$$T_1 = T_2 = T \quad \Rightarrow R_{20} = e^{rT} \frac{R_{10}(1-I_1)}{1-I_2}$$

Example: $e^{rT} = 3,\ R_{10} = 4,\ I_1 = 0.6,\ I_2 = 0.2,\ R_{20} = 3/2R_{10} = 6$

- With neither immunities nor measures nor season effects but $T_2 = 0.5T_1$ ($\gamma = 0.5$), we have $R_{20} = e^{rT_2}\sqrt{R_{10}}$, e.g., for $e^{rT_2} = 2$ and $R_{01} = 4$, we have $R_{02} = R_{01} = 4$
- With measures/season effects and immunities as above, we may have $R_{02} < R_{01}$

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After some manipulations ...

Observed Omicron R_{02}: $$R_{20} = e^{rT_2} (f_\mathrm{m} f_\mathrm{s})^{\gamma-1} \frac{(R_{10}(1-I_1))^\gamma}{1-I_2}, \quad \gamma = \frac{T_2}{T_1} \tag{4}$$

- For equal generation times $T_1 = T_2 = T$, the measures and the seasonal effects drop out and r depends only on the past infection and vacination immunities (remains time dependent since the immunities change):

$$T_1 = T_2 = T \quad \Rightarrow R_{20} = e^{rT} \frac{R_{10}(1-I_1)}{1-I_2}$$

Example: $e^{rT} = 3$, $R_{10} = 4$, $I_1 = 0.6$, $I_2 = 0.2$, $R_{20} = 3/2 R_{10} = 6$

- With neither immunities nor measures nor season effects but $T_2 = 0.5 T_1$ ($\gamma = 0.5$), we have $R_{20} = e^{rT_2}\sqrt{R_{10}}$, e.g., for $e^{rT_2} = 2$ and $R_{01} = 4$, we have $R_{02} = R_{01} = 4$
- With measures/season effects and immunities as above, we may have $R_{02} < R_{01}$

4. Determining the Omicron base reproduction rate from the logistic growth rate

Just use Relation (3) and insert the dependence of the actual growth rates R_1 and R_2 as a function of the base reproduction numbers R_{01}, R_{02} and factors from (1)
After some manipulations ...

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5. Effective infection growth rate and reproduction number

The **effective growth rate r_{eff}** of the infection dynamics (not to be confused with the logistic growth rate r of the Omicron shares p) comes directly from (2):

$$\dot{x} = \dot{x}_1 + \dot{x}_2 = r_1 x_1 + r_2 x_2 = \Big[(1-p)r_1 + pr_2\Big]x \equiv r_{\text{eff}} x$$

Associating r_{eff} with $\ln R_{\text{eff}}/T_1$, we get the

$$\textbf{Effective reproduction number:} \quad R_{\text{eff}} = R_1^{1-p}\ R_2^{p/\gamma} \tag{5}$$

- Because $1/\gamma = T_1/T_2 > 1$, influence factors influencing R_1 and R_2 according to (1) have a more sensitive effect on Omicron than on Delta: If $T_1/T_2 = 1/\gamma = 2$ and measures lead to a factor $1/\sqrt{2} \approx 0.7$ on Delta (R_1), they simultaneously lead to a factor 1/2 on Omicron (R_2)
- If, at a certain time, the true Omicron share p, the effective reproduction number R_{eff}, and the logistic growth rate r are known (all three can be estimated), and $\gamma = T_2/T_1$, I_1, I_2, and the effects of the measures and the season are approximatively known, Eqs (1), (4), and (5) allow for a simultaneous estimation of R_{10} and R_{20}
- Later on, they allow for a simultaneous estimation of the time varying Omicron spreading rate r, the Omicron share p, and the growth rates r_1, r_2, and R_{eff}

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5a. Effective infection growth rate vs Omicron spreading rate

From (3), we obtain

$$e^{rT_1} = R_2^{1/\gamma} R_1^{-1}$$

Eliminating R_2 with (5), we get a relation between the effective infection growth rate R_{eff}, the Omicron spreading rate r, and the actual reproduction number R_1 of the "old" variant:

$$\boxed{R_{\text{eff}} = R_1 e^{rT_1 p}} \tag{6}$$

5b. Determining dynamics based on Dutch reference

- Dutch has excellent time series for Omicron share p from which to determine r and incidences from which to estimate R_{eff}.
- For other countries, only R_{eff} can be estimated while r (which varies with the country as a function of I_1, I_2 and f_{m}) is not are known
- From (1) and (3), we obtain for Holland (prefarably at a time where $p \approx 0.5$) the base reproduction ratio

$$\boxed{\frac{R_{20}}{R_{10}^{\gamma}} = \frac{e^{rT_2}(1-I_1)^{\gamma}(f_{\text{m}} f_{\text{s}})^{\gamma-1}}{1-I_2} \equiv \alpha} \tag{7}$$

- For other countries, R_{01} and R_{02} may be different (because of behavior, pop density etc) but we assume α=const., so using (5), (1), and (7), we get at mutation initialisation, independent from r (but of course we need p)

$$\boxed{\frac{R_{\text{eff}}}{R_{10}} = (\alpha(1-I_2))^{p/\gamma}(1-I_1)^{1-p}(f_{\text{m}} f_{\text{s}})^{p(1/\gamma-1)+1}, \quad R_{20} = \alpha R_{10}^{\gamma}} \tag{8}$$

- After initialisation, just calculate R_1, R_2 from (1), r from (3), the new Omicron share via the odds update $\dot{y} = ry$, $p = y/(1+y)$, and the new effective reproduction number R_{eff} from (5).

6. Immunity I: vaccinations/boosters vs. Delta variant

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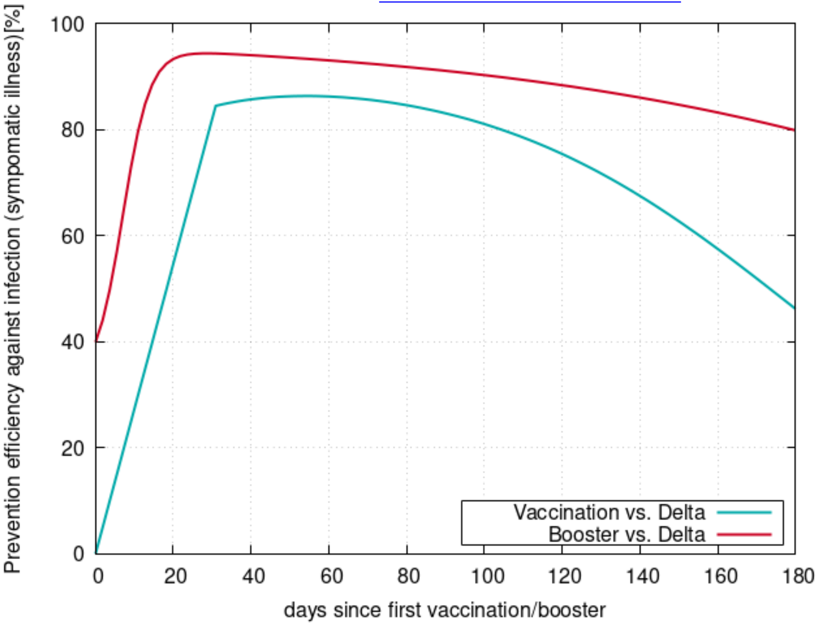
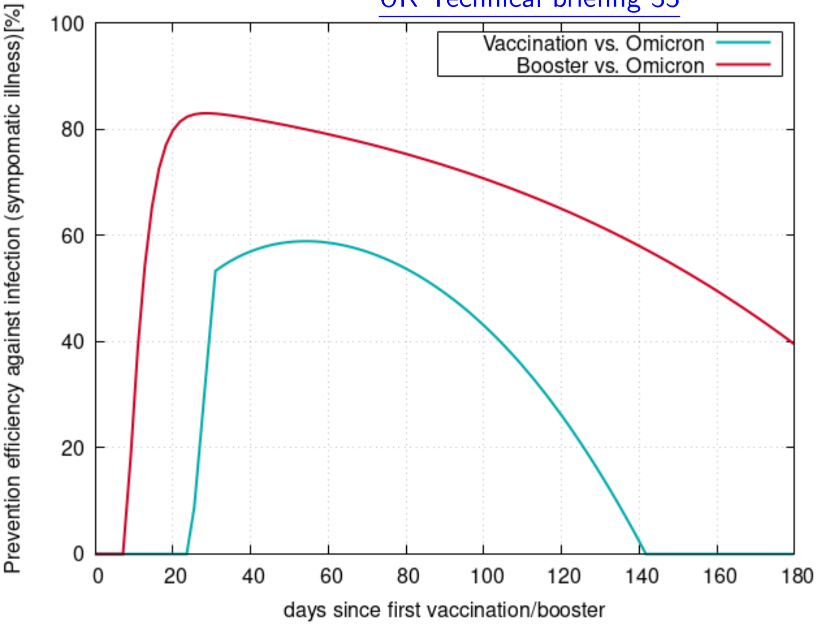


"First vaccinated-first boostered" principle

6. Immunity II: vaccinations/boosters vs. Omicron variant

UK Technical briefing 33



Only fresh full vaccinations or boosters help against Omicron

6. Immunity III: past infections

- 100 % immunity of Delta against Delta reinfections
- 100 % immunity of Omicron against Omicron reinfections
- 100 % no cross immunity (people can get both Delta and Omicron infections)

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7. Determining the population immunities in the simulator I: vaccinations

Here, I make following assumptions

- Vaccination efficiency curves $I_1^{\mathrm{v}}(\tau)$ and $I_2^{\mathrm{v}}(\tau)$ against Delta and Omega as shown,
- corresponding booster efficencies $I_1^{\mathrm{b}}(\tau)$ and $I_2^{\mathrm{b}}(\tau)$,
- *First vaccinated-first boostered*

Since the protection depends on the vaccination times, I sum up the different histories weighted with the past daily vaccination and booster rates $r_{t'}^{v}$ and $r_{t'}^{b}$ (fraction of the population per day):

$$I_1^{\mathrm{vacc}}(t) = \sum_{t'=t_v}^{t} r_{t'}^{v} I_1^{\mathrm{v}}(t-t') + \sum_{t'=t_b}^{t} r_{t'}^{b} I_1^{\mathrm{b}}(t-t')$$

where t_b is the time of the first booster shot, and t_v the oldest time of the first vaccination of any person who is not yet boostered.

The vaccination immunity I_2 is calculated in analogy.

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Determining the population immunities in the simulator II: infections and total

- Everybody can only be infected once with any variant but there is no cross immunity, so the immunity is just equal to the total percentage X_1 and X_2 of people infected with either variant:

$$I_1^x = X_1, \quad I_2^x = X_2$$

Notice: X_i is not just the cumulated number of cases divided by the population because any infection, whether detected or not detected, counts

- There is no correlation between vaccinations and infections:

$$1 - I_1 = (1 - I_1^v)(1 - I_1^x), \quad 1 - I_2 = (1 - I_2^v)(1 - I_2^x) \tag{9}$$

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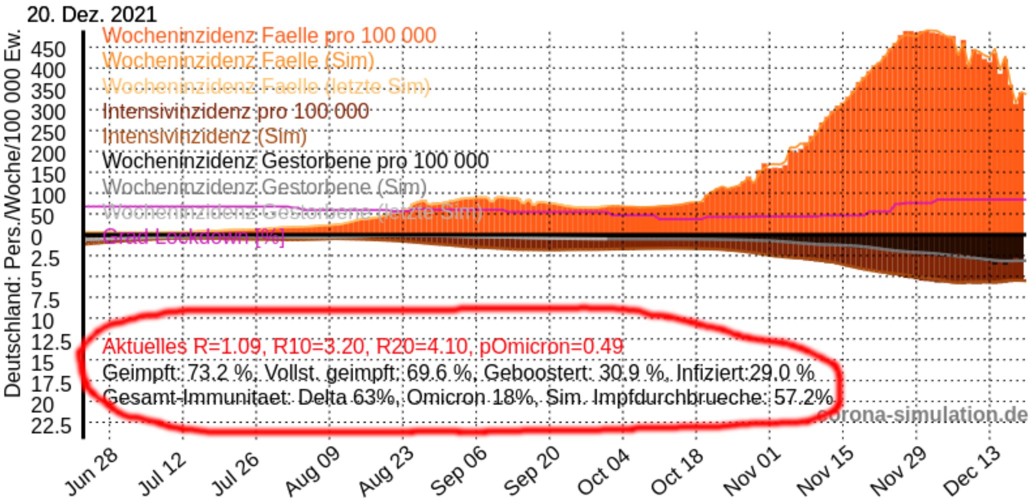
Simulation



All items I_1, I_2, p, R_{10}, R_{20}, f_{season} and $f_{\text{stringency}}$ are displayed in the simulation