# Formulation and validation of a car-following model based on reinforcement learning

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#### Abstract

To be written at the end

Keywords: reinforcement learning, car-following model, stochastic processes, string stability, validation, trajectory data

#### 1. Introduction

[problem statement]

[references for state-of-the art] references RL: [1, 2] references classical, ACC, stochastic CF model: [3, 4, 5] references AR(1), e.g. [6]

[central statement] To our knowledge, no neuronal-network car-following model has been proposed that considers such high safety and comfort standards, that considers the transition between free driving and car following and that shows string-stability. In this contribution, we propose a novel reinforcement learning (RL) car-following model that is trained on leading-vehicle trajectories generated by an AR-1 process with parameters reflecting the kinematics of real leaders. We validate the trained model on experimental and naturalistic trajectory data, and on artifical speed profiles bringing the model to its limits. In all cases, the model proved to be accident free and string stable. Unlike other variants of RL car-following models our approach considers a wider range of possible accelerations in a way, that full-braking scenarios can be successfully mastered.

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Also, unlike other variants of AI models such as LSTM models trained on realistic data, the proposed model is not completely blackbox since the reinforcement learning reward function reflects driving style attributes such as desired time gap and speed, maximum acceleration, and comfortable deceleration.

[short textual enumeration of the sections to come]

#### 2. Model specification

The Follower Vehicle is designed to be controlled by a Reinforcement Learning (RL) agent. By interaction with an environment, the RL agent optimizes a sequential decision making problem. At each time step t the agent observes an environment state  $s_t$ , and based on that state selects an action  $a_t$ . After conducting action  $a_t$ , the RL agent receives a reward  $r(a_t, s_t)$ . The agent aims to learn an optimal state-action mapping policy  $\pi$  that maximizes the expected accumulated discounted reward  $r_t = \sum_{k=0}^{\infty} \gamma^k r_{t+k}$ , with  $\gamma = (0,1]$  describing the discount factor. The crucial elements of the Reinforcement Learning based control strategy are described in detail as follows:

#### 2.1. Action space

In this study, the acceleration of the Follower Vehicle is considered as the action of the RL agent. To maintain comfortable driving and to allow full-brake in safety-critical situations the acceleration is a continuous variable between  $a_{min} = -9m/s^2$  and  $a_{max} = 2m/s^2$ .

### 2.2. State space

The state space defines the observations, the Follower Vehicle can receive from the environment. To compute an optimal acceleration, the Follower Vehicle observes its own acceleration a, its own velocity v, the difference velocity  $\Delta v$  and the spatial distance to the Leading Vehicle s. These observations are normalized to the range [-1,1].

Table 1: RL agent parameters

Parameter	Description	Value
$a_{min}$	Minimum acceleration	$-9m/s^{2}$
$a_{max}$	Maximum acceleration	$2m/s^2$
$b_{comf}$	Comfortable deceleration	$-2m/s^2$
$v_{des}$	Desired velocity	16.6m/s
$T_{gap}$	Desired time gap to Leader	1.5s
$s_{min}$	Desired minimum distance to Leader	2m
$T_{var}$	Time gap to describe the variance of the normal	0.7s
	probability function (see Equation 1 - 4)	
$T_{lim}$	Upper time gap limit for zero reward (see Equation	15s
	1 - 4)	

#### 2.3. Reward Function

The goal of the RL strategy is to reduce the crash risk, while maintaining comfortable driving in non-safety-critical situations. The Reward function is based on a set of parameters, that can be adjusted to realize different driver styles.  $v_{des}$  is the desired velocity, that should not be exceeded.  $a_{min}$  and  $a_{max}$  are the minimum and maximum possible accelerations, as described in Section 2.1. All parameters are described in Table 1.

The reward function consists of four parts, described as follows:

$$r_{1} = \begin{cases} \frac{normpdf(s, s_{opt}, s_{var})}{normpdf(s_{opt}, s_{opt}, s_{var})}, & \text{if } s < s^{*} \\ \frac{normpdf(s^{*}, s_{opt}, s_{var})}{normpdf(s_{opt}, s_{opt}, s_{var})} \left(1 - \frac{s - s^{*}}{s_{lim} - s^{*}}\right) & \text{otherwise} \end{cases}$$
(1)

with

$$s_{opt} = vT_{qap} + s_{min}, (2)$$

$$s_{var} = vT_{var} + 0.5s_{min}, (3)$$

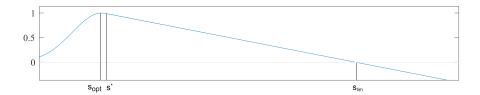


Figure 1: Reward function part 1 maximizes the reward for car following with time gap  $T_{gap}$ .

$$s_{lim} = vT_{lim} + 2s_{min},\tag{4}$$

and  $normpdf(x, \mu, \sigma^2)$  describing a normal probability density function.

The first part of the reward function aims to maintain a reasonable distance to the Leader Vehicle. Figure 1 illustrates the reward function for  $r_1$ , containing the parameter  $s_{opt}$ ,  $s^*$  and  $s_{lim}$ . The reward function is designed in a way, that for high velocities v of the Follower Vehicle the time gap between Follower and Leader Vehicle tends to  $T_{gap}$ , while for low velocities the distance between both tends to  $s_{min}$ . Different values of  $T_{opt}$  result in different driving styles in a way, that for higher values of  $T_{opt}$  the Follower drives up closer, resulting in a more aggressive driving style. The results for different values of  $T_{opt}$  can be found in Section 4.4. Different functions for  $s > s^*$  has been applied, but the best results regarding a smooth and comfortable approaching of the Follower Vehicle has been reached with a linear function. Also, a high value of  $T_{lim}$  results in an early deceleration and comfortable approaching.

$$r_2 = \begin{cases} \tanh\left(\frac{b_{kin} - b_{comf}}{a_{min}}\right), & \text{if } b_{kin} > b_{comf} \\ 0, & \text{otherwise} \end{cases}$$
 (5)

with

$$b_{kin} = \frac{\Delta v^2}{s} \tag{6}$$

The second part of the reward function addresses the vehicle behavior in safetycritical situations. For a deceleration with the delay  $b_{kin}$  the braking distance is equal to the current distance s. Thus, the kinematic deceleration  $b_{kin}$  represents the minimum deceleration necessary to avoid a collision. A situation is considered as "safety-critical", if the kinematic deceleration  $b_{kin}$  is greater than the comfortable deceleration  $b_{comf}$ . Thus, just in safety-critical situations the RL agent is getting penalized, illustrated in figure xy.

$$r_3 = -\left(\frac{da}{dt}\right)^2\tag{7}$$

The third part of the reward function aims to reduce the jerk in order to achieve a comfortable driving.

$$r_4 = \begin{cases} -min\left(1, (v - v_{des})^2\right), & \text{if } v > v_{des} \\ 0, & \text{otherwise} \end{cases}$$
 (8)

The fourth part of the reward function penalizes the RL agent, if the current velocity v is above the desired velocity  $v_{des}$ .

$$r = 0.6r_1 + 1.1r_2 + 0.001r_3 + r_4 \tag{9}$$

The weights of each reward part has been found experimentally and can further be optimized in future studies.

#### 2.4. RL algorithm

In various similar control problems, the Deep Deterministic Policy Gradient (DDPG) Algorithm has been used and proven to perform well on tasks with a continuous action and state space (see xyz). In order to reduce the variance of policy gradients and increase learning speed, DDPG is an actor-critic method. The actor determines the action, while the critic judges about the quality of the action and how the policy should be adjusted. ([-; original paper DDPG])Both, actor and critic, are implemented as neural networks. In this study, both networks are feed-forward neural networks with two layers of hidden neurons and 32 neurons each hidden layer. All DDPG parameters are presented in Table 2.

Table 2: DDPG parameter values

Parameter	Value
Learning rate	0.001
Reward discount factor	0.95
Experience buffer length	100000
Mini batch size	32
Gaussian noise mean	0.15
Gaussian noise variance	0.2
Gaussian noise variance decay	1e-5
Number of hidden layers	2
Neurons per hidden layer	32

#### 2.5. Reward function

learning input (leader speed time series)

[also relate parameters to driving style attributes such as desired speed, accelerations, decelerations, desired time gap, minimum gap]

## 3. Model training

The training of the model comprises two important components, which have to be defined in advance. There is the generation of leading trajectories and the general definition of an training episode, that will be discussed in the following.

## 3.1. Generating synthetic leading trajectories

The leading trajectory is based on an AR(1) process, whose parameters reflect the kinematics of real leaders. The AR(1) process describes the speed of the Leader Vehicle and is defined as

$$v_l(t) = c + \phi v_l(t-1) + \epsilon$$
, with  $E(\epsilon) = 0, Var(\epsilon) = \sigma$  (10)

With reaching of stationarity, this process has

an expected value of 
$$E(v_l) = \frac{c}{1 - \phi}$$
, (11)

the variance 
$$Var(v_l) = \frac{\sigma^2}{1 - \phi^2},$$
 (12)

the autocorrelation 
$$ACF(dt) = \phi^{dt}$$
, (13)

and the correlation time 
$$\tau = -\frac{1}{\ln(\phi)}$$
, (14)

with d corresponding to the simulation step size, which is globally set to 100ms.

To adjust the parameters of the AR(1) process, typical values for real leader trajectories has to be defined: With  $v_{l,des}$  as the desired velocity for the leader, the mean of the AR(1) process is set to be  $v_{l,des}/2$  and the standard deviation is set to be  $v_{l,des}$ . The acceleration  $a_{phys}$  corresponds to typical physical leader accelerations. With these values and by using Equation 11 - 14, the parameters of the AR(1) process can be calculated as:

$$\phi = e^{\left(\frac{-2da_{phys}}{v_{l,des}}\right)} \tag{15}$$

$$c = (1 - \phi) \frac{v_{l,des}}{2} \tag{16}$$

$$\sigma^2 = (1 - \phi^2) \frac{v_{l,des}^2}{4} \tag{17}$$

The assumed typical values for  $v_{l,des}$  and  $a_{phys}$  as well as the resulting values of the AR(1) process parameters can be found in Table 3.

Figure 2 shows an example trajectory of the leading vehicle based on the AR(1) process using the parameters of Table 3. If the velocity exceeds the defined range of  $[0, v_{l,des}]$ , it is manually set to the range limits.

Table 3: Assumed typical values for leading trajectories and the resulting values of the AR(1) process parameters

Real trajectory		AR(1) process	
$v_{l,des}$	15m/s	$\phi$	0.9933
$a_{phys}$	$1m/s^2$	c	0.05m/s
		$\sigma^2$	$0.75m^2/s^2$

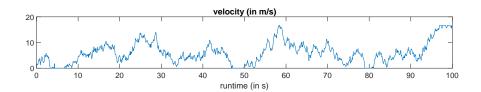


Figure 2: Example of a leading trajectory based on the parametrized AR1 process used to train the RL agent

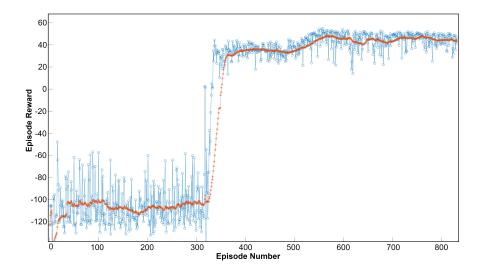


Figure 3: RL training process, one episode containing 500 steps

#### 3.2. Training episode definition

To train the RL agent, a training episode has to be defined. One episode has a simulation time of 50s with a step size of d = 100ms, resulting in a episode length of 500 steps. The initial velocities of Follower and Leader Vehicle is a randomly set in the range  $[0, v_{des}]$  respectively  $[0, v_{l,des}]$ . The initial gap between both is set to 120m.

#### 3.3. Results of the RL training process

Figure 3 shows an example of the training process. The red line shows the moving average reward of the last 30 episodes. After approximately 570 episodes the maximum average reward has been reached. Reaching saturation the learning process has been stopped after 850 episodes.

#### 4. Validation

The goal is not to minimize some error measure as in usual calibration/validation but to check if the driving style is safe, effective, and comfortable. The RL strategy is evaluated with respect to these metrics in different driving scenarios, described in the following.

## 4.1. Response to an external leading vehicle speed profile

The first scenario is designed in order to evaluate the transition between free driving and car-following as well as the Follower behavior in safety-critical situations. Figure 4 shows a driving scenario with an external Leading Vehicle speed profile. The initial gap between Follower and Leader is 200 meters, which refers to a free driving scenario. The Follower accelerates with  $a_{max} = 2m/s^2$  until the desired speed  $v_{des} = 16.6m/s$  is reached and approaches the standing Leader Vehicle. When the gap between both drops below 90 meters, the Follower starts to decelerate with a approximately  $b_{comf} = -2m/s^2$  (transition between free driving and car-following) and comes to a standstill with a final gap of approximately  $s_{min} = 2m$ . In the following the Leader Vehicle makes some

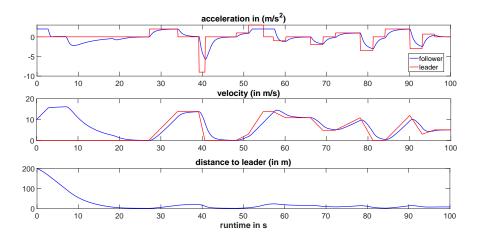


Figure 4: Response to an external leading vehicle speed profile

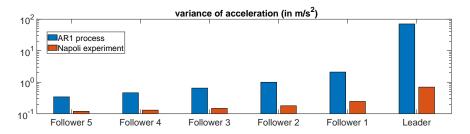


Figure 5: Comparison of the acceleration variance between Leader and Follower for AR(1) and Napoli experiment

random acceleration and deceleration. At the time t=40s the Leading Vehicle makes a full brake, resulting in a comfortable and safe braking of the Follower Vehicle. The transition between different accelerations happens in a comfortable way, reducing the resulting jerk.

## 4.2. String stability

The second scenario, shown in Figure 6, consists of a Leader based on the AR(1) process, followed by five vehicles, each controlled by the trained RL agent. The results show that traffic oscillations can effectively be dampened with a sequence of trained Followers, even if the Leader shows large outliers in acceleration. Figure 5 illustrates the difference of accelerations between Leader

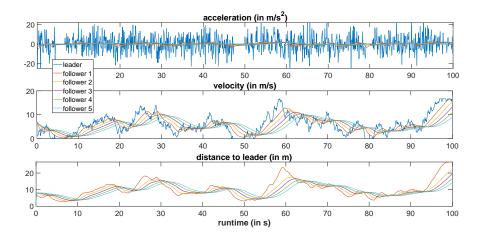


Figure 6: Response to a leader trajectory based on a AR(1) process

and the Followers (blue bars). The last Follower shows the lowest variance of acceleration, which demonstrates the ability of the RL agent to flatten the speed profile, to dampen oscillations and thus to increase comfort.

## 4.3. Response to a real leader trajectory

In a further scenario, the abilities of the RL strategy are evaluated with a real leader trajectory. This trajectory comes from experiments in Napoli, where exact data from Leader and Follower were obtained (reference to Punzo et al.). Figure 7 shows the result of a sequence of five vehicles following a real leader trajectory. Similar to the experiment from Section 4.2 string stability and the reduction of acceleration variance, shown by the red bars in Figure 5, is demonstrated. At time t=140s the Leader stands still, and it can be observed, that all following vehicles are keeping the minimum distance  $s_{min}$  to the Leader.

## 4.4. Response of different driver characteristics

As mentioned in Section 2.3, different driving styles can be achieved by adjusting the parameters of the reward function. Three RL agents has been trained on a reward function, that differs in the desired time gap  $T_{gap}$  between Follower and Leader Vehicle ( $T_{gap,1} = 1.0s$ ,  $T_{gap,2} = 1.5s$ ,  $T_{gap,3} = 2.0s$ ).

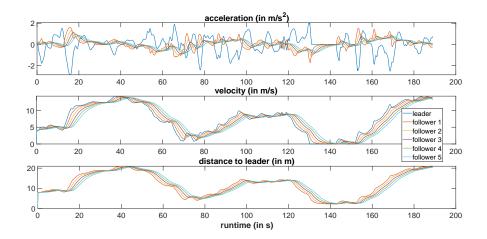


Figure 7: Response to a real leader trajectory

Figure 8 shows the result of these agents, following the real leader trajectory from Napoli. It can be observed, that a lower value for  $T_{gap}$  results in closer driving to the Leader, higher accelerations and decelerations and thus in a more aggressive driving behavior.

## 5. Conclusion/Discussion

evaluation of safety and comfort, comparison to IDM

discussion: adjusting parameters of the reward function to achieve different driving styles

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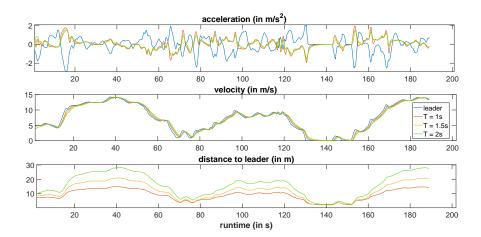


Figure 8: Impact of different parametrized RL agents on driving behavior

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