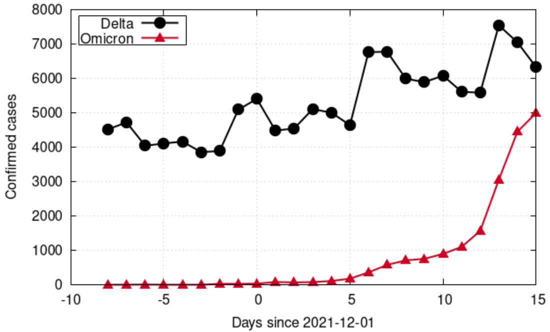
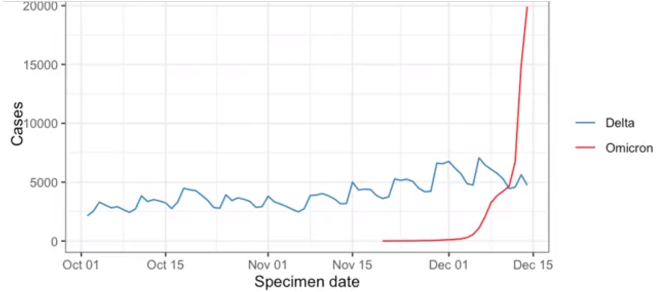
Latest observation 1

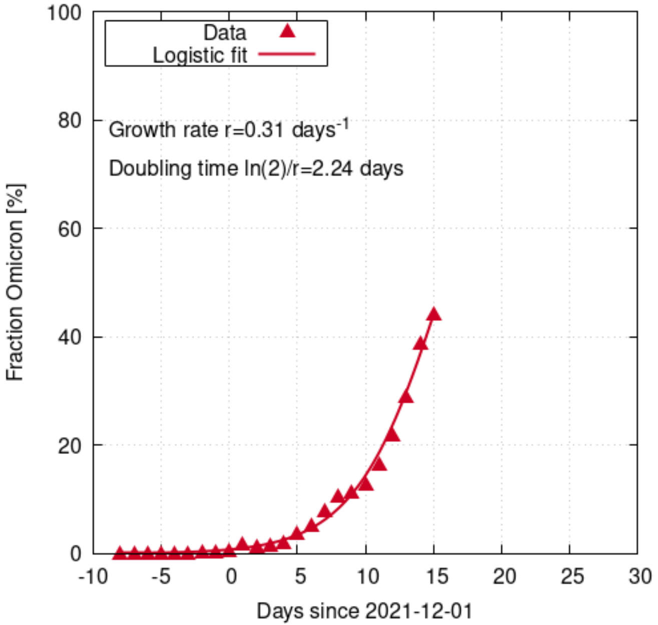




Data: UKHSA. SGTF proportions from daily update combined with case numbers from coronavirus.data.gov.uk.

- Data from Danmark and London
- The Delta and Omicron variants coexist without directly affecting each other
- Indirect interaction via competing for common ressources, i,e., *first come, first served*

Latest observation 2



- The share of Omicron can be well described by a logistic function with growth rate r
- First transform the observed Omicron share p into the **odds ratio** $y = p/(1-p)$
- From Observation 1 (coexistence), it follows that the odds ratio grows exponentially:

$$y(t) = y_0 \, e^{rt}$$

- Transforming back gives the s-shaped predicted Omicron share (logistic function)

$$p(t) = \frac{y(t)}{1 + y(t)}$$

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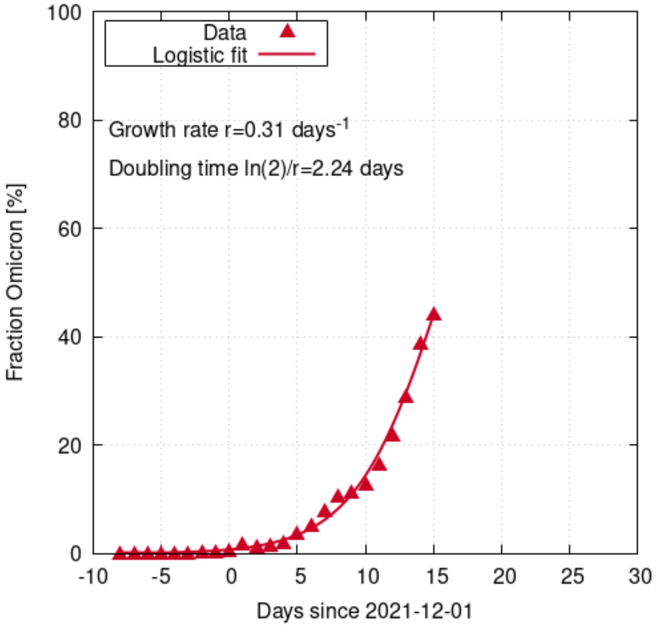

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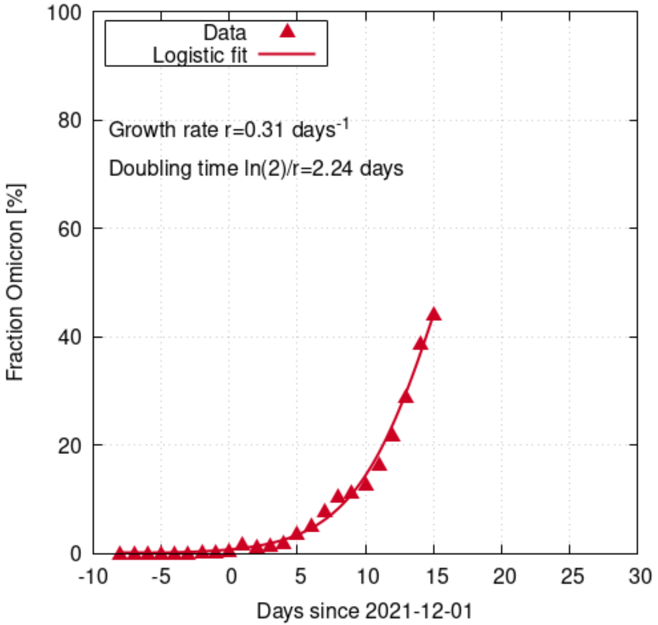

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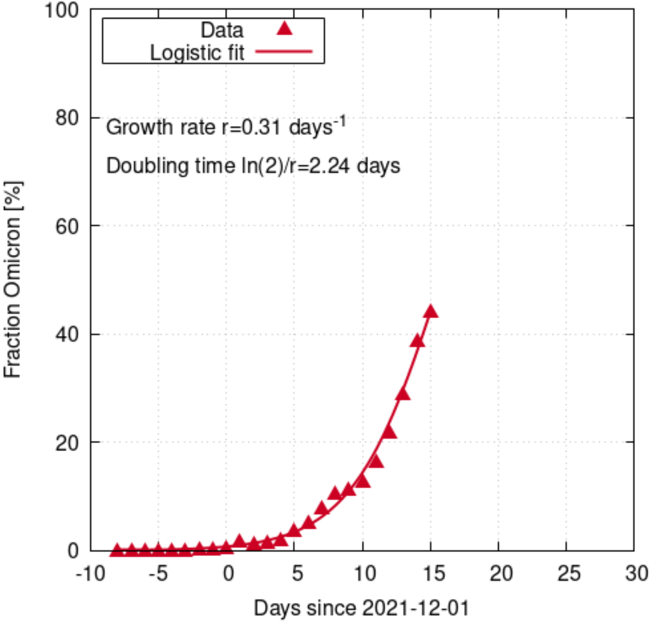

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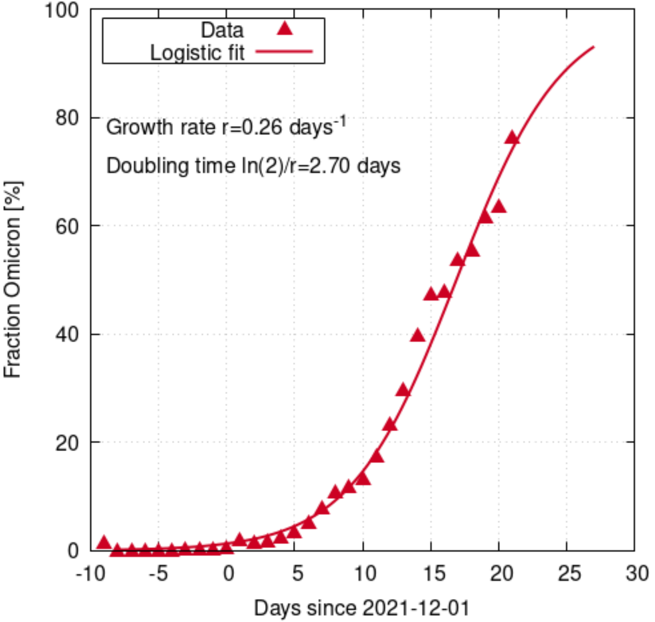

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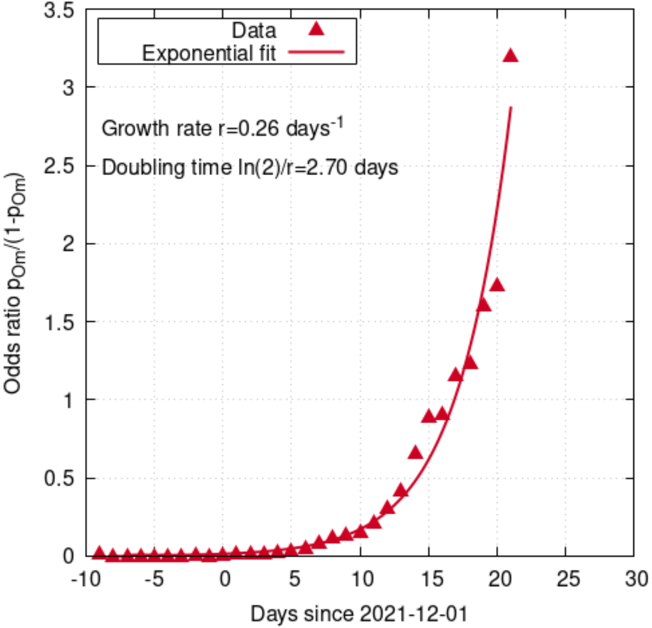

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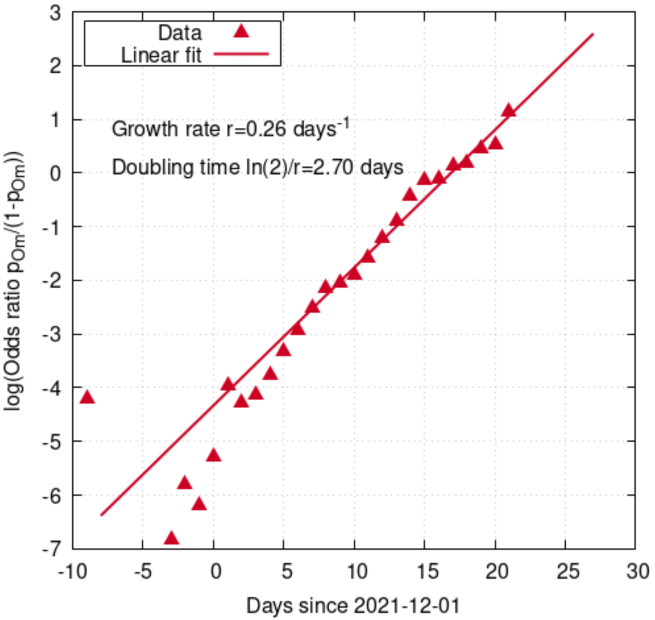

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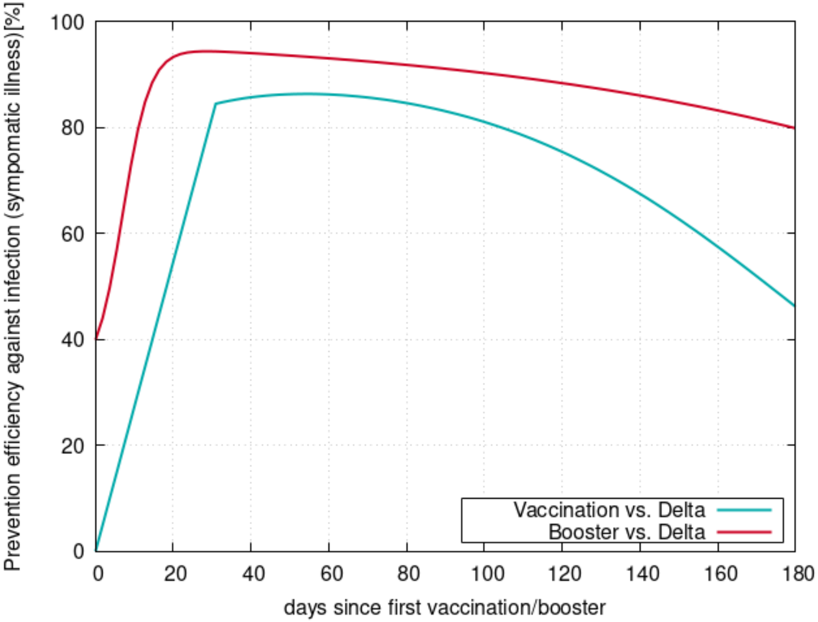
- This means, the **log-odds** are essentially linear in time:

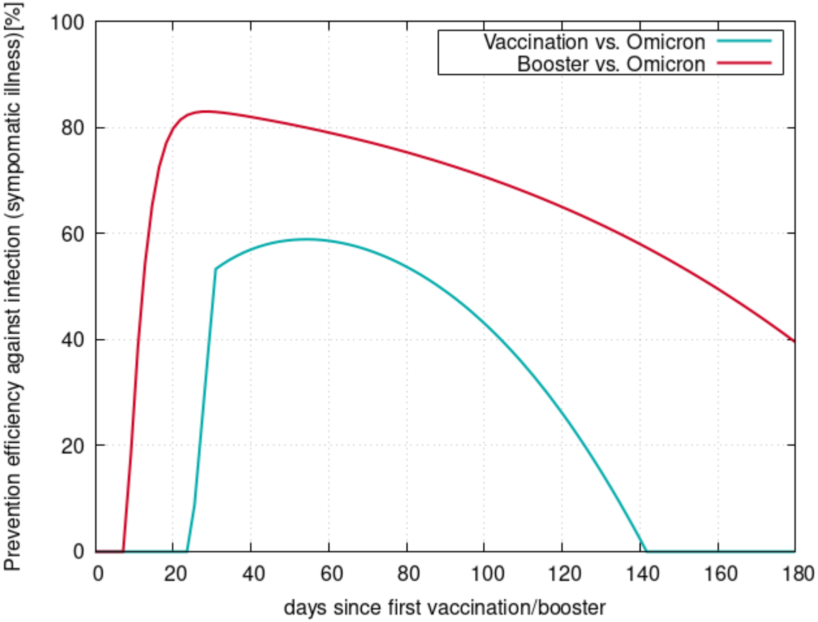
$$\ln y(t) = \ln y_0 + rt$$

Assumed efficiency against Delta infections



"First vaccinated-first boostered" principle

Assumed efficiency against Omicron infections



Only fresh full vaccinations or boosters help against Omicron

Assumed immunity by infections

- 100 % immunity of Delta against Delta reinfections
- 100 % immunity of Omicron against Omicron reinfections
- 100 % no cross immunity (people can get both Delta and Omicron infections)

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Putting it all together

(1) The spreading of the fraction p new variant (Omicron) at the cost of the old variants (Delta) is logistic:

$$\boxed{p(t) = \frac{y_0 e^{rt}}{1 + y_0 e^{rt}}, \quad y_0 = \frac{p_0}{1 - p_0}}$$

(2) The growth rate r of the logistic growth depends on

- the base reproduction numbers R_{10} and R_{20},
- the total population immunities (vaccination, boosters, infections) I_1 and I_2 against each variant,
- the generation time T of the infections (assumed to be equal, 5 days):

according to

$$r = \frac{1}{T}\left[\frac{R_{20}(1 - I_2)}{R_{10}(1 - I_1)} - 1\right] \qquad (1)$$

Notice: Since the I_i are time dependent, so is the growth rate r.

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Putting it all together (ctned)

(3) Assuming no cross effects, the effective R_t value caused by the mixture of the Delta and Omicron viruses depends on

- the Omicron percentage p
- the base reproduction numbers R_{10} and R_{20} of Delta and Omicron, respectively,
- the immunity escape fractions $(1-I_1)$ and $(1-I_2)$,
- the seasonal multiplicator f^{season}
- the stringency (lockdown) multiplicator $f^{\text{stringency}}$:

$$\boxed{R_t = [(1-p)R_{10}(1-I_1) + pR_{20}(1-I_2)]\, f_{\text{season}} f_{\text{stringency}}} \tag{2}$$

- *Notice:* Both the growth rate r of the increase of the Delta share and the growth rate $(R_t-1)/T$ of the actual infection wave depend on $P_1 = R_{10}(1-I_1)$ and $P_2 = R_{20}(1-I_2)$ such that a positive (negative) value of r implies an increase (decrease) of R_t
- A positive Omicron spreading r does not imply that the new variant is more infectious; only the products P_1 and P_2 matter
- A positive spreading r can be related to a negative infection growth $(R_t-1)/T$ both before *and* after the new variant dominates since, for given I_1 and I_2, r is only related to the *ratio* R_{20}/R_{10} of the base immunities $\Rightarrow$ next slide

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Estimating the base reproduction numbers

Assume at time $t = t^*$ given population immunities I_1^* and I_2^* (see later), and estimations of the Omega share p^*, Omicron spreading rate r^*, and effective reproduction number R_t^* of the mixture. Then, we can use Eq (1) to obtain the ratio

$$\boxed{\frac{R_{20}}{R_{10}} = \frac{(r^*T+1)(1-I_1^*)}{1-I_2^*}} \tag{3}$$

and, with Eq (2) determine the base reproduction numbers individually:

$$\boxed{R_{10} = \frac{R_t^*}{(1+p^*r^*T) f_{\text{season}} f_{\text{stringency}}}} \tag{4}$$

For $t > t^*$, I assume fixed base reproduction numbers and calculate the future Omicron spreading and the future wave using (1) and (2) with time varying I_1, I_2, p, f_{season}, and $f_{\text{stringency}}$

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Determining the population immunities I: vaccinations

Here, I make following assumptions

- Vaccination efficiency curves $I_1^v(\tau)$ and $I_2^v(\tau)$ against Delta and Omega as shown,
- corresponding booster efficencies $I_1^b(\tau)$ and $I_2^b(\tau)$,
- *First vaccinated-first boostered*

Since the protection depends on the vaccination times, I sum up the different histories weighted with the past daily vaccination and booster rates $r_{t'}^v$ and $r_{t'}^b$ (fraction of the population per day):

$$I_1^{\text{vacc}}(t) = \sum_{t'=t_v}^{t} r_{t'}^v I_1^v(t-t') + \sum_{t'=t_b}^{t} r_{t'}^b I_1^b(t-t')$$

where t_b is the time of the first booster shot, and t_v the oldest time of the first vaccination of any person who is not yet boostered.

The vaccination immunity I_2 is calculated in analogy.

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Determining the population immunities II: infections and total

- Everybody can only be infected once with any variant but there is no cross immunity, so the immunity is just equal to the total percentage X_1 and X_2 of people infected with either variant:

$$I_1^x = X_1, \quad I_2^x = X_2$$

Notice: X_i is not just the cumulated number of cases divided by the population because any infection, whether detected or not detected, counts

- There is no correlation between vaccinations and infections:

$$1 - I_1 = (1 - I_1^v)(1 - I_1^x), \quad 1 - I_2 = (1 - I_2^v)(1 - I_2^x) \tag{5}$$

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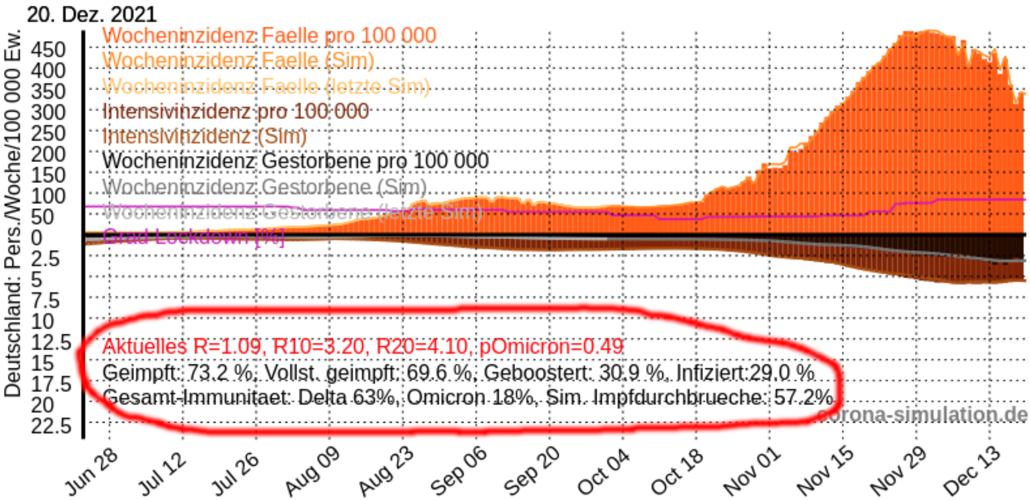
Simulation



All items I_1, I_2, p, R_{10}, R_{20}, f_{season} and $f_{\text{stringency}}$ are displayed in the simulation